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# Sensitivity analysis for a class of robotic grasping quality functionals

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**SUMMARY**

We consider an optimization formulation that captures the quality of a grasp. In this model the object geometry, finger contact points and the magnitude of the external load are given. Perturbation of these parameters from their nominal values might lead to significant variations on the predicted grasping quality measures. In this paper we study the sensitivity of a class of grasping quality functions subject to these perturbations and introduce a global sensitivity measure together with a computational procedure to evaluate it. Our analysis is developed within the framework of sensitivity theory and dual methods of nonlinear programming. Numerical simulations supporting the theoretical analysis are presented.

KEYWORDS: Robotic grasping; Quality functionals; Sensitivity measure; Nonlinear programming; Numerical simulations.

## 1. INTRODUCTION

One of the main problems in the design and control of multi-fingered hands robots is to define a firm grip.[1,2] The basic grasping problem can be approached by studying the mechanics of grasping which characterizes the mechanisms through which a manipulator interacts with its environment. One approach to study the grasping problem is to formulate it as an optimization problem, see e.g., Cutkosky.[3] In a recent work,[4] we formulate the mathematical model that captures the quality of a grasp as a deterministic optimization problem in which nominal values of the parameters defining the geometric configurations, finger positioning and loading are fixed. However, mechanical manipulators have finite positioning accuracy,[5] and the external load does vary while performing a specific task. It is thus natural to ask the following questions:

(i) What are the effects of perturbing the model parameters on the grasping quality functions?
(ii) Can we define and compute a global sensitivity measure for grasping quality functions?

The main goal of this paper is to answer the above questions, for the proposed optimization models. To achieve this task we apply the sensitivity theory and the dual methods of nonlinear programming. After recalling in the next section the optimization formulation of the grasping model that quantify the quality of a grasp, in Section 3 we develop the analytical tools for the sensitivity analysis of the grasping quality functions. In Section 4 we define a sensitivity measure and proposed an explicit computational procedure to evaluate it. Numerical simulations are presented in Section 5 and the last section includes some concluding remarks.

To put in perspective the results obtained in this paper, we conclude this introduction by giving a brief account of results on related research works in sensitivity analysis for robotic grasping problems.

Cheng and Orin[6] use the Compact-Dual Linear Programming formulation to allow real time implementation of linear programming. The concept behind the compact-dual linear programming formulation is the reduction of constraint matrices, thus increasing the speed of convergence to the optimal solution. The technique is developed by first removing the equality constraints from the original LP formulation to obtain the compact-dual primal formulation, and then solving the corresponding dual to obtain the compact-dual. The method is applied to solve the finger force distribution of a grasping problem.

Chung and Cho[7] analyse the sensitivity of the trajectory and input torques of a manipulator subjected to payload variations. This analysis utilizes the sensitivity theory and is applied to three cases: unbalanced, accurately balanced and inaccurately balanced robot configurations. Simulation results demonstrate that an accurately balanced manipulator is not sensitive to payload variations.

Tzes and Yurkovich[8] present a control strategy for driving an N-link manipulator along a specified trajectory while carrying an unknown load. The adaptive component of the controller is computed via the application of the sensitivity theory. This results in a robust adaptive controller that is simple and computationally efficient, and can withstand all payload expected variations.

Bicchi[9] proposes a method for controlling the contact forces applied by manipulators. The controller is formulated as an optimization problem in which the objective function is a weighted combination of the energy consumed by the manipulator and the sensitivity of the contact forces to external load variations.

Khalili[10] designs a passive robotics sysem that is attached to human limbs to estimate its mass and center of gravity. The sensors are positioned such that the parameter estimations are not significantly sensitive to joint angle and torque variations.

Park and Starr[11] analyze a force closure by evaluating the sensitivity of the finger forces, when the contact locations vary. The sensitivity value is defined as the uncertainty

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grasping index. The force closure is modeled by static equilibrium conditions that map the vector of the finger forces into an external load vector. The mapping matrix has a condition number that represents the sensitivity of the finger forces to small variations of grasping geometrical parameters and external loads. To represent the effects of the finger force perturbations an uncertainty grasping index is defined.

We finally note that the above cited list of references is by no means a complete one, but rather reflects some of the works which are mostly related to the present paper. For further reading and references on grasping issues, see e.g., reference 2.

## 2. AN OPTIMIZATION FORMULATION OF THE GRASPING QUALITY PROBLEM

In this section we briefly recall the optimization model for the grasping quality sense recently introduced in reference 4.

A grasp can be characterized by defining the type and location of the finger contacts, and determining the applied force and torque values.[12] A grasp can be formed as a form or a force closure.[13] Nguyen defined a force closure as a grasp in which the contact forces and torques can be adjusted to balance the applied external load[14,15] In [4], we focused on planar force closures, that are generated by $m$ hard (non-soft) contact points. The static equilibrium conditions of such a grasp yield

$$G_n x_n + G_f x_f = F(\theta) \tag{2.1}$$

where $x_n \in \mathbb{R}^m$ is a vector of the applied normal forces, $x_f \in \mathbb{R}^m$ is the frictional force, $G_n$ and $G_f \in \mathbb{R}^3 \times \mathbb{R}^m$ are the equilibrium balancing parameters that depend on the object geometry and contact characterization, and $F(\theta) \in \mathbb{R}^3$ is the generalized external load that is directed along a given $\theta$ direction. The range of $\theta$ in the planar case is $0 \leq \theta \leq 2\pi$.

Hard contact points transmit normal and tangential forces only, and the grasping model is physically realizable when the following two constraints are satisfied. The normal force components ought to be non-negative

$$x_{n_i} \geq 0, \quad i = 1, \ldots, m \tag{2.2}$$

and if a Coulomb friction model is used, the tangential and normal forces have to satisfy

$$|x_{f_i}| \leq \mu x_{n_i}, \quad i = 1, \ldots, m \tag{2.3}$$

where $\mu$ is the static coefficient of friction.

