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A birational map of a projective space whose intermediate dynamical degrees are all transcendental

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Abstract. We construct a birational map of $\mathbb{P}^d$ $(d \geq 6)$ whose intermediate dynamical degrees are all transcendental.

Key words: dynamical degrees, birational maps, rational varieties
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1. *Introduction*

Let X be a smooth projective variety over an algebraically closed field k of characteristic 0. For a birational map $f : X \dashrightarrow X$ and $0 \leq p \leq \dim(X)$, the pth dynamical degree of f is defined by

$$\lambda_p(f) = \lim_{n\to\infty} ((f^n)^*(H^p) \cdot H^{\dim X - p})^{1/n},$$

where H is an ample divisor (cf. [16]).

The pullback map f^* and the pushforward map f_* are defined as follows. For a birational map $f : X \dashrightarrow X$, let Γ_f be the closure of the graph of f (cf. Figure 1). The pullback $f^* : A_l(X) \to A_l(X)$ and the pushforward $f_* : A_l(X) \to A_l(X)$ are defined as $f^*(\gamma) := \pi_*([\Gamma_f] \cdot \pi'^*\gamma)$ and $f_*(\gamma) := \pi'_*([\Gamma_f] \cdot \pi^*\gamma)$ (cf. [12, Definition 16.1.2], [16, §3.1]), where γ is an algebraic cycle on X.

In addition, the dynamical degrees are calculated as

$$\lambda_p(f) = \lim_{n\to\infty} \|(f^n)^* : N^p(X) \otimes_{\mathbb{Z}} \mathbb{R} \to N^p(X) \otimes_{\mathbb{Z}} \mathbb{R}\|^{1/n},$$

where $N^p(X)$ is the group of algebraic cycles of codimension p on X modulo numerical equivalence (cf. [16, 17]).

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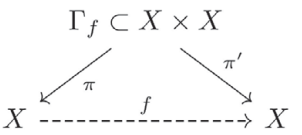


FIGURE 1. The graph of a birational map $f : X \dashrightarrow X$.

When $k = \mathbb{C}$, the projective variety X can be considered as a compact Kähler manifold, the birational map f can be considered as a meromorphic map, and the dynamical degrees can be calculated as

$$\begin{aligned}\lambda_p(f) &= \lim_{n\to+\infty} \left(\int_X (f^n)^*(\omega_X^p) \wedge \omega_X^{\dim X - p} \right)^{1/n} \\ &= \lim_{n\to+\infty} \|(f^n)^* : \mathrm{H}^{p,p}(X) \to \mathrm{H}^{p,p}(X)\|^{1/n},\end{aligned}$$

where ω_X is a normalized Kähler form, $\mathrm{H}^{p,p}(X)$ $(0 \le p \le \dim X)$ is the (p, p)-Dolbeault cohomology, and $\|\cdot\|$ is a norm for linear transformations (cf. [9–11]).

The values of the dynamical degrees are not less than 1 and $\lambda_0(f) = 1$. When f is a morphism, the dynamical degrees of f are algebraic, since $(f^n)^* = (f^*)^n$. In the case of $X = \mathbb{P}^d$, the rational map $f : \mathbb{P}^d \dashrightarrow \mathbb{P}^d$ can be written as $f = [f_0 : f_1 : \cdots : f_d]$ by the homogeneous polynomials (over k) of the same degree. Define $\deg(f) := \deg(f_0)$ and then the first dynamical degree of f is calculated by

$$\begin{aligned}\lambda_1(f) &= \lim_{n\to\infty} ((f^n)^*(H) \cdot H^{d-1})^{1/n} \\ &= \lim_{n\to\infty} (\deg(f^n))^{1/n}.\end{aligned}$$

When $d = 2$, it is known that the first dynamical degree of f is always algebraic by the following theorems.

THEOREM 1.1. (Cf. [7, Theorem 0.1, Proposition 1.18, Corollary 1.19]) *Let X be a compact Kähler surface and $f : X \dashrightarrow X$ be a bimeromorphic map. Then, there exists a proper bimeromorphic morphism $\pi : \tilde{X} \to X$ such that the map $\tilde{f} := \pi^{-1} \circ f \circ \pi$ satisfies $((\tilde{f})^n)^* = (\tilde{f}^*)^n$ on $\mathrm{H}^{1,1}(\tilde{X})$. Moreover, the dynamical degrees $\lambda_1(f)$ and $\lambda_1(\tilde{f})$ are equal, and this is equal to the spectral radius of $\tilde{f}^*$ on $\mathrm{H}^{1,1}(\tilde{X})$.*

