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Spectral fatigue life estimation for components under multiaxial random loading

N. Nguyen[1], M. Bacher-Höchst[1] and C.M. Sonsino[2]

[1] *Robert Bosch GmbH, Corporate Sector Research and Advance Engineering, Schwieberdingen, Germany*
e-mail: NgocBao.Nguyen@de.bosch.com
[2] *Fraunhofer Institute for Structural Durability and System Reliability LBF, Darmstadt, Germany*

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Abstract – This paper presents a probabilistic method for fatigue life estimation within the frequency domain for structural elements subjected to multiaxial random loadings. Multivariate Monte Carlo Simulation is used to account for the correlation between the stress components and their different probability of occurrence and, moreover, enables stochastics during damage analysis to be allowed for and, at the same time, uses any suitable, material dependent multiaxial fatigue criterion known from the time domain. Comparison of the evaluated fatigue damage with experimental results from vibration tests on a demonstrator, chosen from common application fields in the automobile industry, shows good correlation.

In the automobile industry, many vehicle components are submitted to complex vibrations during their service life. Vibrations are generally divided into two main categories: deterministic and stochastic. Harmonic excitations are produced by the unbalance in rotating machinery, for instance generators. Structural elements attached to the car body, such as ESP/ABS hydraulic power units or control devices, are excited by the unevenness of the road surface and are therefore subjected to vibrations that are random and multiaxial in nature, see Figure 1. Since vibration loadings are generally associated with dynamic effects due to structural resonances, leading to early fatigue failure, they need to be treated thoroughly during the consideration of reliability.

Fatigue analyses are usually performed in the time domain. However, it is preferable to treat loadings of random character in the frequency domain, in terms of Power Spectral Density (PSD) functions due to the higher computational efficiency. While there are numerous multiaxial fatigue criteria existing within the time domain, spectral multiaxial methods require a more sophisticated procedure and are still the subject of ongoing basic research. Hence, spectral fatigue life estimation for non-proportional multiaxial random loadings is to be the subject of this paper.

Frequency-domain approaches are based on empirically approximated probability density functions (PDF) of the stress amplitudes and are common for stochastic excitations [1, 2]. However, they are restricted to simple, uniaxial load cases. Important aspects of structural response assessment, e.g. the provision for multiaxial stress states, are not taken into account.

Many efforts in dealing with multiaxial problems within the frequency domain consist of the definition of a spectral multiaxial fatigue criterion, i.e. reformulating known time-domain hypotheses into the frequency domain, and then the application of the equivalent stress to the conventional spectral fatigue failure procedure. Preumont & Piéfort [3] take the von Mises criterion as a basis, whereas Pitoiset [4] uses the criterion proposed by Matake and Crossland. However, these criteria are valid for ductile materials but fail for non-proportional loadings, i.e. changing principal stress directions.

The new method proposed in this paper departs entirely from the conventional

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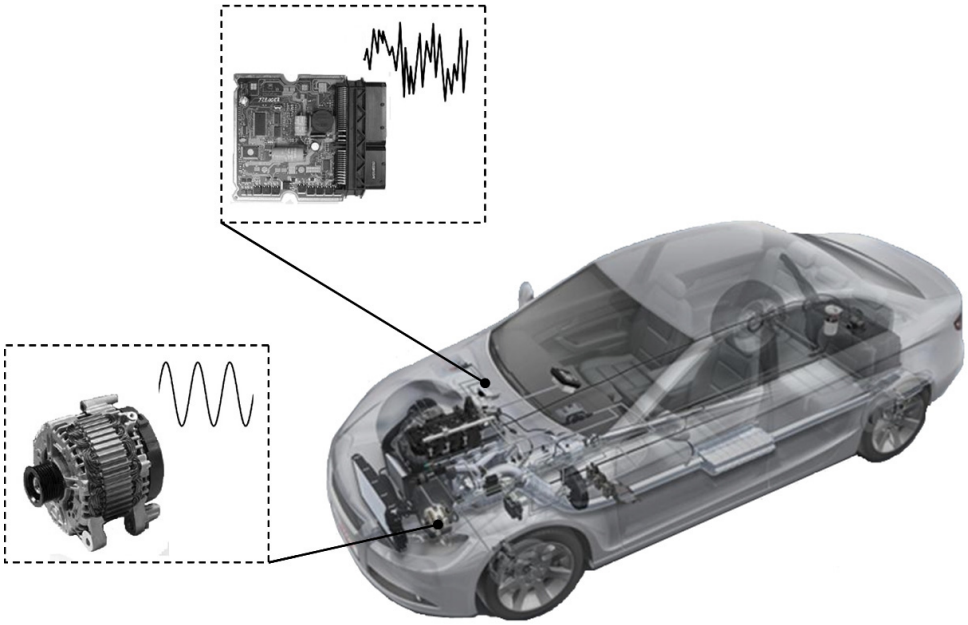


