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Path integral approach in the plasma radiation theory

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Abstract. In this paper, we add a supplementary contribution and support to the path integral method by exploring the theory of radiation in plasma physics. This method, based on the Feynman path integral formalism, has the advantage of treating the electrons and ions on the same physical basis and both time-independent and time-dependent problems on the same footing. Since the main quantity in the problem of plasma radiation is the dipolar autocorrelation function of the radiator, we derive it using the path integral technique in its perturbative form. We show that the dipolar autocorrelation function could be written as a generalized expression taking into account the time-dependency effects.

1. Introduction

The spectral line shapes of the radiative atoms and ions in plasma contain a wealth of information and provide valuable diagnostic tools for a number of physical quantities which are often used by the astrophysicists, such as the density of charged particles, temperature, transported radiative energy and perhaps for measuring electric fields [1, 2]. These line shapes are the result of the interactions between the radiator and all the constituents of the plasma.

The spectral line shapes are then broadened by several causes such as: the Doppler effect, which is produced by the movement of the radiator; natural broadening due to the finite lifetime of the atomic excited state; instrumental broadening; and Stark broadening due to the interaction between the plasma electric field and the radiator. As the notable contribution in the case of Stark broadening is non-far perturbers, it is therefore usually sufficient to keep only the first term in a multipole expansion of the interaction, which is commonly known as the dipole approximation. This problem has largely been studied, using the standard Hamiltonian approach of quantum mechanics. It began with the works of Baranger [3] and Kolb and Griem [4], which are based on Anderson's theory [5]. In these works, the electrons are treated by impact theory and the ions by quasi-static theory. As the path integral technique was well tried in many problems [6–8], here we give a supplementary support by exploring the theory of radiation in plasma physics. This method is based on the Feynman path integral formalism [6], which treats the electrons and ions on the same physical basis and both time-independent and time-dependent problems on the same footing – a real advantage over the standard Hamiltonian-based approaches for solving time-dependent problems. Since the main quantity in

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the problem of spectral line shapes is the dipolar autocorrelation function of the radiator, we derive it using the path integral technique in its perturbative form [7, 9]. In this paper we show that the dipolar autocorrelation function could be written as a generalized expression taking into account the time-dependence effects (the dynamics effects).

2. The dipolar autocorrelation function

As stated in the introduction, the main quantity in plasma radiation theory is the dipolar autocorrelation function; it is defined by [2] as

$$C(s) = \sum_{\alpha\beta\alpha'\beta'} \mathbf{d}_{\alpha\beta}\{\langle\alpha|T_a(s,0)|\alpha'\rangle^*\}_{\text{av}}\, \mathbf{d}^*_{\alpha'\beta'}, \tag{1}$$

where $\mathbf{d}_{\alpha\beta}$ is the dipole matrix elements of the radiator (atom or ion of the plasma), T_a is the evolution operator considered here to only possesses matrix elements between substates $\alpha, \alpha', \ldots$ [10, 11]. The summation runs over all the states $\alpha, \alpha', \ldots$ and $\beta, \beta', \ldots$ of upper and lower levels respectively, $|\ldots\rangle$ is the Dirac notation and $\{\ldots\}_{\text{av}}$ means that we must take the statistical average over the perturbers. In (1), we restrict ourselves to the lower states $\beta = \beta' = \beta'', \ldots$ referring to the degenerate cases, which is of interest in the problem of Lyman series without fine structure. In order to find a path integral formula for the autocorrelation function, we first express the matrix elements of the time-evolution operator in the coordinate basis. To reach that end, we introduce the closure relations in the coordinate space ($\int |y\rangle\langle y|\, dy = 1$) as follows:

$$\langle\alpha|T_a(s,0)|\alpha'\rangle = \iint \langle\alpha|y\rangle\langle y|T_a(s,0)|y'\rangle\langle y'|\alpha'\rangle\, dy\, dy'. \tag{2}$$