THEOREM 1.2. (Cf. [7, Theorem 5.1(1)]) *Let X' be a compact Kähler surface and $g : X' \dashrightarrow X'$ be a bimeromorphic map with $\rho(g^*) > 1$ for the operator g^* on $\mathrm{H}^{1,1}(X')$, where ρ is the spectral radius. Then, the operator g^* has exactly one eigenvalue $\lambda \in \mathbb{R}_{>0}$ of modulus $|\lambda| > 1$, and in fact, $\lambda = \rho(g^*)$.*

These theorems imply that $\lambda_1(f)$ is either 1 or a Pisot number, or a Salem number (see definition in [5]) for the case where $d = 2$. Salem numbers, which can be realized as $\lambda_1(f)$ for some birational map $f : \mathbb{P}^2 \dashrightarrow \mathbb{P}^2$, are all determined in [18, Theorem 1.1]. Pisot numbers, which can be realized as $\lambda_1(f)$, are partially determined (cf. [13, Theorem C], [2, §3], [6, §7]).

In [4], a birational map of $\mathbb{P}^d (d \ge 3)$ with a transcendental first dynamical degree is constructed.

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THEOREM 1.3. (Cf. [4, Theorem 1.1]) *For each $d \geq 3$, there exists a birational map $f : \mathbb{P}^d \dashrightarrow \mathbb{P}^d$, which has a transcendental (first) dynamical degree $\lambda_1(f)$.*

In [4, §7.2], the specific birational map $\phi : \mathbb{P}^3 \dashrightarrow \mathbb{P}^3$ with a transcendental (first) dynamical degree is presented.

Remark 1.4. The birational map f constructed in Theorem 1.3 is of the form $f = L_{B^{-1}} \circ h_{-I} \circ L_B \circ h_A$, where:

- $h_M : \mathbb{P}^d \dashrightarrow \mathbb{P}^d$ is the monomial map corresponding to $M \in \mathrm{GL}_d(\mathbb{Z})$;
- $L_{M'} : \mathbb{P}^d \dashrightarrow \mathbb{P}^d$ is the linear transformation corresponding to $M' \in \mathrm{GL}_{d+1}(k)$

(see the details in [15, §2.3]); and

$$I = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & -1 & 1 & \cdots & (-1)^d \\ 1 & 1 & -1 & \cdots & (-1)^{d-1} \\ -1 & 1 & 1 & \cdots & (-1)^{d-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ (-1)^{d-1} & (-1)^{d-2} & (-1)^{d-3} & \cdots & 1 \end{pmatrix}.$$

In addition, the specific example $\phi : \mathbb{P}^3 \dashrightarrow \mathbb{P}^3$ in [4, §7.2] is the case where $d = 3$ and

$$A = \begin{pmatrix} -3 & -14 & -12 \\ 4 & 11 & 6 \\ -2 & -4 & -1 \end{pmatrix}.$$

In [15, §3], we calculate the second dynamical degree of the map ϕ and confirm that $\lambda_1(\phi) < \lambda_2(\phi)$. In addition, we construct another birational map $\psi : \mathbb{P}^3 \dashrightarrow \mathbb{P}^3$ with a transcendental first dynamical degree such that $\lambda_1(\psi) > \lambda_2(\psi)$ (cf. [15, §4]) by taking

$$A = \begin{pmatrix} 56 & -19 & -17 \\ -16 & 6 & 5 \\ 207 & -71 & -63 \end{pmatrix}.$$

It is confirmed that the second dynamical degree $\lambda_2(\psi)$ is algebraic, as follows.

THEOREM 1.5. (Cf. [15, §4]) *There exists a birational map $\psi : \mathbb{P}^3 \dashrightarrow \mathbb{P}^3$ whose dynamical degrees are*

$$\lambda_0(\psi) = 1, \quad \lambda_1(\psi) = \lambda, \quad \lambda_2(\psi) = a, \quad \lambda_3(\psi) = 1,$$

with $291 \leq \lambda \leq 669$ being a transcendental number and $a = 174.6660\cdots$ being a root of

$$a^9 - 173a^8 - 291a^7 - 2a^6 + 332a^5 + 334a^4 + 238a^3 + 75a + 75 = 0.$$

We use this result to obtain a transcendental arithmetic degree in [15, Corollary 4.5] by combining with the results of [14, Theorem B].

In this paper, we use Theorem 1.5 to resolve the next question by using the algebraicity of a.