Fig. 1. Automotive components subjected to vibrational loading.

spectral methods. On the contrary, any statistical aspects of a random loading are initially dismissed, considering only an arbitrary event in the multidimensional stress signal or the stress tensor, respectively. Equivalent stress calculation and damage analysis are performed on this sample level, whereas, from this non-stochastic point of view, any suitable time-domain multiaxial fatigue criterion is applicable while still based upon statistically correct assumptions. On the basis of the Total Probability Theorem, multivariate Monte Carlo Simulation is then used to account for the probability of occurrence of each of the calculated partial damages in order to determine the total damage.

1 Multivariate Monte Carlo Simulation and fatigue life assessment

In considering the power spectral density of the structural response, only the statistics of the single stress components are provided. The probability distribution of the entire stress tensor itself is, as yet, unknown, as is the distribution of an equivalent uniaxial stress. The computation e.g. of the von Mises stress, as a classical multiaxial fatigue criterion for ductile metal materials, is only straightforward if the applied load is deterministic and proportional. In the case of random, and especially non-proportional, loadings, however, the assessment of the equivalent stress is certainly not straightforward. Due to the nonlinearity of the von Mises function, the steady state root mean square (RMS) values of the stress components which are scalar quantities cannot be directly applied. The von Mises stress computed in this way represents only a certain quantile of the true distribution and the calculation directly from the PSD of response is therefore statistically incorrect. The probability distribution of the equivalent von Mises stress is thus not closed under the distribution of the stress components.

1.1 Multivariate Monte Carlo Simulation

Distribution of the stress tensor can be determined by means of a multivariate Monte Carlo Simulation (MCS) in combination with a preceding random vibration analysis of the system. The required joint probability distribution allows a "directed" generation of

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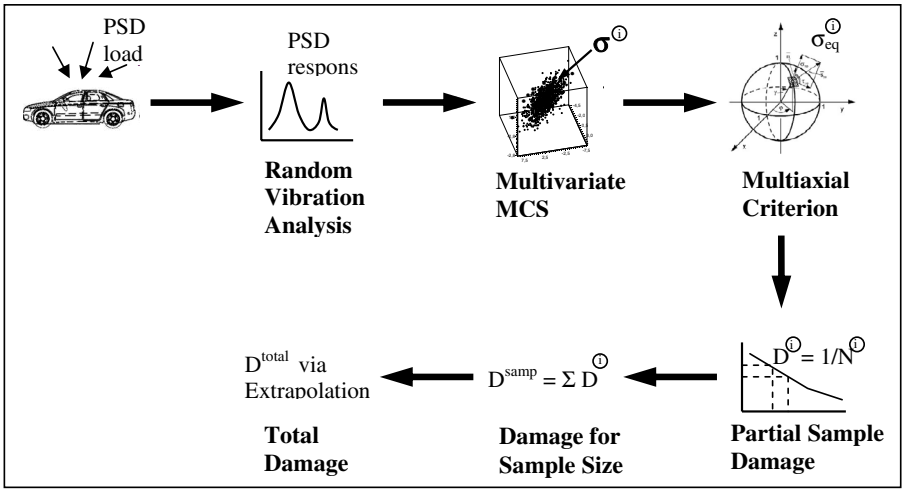


Fig. 2. Fatigue life assessment by means of multivariate MCS.

multivariate random samples of the stress tensor and therefore accounts for the correlation between the stress components [5]. For multivariate MCS, the assumption of a proper joint distribution for the correlated variates of the stress components is necessary. In random vibration analyses, stress components are assumed to be from a normal distribution with zero mean. The multivariate Gaussian distribution can be easily determined by making use of the Nataf model. In this way, multivariate MCS is possible based solely on marginal distribution and correlation coefficients of the random variables, without knowing the complete joint distribution [6,7].

