HALL EFFECTS ON UNSTEADY COUETTE FLOW IN A ROTATING SYSTEM

S. L. Maji

Department of Applied Mathematics
Vidyasagar University
Midnapore, India

S. Das *

Department of Mathematics
University of Gour Bnaga
Malda, India

R. N. Jana

Department of Applied Mathematics
Vidyasagar University
Midnapore, India

ABSTRACT

Hall effects on the unsteady Couette flow between two infinite horizontal parallel plates in a rotating system under the boundary layer approximations have been studied. One of the plate is at rest and the other oscillates in its own plane with the velocity $U(t)$. An exact solution of the governing equations has been obtained by the use of the Laplace transform technique. The numerical results for the velocity components and the shear stresses are depicted graphically. Asymptotic behavior of the solution has analyzed for large values of magnetic parameter, rotation parameter and frequency parameter to gain some physical insight into the flow pattern. It is observed that there exists thin boundary layer (s) is formed near the stationary plate for large values of rotation parameter, magnetic parameter and frequency parameter.

Keywords: Hall effects, Couette flow, Magnetic parameter, Hall parameter, Frequency parameter and boundary layer.

1. INTRODUCTION

Considerable progress has been made recently in the general theory of rotating fluids due to its various applications especially in the geological sciences. The application of rotating fluid are in oceanic, atmospheric and in cosmic fluid dynamics and solar physics (Kinyanjui and Uppal [1]). Geophysicists encounter MHD phenomena in the interactions of conducting fluids and magnetic fields that are present in and around heavenly bodies. Engineers employ MHD principles in the design of heat exchangers, MHD pumps and flow meters, in solving space vehicle propulsion, control and reentry problems; in creating novel power generating systems and in developing confinement schemes for controlled fusion. A number of researchers investigated the hydromagnetic flow of a viscous incompressible electrically conducting fluid in a rotating medium under different conditions and configurations. Mention may be made of the works of of Nanda and Mohanty [2], Jana *et al*. [3], Seth and Maiti [4], Seth *et al*. [5,6], Seth and Ghosh [7], Chandran *et al*. [8], Singh *et al* [9], Ganapathy [10], Singh [11], Seth and Singh [12] and Das *et al*. [13]. When the applied magnetic field is very strong Hall current plays a significant role in determining the flow features of the important problems of geophysical and astrophysical interest and fluid engineering. It may be noted that Hall current and Coriolis force induce secondary flow, therefore, it is of some interest to study the simultaneous effects of Hall current, Coriolis and magnetic forces on such fluid flow problem. Hall effects on the hydromagnetic flow of a viscous incompressible fluid through a parallel plates channel have been studied by Sato [14], Yamanishi [15], Sherman and Sutton [16] and Mandal *et al*. [17,18]. On the other hand, Hall effects on the Couette flow of an electrically conducting fluid between two parallel plates have been studied by Gupta [19]. Hall effect on the hydromagnetic convective flow through a channel with conducting walls have been analyzed by Datta and Jana [20]. The effects of Hall currents on the oscillatory MHD flow past a flat plate have been investigated by Datta and Jana [21]. Majumder [22] has studied the combined effects of Hall current and rotation on hydromagnetic flow over an oscillating porous plate. Jana and Dutta [23] have discussed the Hall effects on unsteady Couette flow when one of the

* Corresponding author (tutusanasd@yahoo.co.in)

DOI : 10.1017/jmech.2013.19
Copyright © 2013 The Society of Theoretical and Applied Mechanics, R.O.C.
https://doi.org/10.1017/jmech.2013.19 Published online by Cambridge University Press

plate started impulsively. Bhaskara and Bathaiah [24] have studied the Hall effects on MHD Couette flow through a porous straight channel. Effects of Hall current and Coriolis force on Hartmann flow under general wall conditions have been investigated by Nagy and Demendy [25]. Pop *et al*. [26] have analyzed the effects of the Hall current on free and forced convective flow in a rotating channel in the presence of an inclined magnetic field. Hall effects on oscillatory hydromagnetic Couette flow in a rotating system have been studied by Seth *et al*. [27].

In the present paper, we have studied the effects of Hall current on the unsteady Couette flow when one of the plate oscillates non-torsionally and the other held at rest. Closed form solution has obtained for the velocity field and shear stresses. It is seen that the primary velocity u_1 decreases and the secondary velocity v_1 increases near the stationary plate while it decreases near the oscillating plate with an increase in rotation parameter K^2. It is also observed that the primary velocity u_1 decreases near the stationary plate while it increases near the oscillating plate with an increase in Hall parameter m. On the other hand, the secondary velocity v_1 increases with an increase in m. For the steady state, the asymptotic behavior of the solution has analyzed for large values of Hartmann number, rotation parameter and frequency parameter to gain some physical insight into the flow pattern. It is observed that in all the three cases there exists a thin boundary layer near the plate $\eta = 0$. It is found that for large magnetic parameter the thickness of the boundary layer decreases with an increase in Hall parameter m. On the other hand, for large rotation parameter as well as for large frequency parameter the boundary layer thicknesses increase with an increase in Hall parameter.

2. MATHEMATICAL FORMULATION AND ITS SOLUTION

Consider the unsteady viscous incompressible electrically conducting fluid bounded by two infinite horizontal parallel plates separated by a distance d. Choose a cartesian co-ordinates system with x- axis along the plate in the direction of the flow, the z-axis is normal to the plates and the y-axis perpendicular to zx-plane shown in Fig. 1. The plates and the fluid are in a state of rigid body rotation with a uniform angular velocity Ω about the z-axis. Initially, at time $t = 0$, the plates and the fluid are at rest. At time $t > 0$, the upper plate at $z = d$ starts to oscillates with the velocity $U(t)$ in its own plane. A uniform magnetic field B_0 is applied perpendicular to the plates. The velocity components are (u, v, w) relative to a frame of reference rotating with the fluid. Since the plates are infinitely long, all physical variables except pressure, depend on z and t only. The equation of continuity then gives $w = 0$ everywhere in the fluid.

Neglecting ion-slip and thermoelectric effects, the generalised Ohm's law for partially ionized gas is [see Cowling [28]]

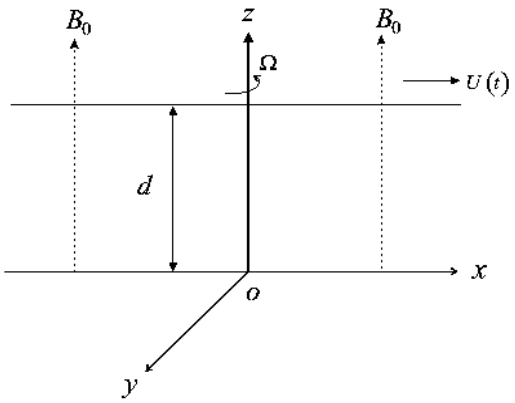


Fig. 1 Geometry of the problem

$$\vec{j} + \frac{\omega_e \tau_e}{B_0}(\vec{j} \times \vec{B}) = \sigma(\vec{E} + \vec{q} \times \vec{B}) \,, \tag{1}$$

where $\vec{B}$, $\vec{E}$, $\vec{q}$, $\vec{j}$, σ, ω_e and τ_e are respectively, the magnetic field vector, the electric field vector, the fluid velocity vector, the current density vector, the conductivity of the fluid, the cyclotron frequency and the electron collision time.