The dipolar autocorrelation function is therefore expressed in term of the Feynmann propagator as

$$C(s) = \sum_{\alpha\beta\alpha'} d^*_{\alpha'\beta}\mathbf{d}_{\alpha\beta} \iint \varphi_\alpha(y)\varphi^*_{\alpha'}(y')\langle y|\{T_a(s,0)\}_{\text{av}}|y'\rangle\, dy\, dy' \tag{3}$$

$$= \sum_{\alpha\beta\alpha'} d^*_{\alpha'\beta}\mathbf{d}_{\alpha\beta} \iint \varphi_\alpha(y)\varphi^*_{\alpha'}(y')\{K(y,s;y',0)\}_{\text{av}}\, dy\, dy' \tag{4}$$

where

$$K(y,s;y',0) = \int_{y(0)=y'}^{y(s)=y} D[\mathbf{y}(s)] \exp\left\{\frac{i}{\hbar}\int_0^s \left(\frac{m}{2}\dot{\mathbf{y}}^2 + \frac{e^2}{\|\mathbf{y}\|} + e\mathbf{y}\mathbf{E}(\tau)\right) d\tau\right\} \tag{5}$$

and $\mathbf{E}(\tau)$ is the electric field due to all component of plasma acting on the radiator and e denote the electron charge. By developing on the basis of the known initial propagator K_0 governed by the initial Lagrangian

$$L_0 = \frac{m}{2}\dot{\mathbf{y}}^2 + \frac{e^2}{\|\mathbf{y}\|}, \tag{6}$$

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it is possible to write

$$K(y,s;y',0)=\sum_{k=0}^{\infty}\left(\frac{i}{\hbar}\right)^k\frac{1}{k!}\int_0^s d\tau_1\cdots\int_0^s d\tau_k\int d\mathbf{y}_1\cdots\int d\mathbf{y}_k(e\mathbf{E}_1\mathbf{y}_1)\cdots(e\mathbf{E}_k\mathbf{y}_k)$$
$$\times\prod_{j=0}^{k}K_0(\mathbf{y}_{j+1},\tau_{j+1};\mathbf{y}_j,\tau_j) \tag{7}$$

where $\mathbf{y}_{k+1}=\mathbf{y}$ and $\mathbf{y}_0=\mathbf{y}'$. Knowing that [8]

$$K_0(\mathbf{y}_{j+1},\tau_{j+1};\mathbf{y}_j,\tau_j)=\sum_n\psi_n^*(\mathbf{y}_{j+1})\psi_n(\mathbf{y}_j)\exp(-iE_n(\tau_{j+1}-\tau_j)), \tag{8}$$

it follows that

$$\begin{aligned}
&K(y,s;y',0)\\
&=\sum_{k=0}^{\infty}\sum_{\alpha_1,\ldots,\alpha_{k+1}}\left(\frac{i}{\hbar}\right)^k\frac{1}{k!}\int_0^s d\tau_1\cdots\int_0^s d\tau_k\int d\mathbf{y}_1\cdots\int d\mathbf{y}_k(e\mathbf{E}_1\mathbf{y}_1)\cdots(e\mathbf{E}_k\mathbf{y}_k)\\
&\times\varphi^*_{\alpha_{k+1}}(\mathbf{y})\varphi_{\alpha_{k+1}}(\mathbf{y}_k)\exp\frac{i}{\hbar}\varepsilon_{\alpha_{k+1}}(s-\tau_k)\\
&\times\varphi^*_{\alpha_k}(\mathbf{y}_k)\varphi_{\alpha_k}(\mathbf{y}_{k-1})\exp\frac{i}{\hbar}\varepsilon_{\alpha_k}(\tau_k-\tau_{k-1})\\
&\quad\vdots\\
&\times\varphi^*_{\alpha_{21}}(\mathbf{y}_2)\varphi_{\alpha_2}(\mathbf{y}_1)\exp\frac{i}{\hbar}\varepsilon_{\alpha_2}(\tau_2-\tau_1)\\
&\times\varphi^*_{\alpha_1}(\mathbf{y}_1)\varphi_{\alpha_1}(\mathbf{y}')\exp\frac{i}{\hbar}\varepsilon_{\alpha_1}(\tau_1-0).
\end{aligned}\tag{9}$$