Equations (2.1)–(2.3) constitute a force closure model. These equations may not have a solution, and when a feasible solution does exist it may not be unique. Such ambiguity is often overcome by introducing a convex functional, that is minimized subject to the constraints expressed in equations (2.1)–(2.3). In our recent work,[4] we considered three specific convex functionals to remove the ambiguity in the problem solution. This provides us with grasping quality information obtained by solving the following opitmization problem:

$$(GR_p) \ \min\{H_p(x_n) : (x_n, x_f) \in C(\theta; m)\} \tag{2.4}$$

where

$$C(\theta; m) := \{(x_n, x_f) \in \mathbb{R}^m \times \mathbb{R}^m : \text{satisfy (2.1), (2.2) and (2.3)}\} \tag{2.5}$$

and $H_p$, $p = 1, 2, 3$ are defined as follows:

$$H_1(x_n) = \sum_{i=1}^{m} x_{n_i}^2 := \|x_n\|^2 \tag{2.6}$$

$$H_2(x_n) = \max_{1 \leq i \leq m} |x_{n_i}| \tag{2.7}$$

$$H_3(x_n) = \sum_{i=1}^{m} (x_{n_i} + 1) \log(x_{n_i} + 1) - \sum_{i=1}^{m} x_{n_i}. \tag{2.8}$$

Here, $H_1(x_n)$ represents the energy level of linear systems, and it has an energy-like character when used in nonlinear applications. Johansson[16] stated that human-beings tend to minimize their muscle effort, when performing force closures. The minimization of the muscle effort may therefore be captured by minimizing the $H_1(x_n)$ function.

$H_2(x_n)$ is an $l_\infty$-norm of the normal finger forces. Clearly, large grasping forces may cause object damage, and human-beings tend to reduce the applied finger forces when grasping soft objects.[16] Thus, it is logical to examine the minimum of the maximum applied finger forces as a grasping quality measure.

In a recent study Lee and Rim[17] observed that, in force closures, the applied contact forces of the individual human fingers maintain fixed ratios. It is postulated that the different contributions of the fingers are due to their physiological variations. It would be logical to assume that mechanical manipulators that possess identical finger structures, generate high quality grasps when their contact forces are uniform. A uniform force distribution may be obtained by minimizing an entropy functional.[18] Therefore, in [4] we suggested to use $H_3(x_n)$ which can be shown to preserve the basic properties of an entropy function. Note that $H_1(x_n)$, $H_3(x_n)$ are strictly convex functions, and thus problems $(GR_1)$, $(GR_3)$ consist of minimizing a strictly convex function over a polyhedral constraints set (2.5). Therefore, any optimal solution (if it exists) will be unique and global.

As shown in reference [4], the solution of problem $(GR_p)$ (where $p = 1$, 2, or 3), allows us to define grasping quality measures. Let $x_n^*(p, \theta)$ be the optimal contact force values, where $\theta$ spans its entire range $0 \leq \theta \leq 2\pi$. For a given external loading direction $\theta$, the three corresponding grasping quality measures are:

$$q_1(x_n^*(1, \theta)) := \|x_n^*(1, \theta)\| \tag{2.9}$$

$$q_2(x_n^*(2, \theta)) := \max_{1 \leq i \leq m} \{x_{n_i}^*(2, \theta)\} \tag{2.10}$$

$$q_3(x_n^*(3, \theta)) := H_3(x_{n_i}^*(3, \theta)) \tag{2.11}$$

where $q_1$ represents the energy level of the gripper, $q_2$ reflects the maximum stress applied on the object, and $q_3$ depicts the uniformity level of the finger contact forces. Equations (2.9)–(2.11) define the quality of a grasp as the optimal value of the corresponding optimization problem, and therefore *low values of $q_1$, $q_2$ and $q_3$ correspond to high quality grasps*.

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## 3. SENSITIVITY ANALYSIS FOR THE GRASPING QUALITY PROBLEM

The formulation of the mathematical model that captures the quality of a grasp (2.4) is a deterministic optimization problem in which the object geometry, the finger contact points, and the magnitude of the external load are known. Nevertheless, mechanical manipulators have finite positioning accuracy,[5] and the external load does fluctuate while performing a task. Hence, it is important to investigate the sensitivity of the obtained optimal solutions, and when the model parameters are subjected to small perturbations.

It is well known (see e.g., Fiacco[19]) that in a nonlinear optimization problem, the sensitivity of the objective function is due to the direct effects of the objective function perturbations, and the indirect effects of the constraints variations. It turns out, that the associated Lagrange multipliers of the optimal solution determine the effects of the latter indirect component. One approach to solve convex optimization problems is via the solution of its dual. The dual method transforms a primal constrained optimization problem into an equivalent dual problem which in many cases is easier to solve. The objective function of the dual problem includes all the primal problem parameters. Moreover, it allows to find the optimal solution of the primal problem from the dual optimal solutions (Lagrange mulitpliers) via an explicit formula. The dual method and related important results for convex optimization can be found in Rockafellar.[20] In this section we combine these two fundamental approaches to evaluate the sensitivity of the proposed grasping quality measures.

The original problem (*GR*) defined in (2.4) is modified to introduce an external load and geometric parameter perturbations ($\varepsilon$):

$$GR(\varepsilon) \quad \text{minimize } H(x_n)$$

subject to

$$G_n(\varepsilon)x_n + G_f(\varepsilon)x_f - F(\theta, \varepsilon) = 0 \tag{3.1}$$

$$\mu x_n - x_f \geq 0 \tag{3.2}$$

$$\mu x_n + x_f \geq 0 \tag{3.3}$$

$$x_n \geq 0 \tag{3.4}$$

where $F(\theta, \varepsilon)$ represents the external load that may be subjected to small $\varepsilon$ variations, and $G_n(\varepsilon)$, $G_f(\varepsilon)$ are the grasping matrices that may be subjected to perturbations due to inaccuracies in the finger contact locations. Note that when $\varepsilon = 0$, $GR(\varepsilon)$ reduces to the original *GR* grasping model. $GR(\varepsilon)$ has a convex objective function and linear constraints. To preserve the convexity of the optimization problem, which is strongly exploited in our approach, here we have to assume that the coefficient of friction is not subject to perturbations and is fixed. Notice that when $\mu > 0$, equations (3.2) and (3.3) force $x_n$ to be nonnegative. Therefore, equation (3.4) is in fact redundant and may be omitted from the subsequent derivations.

