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Parameter estimation and upper bounding adaptation in adaptive-robust control approaches for trajectory control of robots

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SUMMARY

In this paper a new robust adaptive control law for n-link robot manipulators with parametic uncertainties is derived using the Lyapunov theory thus guaranteed the stability of an uncertain system. The novelty of the adaptive robust control algorithm is that manipulator parameters and adaptive upper bounding functions are estimated to control the system properly, and the adaptive robust control law is also updated as an exponential function of manipulator kinematics, inertia parameters and tracking errors. The proposed adaptive control input includes a parameter estimation law as an adaptive controller and an additional control input vector as a robust controller. The developed approach has the advantages of both adaptive and robust control laws, without their discolour tags.

KEYWORDS: Robot manipulators; Adaptive control; Robust control.

1. INTRODUCTION

Since it is difficult to compute or measure dynamic properties of various objects having geometrical complexity, robots still have to face uncertainty of the parameters describing relevant charecteristics of the grasped load, such as mass moment of inertia or exact position of center of mass being handled. In the presence of parametric uncertainty, there are two approaches for the control of the uncertain systems: The adaptive control[1–3] and robust control.[4,5] For combined adaptive-robust-based approaches,[6,7] both adaptive and robust control algorithms are employed. When this approach is compared with the others, adaptive approaches satisfy the learning ability, while the employed robust controller offers the ability to reject disturbances, increases robustness to uncertainty and ensures the desired transient behaviour.

Song et al.[8] developed an integrated adaptive-robust controller based on a sliding mode for uncertain system. When compared with a pure adaptive controller, the approach used improves transient performance, and when compared with pure robust control, it reduces the conservatism of the design. Yu[9] used an adaptive-robust control algorithm for robot manipulators. A combined adaptive and variable structure adaptive control approach is proposed for the tracking control of robot manipulators in an uncertainty environment. Friction compensation in robotic applications has been addressed. Tomei[10] considers the tracking problem for robot manipulators with unknown parameters and dynamic friction. A robust adaptive control algorithm is designed to compensate for disturbance; then trajectory tracking has been achieved. Dawson, Qu and Lewis[11] proposed a hybrid adaptive-robust control approach in which adaptive feedforward terms are designed to compensate for the uncertainties arising from the unknown robot parameters; an additional robust compensator with an adaptive upper bounding function is designed to eliminate the disturbances of viscous and Coulomb frictions. Yao and Tomizuka[12] also proposed adaptive-robust control algorithms for robot tracking control to improve transient behaviour. A smooth adaptive-robust friction compensation strategy is developed by Song et al.[13] Viscous and Coulomb friction can be compensated by a feedforward adaptive controller, while other uncertainties can be compensated by a robust controller with an adaptive upper bounding function. In another approach Song et al.[14] consider the problem of tracking control of robot manipulators having some uncertainties due to joint stick-slip frictions. An integrated adaptive-robust control approach is designed to overcome the uncertainties due to unknown robot parameters and unmodelled friction effects. The integrated adaptive-robust control law includes adaptive feedforward torque and robust compensators with an adaptive upper bounding function.

Slotine and Li,[15] derived a new adaptive control algorithm which consists of a PD feedback part and a full dynamics feedforward compensation part with the unknown manipulator and payload parameters; a convergence of position and velocity errors to zero has been shown. Spong[16] proposed a simple robust nonlinear control similar to the adaptive control algorithm developed by Slotine and Li.[15] Here, the Leitman[17] or the Corless-Leitmann[5] approach was used to design a robust controller. For this purpose, the upper bound on the parametric uncertainty should be known a priori. Koo and Kim,[18] present a robust control law for n-link robot manipulators with parametric uncertainty. The basic control structure is the same as that of Spong's study,[16] but the proposed control law needs no prior information about the upper bound on the uncertainty.