The selection rule for the Lyman-α line requires [12]

$$\begin{aligned}
C(s)&=\tfrac{1}{3}|\langle 21\|d^1\|10\rangle|^2\{\langle 210|T|210\rangle+2\langle 211|T|211\rangle\}_{\mathrm{av}}\\
&=\tfrac{1}{3}|\langle 21\|d^1\|10\rangle|^2\exp\{C_0(s)+2C_1(s)\}_{\mathrm{av}}
\end{aligned}\tag{10}$$

where the operator T acts only on the subspace $\{|2,l,m\rangle\}$. Let us first compute $C_1(s)$:

$$C_1(s)=\iint d\mathbf{y}\,d\mathbf{y}'\,\varphi_{211}(\mathbf{y})\varphi^*_{211}(\mathbf{y}')K(\mathbf{y},s;\mathbf{y}',0). \tag{11}$$

Replacing the propagator K by its expression and integrating over $\mathbf{y}$ and $\mathbf{y}'$, and using the orthogonality of the wave function we get

$$\begin{aligned}
C_1(s)=&\sum_{k=0}^{\infty}\sum_{\alpha_2,\alpha_3,\ldots,\alpha_k}\left(\frac{i}{\hbar}\right)^k\frac{1}{k!}\int_0^s\cdots\int_0^s d\tau_1\cdots d\tau_k\int d\mathbf{y}_1\cdots\int d\mathbf{y}_k\\
&\times(e\mathbf{E}_1\mathbf{y}_1)\cdots(e\mathbf{E}_k\mathbf{y}_k)\varphi_{211}(\mathbf{y}_k)\varphi^*_{\alpha_k}(\mathbf{y}_k)\cdots\varphi_{\alpha_2}(\mathbf{y}_1)\varphi^*_{211}(\mathbf{y}_1)\\
&\times\exp\left[\frac{i}{\hbar}\varepsilon_{211}(s-\tau_k)+\frac{i}{\hbar}\varepsilon_{\alpha_k}(\tau_k-\tau_{k-1})+\cdots+\frac{i}{\hbar}\varepsilon_{\alpha_2}(\tau_2-\tau_1)+\cdots\right].
\end{aligned}\tag{12}$$

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and also

$$C_1(s) = \sum_{k=0}^{\infty} \sum_{\alpha_2,\alpha_3\ldots\alpha_k} \left(\frac{i}{\hbar}\right)^k \frac{1}{k!} \int_0^s d\tau_1 \cdots d\tau_k \int d\mathbf{y}_1 \cdots \int d\mathbf{y}_k \times (e\mathbf{E}_1\mathbf{y}_1)\cdots(e\mathbf{E}_k\mathbf{y}_k)\varphi_{211}(\mathbf{y}_k)\varphi^*_{\alpha_k}(\mathbf{y}_k)\cdots\varphi_{\alpha_2}(\mathbf{y}_1)\varphi^*_{211}(\mathbf{y}_1). \quad (13)$$

For the first terms, we have the following.

- Case $k=0$:

$$C_1^{(0)} \equiv C_1^{k=0} = 1. \quad (14)$$

- Case $k=1$: by using the selection rule for the orbital moment l, it is easy to see that:

$$C_1^{(1)}(s) = \frac{i}{\hbar} e \int_0^s d\tau_1\, \mathbf{E}(\tau_1)\left[\int d\mathbf{y}_1\, \varphi_{211}(\mathbf{y}_1)\mathbf{y}_1\varphi^*_{211}(\mathbf{y}_1)\right] = 0. \quad (15)$$

- Case $k=2$:

$$C_1^{(2)}(s) = \frac{1}{2!}\left(\frac{ie}{\hbar}\right)^2 \sum_{\alpha_2} \int_0^s d\tau_1 \int_0^s d\tau_2 \left[\mathbf{E}(\tau_2)\int d\mathbf{y}_2\, \varphi_{211}(\mathbf{y}_2)\mathbf{y}_2\varphi^*_{\alpha_2}(\mathbf{y}_2)\right] \times \left[\mathbf{E}(\tau_1)\int d\mathbf{y}_1\, \varphi_{\alpha_2}(\mathbf{y}_1)\mathbf{y}_1\varphi^*_{211}(\mathbf{y}_1)\right]. \quad (16)$$