The grasping quality problem (*GR*) is a special case of a general nonlinear programming formulation that contains equality and inequality constraints with generic form:

$$P(\varepsilon) \quad \text{minimize } f(x, \varepsilon)$$

subject to

$$\begin{aligned} g_i(x, \varepsilon) &\geq 0 \quad i = 1, \ldots, m \\ h_j(x, \varepsilon) &= 0 \quad j = 1, \ldots, p \\ x &\in \mathbb{R}^n \end{aligned} \tag{3.5}$$

where $x \in \mathbb{R}^n$ is a vector of the optimization variables, $\varepsilon \in \mathbb{R}^k$ is a vector of the perturbations of selected parameters, and $f$, $g_i$, $h_j$ are given functions. The optimal solution to the $P(\varepsilon)$ problem depends on the values of the $\varepsilon$ vector.

A fundamental theorem (First-order changes in the optimal value function of $P(\varepsilon)$) and basic sensitivity results, established by Fiacco[21] will be used to derive the sensitivity of the optimal solutions to small perturbations. For the reader's convenience, these basic results are collected in the appendix.

Throughout the rest of this section, our results are developed for problem (*GR*) where the objective function *H* is either the energy or the entropy-like functions as defined in (2.6) and (2.8) respectively. The case of the $l_\infty$-norm can easily be seen to reduce to a linear programming problem and thus will be handled separately (See Section 4 for details).

We start our analysis by deriving a dual formulation for the constrained primal problem $GR(\varepsilon)$. As shown below, the dual problem obtained is a concave maximization problem with simple non-negative constraints. The concavity of the dual objective is a generic property of *every* dual,[20] and thus guarantees that the optimal solution to the dual problem is a global one. Strong duality relations between $GR(\varepsilon)$ and $D(\varepsilon)$ are established, and an explicit expression for the primal optimal solution is obtained from its corresponding dual one. Furthermore, the sensitivity of the optimal primal objective function, is derived from the dual solution.

In the sequel, to simplify notation, $\varepsilon$ is dropped from the problem $GR(\varepsilon)$. Let $y \in \mathbb{R}^3$ (for the planar case), $u \in \mathbb{R}^m_+$, and $v \in \mathbb{R}^m_+$ be the Lagrange multipliers associated with the equality constraints (3.1), the inequality $\mu x_n - x_f \geq 0$ constraints, and $\mu x_n + x_f \geq 0$ constraints, respectively. $\mathbb{R}^m_+$ denotes the *m* dimensional space of nonnegative vectors. The Lagrangian for problem *GR* is

$$\begin{aligned} L(x_n, x_f; y, u, v) := {} & H(x_n) + y^T F(\theta) - y^T G_n x_n - y^T G_f x_f \\ & + u^T x_f - \mu u^T x_n - v^T x_f - \mu v^T x_n \end{aligned} \tag{3.6}$$

The objective function of the dual problem is

$$D(y, u, v) = \min_{x_n, x_f} L(x_n, x_f; y, u, v), \tag{3.7}$$

and the dual problem (*D*) is formulated as

$$\max_{y, u, v} \{D(y, u, v)\}$$

$$\begin{aligned} y &\in \mathbb{R}^k \\ u &\in \mathbb{R}^m_+ \\ v &\in \mathbb{R}^m_+ \end{aligned} \tag{3.8}$$

The objective function of the dual problem can be separated in the $x_n$ and $x_f$ variables as follows:

$$\begin{aligned} & y^T F(\theta) + \min_{x_n} \{(H(x_n) - [y^T G_n + \mu u^T + \mu v^T] x_n\} \\ & + \min_{x_f} \{[-y^T G_f + u^T - v^T] x_f\} \end{aligned} \tag{3.9}$$

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The solution of the third term of (3.9), $\min_{x_f}$ is

$$\min_{x_f}\{[-y^TG_f+u^T-v^T]x_f\}=\begin{cases}0 & \text{if } G_f^Ty=u-v\\ -\infty & \text{otherwise}\end{cases} \quad (3.10)$$

In solving the second term in (3.9) ($\min_{x_n}$), we exploit the fact that $H(x_n)$ is separable in $x_{n_i}$ for the energy-like and entropy functionals respectively (equations (2.6) and (2.8)), i.e.,

$$H(x_n)=\sum_{i=1}^{m} h(x_{n_i}) \quad (3.11)$$

where $h$ is a single valued real function. Hence, the second term of equation (3.9) can be recast as

$$\tilde{h}(s):=\min_t[h(t)-st\} \quad (3.12)$$

where $s\in\mathbb{R}$. Note that $t$ denotes $x_{n_i}$ in (3.11), and $s$ substitutes the expression $[y^TG_n+\mu uu^T+\mu vv^T]$ in (3.9) to obtain a solution for the second term of (3.9) as

$$\min_{x_n}\{H(x_n)-[y^TG_n+\mu uu^T+\mu vv^T]x_n\}=\sum_{i=1}^{m}\tilde{h}([G_n^Ty+\mu(u+v)]_i) \quad (3.13)$$

Equations (3.9), (3.10), (3.13) are utilized to form the dual problem (*D* as

$$\max_{y,\,u,\,v}\left\{y^TF(\theta)+\sum_{i=1}^{m}\tilde{h}([G_n^Ty+\mu(u+v)]_i)\right\} \quad (3.14)$$

subject to:

$$G_f^Ty=u-v \quad (3.15)$$

where $y\in\mathbb{R}^k$, $u\in\mathbb{R}_+^m$, $v\in\mathbb{R}_+^m$. To solve (3.14), the columns of $G_f\in\mathbb{R}^{k\times m}$ are assumed to be linearly independent. This assumption in our case is valid, and therefore the equality constraint in (3.15) is

$$y=[G_f^T]^\dagger(u-v) \quad (3.16)$$

where $[G_f^T]^\dagger$ is the generalized inverse of $G_f^T$ and is equal to $(G_fG_f^T)^{-1}G_f$. Note that in a planar three finger grasp $G_f^T\in\mathbb{R}^{3\times 3}$, and in this case $[G_f^T]^\dagger=[G_f^T]^{-1}$.