In this paper, for the case of uncertainties of dynamic models, the problem of tracking control of robot manipulator is considered. For that purpose, an adaptive-robust

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control approach is designed to compensate for the uncertainties arising from unknown robot parameters and unknown payload. This adaptive-robust control law includes a PD feedforward part, an adaptive dynamics compensation and robust feed forward compensators with an adaptive upper bounding function. The presented method has the capability of learning uncertain parameters through a parameter estimation law and, as a result of robust control action, an uncertain system learns through the robust adaptation of the upper bounding function. The control structure is similar to those of the adaptive control law proposed by Slotine[15] and Sciavicco.[19] The robust input of the proposed control law has been developed considering the approaches of the Spong[16] and Koo and Kim.[18] The robust adaptive control law has a parameter estimation law like an adaptive controller, and besides it has a robust adaptation of the upper bounding function of a feedforward robust compensator. Apart from smilar studies, the parameter estimation law and upper bounding function are written as an exponential function of manipulator kinematics, inertia parameters and tracking errors. The upper bounding function is also updated in time. The adaptive controller learns the procedure experimentally, while the robust controller rejects external disturbances and increases robustness to uncertainty to ensure the transient response behaviour.

2. ADAPTIVE CONTROL LAW

In the absence of friction or other disturbances, the dynamic model of an n-link manipulator can be written as:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = T \tag{1}$$

where q is generalised coordinate, T is the n × 1 vector of applied torques (or forces), M(q) is the $n \times n$ symmetric positive definite inertia matrix, $C(q,\dot{q})\dot{q}$ is the n × 1 vector of centripetal and Coriolis terms and G(q) is the $n \times 1$ vector of gravitational terms.

Eq. (l) can also be expressed in the following form:[20]

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = Y(q,\dot{q},\ddot{q})\,\pi = u \tag{2}$$

where π is a constant $(p \times 1)$ dimensional vector of inertia parameters and Y is an $n \times p$ matrix of known functions of the joint position, velocity and acceleration.

Consider the control law:

$$u = M(q)\ddot{q}_r + C(q,\dot{q})\dot{q}_r + G(q) + K\sigma \tag{3}$$

with K is a positive definite matrix. The other quantities are given by:

$$\tilde{q} = q_d - q \qquad \dot{q}_r = \dot{q}_d + \Lambda\tilde{q} \qquad \ddot{q}_r = \ddot{q}_d + \Lambda\dot{\tilde{q}} \tag{4}$$

where q̃ is the error between the desired and the actual position, Λ is a positive definite matrix describes the nonlinear compensation and decoupling terms as a function of the desired velocity and acceleration, corrected by the current state (q and q̇) of the manipulator. The term Kσ shows PD action on the error, the error σ is taken as:

$$\sigma = \dot{q}_r - \dot{q} = \dot{\tilde{q}} + \Lambda\tilde{q} \tag{5}$$

Suppose that the computational model has the same structure as that of the manipulator dynamic model, but its parameters are not known exactly. The control law (3) is then modified into:

$$\begin{aligned} u &= \hat{M}(q)\ddot{q}_r + \hat{C}(q,\dot{q})\dot{q}_r + \hat{G} + K\sigma \\ &= Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\,\hat{\pi} + K\sigma \end{aligned} \tag{6}$$

where $\hat{\pi}$ represents the available estimate on the parameters, and accordingly, $\hat{M}$, $\hat{C}$, $\hat{G}$ denote the estimated terms in the dynamic model. Substituting (6) into (2) gives:

$$\begin{aligned} M(q)\dot{\sigma} + C(q,\dot{q})\dot{q}_r + K\sigma &= -\tilde{M}(q)\ddot{q}_r \\ -\tilde{C}(q,\dot{q})\dot{q}_r - \hat{G} &= -Y(q,\dot{q},q_r,\dot{\tilde{q}}_r)\,\tilde{\pi} \end{aligned} \tag{7}$$

where $\tilde{\pi} = \hat{\pi} - \pi$ is the property of linearity in the parameter error. Error quantities concerning system parameters are characterised by:

$$\tilde{M} = \hat{M} - M, \tilde{C} = \hat{C} - C, \tilde{G} = \hat{G} - G \tag{8}$$