We shall assume that the magnetic Reynolds number for the flow is small so that the induced magnetic field can be neglected. This assumption is justified since the magnetic Reynolds number is generally very small for partially ionized gases. The solenoidal relation $\nabla \cdot \vec{B} = 0$ for the magnetic field gives $B_z = B_0 =$ constant everywhere in the fluid where $\vec{B} \equiv (0, 0, B_0)$. The equation of the conservation of the charge $\nabla \cdot \vec{j} = 0$ gives $j_z =$ constant. This constant is zero since $j_z = 0$ at the plate which is electrically non-conducting. Thus $j_z = 0$ everywhere in the flow. Since the induced magnetic field is neglected, the Maxwell's equation $\nabla \times \vec{E} = -\partial \vec{B} / \partial t$ becomes $\nabla \times \vec{E} = 0$ which gives $\partial E_x / \partial z = 0$ and $\partial E_y / \partial z = 0$. This implies that E_x = constant and E_y = constant everywhere in the flow.

In view of the above assumption, Eq. (1) gives

$$j_x + m j_y = \sigma(E_x + v B_0) \,, \tag{2}$$

$$j_y - m j_x = \sigma(E_y - u B_0) \,, \tag{3}$$

where $m = \omega_e \tau_e$ is the Hall parameter.

Solving for j_x and j_y, we get

$$j_x = \frac{\sigma}{1+m^2}\left[(E_x + v B_0) - m(E_y - u B_0)\right] \,, \tag{4}$$

$$j_y = \frac{\sigma}{1+m^2}\left[m(E_x + v B_0) + (E_y - u B_0)\right] \,. \tag{5}$$

On the use of Eqs. (4) and (5), the equations of motion along x, y and z-directions are

https://doi.org/10.1017/jmech.2013.19 Published online by Cambridge University Press

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p^*}{\partial x} + \nu\frac{\partial^2 u}{\partial z^2} + 2\Omega v - \frac{\sigma B_0}{\rho(1+m^2)}\left[m(E_x + vB_0) + (E_y - uB_0)\right], \quad (6)$$

$$\frac{\partial v}{\partial t} = -\frac{1}{\rho}\frac{\partial p^*}{\partial y} + \nu\frac{\partial^2 v}{\partial z^2} - 2\Omega u - \frac{\sigma B_0}{\rho(1+m^2)}\left[(E_x + vB_0) - m(E_y - uB_0)\right], \quad (7)$$

$$0 = -\frac{1}{\rho}\frac{\partial p^*}{\partial z}, \quad (8)$$

where ρ, v and p^* are respectively the fluid density, the kinematic viscosity and the modified fluid pressure including centrifugal force.

The initial and boundary conditions are respectively

$$u = v = 0 \text{ for } t \le 0,\ 0 \le z \le d, \quad (9)$$

$$u = v = 0 \text{ at } z = 0 \text{ for } t > 0,$$

$$u = U(t),\ v = 0 \text{ at } z = d \text{ for } t > 0. \quad (10)$$

Under the usual boundary layer approximations, Eqs. (6) and (7) become

$$\frac{\partial u}{\partial t} = \frac{\partial U}{\partial t} + \nu\frac{\partial^2 u}{\partial y^2} + 2\Omega v - \frac{\sigma B_0^2}{\rho(1+m^2)}\left[(u - U) - mv\right], \quad (11)$$

$$\frac{\partial v}{\partial t} = \nu\frac{\partial^2 v}{\partial y^2} - 2\Omega(u - U) - \frac{\sigma B_0^2}{\rho(1+m^2)}\left[v + m(u - U)\right]. \quad (12)$$

Introduce the non-dimensional variables

$$\eta = \frac{z}{d},\ u_1 = \frac{u}{U_0},\ v_1 = \frac{v}{U_0},\ \tau = \frac{\nu t}{d^2},\ U = U_0 F(\tau), \quad (13)$$

where U_0 being a constant mean velocity in the x-direction.

On the use of (13), Eqs. (11) and (12) become

$$\frac{\partial u_1}{\partial \tau} - 2K^2 v_1 = \frac{\partial F}{\partial \tau} + \frac{\partial^2 u_1}{\partial \eta^2} - \frac{M^2}{1+m^2}\left[(u_1 - F) - mv_1\right], \quad (14)$$

$$\frac{\partial v_1}{\partial \tau} + 2K^2(u_1 - F) = \frac{\partial^2 v_1}{\partial \eta^2} - \frac{M^2}{1+m^2}\left[v_1 + m(u_1 - F)\right], \quad (15)$$

where $M^2 = \dfrac{\sigma B_0^2 d^2}{\rho\nu}$ is the magnetic parameter and $K^2 = \dfrac{\Omega d^2}{\nu}$ is the rotation parameter.

Equations (14) and (15) can be combined into the following equation

$$\frac{\partial q}{\partial \tau} + 2iK^2(q - F) = \frac{\partial F}{\partial \tau} + \frac{\partial^2 q}{\partial \eta^2} - \frac{M^2(1+im)}{1+m^2}(q - F), \quad (16)$$

where

$$q = u_1 + iv_1,\ i = \sqrt{-1}. \quad (17)$$

The initial and boundary conditions for $q(\eta, \tau)$ are

$$q = 0 \text{ for } \tau \le 0 \text{ and } 0 \le \eta \le 1, \quad (18)$$

$$q = 0 \text{ at } \eta = 0 \text{ and } q = F(\tau) \text{ at } \eta = 1 \text{ for } \tau > 0. \quad (19)$$

The non-dimensional oscillatory velocity $F(\tau)$ of the upper plate is assumed the following form (non-torsional oscillations)

$$F(\tau) = 1 + [ae^{\Omega^*} + be^{-\Omega^*}], \quad (20)$$

where a and b are complex constants. Also, $\omega(= \omega^* d^2 / \nu$, ω^* being the frequency of oscillations) is the non-dimensional frequency of oscillations.