Equation (3.16) is substituted into the argument of $\tilde{h}$ in (3.13) to obtain

$$[G_n^Ty+\mu(u+v)]_i=[[G_n^T(G_f^T)^\dagger+\mu I]u-[G_n^T(G_f^T)^\dagger-\mu I]v]_i, \quad (3.17)$$

where $I\in\mathbb{R}^{m\times m}$ is the identity matrix. To simplify the notation in the following derivations, $\Gamma^+$ and $\Gamma^-$ are defined as

$$\Gamma^+=G_n^T(G_f^T)^\dagger+\mu I \quad (3.18)$$

$$\Gamma^-=G_n^T(G_f^T)^\dagger-\mu I \quad (3.19)$$

Using (3.14), (3.15), (3.17), (3.18) and (3.19) we have proved the following proposition.

**Proposition 3.1.** *The dual formulation of the primal GR problem is the following concave maximization problem with non-negativity constraints:*

$$(D)\ \max\left\{F(\theta)^T(G_f^T)^\dagger(u-v)+\sum_{i=1}^{m}\tilde{h}[(\Gamma^+u)_i-(\Gamma^-v)_i]:\ u\in\mathbb{R}_+^m,\ v\in\mathbb{R}_+^m\right\} \quad (3.20)$$

*where* $(\Gamma^+u)_i$ *and* $(\Gamma^-v)_i$ *denote the i-th component of* $(\Gamma^+u)$ *and* $(\Gamma^-v)$, *respectively.*

Notice that all the perturbation parameters now appear only in the objective function, which implies a simpler sensitivity analysis. The relations between the problems (*GR*) and (*D*) are summarized in the following result.

**Proposition 3.2.** *The optimal value V(GR), of the primal problem (GR) is equal to its corresponding optimal dual value V(D). Moreover, if (u*, v*) solves problem (D), then the following* $x_{n_i}^*$ *and* $x_f^*$ *solve the primal (GR) problem:*

$$x_{n_i}^*=[h']^{-1}([\Gamma^+u^*]_i-[\Gamma^-v^*)_i]) \quad i=1,\ldots,m \quad (3.21)$$

$$x_f^*=[G_f]^\dagger[F(\theta)-G_nx_n^*] \quad (3.22)$$

*where* $h'$ *denotes the derivative of* $h$, *and* $[h']^{-1}$ *denotes the inverse function of* $[h']$.

**Proof:** Since the primal problem [*GR*] is convex with linear constraints, strong duality holds,[20] and we have $\min(GR)=\max(D)$, i.e., $V(GR)=V(D)$. The optimal $x_n^*$ is simply obtained from the minimizer in (3.12) (recall that the problem is here separable), i.e., solving $h'(t^*)=s\Leftrightarrow t^*=[h']^{-1}(s)$. Note that $[h']^{-1}(s)$ exists in our case. Substituting the appropriate value for $s$ gives formula (3.21). For $x_f^*$, one must satisfy the constraint $G_nx_n+G_fx_f=F(\theta)$ for $x_n=x_n^*$, hence the result (3.22) follows.

## 4. A SENSITIVITY MEASURE FOR THE GRASPING QUALITY FUNCTIONS

The dual problem (3.20) can be solved by any standard optimization algorithm that maximizes a concave function subject to non-negative constraints (see e.g., reference 22 for example). If $u^*$ and $v^*$ solve the dual (*D*) problem, then $y^*$ can be obtained from (3.16) where $u^*$, $v^*$ and $y^*$ are the optimal Lagrange multipliers of the primal problem (*GR*). Furthermore, equations (3.21) and (3.22) can be used for deriving $x_n^*$ and $x_f^*$ that solve the primal problem (*GR*) By applying Fiacco's theorem (see appendix) to the grasping quality functions $H_p$, proposed in Section 2, one can obtain the sensitivity of the optimal $H_p$ to $\varepsilon$ perturbations, $\nabla_\varepsilon H_p^*(\varepsilon)$, where $\nabla_\varepsilon H_p^*(\varepsilon)$ is a function of the perturbation value (see eqs. (4.10–4.11) below).

If the perturbations are in the range $\varepsilon^-\le\varepsilon\le\varepsilon^+$ then an overall sensitivity measure for a given external load direction $\theta$ can be defined as

$$S_{m_p}(\theta)=\frac{1}{\varepsilon^--\varepsilon^+}\int_{\varepsilon^-}^{\varepsilon^+}|\nabla_\varepsilon H_p^*(\varepsilon)|\,d\varepsilon. \quad (4.1)$$

The sensitivity measure $S_{m_p}$ which is computed from the

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perturbed optimal grasping quality functions can be interpreted as an average measure over all possible variations of the perturbed parameters. Note that in equation (4.1) one is concerned with the magnitude of the sensitivity rather than its sign, as also suggested by Khalili.[10] Note also that low $S_m$ values correspond to low sensitivity to the specific $\varepsilon$ perturbations.

Using the results developed in Section 3, we are now ready to present an algorithm that evaluates the sensitivity measures of the grasping quality functions.

**Computational procedure for evaluating the sensitivity measure $S_m$(u)**

(i) For a given force closure, $G_n$, $G_f$, $F(\theta)$ and $\mu$ are known. Select a perturbation parameter $\varepsilon$ to formulate equations (3.1) to (3.3).

(ii) For the energy $H_1(x_n)$ (equation 2.6), and the entropy $H_3(x_n)$ functions (2.12), separate $H_p(x_n)$ into $\Sigma h_p(x_{n_i})$ utilizing (3.11), and evaluate the single valued real function $\tilde{h}(s)$ using equation 3.12.