The Lyapunov function candidate is defined as,

$$V(\sigma,\tilde{q},\tilde{\pi}) = \frac{1}{2}\sigma^T M(q)\sigma + \frac{1}{2}\tilde{q}^T B\tilde{q}$$

$$+ \frac{1}{2}\tilde{\pi}^T K_\pi \tilde{\pi} > 0 \tag{9}$$

where π is a p dimensional vector containing the unknown manipulators and load parameters, and $\hat{\pi}$ is its estimate, $\tilde{\pi} = \hat{\pi} - \pi$ denotes the parameter estimation error vector. B and K_π are the positive definite, usually diagonal, matrix. Using the property $\sigma^T[\dot{M}(q) - 2C(q,\dot{q})]\sigma = 0 \;\forall\; \sigma \in R^n$ and choosing $B = 2\Lambda K$, the time derivative of V along the trajectory of system (7) is

$$\begin{aligned} \dot{V} = &-\dot{\tilde{q}}^T K\dot{\tilde{q}} - \tilde{q}^T\Lambda K\Lambda\tilde{q} + \tilde{\pi}^T \\ &(K_\pi\dot{\tilde{\pi}} - Y^T(q,\dot{q},\dot{q}_r,\ddot{q}_r)\,\sigma) \end{aligned} \tag{10}$$

If the estimate of the parameter vector is updated as the adaptive law

$$\dot{\hat{\pi}} = K_\pi^{-1} Y^T(q,\dot{q},\dot{q}_r\ddot{q}_r)\,\sigma \tag{11}$$

Eq. (10) becomes

$$\dot{V} = -\dot{\tilde{q}}^T K\dot{\tilde{q}} - \tilde{q}^T\Lambda K\Lambda\tilde{q} \tag{12}$$

It should be noted that $\dot{\hat{\pi}} = \dot{\tilde{\pi}}$ (π is constant).[19] The parameter estimation law (11) can also be written as

$$\hat{\pi} = \int K_\pi^{-1} Y^T(q,\dot{q},\ddot{q}_r q_r)\,\sigma dt \tag{13}$$

3. ROBUST CONTROL LAW

Consider the nominal control vector for the model system described by Eqs. (I) and (2).

$$\begin{aligned} u_0 &= M_0(q)\ddot{q}_r + C_0(q,\dot{q})\dot{q}_r + G_0(q) - K\sigma \\ &= Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\,\pi_0 - K\sigma \end{aligned} \tag{14}$$

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The nominal control vector u_0 in Eq. (14) is defined in terms of fixed parameters given by π_0, and, accordingly, M_0, C_0, and G_0 denotes the estimated terms in dynamic model. The other quantities as distinct from adaptive law are given by:

$$\tilde{q} = q - q_d, \quad \dot{q}_r = \dot{q}_d - \Lambda\tilde{q}, \quad \ddot{q}_r = \ddot{q}_d - \Lambda\dot{\tilde{q}} \tag{15}$$

The control input $\boldsymbol{u}$ can be defined in terms of the nominal control vector $\boldsymbol{u}_0$

$$\begin{aligned} u &= u_0 + Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\,\tau \\ &= Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\,(\pi_0 + \tau) - K_D\,\sigma \end{aligned} \tag{16}$$

It is assumed that both π_0 and ρ are known, the parametric uncertainty $\tilde{\pi}$ is,

$$\|\tilde{\pi}\| = \|\pi - \pi_0\| \le \rho \tag{17}$$

Let $\varepsilon > 0$ and the additional control vector as defined by Spong[16] as:

$$\tau = \begin{cases} -\rho\dfrac{Y^T\sigma}{\|Y^T\sigma\|} & \text{if} \quad \|Y^T\sigma\| > \varepsilon \\ -\rho\dfrac{Y^T\sigma}{\varepsilon} & \text{if} \quad \|Y^T\sigma\| \le \varepsilon \end{cases} \tag{18}$$