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Question 1.6. [4, Question 1.6] Does there exist a birational map $f : \mathbb{P}^d \dashrightarrow \mathbb{P}^d$ for which all intermediate dynamical degrees $\lambda_1(f), \ldots, \lambda_{d-1}(f)$ are transcendental?

It is known that the intermediate dynamical degrees of an automorphism f of a compact Kähler manifold (or a projective variety over $\mathbb{C}$) are either irrational or equal to 1 [1, Theorem 6.1].

THEOREM A. *For $d \geq 6$, there exists a birational map $f : \mathbb{P}^d \dashrightarrow \mathbb{P}^d$ (over an algebraically closed field k of characteristic 0), whose intermediate dynamical degrees are all transcendental.*

This theorem is proved in §3. In the counterexample that we found, all intermediate dynamical degrees are transcendental, but they are algebraically dependent.

Another question about dynamical degrees is presented in [4, Question 1.6].

Question 1.7. [4, Question 1.6] Does there exist a birational map $f : \mathbb{P}^d \dashrightarrow \mathbb{P}^d$ for which $\lambda_1(f)$ is transcendental and also maximal among intermediate dynamical degrees $\lambda_i(f)$?

This question is resolved for the case $d \geq 3$ in [15, Remark 4.6]. In addition, for the case where $d = 3$, we construct an example where the first dynamical degree $\lambda_1(f)$ is strictly maximal among the dynamical degrees in Theorem 1.5. However, for the case where $d > 3$, the first dynamical degree is not strictly maximal among the dynamical degrees in our example (see [15, Remark 4.6]).

2. *Preliminaries*

There are some properties of dynamical degrees.

PROPOSITION 2.1. *Let X be an N-dimensional smooth projective variety over an algebraically closed field of characteristic 0 and let $f : X \dashrightarrow X$ be a birational self-map. Then,*

$$\lambda_p(f) = \lambda_{N-p}(f^{-1})$$

for $0 \leq p \leq N$.

This result is well known, but we would like to give an outline of the proof here for the convenience of the reader (cf. [15, §2.1] in the case where $\dim(X) = 3$).

Proof of Proposition 2.1. The proposition is shown by the following equations:

$$\begin{aligned}
\lambda_p(f) &= \lim_{n\to\infty} ((f^n)^*(H^p) \cdot H^{N-p})^{1/n} \\
&= \lim_{n\to\infty} (H^p \cdot (f^n)_*(H^{N-p}))^{1/n} \\
&= \lim_{n\to\infty} (H^p \cdot (f^{-n})^*(H^{N-p}))^{1/n} \\
&= \lambda_{N-p}(f^{-1}).
\end{aligned}$$

The first equation and the fourth equation are derived from the definition of the dynamical degrees. The third equation is the definition of the pushforward and the pullback of cycles.

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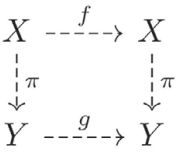


FIGURE 2. Birational conjugacy of maps f and g.

It remains to confirm the second equation. Denote $\pi, \pi' : X \times X \to X$ by the projections to the first and the second components as in Figure 1. Let $\Gamma_f \subset X \times X$ be the closure of the graph of f and let $\pi_1 : X \to \mathrm{Spec}(k)$ be the structure map. Let $\alpha \in A_p(X)$ and $\beta \in A_q(X)$ be cycles with $p + q = \dim(X)$. We have $f^*\beta := \pi_*([\Gamma_f] \cdot \pi'^*\beta)$ and $f_*\alpha := \pi'_*([\Gamma_f] \cdot \pi^*\alpha)$ by the definitions of the pullback and the pushforward (cf. [12, Definition 16.1.2]). By applying the projection formula to $\pi : X \times X \to X$ (cf. [12, Proposition 8.3]), the intersection of cycles can be calculated as

$$\begin{aligned}
\pi_{1*}(f^*(\beta) \cdot \alpha) &= \pi_{1*}(\pi_*([\Gamma_f] \cdot \pi'^*\beta) \cdot \alpha) \\
&= (\pi_{1*} \circ \pi_*)(([\Gamma_f] \cdot \pi'^*\beta) \cdot \pi^*\alpha) \\
&= (\pi_{1*} \circ \pi'_*)(\pi'^*\beta \cdot ([\Gamma_f] \cdot \pi^*\alpha)) \\
&= \pi_{1*}(\beta \cdot \pi'_*([\Gamma_f] \cdot \pi^*\alpha)) \\
&= \pi_{1*}(\beta \cdot f_*(\alpha)),
\end{aligned}$$

and thus the second equation in the proposition follows. □

PROPOSITION 2.2. (Cf. [8, Corollary 1.2], [10, Corollaire 7], [16, Lemma 3.7]) *Let X and Y be smooth projective varieties of the same dimension n over an algebraically closed field of characteristic 0. Let $f : X \dashrightarrow X$ and $g : Y \dashrightarrow Y$ be dominant rational maps, and let $\pi : X \dashrightarrow Y$ be a birational map with $\pi \circ f = g \circ \pi$ (cf. Figure 2).*