1.2 Equivalent stress and fatigue life assessment

On the sample level, equivalent stress calculation is trivial. For every sample of the stress tensor σ^i, i.e. for every set i of stress components, any suitable multiaxial fatigue criterion can be applied in order to calculate the equivalent stress σ^i_{eq} for that sample.

Similarly, each sample corresponds to a cycle and induces a certain amount of fatigue damage on the component. Thus, for every set i, a partial damage D^i can be computed. This partial damage D^i is associated with a corresponding probability of occurrence $P\left(D^i\right)$. According to the Theorem of Total Probability, total damage D^{samp} for the whole sample size can now be considered as a "probability-weighted average" summation of the single damages [8].

Finally, total damage for a certain life time is determined through extrapolation on the total spectrum size. The time relation between spectrum size and life time is given by the rate of peaks, which can be directly computed from spectral moments of the PSD response. Figure 2 shows a schematic illustration of the described method.

1.3 Analysis flow chart

Figure 3 shows the analysis flow chart with the essential procedural steps for the proposed approach. Starting with a modal analysis, resonance frequencies and mode shapes are determined, describing the overall dynamic behaviour of the structure. Structural response for a critical location is then computed using modal random vibration (PSD) analysis, based on the linear transfer function and a given load spectrum. Modal analysis and PSD analysis are processed during an FE analysis, e.g. ANSYS. From the local PSD stress response, spectral moments and the covariance matrix can be directly be determined. The nth spectral moments of the one-sided PSD response $G(f)$ are defined as

$$m_n = \int_0^\infty f^n G(f)\,\mathrm{d}f \qquad (1)$$

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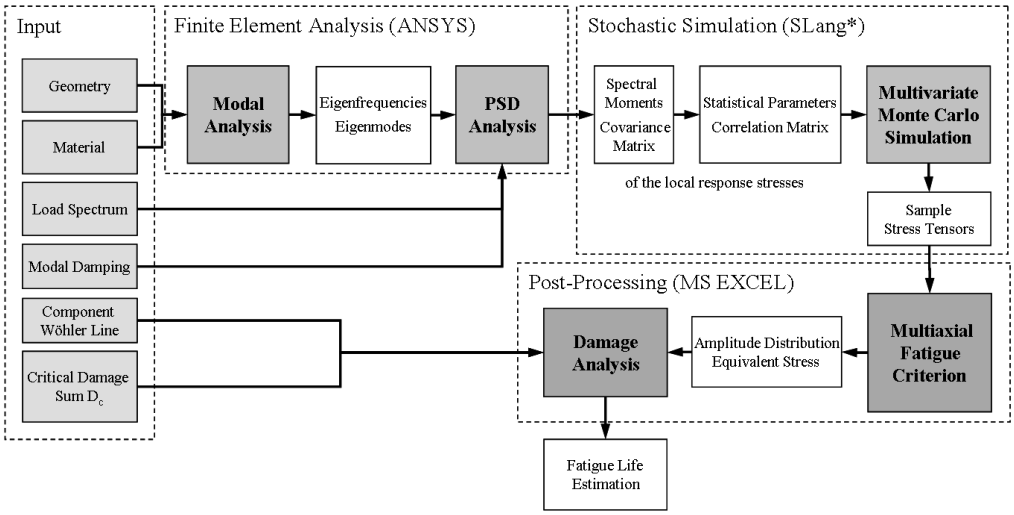


Fig. 3. Analysis flow chart for the proposed method.

where the spectral moments m_0, m_2 and m_4 are

$$m_0 = \int_0^\infty G(f)\,\mathrm{d}f \quad m_2 = \int_0^\infty f^2 G(f)\,\mathrm{d}f$$
$$m_4 = \int_0^\infty f^4 G(f)\,\mathrm{d}f \tag{2}$$

with, in particular, m_0 denoting the variance; the RMS value of the zero mean process can be derived from the square root of m_0.