As a measure of parameter uncertainty on which the additional control input is based, ρ can be defined as:

$$\rho = \left(\sum_{i=1}^{p} \rho_i\right)^{1/2} \tag{19}$$

An individual unique bound is assigned for each parameter separately as:

$$\lfloor \tilde{\pi}_i \rfloor \le \rho_i \qquad i = 1, 2, \ldots, p \tag{20}$$

Let υ_i denote the ith component of the vector $Y^T\sigma$, ε_i represent the ith component of ε, and the ith component of the control input τ_i as,[16] then:

$$\tau_i = \begin{cases} -\rho_i\,\upsilon_i/|\upsilon_i| & \text{if} \quad |\upsilon_i| > \varepsilon_i \\ -(\rho_i/\varepsilon_i)\,\upsilon_i & \text{if} \quad |\upsilon_i| \le \varepsilon_i \end{cases} \tag{21}$$

4. ADAPTIVE ROBUST CONTROL LAW

Considering the dynamic model of an n-link robot manipulator given by Eqs. (l) and (2), the control input vector that comprises the parameter estimation, and the additional control input defined as:

$$u = Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\,(\hat{\pi} + \delta(t)) + K\sigma \tag{22}$$

Substituting (22) into (l), we obtain after some rearrangement:

$$\begin{aligned} M(q)\,\dot{\sigma} + C(q, \dot{q})\,\sigma + K\sigma = &- Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\,\tilde{\pi} \\ &- Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\,\delta(t) \end{aligned} \tag{23}$$

Adaptive robust parameters are identical with the adaptive control law in the known parameter case such as σ, q_r, Λ and K. It is assumed that the parameter error is unknown such that:

$$\tilde{\pi} = \hat{\pi} - \pi = \rho(t) \tag{24}$$

where $\hat{\pi}$ is the estimate of the available parameters and updated in time; the fixed parameter π shows the lower bounds of the parameters. The upper bounding function $\hat{\rho}(t)$ is assumed to be unknown; it should be determined using the estimation law to control the system properly. The error $\tilde{\rho}(t)$ shows the difference between the parameter error and upper bounding function as:

$$\tilde{\rho}(t) = \rho(t) - \hat{\rho}(t) = \hat{\pi} - \pi - \hat{\rho}(t) \tag{25}$$

Theorem:

π shows the unloaded and lower bound of the parameter, ρ is the upper uncertainity bound, and both of them are supposed to be known initially. The estimate of parameter $\hat{\pi}$ and the additional control input $\delta(t)$ in the control law (22) are defined, respectively as:

$$\begin{aligned} \hat{\pi} &= 2\,(e^{-1/2\int Y^T\sigma dt} - e^{-\int Y^T\sigma dt}) + \pi; \\ \delta(t) &= \rho e^{-1/2\int Y^T\sigma dt} \end{aligned} \tag{26}$$

If the above control law (26) is used in the control input (22) for the trajectory control of the model manipulator, the tracking errors $\tilde{q}$ and $\dot{\tilde{q}}$ converge to zero.

Proof:

By taking into account the above parameters and control algorithm, the Lyapunov function candidate is defined as:

$$\begin{aligned} V(\sigma, \tilde{q}, \tilde{\rho}(t)) = &\frac{1}{2}\sigma^T M(q)\,\sigma + \frac{1}{2}\tilde{q}^T B\tilde{q} \\ &+ \frac{1}{2}\tilde{\rho}(t)^T\Gamma\tilde{\rho}(t) \end{aligned} \tag{27}$$

Apart from the similar studies, Γ varies in time. The time derivative of Eq. (27) is written as:

$$\begin{aligned} \dot{V} = &\sigma^T M(q)\,\dot{\sigma} + \frac{1}{2}\sigma^T\dot{M}(q)\,\sigma + \tilde{q}^T B\dot{\tilde{q}} \\ &+ \frac{1}{2}\tilde{\rho}(t)^T\dot{\Gamma}\,\tilde{\rho}(t) + \tilde{\rho}(t)^T\Gamma\,\dot{\tilde{\rho}}(t) \end{aligned} \tag{28}$$

where:

$$\begin{aligned} \tilde{\rho}(t) &= \rho(t) - \hat{\rho}(t) = \hat{\pi} - \pi - \hat{\rho}(t); \\ \dot{\tilde{\rho}} &= \dot{\rho}(t) - \dot{\hat{\rho}}(t) = \dot{\hat{\pi}} - \dot{\hat{\rho}}(t) \end{aligned} \tag{29}$$