Taking Laplace transform of Eq. (16) and using initial conditions (18), we get

$$\frac{d^2 q^*}{d\eta^2} - \left[s + \frac{M^2(1+im)}{1+m^2} + 2iK^2\right]q^* = -\left[s + \frac{M^2(1+im)}{1+m^2} + 2iK^2\right]F^*, \quad (21)$$

where

$$q^*(\eta, s) = \int_0^\infty q(\eta, \tau)\, e^{-s\tau} d\tau \text{ and } F^*(s) = \int_0^\infty F(\tau)\, e^{-s\tau} d\tau. \quad (22)$$

On the use of (20), the boundary conditions for $q^*(\eta, s)$ are

$$q^* = 0 \text{ at } \eta = 0 \text{ and } q^* = F^*(s) \text{ at } \eta = 1, \quad (23)$$

where

$$F^*(s) = \frac{1}{s} + \frac{a}{s - i\omega} + \frac{b}{s + i\omega} \quad (24)$$

The solution of the Eq. (21) subject to the boundary conditions (23) is

$$q^*(\eta, s) = \left[\frac{1}{s} + \frac{a}{s - i\omega} + \frac{b}{s + i\omega}\right]\left[1 - \frac{\sinh\sqrt{s + r}(1 - \eta)}{\sinh\sqrt{s + r}}\right], \quad (25)$$

where

$$r = \frac{M^2(1+im)}{1+m^2} + 2iK^2. \quad (26)$$

The inverse Laplace transform of Eq. (25) gives

https://doi.org/10.1017/jmech.2013.19 Published online by Cambridge University Press

$$q(\eta,\ \tau)=\begin{cases}\left[1-\dfrac{\sinh(\alpha+i\beta)(1-\eta)}{\sinh(\alpha+i\beta)}\right]+ae^{i\omega\tau}\left[1-\dfrac{\sinh(\alpha_1+i\beta_1)(1-\eta)}{\sinh(\alpha_1+i\beta_1)}\right]\\ +be^{-i\omega\tau}\left[1-\dfrac{\sinh(\alpha_2\pm i\beta_2)(1-\eta)}{\sinh(\alpha_2\pm i\beta_2)}\right]\\ -2\pi\sum_{n=1}^{\infty}n\left[\dfrac{1}{s_1}+\dfrac{a}{s_1-i\omega}+\dfrac{b}{s_1+i\omega}\right]\sin n\pi\eta\, e^{s_1\tau} \qquad \text{for } 2K^2+\dfrac{mM^2}{1+m^2}\neq\omega\ ,\\ \left[1-\dfrac{\sinh(\alpha+i\beta)(1-\eta)}{\sinh(\alpha+i\beta)}\right]+ae^{i\omega\tau}\left[1-\dfrac{\sinh(\alpha_1+i\beta_1)(1-\eta)}{\sinh(\alpha_1+i\beta_1)}\right]\\ +be^{-i\omega\tau}\left[1-\dfrac{\sinh\alpha_3(1-\eta)}{\sinh\alpha_3}\right]\\ -2\pi\sum_{n=1}^{\infty}n\left[\dfrac{1}{s_1}+\dfrac{a}{s_1-i\omega}+\dfrac{b}{s_1+i\omega}\right]\sin n\pi\eta\, e^{s_1\tau} \qquad \text{for } 2K^2+\dfrac{mM^2}{1+m^2}=\omega\end{cases} \tag{27}$$

here

$$\alpha,\ \beta=\frac{1}{\sqrt{2}}\left[\left\{\left(\frac{M^2}{1+m^2}\right)^2+\left(2K^2+\frac{mM^2}{1+m^2}\right)^2\right\}^{\frac{1}{2}}\pm\frac{M^2}{1+m^2}\right]^{\frac{1}{2}},$$

$$\alpha_1,\ \beta_1=\frac{1}{\sqrt{2}}\left[\left\{\left(\frac{M^2}{1+m^2}\right)^2+\left(\omega+2K^2+\frac{mM^2}{1+m^2}\right)^2\right\}^{\frac{1}{2}}\pm\frac{M^2}{1+m^2}\right]^{\frac{1}{2}},$$

$$\alpha_2,\ \beta_2=\frac{1}{\sqrt{2}}\left[\left\{\left(\frac{M^2}{1+m^2}\right)^2+\left(\omega-2K^2-\frac{mM^2}{1+m^2}\right)^2\right\}^{\frac{1}{2}}\pm\frac{M^2}{1+m^2}\right]^{\frac{1}{2}}, \tag{28}$$

$$\alpha_3=\frac{M}{\sqrt{1+m^2}},\ s_1=-\left[n^2\pi^2+\frac{M^2(1+im)}{1+m^2}+2iK^2\right]$$

In the Eq. (27), the upper sign is for $2K^2+\dfrac{mM^2}{1+m^2}>\omega$ and the lower sign is for $2K^2+\dfrac{mM^2}{1+m^2}<\omega$. If $M=0$, then the Eq. (27) coincides with the Eq. (21) of Das *et al.* [13]. The Eq. (27) describes the fluid velocities in the general case. Many particular results can be derived easily from the Eq. (27) which are given in section 4 -6.

3. RESULTS AND DISCUSSION

In order to study the effects of rotation parameter, frequency parameter and Hall parameter on the velocity distributions we have presented the non-dimensional velocities u_1 and v_1 against η for $a = b = 1/2$, $\tau = 2$ and $M^2 = 10$ in Figs. 2 ~ 5 for several values of phase angle $\omega\tau$, rotation parameter K^2, Hall parameter m and frequency parameter ω. It is seen from Fig. 2 that both the primary velocity u_1 and the secondary velocity v_1 decrease with an increase in $\omega\tau$. Figure 3 reveals that the primary velocity u_1 increases with an increase in rotation parameter K^2 and the secondary velocity v_1 increases near the stationary plate while it decreases near the oscillating plate with an increase in rotation parameter K^2. It is observed from Fig. 4 that the primary velocity u_1 decreases near the stationary plate while it increases near the oscillating plate with an increase in Hall parameter m. On the other hand, the secondary velocity v_1 increases with an increase in m. Figure 5 shows that both the primary velocity u_1 and the secondary velocity v_1 decrease with an increase in frequency parameter ω. Thus, the frequency parameter has accelerating influence on the fluid flow.