Then, the equation $\lambda_p(f) = \lambda_p(g)$ holds for all $0 \leq p \leq n$.

By Proposition 2.2, the dynamical degrees are invariant under a birational conjugacy and, therefore, the set

$$\mathrm{DynBir}_p(X) := \{\lambda_p(f) \mid f : X \dashrightarrow X \text{ is a birational map}\} \subset \mathbb{R}_{\geq 1}$$

is a birationally invariant set for projective varieties for each $0 \leq p \leq \dim(X)$.

PROPOSITION 2.3. (Cf. [8, Example 3.7], [16, Theorem 1.4]) *Let $X = X_1 \times X_2$ be the product of two smooth projective varieties over an algebraically closed field k of characteristic 0, with $\dim(X_i) = n_i$. Let f_1, f_2 be dominant rational self-maps of X_1, X_2 and define $f := f_1 \times f_2 : X_1 \times X_2 \dashrightarrow X_1 \times X_2$.*

Then,

$$\lambda_l(f) = \max_{\substack{p+q=l \\ 0 \leq p \leq n_1 \\ 0 \leq q \leq n_2}} \lambda_p(f_1)\lambda_q(f_2).$$

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3. *Proof of Theorem A*

In this section, we prove Theorem A. First, we prove the following proposition.

PROPOSITION 3.1. *There exists a birational map* $\Psi : \mathbb{P}^6 \dashrightarrow \mathbb{P}^6$ *whose intermediate dynamical degrees are all transcendental.*

Proof. Let $\psi : \mathbb{P}^3 \dashrightarrow \mathbb{P}^3$ be a birational map, which is presented in Theorem 1.5. Then, we have

$$\lambda_0(\psi) = 1, \quad \lambda_1(\psi) = \lambda, \quad \lambda_2(\psi) = a, \quad \lambda_3(\psi) = 1,$$

where λ is transcendental, a is algebraic, and $\lambda > a$.

The inverse $\psi^{-1} : \mathbb{P}^3 \dashrightarrow \mathbb{P}^3$ is also a birational map, and

$$\lambda_0(\psi^{-1}) = 1, \quad \lambda_1(\psi^{-1}) = a, \quad \lambda_2(\psi^{-1}) = \lambda, \quad \lambda_3(\psi^{-1}) = 1,$$

by Proposition 2.1.

Define the birational map $\Psi' := \psi \times \psi^{-1} : \mathbb{P}^3 \times \mathbb{P}^3 \dashrightarrow \mathbb{P}^3 \times \mathbb{P}^3$. Then, by Proposition 2.3, we have

$$\begin{aligned} &\lambda_0(\Psi') = 1, \quad \lambda_1(\Psi') = \lambda, \quad \lambda_2(\Psi') = a\lambda, \quad \lambda_3(\Psi') = \lambda^2, \\ &\lambda_4(\Psi') = a\lambda, \quad \lambda_5(\Psi') = \lambda, \quad \lambda_6(\Psi') = 1 \end{aligned}$$

from $\lambda > a$.

Define the birational map

$$\Phi : \mathbb{P}^3 \times \mathbb{P}^3 \dashrightarrow \mathbb{P}^6$$

$$([x_0 : x_1 : x_2 : x_3],\ [y_0 : y_1 : y_2 : y_3]) \mapsto [x_0y_0 : x_1y_0 : x_2y_0 : x_3y_0 : x_3y_1 : x_3y_2 : x_3y_3]$$

and define $\Psi := \Phi \circ \Psi' \circ \Phi^{-1} : \mathbb{P}^6 \dashrightarrow \mathbb{P}^6$.