For the *energy functional* $h(t)=t^2$, and

$$\tilde{h}(s):=\min_t\{t^2-st\}, \tag{4.2}$$

which yields

$$\tilde{h}(s)=-\frac{1}{4}s^2. \tag{4.3}$$

On the other hand, for the *entropy functional* $h(t)=(t+1)\log(t+1)-t$ and

$$\tilde{h}(s):=\min_t\{(t+1)\log(t+1)-t-st\}, \tag{4.4}$$

which yields

$$\tilde{h}(s)=s-e^s+1. \tag{4.5}$$

In the case of the *min-max* functional $H_2(x_n)$ (equation 2.7), the problem can be reduced to a linear programming formulation, and solved via the Simplex method. Utilizing the Simplex method, the solution directly produces the optimal Lagrange multipliers, the optimal values of the primal variables $x_n^*$, $x_f^*$, and the $S_m(\theta)$ sensitivity measure. Hence, the following steps in the algorithm are required for evaluating $S_m(\theta)$ for the energy and entropy functionals only.

(iii) Compute $\Gamma^+$ from (3.18) and $\Gamma^-$ from (3.19).

(iv) Solve the dual problem (3.20) for a specific $\varepsilon$ value. Use any efficient algorithm that maximizes a concave function subjected to non-negative constraints. The dual optimal solution is $u^*$ and $v^*$.

Note that in the case of the *energy functional*, the dual problem is

$$\max_{u\geq 0,\, v\geq 0}\left\{F(\theta)^T(G_f^T)^\dagger(u-v)-\frac{1}{4}\sum_{i=1}^{m}[(\Gamma^+u)_i-(\Gamma^-v)_i)^2\right\} \tag{4.6}$$

In the case of the *entropy functional*, the dual problem becomes

$$\max_{u\geq 0,\, v\geq 0}\left\{F(\theta)^T(G_f^T)^\dagger(u-v)+\sum_{i=1}^{m}[(\Gamma^+u)_i-(\Gamma^-v)_i]-\sum_{i=1}^{m}e^{(\Gamma^+u)_i-(\Gamma^-v)_i}+(m)\right\}. \tag{4.7}$$

(v) Compute $y^*$ from (3.16), $x_n^*$ and $x_f^*$ from (3.21) and (3.22), respectively. The pair $(x_n^*, x_f^*)$, then solve the primal problem (*GR*).

Note that for the *energy functional*

$$x_{n_i}^*(\varepsilon)=\frac{1}{2}[(\Gamma^+u)_i-(\Gamma^-v)_i], \tag{4.8}$$

and for the *entropy functional*

$$x_{n_i}^*(\varepsilon)=e^{(\Gamma^+u)_i-(\Gamma^-v)_i}-1. \tag{4.9}$$

(vi) Compute $\nabla_\varepsilon H(x_n^*, \varepsilon)$ for the selected perturbation parameter. When the perturbations of the external load $F(\theta, \varepsilon)$ are considered, $\nabla_\varepsilon H(\varepsilon)$ is expressed as

$$\nabla_\varepsilon H(x_n^*, \varepsilon)=(G_f^T)^\dagger(u^*-v^*)\frac{d}{d\varepsilon}F(\theta, \varepsilon). \tag{4.10}$$

On the other hand, when the perturbations of contact finger locations are of interest, the grasping matrices $G_n$ and $G_f$ are functions of $\varepsilon$, and $\nabla_\varepsilon H^*(\varepsilon)$ is expressed as

$$\nabla_\varepsilon H(x_n^*, \varepsilon)=-(G_f^T)^\dagger(u^*-v^*)(\nabla G_n x_n^*+\nabla_\varepsilon G_f x_f^*). \tag{4.11}$$

(vii) For a given $\theta$ direction, the sensitivity measure $S_m(\theta)$ is evaluated via equation (4.1). Note that steps 3 to 6 are repeated to cover the range $\varepsilon^-\leq\varepsilon\leq\varepsilon^+$.

## 5. NUMERICAL SIMULATIONS

The procedure presented in Section 4 is utilized to evaluate the sensitivity of two and three finger grasps of a planar trapezoidal shown in Figure 1. The free body diagrams are

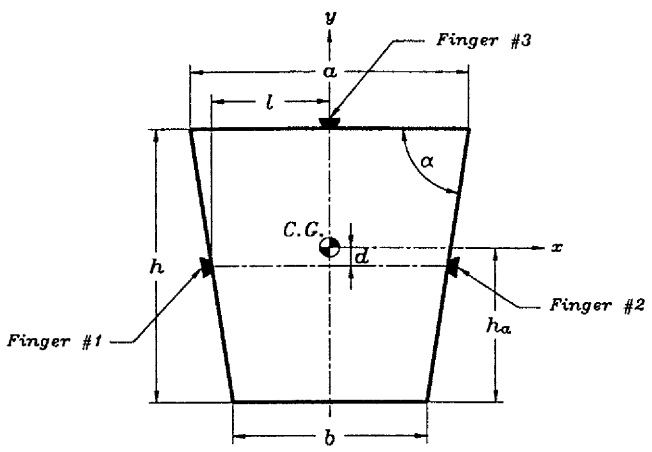



Fig. 1. The planar trapezoidal object.