The non-dimensional shear stresses due to the primary and secondary flows at the plate $\eta = 0$ are given by

https://doi.org/10.1017/jmech.2013.19 Published online by Cambridge University Press

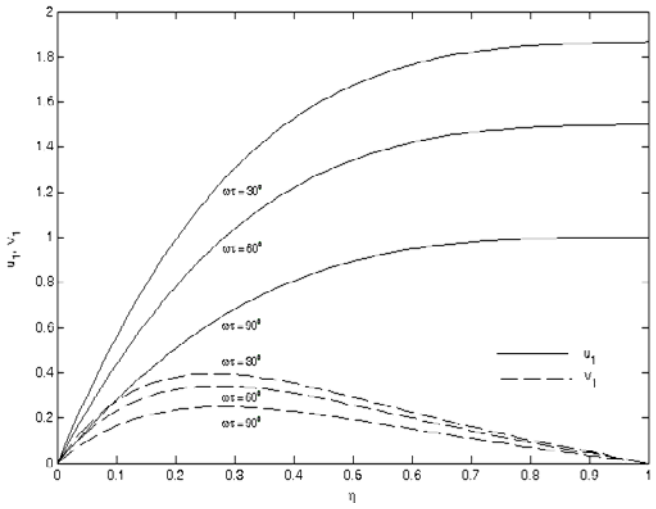


Fig. 2 Variations of u_1 and v_1 for different $\omega\tau$ when $m = 0.5$, $\omega = 4$ and $K^2 = 5$

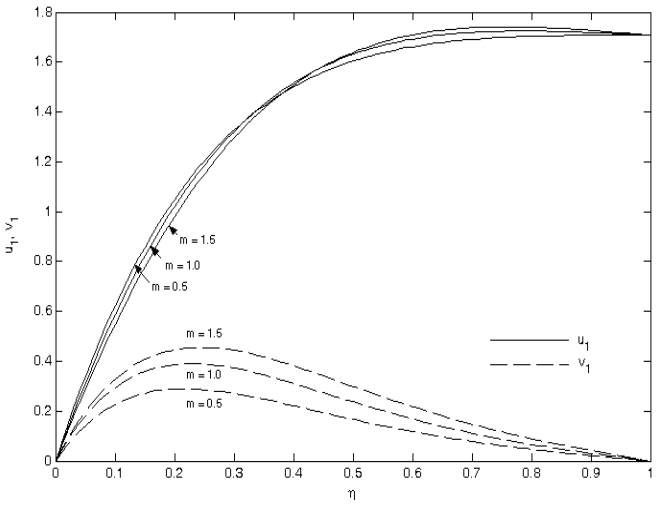


Fig. 4 Variations of u_1 and v_1 for different m when $\omega = 4$, $\omega\tau = \pi/4$ and $K^2 = 5$

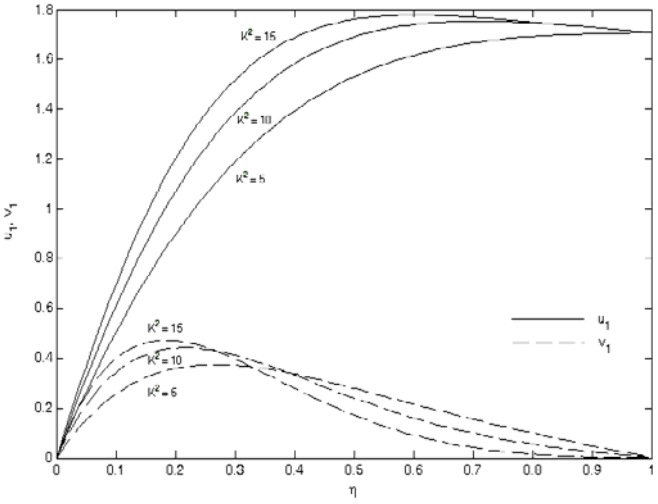


Fig. 3 Variations of u_1 and v_1 for different K^2 when $m = 0.5$, $\omega = 4$ and $\omega\tau = \pi/4$

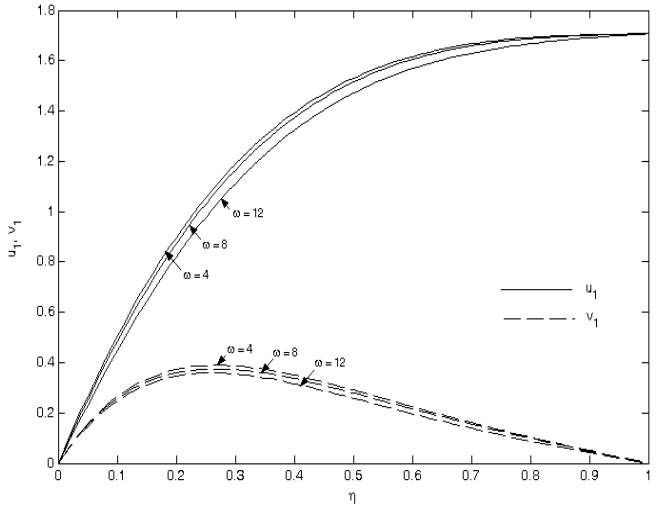


Fig. 5 Variations of u_1 and v_1 for different ω when $m = 0.5$, $\omega\tau = \pi/4$ and $K^2 = 5$

$$\tau_x + i\tau_y = \left[\frac{\partial q}{\partial \eta}\right]_{\eta=0} = \begin{cases} (\alpha+i\beta)\dfrac{\cosh(\alpha+i\beta)}{\sinh(\alpha+i\beta)} + ae^{i\omega\tau}(\alpha_1+i\beta_1)\dfrac{\cosh(\alpha_1+i\beta_2)}{\sinh(\alpha_1+i\beta_1)} \\ +be^{-i\omega\tau}(\alpha_2\pm i\beta_2)\dfrac{\cosh(\alpha_2\pm i\beta_2)}{\sinh(\alpha_2\pm i\beta_2)} \\ -2\pi^2\sum_{i=1}^{\infty} n^2\left[\dfrac{1}{s_1}+\dfrac{a}{s_1-i\omega}+\dfrac{b}{s_1+i\omega}\right]e^{s_1\tau} \qquad \text{for } 2K^2+\dfrac{mM^2}{1+m^2}\neq\omega\,, \\ (\alpha+i\beta)\dfrac{\cosh(\alpha+i\beta)}{\sinh(\alpha+i\beta)} + ae^{i\omega\tau}(\alpha_1+i\beta_1)\dfrac{\cosh(\alpha_1+i\beta_2)}{\sinh(\alpha_1+i\beta_1)} \\ +e^{-i\omega\tau}\dfrac{\alpha_2\cosh\alpha_3}{\sinh\alpha_3} \\ -2\pi^2\sum_{i=1}^{\infty} n^2\left[\dfrac{1}{s_1}+\dfrac{a}{s_1-i\omega}+\dfrac{b}{s_1+i\omega}\right]e^{s_1\tau} \qquad \text{for } 2K^2+\dfrac{mM^2}{1+m^2}=\omega \end{cases} \tag{29}$$

where s_1, α's and β's are given by (28).

Similarly, from Eqs. (22) and (27), one can easily obtain the shear stresses at the oscillating plate $\eta = 1$.

