Therefore from (8) we have

$$t^2 = \frac{2h}{g}\left(1 + \frac{5h}{3R}\right),$$

or taking square roots and using the first order approximation

$$t \approx \sqrt{\frac{2h}{g}}\left(1 + \frac{5h}{6R}\right). \tag{9}$$

The time to reach the ground assuming a constant force is $t \approx \sqrt{\frac{2h}{g}}$. This last relation tells us that the relative error is $\frac{5h}{6R}$. If the earth had no atmosphere, then a one percent error would occur when the height h was 48 miles!

Reference

1. T. J. Osler, A higher look at vertical motion, *The Mathematical Scientist*, **39** (2014) pp. 138-142.

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100.37 Generalising an SMC problem

The Senior Mathematical Challenge (SMC) is a multiple-choice competition for senior school students. (For more information, see [1].)

Question 21 from the SMC of November 2015 was the following problem (where the names of points have been changed and the multiple-choice options have been removed).

Problem

The diagram shows a triangle ABC. The sides AB, BC and CA have lengths 4, 3 and 2 respectively. The lines GMD, IMF and EMH are drawn parallel to the sides of triangle ABC so that DE, FG and HI are of equal length. What is the length of DE?

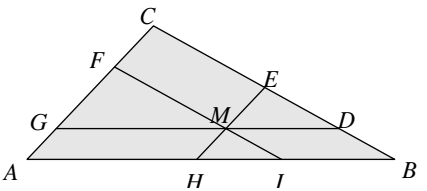


FIGURE 1

In this Note we derive results for a general triangle, not just one with sides 2, 3 and 4. More importantly, we generalise the problem by considering what happens when the point M is replaced by a triangle.

The reader might like to try the above problem before reading on.

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The result

Three lines are drawn parallel to the sides of triangle ABC, as shown, forming a similar triangle JKL, and meeting the sides in D, E, F, G, H and I in such a way that $DE = FG = HI$.

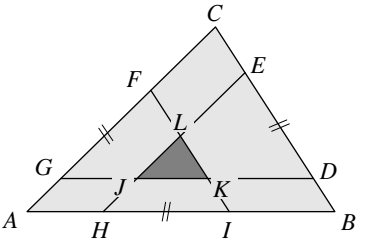


FIGURE 2

Let the sides of triangle ABC have lengths a, b and c, and let the sides of triangle JKL have lengths ka, kb and kc. Then

$$\frac{1 + 2k}{DE} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}.$$

Proof: Draw the line XY through J parallel to side BC, and let $DE = FG = HI = d$, as shown.

It follows from the parallel lines that triangles HXJ and GJY are similar to triangle ABC, and that JXK, $JXBD$, $YJLF$ and $YJEC$ are parallelograms.

Hence, using the fact that

$$BC = BD + DE + EC,$$

we obtain

$$a = \frac{a}{c} \times (d - kc) + d + \frac{a}{b} \times (d - kb). \tag{1}$$

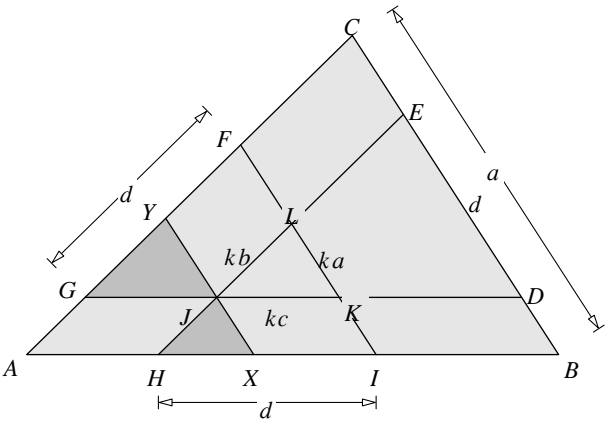


FIGURE 3

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Dividing each term by a and rearranging, we get

$$1 + 2k = \frac{d}{c} + \frac{d}{a} + \frac{d}{b}$$

and the result follows.

Solution to the SMC problem

Using the values $a = 2$, $b = 3$, $c = 4$ and $k = 0$, we get $DE = \frac{12}{13}$. So the solution to the SMC problem is that the length of AP is $\frac{12}{13}$.

Where is the triangle JKL?

The position of the triangle JKL is clearly determined by the other information, but where is it exactly?

When $k = 0$, the triangle JKL has shrunk to a point J, corresponding to the point M in the SMC problem. In that case it is not too hard to show that

$$BD : DE : EC = \frac{1}{c} : \frac{1}{a} : \frac{1}{b}, \tag{2}$$

with similar results for the positions of F, G, H and I. Then the point J is the point of intersection of DG and EH (and FI).

The general result is a little more complicated, but it follows from (1) that

$$BD : DE : EC = \frac{1}{c} - \frac{k}{d} : \frac{1}{a} : \frac{1}{b} - \frac{k}{d}, \tag{3}$$

where d is given by

$$\frac{1 + 2k}{d} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}.$$

Note that (3) agrees with (2) when $k = 0$. As before, there are similar results for the positions of F, G, H and I.

The position of the triangle JKL may now be found. For example, J is the point of intersection of DG and EH.

Reference

1. The United Kingdom Mathematics Trust, https://www.ukmt.org.uk

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