European Journal of Applied Mathematics (2026), **37**, pp. 551–552
doi:10.1017/S0956792526100333

CAMBRIDGE UNIVERSITY PRESS

EDITORIAL ANNOUNCEMENT

Evolution equations on graphs: analysis and applications

Katy Craig[1], Antonio Esposito[2], Dimitrios Giannakis[3] and André Schlichting[4]

[1]Department of Mathematics, University of California, Santa Barbara, USA
[2]Department of Information Engineering, Computer Science and Mathematics, University of L'Aquila, Italy
[3]Department of Mathematics, Dartmouth College, USA
[4]Institute of Applied Analysis, University of Ulm, Germany
Corresponding author: Antonio Esposito; Email: antonio.esposito3@univaq.it

Received: 16 January 2026; **Revised:** 18 January 2026; **Accepted:** 19 January 2026

Evolutionary models on graphs arise throughout the sciences, from physics to economics, biology and machine learning. Such models describe a range of phenomena, expressing how attributes of nodes or even the graph itself change over time. On one hand, many evolutionary models on graphs are inspired by analogues in the spatially continuous setting and can be shown to converge to these continuum limits as the graph is refined. On the other hand, the intrinsically discrete nature of graphs often requires new notions of differential equations, which can be flexible enough to apply to a wide range of graph structures, while still recovering the correct behaviour in the continuum limit.

This special issue highlights recent research in the analysis and applications of evolutionary equations on graphs, as well as extensions to more general settings. The collected works cover diverse topics, from homogenisation of gradient flows to differential equations describing the flow of a gas through a network or heterogeneous interactions in multi-agent systems. Furthermore, the issue also considers a broad range of applications, including energy infrastructure, community detection in networks and data clustering methods. Whether describing the flow of gas through pipeline networks, the spread of information or disease across social networks, the evolution of opinions in adaptive communities or the geometry of high-dimensional data, graphs provide a flexible language for formulating evolutionary equations that encode both discrete interactions and spatial organisation.

In detail, a group of articles focuses on evolutionary equations and optimal transport on graphs, emphasising geometric and variational structures. In [1], Fazeny, Burger, and Pietschmann introduce a dynamical optimal transport formulation for gas flow on networks, providing a distance adapted to conservation laws on graphs. Portinale and Quattrocchi [8] study discrete-to-continuum limits of dynamical optimal transport problems with cost functional having linear growth at infinity on periodic graphs, rigorously identifying the effective continuum transport model arising from graph-based formulations. Gao and Yip [2] analyse the homogenisation of Wasserstein gradient flows in heterogeneous settings, establishing convergence to effective macroscopic dynamics and illustrating their theory with applications to data analysis.

Another theme concerns interacting particle systems on graphs and adaptive networks, where both node states and network structure may evolve. Kuehn and Wöhrer [4] provide global stability results for McKean–Vlasov equations on large networks, revealing how the network heterogeneity influences the long-time behaviour. In [3], Gkokas, Kuehn, and Xu derive mean-field limits for co-evolutionary signed heterogeneous networks, focusing on Kuramoto oscillators. Throm develops continuum limits for interacting systems on adaptive networks, providing a systematic passage from discrete co-evolving graphs to macroscopic models, cf. [9]. Complementing these works, in [7], Nurisso, Raviola, and Tosin introduce network-based kinetic models in which a statistical description of the graph topology emerges from microscopic interaction rules in multi-agent systems.

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https://doi.org/10.1017/S0956792526100333 Published online by Cambridge University Press

Finally, the issue highlights applications in data science, where evolution equations on graphs underpin modern computational methods. Li, van Gennip, and John propose an MBO-type diffusion–thresholding scheme for modularity optimisation based on a formulation in terms of total variation on the graph and signless total variation on the null model, offering an efficient and analytically grounded approach to community detection in complex networks, see [6]. The community detection problem is also considered by Li, in [5], on a Gaussian mixture model, with applications on hypergraphs.

Alongside this breadth of topics and applications, several key themes have emerged. Extensions of optimal transport techniques to the graph setting provide new metrics to compare probability measures on graphs and methods for modelling the flow of densities on a graph. Geometric flows – from Wasserstein gradient flows to mean curvature flows – possess rich analogues on the graph, and the graph setting provides a framework both for understanding how dynamics depend on the underlying geometry as well as how long time behaviour depends on community structure in the graph. Finally, several works analysed interacting particle systems on graphs, including steady states and mean field limits, particularly when the graph itself is also changing in time. In summary, although it is not exhaustive, our special issue captures both the breadth and key themes of recent developments in the study of graph evolution equations – a challenging and growing research direction bridging discrete and continuous perspectives, theory and application and mathematical analysis with real-world systems.

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Cite this article: Craig K., Esposito A., Giannakis D. and Schlichting A. (2026). Evolution equations on graphs: analysis and applications. *European Journal of Applied Mathematics*, **37**, 551–552. https://doi.org/10.1017/S0956792526100333

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