The non-dimensional shear stress τ_x due to the primary flow and the shear stress τ_y due to the secondary flow at

https://doi.org/10.1017/jmech.2013.19 Published online by Cambridge University Press

the plate $\eta = 0$ have been plotted against Hall parameter m for $a = b = 1/2$, $\tau = 2$ and $M^2 = 10$ for several values of rotation parameter K^2, frequency parameter ω and phase angle $\omega\tau$ in Figs. 6 ~ 7. It is observed from Figs. 6 ~ 7 that both the shear stresses τ_x and τ_y increase with an increase in rotation parameter K^2 while they decrease with increase in either frequency parameter ω or phase angle $\omega\tau$. It is also observed that the shear stress τ_x first increases, reach a maximum and then decreases with increase in Hall parameter m. On the other hand, the shear stress τ_y due to the secondary flow steadily increases with increase in Hall parameter m.

4. STEADY STATE SOLUTION

In the limit $\tau \to \infty$, the transient component of Eq. (27) decays very rapidly and ultimate the steady state is reached. The fluid velocity then becomes

$$q(\eta,\ \tau) = \begin{cases} \left[1-\dfrac{\sinh(\alpha+i\beta)(1-\eta)}{\sinh(\alpha+i\beta)}\right] + ae^{i\omega\tau}\left[1-\dfrac{\sinh(\alpha_1+i\beta_1)(1-\eta)}{\sinh(\alpha_1+i\beta_1)}\right] \\ +be^{-i\omega\tau}\left[1-\dfrac{\sinh(\alpha_2\pm i\beta_2)(1-\eta)}{\sinh(\alpha_2\pm i\beta_2)}\right] \qquad \text{for } \ 2K^2+\dfrac{mM^2}{1+m^2} \neq \omega\ , \\ \left[1-\dfrac{\sinh(\alpha+i\beta)(1-\eta)}{\sinh(\alpha+i\beta)}\right] + ae^{i\omega\tau}\left[1-\dfrac{\sinh(\alpha_1+i\beta_1)(1-\eta)}{\sinh(\alpha_1+i\beta_1)}\right] \\ +be^{-i\omega\tau}\left[1-\dfrac{\sinh\alpha_3(1-\eta)}{\sinh\alpha_3}\right] \qquad \text{for } \ 2K^2+\dfrac{mM^2}{1+m^2} = \omega\ , \end{cases} \tag{30}$$

where α, β, α_1 and β_1 are given by (28).

Asymptotic behavior of the solution for fluid velocity field is analyzed for small and large values of rotation parameter K^2, magnetic parameter M^2 and frequency parameter ω to gain further insight into the flow pattern. We shall now discuss the following cases.

Case(i): When $M^2 >> 1$, $K^2 << 1$, $\omega << 1$ and $a = b = \varepsilon/2$, where ε is arbitrary small real number.

In this case, we have from Eq. (30), on separating real and imaginary parts

$$u_1(\eta,\ \tau) = 1-e^{-\alpha\eta}\cos\beta\eta+\varepsilon\left[\cos\omega\tau-\frac{1}{2}e^{-\alpha\eta}\left\{\cos(\omega\tau-\beta_1\eta)+\cos(\omega\tau-\beta_2\eta)\right\}\right], \tag{31}$$

$$v_1(\eta,\ \tau) = e^{-\alpha\eta}\sin\beta\eta+\frac{\varepsilon}{2}e^{-\alpha\eta}\left[\sin(\omega\tau+\beta_2\eta)-\sin(\omega\tau-\beta_1\eta)\right], \tag{32}$$

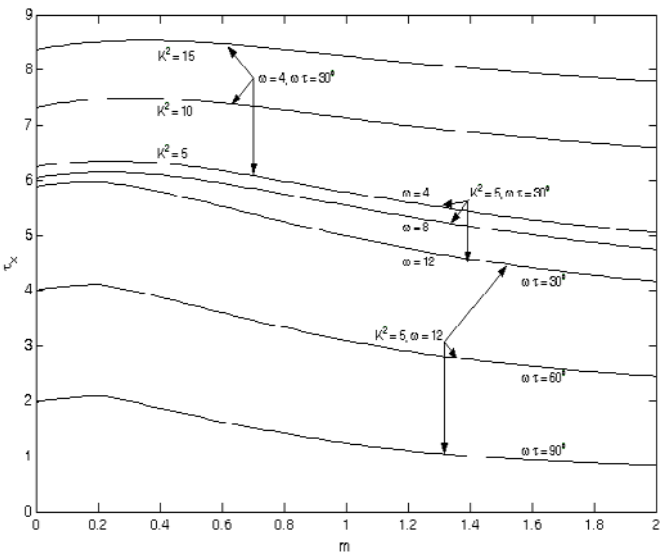


Fig. 6 Variation of shear stress τ_x

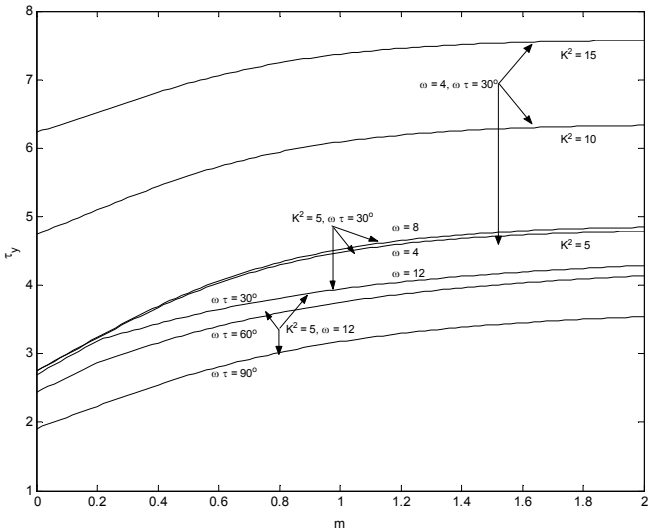


Fig. 7 Variation of shear stress τ_y

https://doi.org/10.1017/jmech.2013.19 Published online by Cambridge University Press

where

$$\alpha = \frac{M}{\sqrt{1+m^2}}\ ,\ \beta = \frac{\sqrt{1+m^2}}{2M}\left(2K^2 + \frac{mM^2}{1+m^2}\right),$$

$$\beta_1 = \frac{\sqrt{1+m^2}}{2M}\left(2K^2 + \frac{mM^2}{1+m^2} + \omega\right),$$

$$\beta_2 = \frac{\sqrt{1+m^2}}{2M}\left(2K^2 + \frac{mM^2}{1+m^2} - \omega\right). \tag{33}$$

It is seen from Eqs. (31) and (32) that there exists a single-deck boundary layer of thickness of the order $O(\alpha^{-1})$ where α is given by (33). It is interesting to note that when magnetic field is very strong then the boundary layer is independent of rotation as well as the frequency of oscillations. It is also seen that the thickness of this boundary layer increases with increase in Hall parameter m but it decreases with an increase in Hartmann number M.

Case(ii): When $K^2 >> 1$, $M^2 << 1$, $\omega << 1$ and $a = b = \varepsilon/2$.