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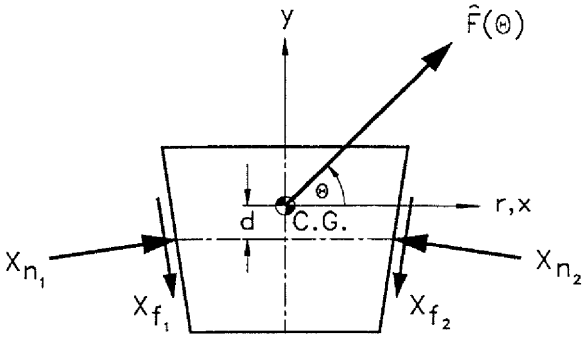



Fig. 2. Two finger grasp of a planar trapezoidal object.

depicted in Figures 2 and 3. The object is subject to a unit external force $\hat{F}(\theta)$ that is applied at the center of gravity along a given $\theta$-direction. When $\theta$ spans its entire planar space, the grasping quality of a trapezoid possesses unique features that are captured by the proposed grasping quality measures. In some $\theta$ directions the grasp is mechanically locked, and in other $\theta$ values the grasp is maintained mainly by frictional forces. The sensitivity analysis in this section, examines how the quality of a grasp is affected by (a) external force perturbations, and (b) variations of the finger contact locations. In both cases the quality of the grasp is chosen to be calculated by the energy $H_1(x_n, \varepsilon)$ and the entropy $H_3(x_n, \varepsilon)$ functionals. Note that the computational procedure can be applied to any of the $H_p$ functions.

In the case of external force perturbations, $F(\theta, \varepsilon)$ is

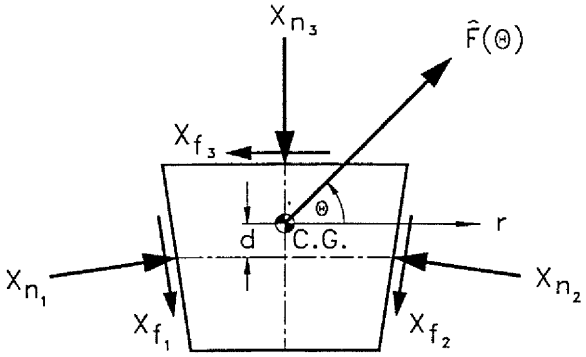



Fig. 3. Three finger grasp of a planar trapezoidal object.

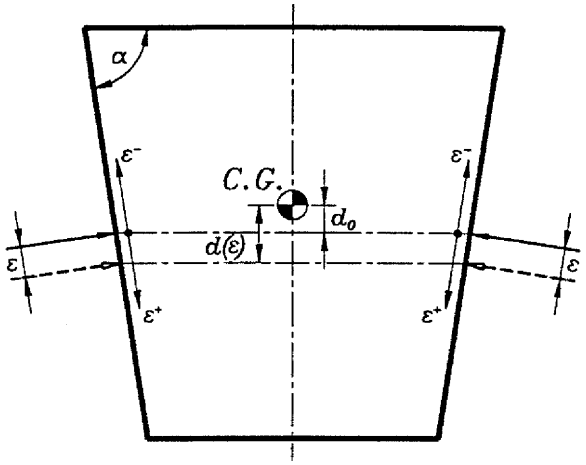



Fig. 4. Perturbations of the finger contact locations.

defined as

$$F(\theta, \varepsilon) = F_0(\theta) + \varepsilon, \tag{5.1}$$

where $-0.1F_0(\theta) \le \varepsilon \le 0.1F_0(\theta)$, and $[\nabla_\varepsilon H(x_n^*, \varepsilon)$ is

$$\nabla_\varepsilon H(x_n^*, \varepsilon) = [(G_f^T)^\dagger (u^* - v^*)]^T F_0(\theta). \tag{5.2}$$

To examine the effects of the finger contact locations, finger 1 and 2 are positioned such that the distance $d(\varepsilon)$ between the horizontal line connecting the two contact points and the center of gravity (shown in Figure 4) is

$$d(\varepsilon) = d_o + \varepsilon \sin \alpha. \tag{5.3}$$

When $\varepsilon = 0$ the fingers are positioned in the middle of the trapezoidal sides. In our case we choose $-0.2d_o \le \varepsilon \le 0.2d_o$, and the corresponding $\nabla_\epsilon H(x_n^*, \varepsilon)$ is

$$\nabla_\varepsilon H(x_n^*, \varepsilon) = -(G_f^T)^\dagger (u^* - v^*)[\nabla_\varepsilon G_n(\varepsilon) x_n^* + \nabla_\varepsilon G_f(\varepsilon) x_f^*]. \tag{5.4}$$

Figures 5 and 6 depict the grasping quality functions and the

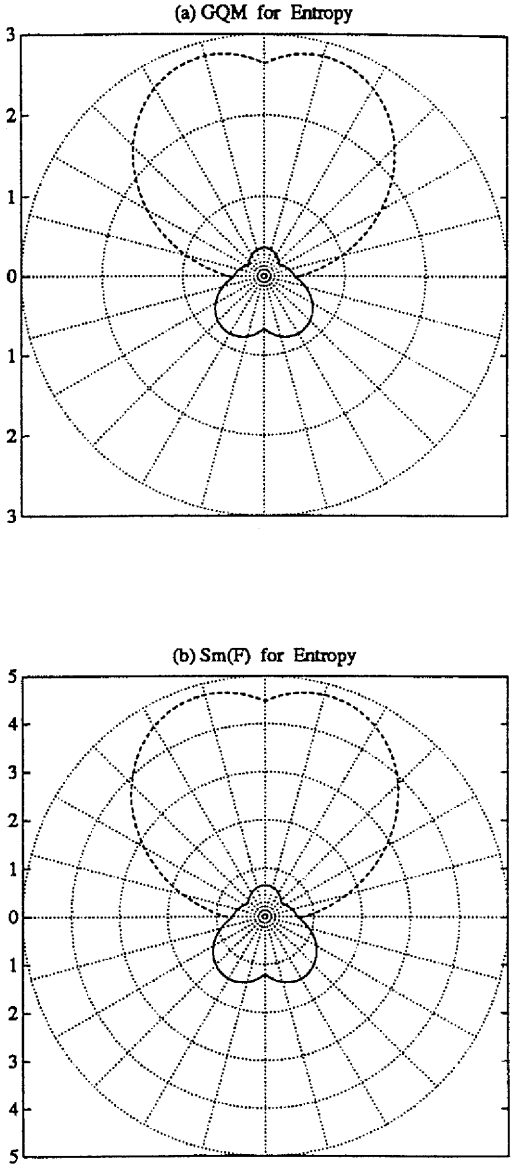



Fig. 5. (a) GQM and, (b) sensitivity measure that reflects the effect of perturbation in the external force $S_m(F)$, for two (dashed line) and, three (solid line) finger grasps when entropy is the objective function.