In this formula, if we neglect the coupling effect of the states $|n\neq 2, lm\rangle$ with the state $\alpha_2 = |211\rangle$, only the state $\alpha_2 = |200\rangle$ contributes in the summation. Putting

$$e\int d\mathbf{y}_1\, \varphi_{200}(\mathbf{y}_1)\mathbf{y}_1\varphi^*_{211}(\mathbf{y}_1) \equiv \mathbf{D}_+ \quad (17)$$

then

$$C_1^{(2)}(s) = \frac{1}{2!}\left(\frac{i}{\hbar}\right)^2 \left[\int_0^s d\tau\, \mathbf{E}(\tau)\mathbf{D}_+\right]\left[\int_0^s d\tau\, \mathbf{E}(\tau)\mathbf{D}_+\right]^*. \quad (18)$$

Following step by step, we show that the terms of odd order k vanish, and only the terms of even order contribute to the dipolar autocorrelation function. For example, the term of order $k=4$ is equal to

$$\begin{aligned} C_1^{(4)}(s) &= \frac{1}{4!}\left(\frac{i}{\hbar}\right)^4 \int_0^s d\tau_1 \int_0^s d\tau_2 \int_0^s d\tau_3 \int_0^s d\tau_4 [\mathbf{E}(\tau_4)\mathbf{D}_+][\mathbf{E}(\tau_1)\mathbf{D}_+]^* \\ &\quad \times \sum_{\alpha_3}\left[\mathbf{E}(\tau_3)\int d\mathbf{y}_3\, \varphi_{200}(\mathbf{y}_3)\mathbf{y}_3\varphi^*_{\alpha 3}(\mathbf{y}_3)\right] \\ &\quad \times \left[\mathbf{E}(\tau_2)\int d\mathbf{y}_2\, \varphi_{\alpha_3}(\mathbf{y}_2)\mathbf{y}_2\varphi^*_{200}(\mathbf{y}_2)\right] \\ &= \frac{1}{4!}\left(\frac{i}{\hbar}\right)^4 \left|\int_0^s d\tau\, \mathbf{E}(\tau)\mathbf{D}_+\right|^2 \\ &\quad \times \left[\left|\int_0^s d\tau\, \mathbf{E}(\tau)\mathbf{D}_+\right|^2 + \left|\int_0^s d\tau\, \mathbf{E}(\tau)\mathbf{D}_0\right|^2 + \left|\int_0^s d\tau\, \mathbf{E}(\tau)\mathbf{D}_-\right|^2\right] \\ &= \frac{1}{4!}\left(\frac{i}{\hbar}\right)^4 \times A_+(s)^2[A_+(s)^2 + A_0(s)^2 + A_-(s)^2]. \end{aligned} \quad (19)$$

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Similarly, for $k=6$

$$C_1^{(6)}(s) = \frac{1}{6!}\left(\frac{i}{\hbar}\right)^6 A_+(s)^2[A_+(s)^2 + A_0(s)^2 + A_-(s)^2]^2 \tag{20}$$

where

$$e\int d\mathbf{y}_1\, \varphi_{200}(\mathbf{y}_1)\mathbf{y}_1\varphi^*_{210}(\mathbf{y}_1) \equiv \mathbf{D}_0 \tag{21}$$

$$e\int d\mathbf{y}_1\, \varphi_{200}(\mathbf{y}_1)\mathbf{y}_1\varphi^*_{21-1}(\mathbf{y}_1) \equiv \mathbf{D}_- \tag{22}$$

$$e\int d\mathbf{y}_1\, \varphi_{200}(\mathbf{y}_1)\mathbf{y}_1\varphi^*_{211}(\mathbf{y}_1) \equiv \mathbf{D}_+ \tag{23}$$

and

$$A_+(s) = \left|\int_0^s d\tau\, \mathbf{E}(\tau)\mathbf{D}_+\right| \tag{24}$$

$$A_0(s) = \left|\int_0^s d\tau\, \mathbf{E}(\tau)\mathbf{D}_0\right| = \int_0^s d\tau\, \mathbf{E}(\tau)\mathbf{D}_0 \tag{25}$$

$$A_-(s) = \left|\int_0^s d\tau\, \mathbf{E}(\tau)\mathbf{D}_-\right|. \tag{26}$$