In this case, we have from Eq. (30), on separating real and imaginary parts

$$u_1(\eta,\ \tau) = 1 - e^{-\alpha\eta}\cos\beta\eta + \varepsilon\left[\cos\omega\tau - \frac{1}{2}\left\{e^{-\alpha_1\eta}\cos(\omega\tau - \beta_1\eta) + e^{-\alpha_2\eta}\cos(\omega\tau + \beta_2\eta)\right\}\right], \tag{34}$$

$$v_1(\eta,\ \tau) = e^{-\alpha\eta}\sin\beta\eta + \frac{\varepsilon}{2}\left[e^{-\alpha_2\eta}\sin(\omega\tau + \beta_2\eta) - e^{-\alpha_1\eta}\sin(\omega\tau - \beta_1\eta)\right], \tag{35}$$

where

$$\alpha = K\left[1 + \frac{(1+m)M^2}{4K^2(1+m^2)}\right],\ \beta = K\left[1 + \frac{(1-m)M^2}{4K^2(1+m^2)}\right],$$

$$\alpha_1 = K\left[1 + \frac{\omega}{4K^2} + \frac{(1+m)M^2}{4K^2(1+m^2)}\right],\ \beta_1 = K\left[1 + \frac{\omega}{4K^2} - \frac{(1-m)M^2}{4K^2(1+m^2)}\right],$$

$$\alpha_2 = K\left[1 - \frac{\omega}{4K^2} + \frac{(1+m)M^2}{4K^2(1+m^2)}\right],\ \beta_2 = K\left[1 - \frac{\omega}{4K^2} - \frac{(1-m)M^2}{4K^2(1+m^2)}\right]. \tag{36}$$

Equations (34) and (35) reveal the existence of the triple-decker boundary layers of thicknesses of the order $O(\alpha^{-1})$, $O(\alpha_1^{-1})$ and $O(\alpha_2^{-1})$ near the plate $\eta = 0$, where α, α_1 and α_2 are given by (36). It is seen from (36) that α, α_1 and α_2 decrease with an increase in Hall parameter m and they increase with an increase in either rotation parameter K^2 or magnetic parameter M^2. Hence we conclude that the thickness of the boundary layers increase with an increase in Hall parameter m while it decreases with an increase in either rotation parameter K^2 or magnetic parameter M^2. It is also seen from (34) and (35) that the unsteady parts of the velocity profile consists of two parts, one oscillates with amplitude $1/2\ \varepsilon e^{-\alpha_1\eta}$ and the other with amplitude $1/2\ \varepsilon e^{-\alpha_2\eta}$. The boundary layer corresponding to the former part oscillates with phase lag $\beta_1\eta$ and the layer corresponding to the later part oscillates with phase advance of $\beta_2\eta$.

Case(iii): When $\omega >> 1$, $K^2 << 1$, $M^2 << 1$ and $a = b = \varepsilon/2$.

In this case, the velocity distributions are obtained from (30) as

$$u_1(\eta,\ \tau) = \eta + \frac{1}{6}\cdot\frac{M^2}{1+m^2}(\eta-1)(\eta^2-2\eta) + \cdots + \varepsilon\left[\cos\omega\tau - \frac{1}{2}\left\{e^{-\alpha_1\eta}\cos(\omega\tau - \beta_1\eta) + e^{-\alpha_2\eta}\cos(\omega\tau - \beta_2\eta)\right\}\right], \tag{37}$$

$$v_1(\eta,\tau) = -\frac{1}{6}\left(2K^2 + \frac{mM^2}{1+m^2}\right)(1-\eta)(\eta^2-2\eta) + \cdots - \frac{\varepsilon}{2}\left[e^{-\alpha_1\eta}\sin(\omega\tau - \beta_1\eta) - e^{-\alpha_2\eta}\sin(\omega\tau - \beta_2\eta)\right], \tag{38}$$

where

$$\alpha_1 = \left(\frac{\omega}{2}\right)^{\frac{1}{2}}\left[1 + \frac{K^2}{\omega} + \frac{(1+m)M^2}{2\omega(1+m^2)}\right],\quad \beta_1 = \left(\frac{\omega}{2}\right)^{\frac{1}{2}}\left[1 + \frac{K^2}{\omega} - \frac{(1-m)M^2}{2\omega(1+m^2)}\right],$$

$$\alpha_2 = \left(\frac{\omega}{2}\right)^{\frac{1}{2}}\left[1 - \frac{K^2}{\omega} + \frac{(1+m)M^2}{2\omega(1+m^2)}\right],\quad \beta_2 = \left(\frac{\omega}{2}\right)^{\frac{1}{2}}\left[1 - \frac{K^2}{\omega} - \frac{(1-m)M^2}{2\omega(1+m^2)}\right]. \tag{39}$$

https://doi.org/10.1017/jmech.2013.19 Published online by Cambridge University Press

Expressions (37) and (38) show the existence of double-decker boundary layers of thicknesses of the order $O(\alpha_1^{-1})$ and $O(\alpha_2^{-1})$ where α_1 and α_2 are given by (39). The thicknesses of these layers increase with an increase in Hall parameter m while they decrease with an increase in either frequency parameter ω or magnetic parameter M^2. It is observed from Eqs. (37) and (38) that the unsteady part of the velocity profile consists of two parts, one oscillates with amplitude $1/2\ \varepsilon e^{-\alpha_1\eta}$ and the other with amplitude $1/2\ \varepsilon e^{-\alpha_2\eta}$. The layers corresponding to these amplitudes oscillate with phase lags of $\beta_1\eta$ and $\beta_2\eta$ respectively.

5. SINGLE OSCILLATING PLATE

In the limit $d \to \infty$, when the oscillating plate is at an infinite distance, the steady state solution (30) becomes, on using (13)

$$u + iv = U_0 \left\{ \left[1 - e^{-(\alpha^* + i\beta^*)\frac{z}{\sqrt{2\nu}}} \right] + ae^{i\omega^* t} \left[1 - e^{-(\alpha_1^* + i\beta_1^*)\frac{z}{\sqrt{2\nu}}} \right] + be^{-i\omega^* t} \left[1 - e^{-(\alpha_2^* \pm i\beta_2^*)\frac{z}{\sqrt{2\nu}}} \right] \right\}, \tag{40}$$

where

$$\alpha^*,\ \beta^* = \left[\left\{ \left(\frac{\sigma B_0^2}{\rho(1+m^2)} \right)^2 + \left(2\Omega + \frac{m\sigma B_0^2}{\rho(1+m^2)} \right)^2 \right\}^{1/2} \pm \frac{\sigma B_0^2}{\rho(1+m^2)} \right]^{1/2},$$

$$\alpha_1^*,\ \beta_1^* = \left[\left\{ \left(\frac{\sigma B_0^2}{\rho(1+m^2)} \right)^2 + \left(\omega^* + 2\Omega + \frac{m\sigma B_0^2}{\rho(1+m^2)} \right)^2 \right\}^{1/2} \pm \frac{\sigma B_0^2}{\rho(1+m^2)} \right]^{1/2},$$

$$\alpha_2^*,\ \beta_2^* = \left[\left\{ \left(\frac{\sigma B_0^2}{\rho(1+m^2)} \right)^2 + \left(\omega^* - 2\Omega - \frac{m\sigma B_0^2}{\rho(1+m^2)} \right)^2 \right\}^{1/2} \pm \frac{\sigma B_0^2}{\rho(1+m^2)} \right]^{1/2}. \tag{41}$$