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sensitivity values for perturbation in the external force and finger contact locations, for two and three finger grasps. These polar plots depict $q_p(\theta)$ and $S_{m_p}(\theta)$ for $0° \leq \theta \leq 360°$.

Figures 5a and 6a depict the quality $q_3(\theta)$ of two (dashed line) and three (solid line) finger grasps, when the selected objective function is entropy ($H_3(x_n)$), and $0° \leq \theta \leq 360°$. The corresponding sensitivity measures $S_{m_3}(\theta)$, for perturbations in the external force and the finger contact locations are shown in Figures 5b and 6b respectively. In the latter two Figures, dashed line corresponds to a two finger grasp, and solid line to a three finger grasp. The information in Figures 5a, 5b and 6b enable one to assess the quality of a specific grasping configuration (given geometry, and force direction) and determine its robustness. For example, the qualities of the two finger grasp when $\theta = 90°$ and $\theta = 53°$ are identical, $q_3(90°) = q_3(53°) = 2.65$. *At* $\theta = 53°$ the sensitivity to force variations and finger contact locations are 4.8, and 4.9, respectively. Whereas at $\theta = 90°$, the corresponding sensitivity values are 4.5 and 0. Hence the grasp at $\theta = 90°$ and at $\theta = 53$, have identical qualities, yet the former is less sensitive to variation in force and finger positions, and therefore it is more robust in nature. The information presented in Figures 5 and 6 is therefore more complete and useful when evaluating various grasping configurations.

Figures 7 and 8 illustrate the quality and sensitivity information for the case where the objective function is energy-like, $H_1(x_n)$.

## 6. CONCLUDING REMARKS

This paper formulates a sensitivity analysis for the quality of a grasp, when the external load or finger contact locations are perturbed. The sensitivity analysis is carried out by transforming the primal, constrained optimization problem into a dual unconstrained formulation. The Lagrange multipliers of the primal problem are the independent variables of the dual one, and their optimal values

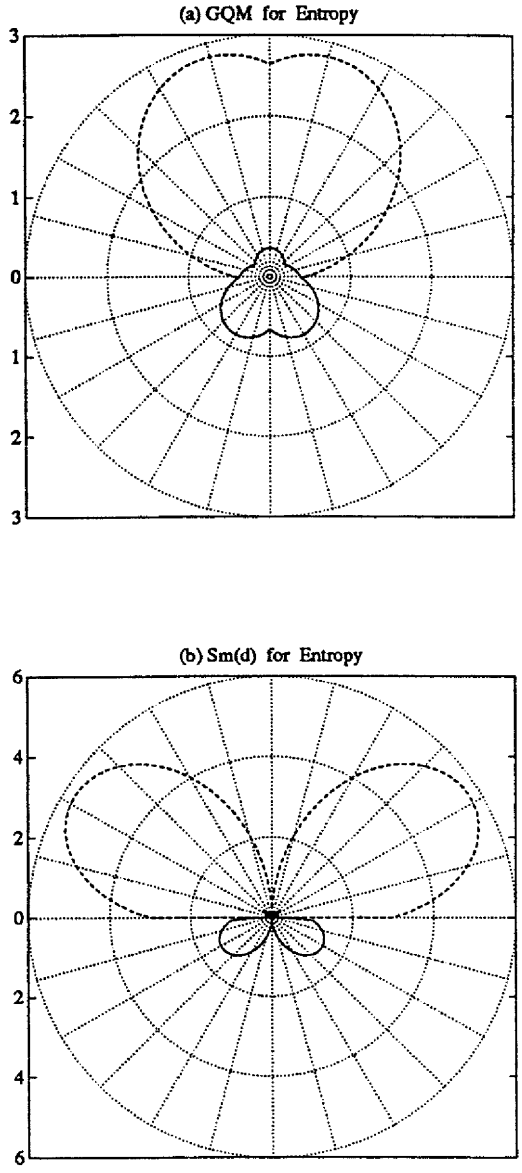



Fig. 6. (a) GQM and, (b) sensitivity measure that reflects the effect of perturbation in the finger contact locations $S_m(d)$, for two (dashed line) and, three (solid line) finger grasps when entropy is the objective function.

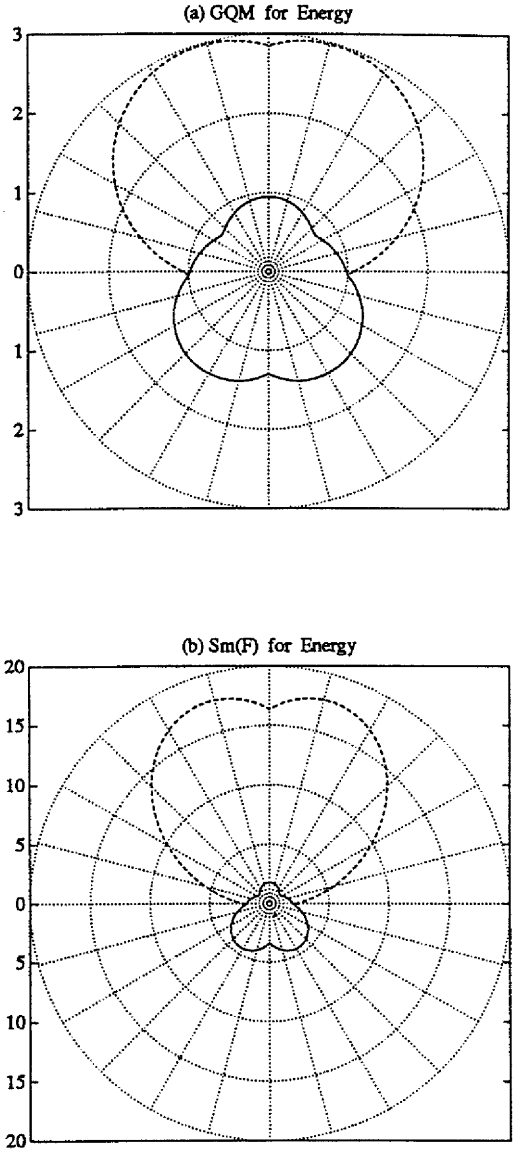



Fig. 7. (a) GQM and, (b) sensitivity measure that reflects the effect of perturbation in the external force $S_m(F)$, for two (dashed line) and, three (solid line) finger grasps when energy is the objective function.