Summing the different terms, the result of $C_1(s)$ can be written in a compact form

$$\begin{aligned} C_1(s) &= 1 + A_+(s)^2\sum_{p=1}^{\infty}\left(\frac{i}{\hbar}\right)^{2p}\frac{1}{(2p)!}[A_+(s)^2 + A_0(s)^2 + A_-(s)^2]^{p-1} \\ &= \frac{A_0(s)^2 + A_-(s)^2}{A_+(s)^2 + A_0(s)^2 + A_-(s)^2} \\ &\quad + \frac{A_+(s)^2\cos(1/\hbar)\sqrt{A_+(s)^2 + A_0(s)^2 + A_-(s)^2}}{A_+(s)^2 + A_0(s)^2 + A_-(s)^2}. \end{aligned} \tag{27}$$

$C_0(s)$ can be computed in a similar way

$$\begin{aligned} C_0(s) &= 1 + A_0(s)^2\sum_{p=1}^{\infty}\left(\frac{i}{\hbar}\right)^{2p}\frac{1}{(2p)!}[A_+(s)^2 + A_0(s)^2 + A_-(s)^2]^{p-1} \\ &= \frac{A_+(s)^2 + A_-(s)^2}{A_+(s)^2 + A_0(s)^2 + A_-(s)^2} \\ &\quad + \frac{A_0(s)^2\cos(1/\hbar)\sqrt{A_+(s)^2 + A_0(s)^2 + A_-(s)^2}}{A_+(s)^2 + A_0(s)^2 + A_-(s)^2}. \end{aligned} \tag{28}$$

By gathering the terms $C_0(s)$ and $C_1(s)$, we get the final electric dipolar autocorrelation function as

$$\begin{aligned} C(s) &= \frac{1}{3}|\langle 21\|d^1\|10\rangle|^2\exp\left(\frac{i}{\hbar}\varepsilon_2 s\right)\{C_0(s) + 2C_1(s)\}_{\text{av}} \\ &= \frac{1}{3}|\langle 21\|d^1\|10\rangle|^2\exp\left(\frac{i}{\hbar}\varepsilon_2 s\right) \end{aligned}$$

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$$\times \left\{ \frac{3A_-(s)^2 + 2A_0(s)^2 + A_+(s)^2}{A_+(s)^2 + A_0(s)^2 + A_-(s)^2} \right.$$

$$\left. + [2A_+(s)^2 + A_0(s)^2] \frac{\cos(1/\hbar)\sqrt{A_+(s)^2 + A_0(s)^2 + A_-(s)^2}}{A_+(s)^2 + A_0(s)^2 + A_-(s)^2} \right\}_{\text{av}}. \quad (29)$$

This important result, as mentioned in the abstract, contains the time-dependency effects via the functions $A(\mathbf{E}(s))$ and leads, in principle, to the calculation of the radiation in the plasma: it suffices to make the Fourier transform of the electric dipolar autocorrelation function $C(s)$. We have not done this for several reasons. First, we do not possess the statistics of the collective electrical microfield $E(t)$, second this question is not the aim of this paper, since we are only interested in showing how we use path integral method in the theory of radiation in the plasma and in supporting it. However, this does not prevent us from testing this method in the following simple case. Indeed if we consider the case of a static field oriented towards the $(z'Oz)$ axis, only A_0 contribute (A_+ and A_- are zero) and we find the same result as developed earlier by the standard Hamiltonian approach [12]:

$$C(s) = \frac{1}{3}|\langle 21\|d^1\|10\rangle|^2 \exp\left(\left(\frac{i}{\hbar}\varepsilon_2\right)s\right)\{C_0(s) + 2C_1(s)\}_{\text{av}}$$

$$= \frac{1}{3}|\langle 21\|d^1\|10\rangle|^2 \exp\left(\left(\frac{i}{\hbar}\varepsilon_2\right)s\right)\left\{2 + \cos\left[\frac{DEs}{\hbar}\right]\right\}_{\text{av}}. \quad (30)$$