If $a = b = \varepsilon/2$, then we have, on separating real and imaginary parts

$$u(z,\ t) = U_0 \left\{ 1 - e^{-\frac{\alpha^* z}{\sqrt{2\nu}}} \cos\left(\frac{\beta^* z}{\sqrt{2\nu}} \right) + \varepsilon \left[\cos\omega^* t - \frac{1}{2} \left\{ e^{-\frac{\alpha_1^* z}{\sqrt{2\nu}}} \cos\left(\omega^* t - \frac{\beta_1 * z}{\sqrt{2\nu}} \right) + e^{-\frac{\alpha_2^* z}{\sqrt{2\nu}}} \cos\left(\omega^* t \pm \frac{\beta_2^* z}{\sqrt{2\nu}} \right) \right\} \right] \right\}, \tag{42}$$

$$v(z,\ t) = U_0 \left\{ e^{-\frac{\alpha^* z}{\sqrt{2\nu}}} \sin\left(\frac{\beta^* z}{\sqrt{2\nu}} \right) - \frac{\varepsilon}{2} \left[e^{-\frac{\alpha_1^* z}{\sqrt{2\nu}}} \sin\left(\omega^* t - \frac{\beta_1^* z}{\sqrt{2\nu}} \right) - e^{-\frac{\alpha_2^* z}{\sqrt{2\nu}}} \sin\left(\omega^* t \pm \frac{\beta_2^* z}{\sqrt{2\nu}} \right) \right] \right\} \tag{43}$$

which are the velocity components due to free stream oscillations in the presence of a uniform transverse magnetic field and Hall currents when both the plate and the fluid rotate in unison with constant angular velocity Ω about an axis normal to the plate and the free-stream velocity oscillates with velocity $U_0(1 + \varepsilon \cos \omega^* t)$.

It is seen from Eqs. (42) and (43), that the unsteady part of the velocity profile consists of two parts one part oscillates with amplitude $1/2\ \varepsilon e^{-\alpha_1^* z/\sqrt{2\nu}}$ and the other one with amplitude $1/2\ \varepsilon e^{-\alpha_2^* z/\sqrt{2\nu}}$, where α_1^* and α_2^* are given by (41). The layer corresponding to the former part at a distance z from the plate oscillates with phase lag of $\beta_1^* z/\sqrt{2\nu}$ while the layer corresponding to the latter part oscillates with phase advance of $\beta_2^* z/\sqrt{2\nu}$ when $2\Omega + \sigma B_0^2/\rho(1+m^2) > \omega^*$ and with a phase lag of $\beta_2^* z/\sqrt{2\nu}$ when $\omega^* > 2\Omega + \sigma B_0^2/\rho(1+m^2)$. The depths of penetration or the wavelengths of the layers are $2\pi(2\nu)^{1/2}/\beta_1^*$ and $2\pi(2\nu)^{1/2}/\beta_2^*$.

Finally, if $M = 0$, $m = 0$ and $\varepsilon = 0$, then the Eqs. (42) and (43) become

$$u(z,\ t) = U_0 \left[1 - e^{-\sqrt{\frac{\Omega}{\nu}} z} \cos\sqrt{\frac{\Omega}{\nu}} z \right], \tag{44}$$

$$v(z,\ t) = U_0 \left[e^{-\sqrt{\frac{\Omega}{\nu}} z} \sin\sqrt{\frac{\Omega}{\nu}} z \right]. \tag{45}$$

Equations (44) and (45) coincide with the result obtained by Batchelor [29].

https://doi.org/10.1017/jmech.2013.19 Published online by Cambridge University Press

6. SOLUTION FOR NON-OSCILLATING CASE

In this case, substituting $\omega = 0$ in Eq. (27), we get the velocity distribution

$$q(\eta, \tau) = (1+a+b)\left[1-\frac{\sinh(\alpha+i\beta)(1-\eta)}{\sinh(\alpha+i\beta)}-2\pi\sum_{i=1}^{\infty}\frac{n\sin n\pi\eta}{s_1}e^{s_1\tau}\right]. \quad (46)$$

The transient effect represented by the infinite series decay in time of the order $O\left\{\left(n^2\pi^2+\frac{M^2}{1+m^2}\right)^{-1}\right\}$ which increases with an increase in Hall parameter m. It is observed that this time is independent of the rotation parameter K^2. Further, it is observed that Hall currents take more time to reach the steady state than the absence of Hall currents.

7. CONCLUSIONS

Hall effects on unsteady Couette flow between infinite horizontal parallel plates in a rotating system under boundary layer approximation have been studied. It is observed that the velocity components are significantly affected by Hall parameter. It is found that for large values of magnetic parameter there exists a single- deck boundary layer. On the other hand, for large values of rotation parameter and frequency parameter there exists triple-deck and double-deck boundary layers near the plate $\eta = 0$ respectively. Further, when the oscillatory plate is placed at infinity ($d \to \infty$) then we arrive at the problem of the conducting fluid past an infinite flat plate and the free-stream velocity oscillates with velocity $U_0(1 + \varepsilon \cos \omega^* t)$. Further, it is interesting to note that Hall currents take more time to reach the steady state than the absence of Hall currents.

ACKNOWLEDGMENTS

The authors wish to express their sincere thanks to the reviewers for their valuable comments and suggestions.