Since Ψ is a birational map and by Proposition 2.2, the dynamical degrees are the same as Ψ',

$$\begin{aligned} \lambda_0(\Psi) = 1, \quad \lambda_1(\Psi) = \lambda, \quad \lambda_2(\Psi) = a\lambda, \quad \lambda_3(\Psi) = \lambda^2, \\ \lambda_4(\Psi) = a\lambda, \quad \lambda_5(\Psi) = \lambda, \quad \lambda_6(\Psi) = 1. \end{aligned}$$

Now, λ is a transcendental number and a is an algebraic integer, and thus the intermediate dynamical degrees of Ψ are all transcendental numbers. □

Proof of Theorem A. We prove this theorem by induction on n.

If $n = 6$, the theorem is proved by Proposition 3.1.

Assume that the theorem holds for some integer $n = n_0$. Let $g : \mathbb{P}^{n_0} \dashrightarrow \mathbb{P}^{n_0}$ be a birational map, whose intermediate dynamical degrees are all transcendental.

Denote $\lambda_i(g) = \lambda_i$ $(0 \le i \le n_0)$. The product map $g \times \mathrm{id}$ is a birational map and the dynamical degrees of $g \times \mathrm{id} : \mathbb{P}^{n_0} \times \mathbb{P}^1 \dashrightarrow \mathbb{P}^{n_0} \times \mathbb{P}^1$ can be calculated by using Proposition 2.3 as

$$\begin{aligned} &\lambda_0(g \times \mathrm{id}) = \lambda_0 = 1, \\ &\lambda_1(g \times \mathrm{id}) = \max\{\lambda_0, \lambda_1\} = \lambda_1, \end{aligned}$$

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$$\begin{aligned}
\lambda_2(g \times \mathrm{id}) &= \max\{\lambda_1, \lambda_2\},\\
&\vdots\\
\lambda_i(g \times \mathrm{id}) &= \max\{\lambda_{i-1}, \lambda_i\},\\
&\vdots\\
\lambda_{n_0-1}(g \times \mathrm{id}) &= \max\{\lambda_{n_0-2}, \lambda_{n_0-1}\},\\
\lambda_{n_0}(g \times \mathrm{id}) &= \max\{\lambda_{n_0-1}, \lambda_{n_0}\} = \lambda_{n_0-1},\\
\lambda_{n_0+1}(g \times \mathrm{id}) &= \lambda_{n_0} = 1.
\end{aligned}$$

Thus, the intermediate dynamical degrees of $g \times \mathrm{id}$ are all transcendental. Define the birational map

$$\begin{aligned}
h : \mathbb{P}^{n_0} \times \mathbb{P}^1 &\dashrightarrow \mathbb{P}^{n_0+1}\\
([x_0 : x_1 : \cdots : x_{n_0}],\ [y_0 : y_1]) &\mapsto [x_0y_0 : x_1y_0 : \cdots : x_{n_0}y_0 : x_{n_0}y_1]
\end{aligned}$$

and define $g' = h \circ (g \times \mathrm{id}) \circ h^{-1} : \mathbb{P}^{n_0+1} \dashrightarrow \mathbb{P}^{n_0+1}$. Now, g' is a birational map and by Proposition 2.2, the dynamical degrees are the same as $g \times \mathrm{id}$, and this is the case for $n = n_0 + 1$. Therefore, all intermediate dynamical degrees of g' are transcendental and the proof is complete. □

Remark 3.2. By using the induction in the proof of Theorem A for the birational map $\Psi : \mathbb{P}^6 \dashrightarrow \mathbb{P}^6$ constructed in the proof of Proposition 3.1, we get the birational map $g : \mathbb{P}^d \dashrightarrow \mathbb{P}^d$ $(d \geq 6)$ with

$$\begin{aligned}
&\lambda_0(g) = 1, \quad \lambda_1(g) = \lambda, \quad \lambda_2(g) = a\lambda, \quad \lambda_3(g) = \cdots = \lambda_{d-3}(g) = \lambda^2,\\
&\lambda_{d-2}(g) = a\lambda, \quad \lambda_{d-1}(g) = \lambda, \quad \lambda_d(g) = 1.
\end{aligned}$$

Remark 3.3. The paper [**3**, §1] shows that there exists a dominant rational map $f : \mathbb{P}^2 \dashrightarrow \mathbb{P}^2$ such that

$$\lambda_0(f) = 1, \quad \lambda_1(f) = 6.8575574092\cdots, \quad \lambda_2(f) = 5$$

with transcendental $\lambda_1(f)$. By applying induction in the proof of Theorem A to the map f, there exists a dominant rational map $\mathbb{P}^d \dashrightarrow \mathbb{P}^d$ such that its intermediate dynamical degrees are all transcendental for any $d \geq 2$.

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