The statistical parameters necessary for the subsequent fatigue life analysis can now be computed from the spectral moments. The rate of zero crossings ν_{0^+} and peaks ν_p as well as the irregularity factor α, representing the bandwidth of the structural response, are given as

$$\nu_{0^+} = \frac{1}{2\pi}\sqrt{\frac{m_2}{m_0}} \quad \nu_p = \frac{1}{2\pi}\sqrt{\frac{m_4}{m_2}}$$
$$\alpha = \frac{\nu_{0^+}}{\nu_p} = \frac{m_2}{m_0 m_4}. \tag{3}$$

Multivariate Monte Carlo Simulation is then performed using SLang (Structural Language), a freeware program for probabilistic and structural analysis, developed at the Bauhaus-University Weimar [9]. SLang bases on the Nataf model [7], and, hence, MCS requires only the input of the marginal distributions of the stress components and the correlation matrix. The correlation matrix is computed from the covariance matrix from the PSD response by

$$\rho_{\sigma_i,\sigma_j} = \frac{Cov(\sigma_i,\sigma_j)}{\sqrt{Var(\sigma_i)}\,\sqrt{Var(\sigma_j)}}, \quad \left|\rho_{\sigma_i,\sigma_j}\right| \le 1 \tag{4}$$

representing the correlation coefficient between two stress components σ_i and σ_j, with $Cov(\sigma_i,\sigma_j)$ as the covariance between σ_i and σ_j.

Next, the equivalent stress, e.g. von Mises stress, is determined and damage analysis is performed for every realization of the simulated random stress tensors. Summation over the sample size and extrapolation for a given spectrum size finally leads to the total damage. The corresponding fatigue life is given by means of the critical damage sum and the rate of peaks, see Equation (3).

2 Example

The principal procedural steps of the proposed method of fatigue life assessment are shown by means of an example problem. Assessment is based on the formulae given in Section 1.

2.1 Demonstrator

The demonstrator for the validation of the method is of simple geometry and mildly

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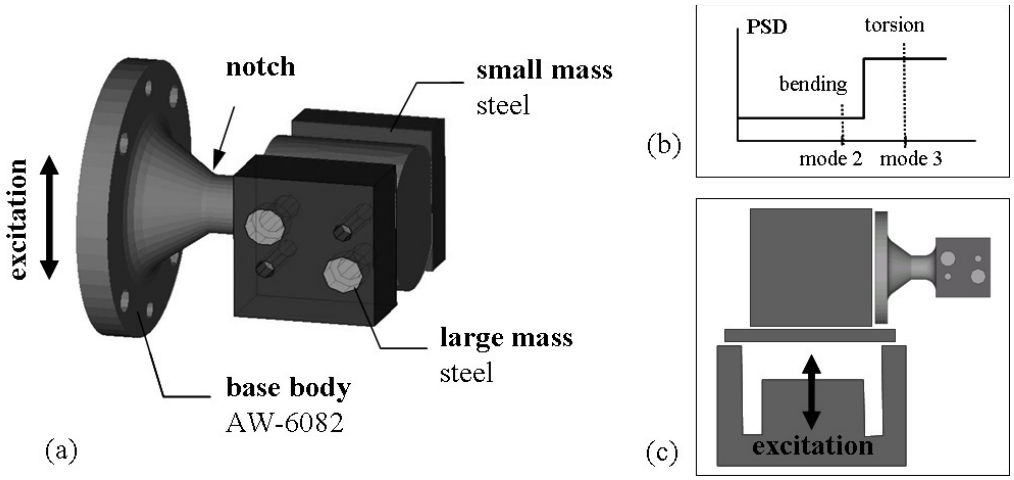


Fig. 4. (a) Demonstrator, (b) PSD load spectrum, (c) experimental setup.

notched, allowing strain gauge measurements on the specimen. Since aluminium is a common and widely used material in lightweight design, an aluminium extrusion profile AW-6082 T6 has been chosen as the base material. Eigenfrequencies and mode shapes lie within a range relevant to the automobile industry, i.e. up to 1000 Hz.