NOMENCLATURES

a, b	complex constants
B_0	applied magnetic field
$\vec{B}$	magnetic field vector
(B_x, B_y, B_Z)	magnetic field components
d	distance between two plates
$\vec{E}$	electric field vector
(E_x, E_y)	electric field components
$\vec{j}$	current density vector
(j_x, j_y)	current density components
$K^2\left(=\frac{\Omega d^2}{\nu}\right)$	rotation parameter
$M^2\left(=\frac{\sigma B_0^2 d^2}{\rho\nu}\right)$	magnetic parameter
$m(=\omega_e\tau_e)$	Hall parameter
p^*	modified fluid pressure
$\vec{q}$	fluid velocity vector
s	Laplace transform variable
t	time
$U(t)$	velocity of oscillations of upper plate
(u, v, w)	fluid velocity components
(u_1, v_1)	non-dimensional fluid velocity components
U_0	constant mean velocity
x, y, z	co-ordinate axes

Greek symbols

α, β	defined by (28)
α^*, β^*	defined by (41)
η	non-dimensional distance
ν	kinematic coefficient of viscosity
σ	fluid conductivity
ρ	fluid density
ε	real constant
$\omega\left(=\frac{\omega^* d^2}{\nu}\right)$	frequency parameter
ω_e	cyclotron frequency
ω^*	frequency of the oscillations
Ω	uniform angular velocity
τ	non-dimensional time
τ_e	electron collision time
τ_x, τ_y	shear stress components

REFERENCES

1. Kinyanjui, C. N. and Uppal, S. M., "MHD Stokes Problem for a Vertical Infinite Plate in a Dissipative Rotating Fluid with Hall Current," *Journal of Magnetohydrodynamics, Plasma and Space Research*, **8**, pp. 15–30 (1998).
2. Nanda, R. S. and Mohanty, H. K., "Hydromagnetic Flow in a Rotating Channel," *Applied Science Research*, **24**, p. 65 (1971).
3. Jana, R. N., Datta, N. and Mazumder, B. S., "Magnetohydrodynamic Couette Flow and Heat Transfer in a Rotating System," *Journal of the Physical Society of Japan*, **42**, p. 1034 (1977).
4. Seth, G. S. and Maiti, M. K., "MHD Couette Flow and Heat Transfer in a Rotating System," *Indian Journal for Pure and Applied Mathematics,* **13**, p. 931 (1982).
5. Seth, G. S., Jana, R. N. and Maiti, M. K., "Unsteady Hydromagnetic Couette Flow in a Rotating System," *International Journal of Engineering Science*, **20**, p. 989 (1982).
6. Seth, G. S., Nandkeolyar, R., Mahto, N. and Singh, S. K., "MHD Couette Flow in a Rotating System in

https://doi.org/10.1017/jmech.2013.19 Published online by Cambridge University Press

the Presence of Inclined Magnetic Field," *Applied Mathematical Science*, **3**, p. 2919 (2009).
7. Seth, G. S. and Ghosh, S. K., "Unsteady Hydromagnetic Flow in a Rotating Channel in the Presence of Oblique Magnetic Field," *International Journal of Engineering Science*, **24**, p. 1183 (1986).
8. Chandran, P., Sacheti, N. C. and Singh, A. K., "Effect of Rotation on Unsteady Hydromagnetic Couette Flow," *Journal of Astrophysics and Space Science*, **202**, p. 1 (1993).
9. Singh, A. K., Sacheti, N. C. and Chandran, P., "Transient Effects on Magnetohydrodynamic Couette Flow with Rotation," *International Journal of Engineering Science*, **32**, p. 133 (1994).
10. Ganapathy, R., "A Note on Oscillatory Couette Flow in a Rotating System," *ASME Journal of Applied Mechanics*, **61**, p. 208 (1994).
11. Singh, K. D., "An Oscillatory Hydromagnetic Couette Flow in a Rotating System," *ZAMM*, **80**, p. 429 (2000).
12. Seth, G. S. and Singh, M. K., "Combined Free and Forced Convection MHD Flow in a Rotating Channel with Perfectly Conducting Walls," *Indian Journal of Theoretical Physics*, **56**, p. 203 (2008).
13. Das, S., Maji, S. L., Guria, M. and Jana, R. N., "Unsteady MHD Couette Flow in a Rotating System," *Mathematical and Computer Modelling*, **30**, pp. 1211–1217 (2009).
14. Sato, H., "Hall Effect in the Viscous Flow of Ionised Gas Between Parallel Plates Under Transverse Magnetic Field," *Journal of the Physical Society of Japan*, **16**, p. 1427 (1961).
15. Yamanishi, T., "Hall Effect in Viscous Flow of Ionised Gases Through Straight Channels, Preprint, 17 Annual Meeting," *Physics Society Japan Osaka*, **5**, p. 29 (1962).
16. Sherman, A. and Sutton, G. W, *Magnetohydrodynamics,* Evanston, p. 173 (1971).
17. Mandal, G., Mandal, K. K. and Choudhury, G., "On Combined Effects of Coriolis Force and Hall Current on Steady MHD Couette Flow and Heat Transfer," *Journal of the Physical Society of Japan*, **51**, pp. 2010–2015 (1982).
18. Mandal, G. and Mandal, K. K., "Effects of Hall Current on MHD Couette Flow Between Thick Arbitrarily Conducting Plates in a Rotaing System," *Journal of the Physical Society of Japan*, **52**, pp. 470–477 (1983).
19. Gupta, A. S., "Heat Transfer in Hydromagnetic Couette Flow with Hall Effects," *The Mathematics Student*, XL, p. 103 (1972).
20. Datta, N. and Jana, R. N., "Hall Effects on Hydromagnetic Convective Flow Though a Channel with Conducting Walls," *International Journal of Engineering Science*, **15**, p. 561 (1977).
21. Datta, N. and Jana, R. N., "Oscillatory Magnetohydrodynamic Flow Past a Flat Plate with Hall Effects," *Journal of the Physical Society of Japan*, **40**, p. 1469 (1976).
22. Majumder, B. S., "Combined Effects of Hall Currents and Ratation on Hydromagnetic Flow over an Oscillating Porous Plate," *International Journal of Engineering Science*, **15,** p. 601 (1977).
23. Datta, N. and Jana, R. N., "Hall Effects on Unsteady Couette Flow," *International Journal of Engineering Science*, **15**, pp. 35–43 (1977).
24. Bhaskara, R. N. and Bathaiah, D., "Hall Effects on MHD Couette Flow Through a Porous Straight Channel," *Defence Science Journal*, **32**, pp. 313–326 (1982).
25. Nagy, T. and Demendy, Z., "Effects of Hall Current and Coriolis Force on Hartmann Flow Under General Wall Conditions," *Acta Mechanica*, **113**, p. 77 (1995).
26. Pop, I., Ghosh, S. K. and Nandi, D. K., "Effects of the Hall Current on Free and Forced Convective Flow in a Rotating Channel in the Presence of an Inclined Magnetic Field," *Magnetohydrodynamics*, **37**, p. 348 (2001).
27. Seth, G. S., Nandkeolyarb, R. and Ansaric, Md. S., "Hall Effects on Oscillatory Hydromagnetic Couette Flow in a Rotating System," *International Journal of Academy Research*, **1**, pp. 6–17 (2009).
28. Cowling, T. G., *Magnetohydrodynamics*, New York, Interscience (1957).
29. Batchelor, G. K., *An Introduction to Fluid Dynamics*, Ist Edition, Cambridge Press, Cambridge, UK (1967).

(Manuscript received June 15, 2012,
accepted for publication November 18, 2012.)

https://doi.org/10.1017/jmech.2013.19 Published online by Cambridge University Press