For the electrical microfield obeying the Gauss distribution law (the case of the strong coupling limit), the statistical average easily gives

$$C(s) = \frac{1}{3}|\langle 21\|d^1\|10\rangle|^2 \exp\left(\left(\frac{i}{\hbar}\varepsilon_2\right)s\right)\int_0^\infty dE\, W(E)\left[2 + \cos\left(\frac{DEs}{\hbar}\right)\right]$$

$$= \frac{1}{3}|\langle 21\|d^1\|10\rangle|^2 \exp\left(\left(\frac{i}{\hbar}\varepsilon_2\right)s\right)$$

$$\times \left[2 + \frac{1}{Q}\int_0^\infty E^2 \cos\left(\frac{DEs}{\hbar}\right) dE \exp\left(-\frac{E^2}{\chi^2}\right)\right] \quad (31)$$

where χ is a parameter having the dimensions of the electric field ($\chi^2 = \frac{2}{3}\langle E^2\rangle$) and Q is the normalization factor of the microfield distribution:

$$Q = \int_0^\infty E^2\, dE \exp\left(\frac{-E^2}{\chi^2}\right) = \frac{\sqrt{\pi}}{4}\chi^3.$$

An elementary calculation for strong coupling limit gives

$$\frac{1}{Q}\int_0^\infty E^2\, dE \exp\left(-\frac{E^2}{\chi^2}\right)\cos\left(\frac{DEs}{\hbar}\right) = \left[1 - \frac{D^2s^2\chi^2}{2\hbar^2}\right]\exp\left(-\frac{D^2s^2\chi^2}{4\hbar^2}\right)$$

and, finally, we get the expression of the dipolar autocorrelation function, which is the same as found in [12]:

$$C(s) = \frac{1}{3}|\langle 21\|d^1\|10\rangle|^2 \exp\left(\left(\frac{i}{\hbar}\varepsilon_2\right)s\right)$$

$$\times \left[2 + \left(1 - \frac{D^2s^2\langle E^2\rangle}{3\hbar^2}\right)\exp\left(-\frac{D^2s^2\langle E^2\rangle}{6\hbar^2}\right)\right]. \quad (32)$$

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3. Discussion

In this paper, we have added a supplementary contribution of the path integral method by exploring the theory of radiation in plasma physics. This method is based on the perturbative scheme of path integrals and allows us to derive the dipolar autocorrelation function of the radiator since it is the main quantity in the problem of plasma radiation. We restricted ourselves to the case of Lyman-alpha radiation, which is the simplest in this problem, although all Lyman terms can in principle be derived in the same manner. Several questions need to be treated in the future: the first is how to extend the method when non-degenerate states are present in the problem (say Balmer terms for example); another is a statistical feature – how to obtain the average over the collective time-dependent electrical microfield.

References

[1] Griem, H. R., Blaha, M. and Kepple, P. C. 1990 *Phys. Rev.* A **41**, 5600.

[2] Baranger, M. 1962 *Atomic and Molecular Processes* (ed. D. R. Bates). New York: Academic Press.

[3] Baranger, M. 1958 *Phys. Rev.* **111**, 481.
Baranger, M. 1958 *Phys. Rev.* **111**, 494.
Baranger, M. 1959 *Phys. Rev.* **112**, 855.

[4] Kolb, A. C. and Griem, H. R. 1958 *Phys. Rev.* **111**, 514.

[5] Anderson, P. W. 1949 *Phys. Rev.* **76**, 647.

[6] Feynman, R. and Hibbs, A. R. 1965 *Quantum Mechanics and Path Integrals*. New York: McGraw-Hill.

[7] Shulmann, L. S. 1982 *Techniques and Applications of Path Integration*. New York: Wiley–Interscience.

[8] Kleinert, H. 1990 *Path Integrals in Quantum Mechanics Statistics and Polymer Physics*. Singapore: World Scientific.

[9] Khandekar, D. C., Lawande, S. V. and Bhagwat, K. V. 1993 *Path-Integral Methods and their Applications*. Singapore: World Scientific.

[10] Griem, H. R., Kolb, A. and Shen, Y. 1959 *Phys. Rev.* **116**.

[11] Griem, H. R., Baranger, M., Kolb, A. C. and Oertel, G. 1962 *Phys. Rev.* **125**, 177.

[12] Stamm, R., Talin, B., Pollock, E. L. and Iglesias, C. A. 1986 *Phys. Rev.* A **34**, 4144.

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