Figure 4a shows the cantilever-like demonstrator, consisting of a notched base body (notch radius 2 mm) and two additional masses of different weight mounted at the free end [10]. For non-proportional multiaxial loading, base excitation is effected by a PSD acceleration spectrum inducing eigenmodes of the structure with changing principal stress directions; in this case, bending and torsion, Figure 4b.

Finite Element analysis is performed using ANSYS and probabilistic simulation using SLang. Experimental verification of the new method is carried out by means of shaker tests. Figure 4c shows a schematic presentation of the experimental setup.

2.2 Multiaxial criterion and cumulative damage

Fatigue analysis is performed using the local stress concept. For semi-ductile materials, like aluminium extrusion profiles, with low ductility, the strength hypothesis Combined Normal and Shear Stress (KoNoS) proposed by Küppers & Sonsino [11] has been found to be an applicable multiaxial fatigue failure criterion. KoNoS is based on a critical plane approach where damage is caused by the critical combination of local normal and shear stress in the interference planes. The critical plane is the one where the combination of the local normal and shear stresses becomes a maximum. The equivalent stress calculated from the most critical combination is able to consider phase shifts caused by non-proportional loading. In the example case, loading due to the excited eigenmodes is non-proportional, but the local multiaxial stress state within a certain mode remains proportional. Hence, for in-phase loading, the equivalent stress according to KoNoS reduces to the special case of the conventional von Mises stress, Figure 5.

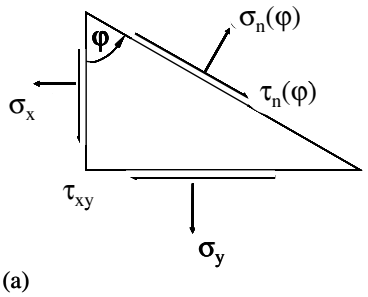
Linear cumulative damage calculation [12] is performed using the Palmgren-Miner rule elementar at $N \geq 5 \times 10^6$ cycles, in combination with an allowable damage sum of $D_{al} = 0.3$ [13].

2.3 Results and discussion

The first three mode shapes, eigenfrequencies and modal damping factors from modal analysis are presented in Figure 6. Modes 2 and 3 represent bending and torsional mode causing non-proportional modal displacement.

Figure 7 shows the stress response PSD for normal stress sx and shear stress sxz at the critical location in the notch. Spectral moment computation gives an irregularity

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(a)

$$S(\varphi,\delta)=\sqrt{\sigma_n^2(\varphi,\delta)+3\tau_n^2(\varphi,\delta)}$$

$$\sigma_n(\varphi)=\frac{\sigma_x+\sigma_y}{2}+\frac{\sigma_x-\sigma_y}{2}\cos(2\varphi)+\tau_{xy}\sin(2\varphi)$$

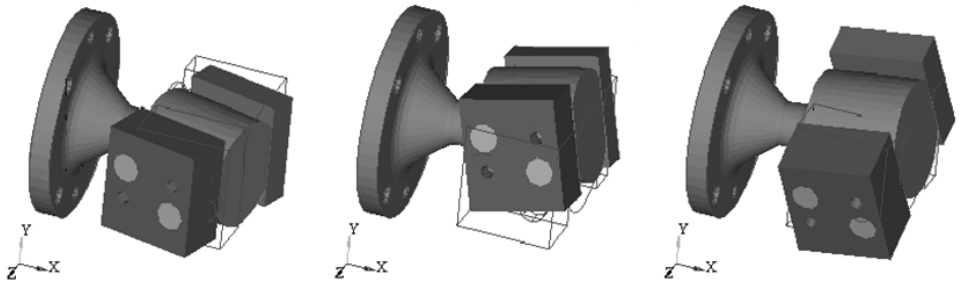
$$\tau_n(\varphi)=\frac{\sigma_x-\sigma_y}{2}\sin(2\varphi)-\tau_{xy}\cos(2\varphi)$$