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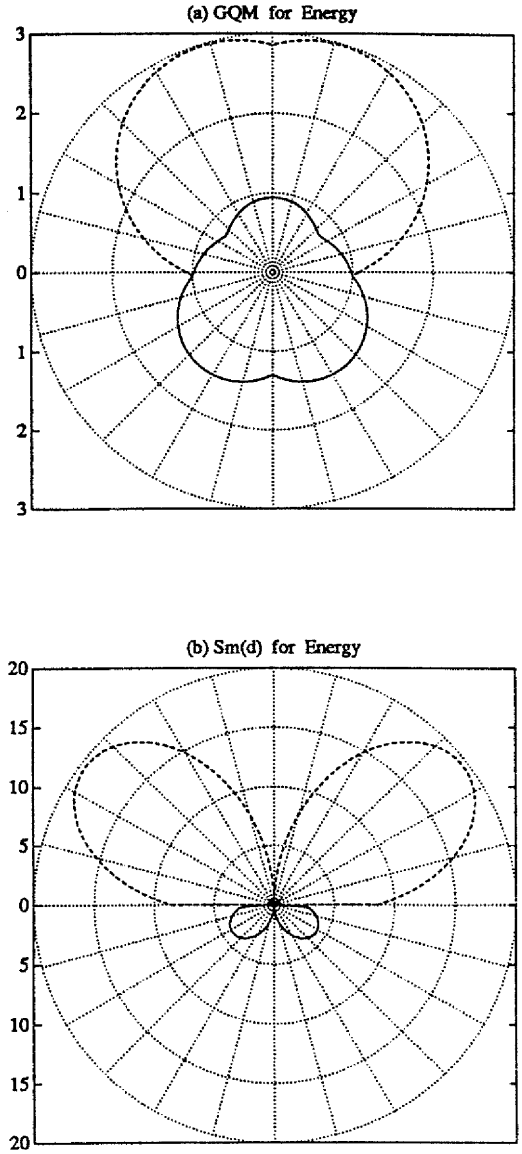



Fig. 8. (a) GQM and, (b) sensitivity measure that reflects the effect of perturbation in the finger contact locations $S_m(d)$, for two (dashed line) and, three (solid line) finger grasps when energy is the objective function.

correspond to the sensitivity of the primal optimal solutions. Furthermore, from the optimal solution of the dual problem one can obtain the optimal finger forces and the grasping quality values, i.e., the optimal solution of the primal problem.

We have suggested an overall sensitivity measure $S_m(\theta)$ which is computed from the perturbed optimal grasping quality functions. The values of $S_m$ indicate the degree of sensitivity to system parameters perturbation, with low values of $S_m$ corresponding to low sensitivity of the grasp. Thus combined with the proposed grasping quality measures, we have a tool for evaluating grasping quality information.

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## 7. APPENDIX

**Theorem.** *If $f$, $g_i$, $h_j$ are twice continuously differentiable in $(x, \varepsilon)$ near the optimal variables $(x^*, 0)$, and if conditions (i)–(iv) (discussed below) are valid, then, in the neighborhood of $\varepsilon = 0$ the following explicit relations hold:*

$$f^*(\varepsilon) = L^*(\varepsilon) \tag{7.1}$$

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$$\nabla_\varepsilon f^*(\varepsilon)=\nabla_\varepsilon f-\sum_{i=1}^{m} u_i(\varepsilon)\nabla_\varepsilon g_i(\varepsilon)+\sum_{j=1}^{p} w_j(\varepsilon)\nabla_\varepsilon h_j(\varepsilon) \quad (7.2)$$

*where*

$$f^*(\varepsilon)=f(x^*(\varepsilon), \varepsilon).$$

*L is the Lagrangian defined as:*

$$L(x, u, w)=F(x)-\sum_{i=1}^{m} u_i g_i(x)+\sum_{j=1}^{p} w_j h_j(x),$$

*where u, and w are the Lagrange multipliers associated with $g_i$ and $h_j$ constraints, respectively. In addition, $L^*(\varepsilon)$ is the optimal value of the Lagrangian that is defined as:*

$$L^*(\varepsilon)=L(x^*(\varepsilon), u^*(\varepsilon), w^*(\varepsilon), \varepsilon).$$

The four conditions that have to be satisfied are:

(i) The functions $f$, $g_i$, $h_j$ are twice continuously differentiable with respect to $x$, and the gradients $\nabla_x f$, $\nabla_x g_i$, $\nabla_x h_j$ are continuously differentiable with respect to $\varepsilon$ in the neighborhood of $(x^*, 0)$.

(ii) $\nabla g_i(x^*, 0)$ and $\nabla h_j(x^*, 0)$ are linearly independent.

(iii) Second order sufficient conditions for a local minimum of $P(0)$ hold at $x^*$, with associated Lagrange multipliers $u^*$ and $w^*$.

(iv) $u_i^*>0$ when $g_i(x^*, 0)=0$ (i.e., strict complementary holds).

The above results are very general. In the special case of problems with right hand side perturbations (i.e., $\varepsilon$ is separated from the optimization variables and can be transformed into the right hand side of equality and inequality constraints), then equation (7.2) reduces to

$$\nabla_\varepsilon f^{*T}(\varepsilon)=\begin{bmatrix} u^*(\varepsilon) \\ -w^*(\varepsilon) \end{bmatrix}. \quad (7.3)$$

Hence, in this case, the optimal Lagrange multipliers capture the sensitivity of the objective function under small $\varepsilon$ perturbation.

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