(b)

$$\sigma_{eq}(\delta)=\sigma_{eq}(\delta=0^\circ)\frac{S(\varphi^{**},\delta)}{S(\varphi^{*},\delta=0^\circ)} \qquad \sigma_{eq}(\delta=0^\circ)=\sqrt{\sigma_x^2+\sigma_y^2-\sigma_x\sigma_y+3\tau_{xy}^2}$$

Fig. 5. Strength hypothesis KoNoS: (a) interference plane, local stresses and damage parameter, (b) equivalent stress.

(a)



Mode 1 Mode 2 Mode 3

(b)

Mode	Eigenfrequency (Hz)	Modal Damping (-)
1	499	0.00360
2	547	0.00105
3	767	0.00094

Fig. 6. (a) Mode shapes, (b) modal parameters.

factor of 0.95, indicating a narrow band process. From the output PSD, 1σ stress values and the covariance matrix are used directly for multivariate Monte Carlo Simulation. The simulated stress tensors for a sample size of 65 000 are visualized as a two-dimensional scatter plot in Figure 8a. Comparing the simulated amplitude distribution of the equivalent von Mises stress with the distribution, calculated directly from RMS values of the stress components by means of the narrow band approach, shows that the narrow band approach is too conservative, see Figure 8b.

Finally, the damage analysis is carried out by means of the component *S-N* curve from shaker test – firstly for every sample set and then extrapolated for the entire

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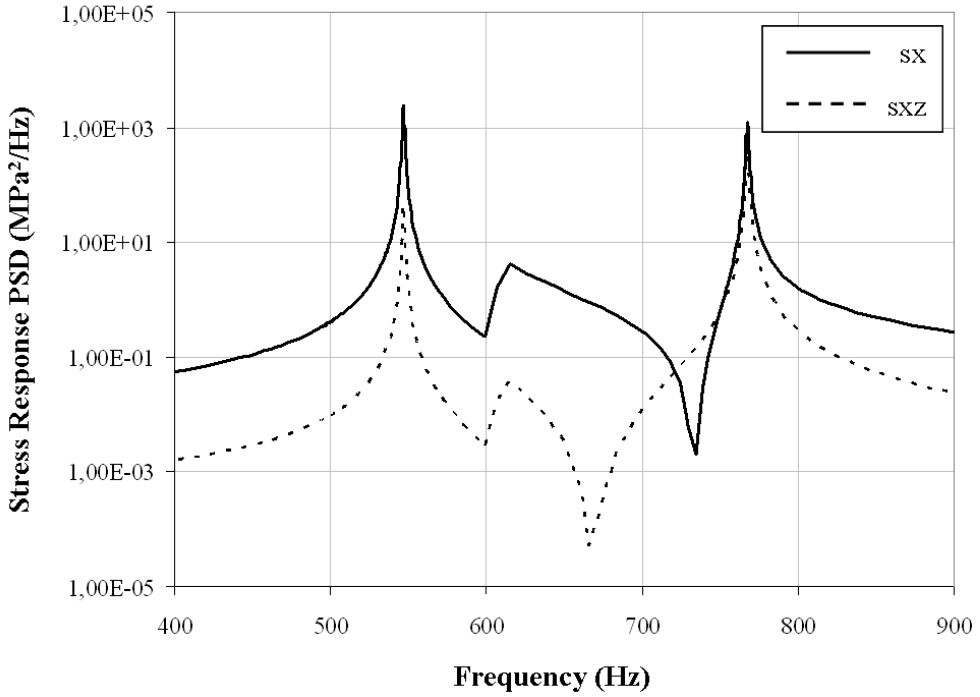


Fig. 7. Stress response PSD at the critical location.

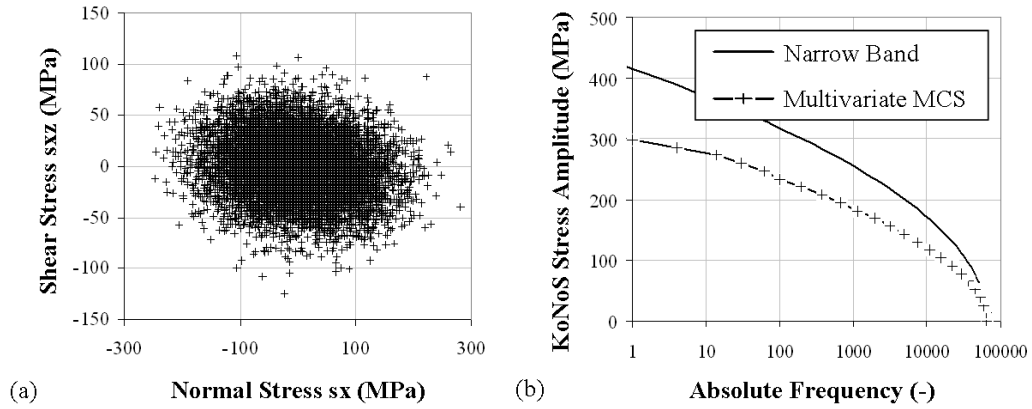


Fig. 8. (a) Stress tensor scatter plot, (b) amplitude distribution.

spectrum size. Figure 9 compares Gassner curves according to the Palmgren-Miner rule elementary, the Palmgren-Miner original and the modification according to Haibach. The Palmgren-Miner rule elementary shows the best correlation with experimental results. Furthermore, Gassner curves computed from conventional spectral uniaxial methods such as the narrow band approach or the wide band approach according to Dirlik [1] turn out to be five times more conservative than the results from the proposed method derived by means of multivariate MCS.

3 Conclusion

Probabilistic fatigue life assessment by means of multivariate Monte Carlo Simulation enables computationally efficient multiaxial spectral analysis within the frequency

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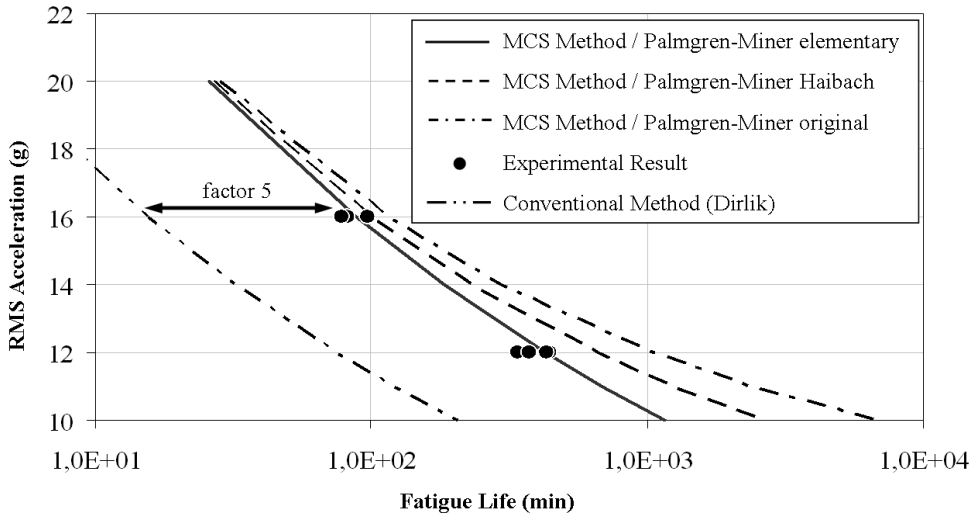


Fig. 9. Gassner curves for the proposed and the conventional method.

domain. Any suitable, material and ductility dependent multiaxial fatigue criterion known from the time domain can be applied under statistically correct assumptions. Comparison of the evaluated fatigue damage with experimental results shows good correlation for the proposed example problem.

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