Let $B = 2\Lambda K$ and use the property $\sigma^T\,[\dot{M}(q) - 2C\,(q, \dot{q})]\,\sigma = 0$, $\forall\,\sigma \in R^n$, if the time derivative of V for system (23) is taken:

$$\begin{aligned} \dot{V} = &-\dot{\tilde{q}}^T K\dot{\tilde{q}} - \tilde{q}^T\Lambda K\Lambda\tilde{q} + \sigma^T Y\delta(t) - \sigma^T Y\rho(t) \\ &+ \frac{1}{2}\tilde{\rho}(t)^T\dot{\Gamma}\tilde{\rho}(t) + \tilde{\rho}(t)^T\Gamma\dot{\tilde{\rho}}(t) \end{aligned} \tag{30}$$

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where:

$$-\dot{\tilde{q}}^T K\dot{\tilde{q}} - \tilde{q}^T \Lambda K \Lambda \tilde{q} < 0 \tag{31}$$

If the rest of Eq. (30) is equal to or less than zero, the system will be stable. The remaining terms in Eq. (30) are:

$$\begin{gathered} -\sigma^T Y\delta(t) - \sigma^T Y\rho(t) + \frac{1}{2}[\rho(t) - \hat{\rho}(t)]^T \\ \dot{\Gamma}[\rho(t) - \hat{\rho}(t)] + [\rho(t) - \hat{\rho}(t)]^T \\ \Gamma[\dot{\rho}(t) - \dot{\hat{\rho}}(t)] = 0 \end{gathered} \tag{32}$$

If $\delta(t)$ is considered as an estimated term of the uncertainty bound, that is, $\delta(t) = -\hat{\rho}(t)$ then Eq. (32) is written as:

$$\begin{gathered} \sigma^T Y\hat{\rho}(t) - \sigma^T Y\rho(t) + \frac{1}{2}[\rho(t) - \hat{\rho}(t)]^T \\ \dot{\Gamma}[\rho(t) - \hat{\rho}(t)] + [\rho(t) - \hat{\rho}(t)]^T \\ \Gamma[\dot{\rho}(t) - \dot{\hat{\rho}}(t)] = 0 \end{gathered} \tag{33}$$

When $[\rho(t) - \rho(t)]$ is considered as a common multiplier:

$$\begin{gathered} -\sigma^T Y[\rho(t) - \hat{\rho}(t)] + \frac{1}{2}[\rho(t) - \hat{\rho}(t)]^T \\ \dot{\Gamma}[\rho(t) - \hat{\rho}(t)] + [\rho(t) - \hat{\rho}(t)]^T \\ \Gamma[\dot{\rho}(t) - \dot{\hat{\rho}}(t)] = 0 \end{gathered} \tag{34}$$

Equation (34) can be arranged as:

$$\begin{gathered} [(\rho(t) - \hat{\rho}(t)]^T[-Y^T\sigma + \frac{1}{2}\dot{\Gamma}[\rho(t) - \hat{\rho}(t)) \\ + \Gamma(\dot{\rho}(t) - \dot{\hat{\rho}}(t)] = 0 \end{gathered} \tag{35}$$

The terms constituting the above equation are expressed as:

$$\begin{gathered} \tilde{\rho}(t) = \rho(t) - \hat{\rho}(t); \quad \tilde{\pi} = \rho(t) = \hat{\pi} - \pi; \\ \dot{\rho}(t) = \dot{\tilde{\pi}}; \quad \tilde{\rho} = \dot{\rho}(t) - \dot{\hat{\rho}}(t) = \dot{\hat{\pi}} - \dot{\hat{\rho}}(t) \end{gathered} \tag{36}$$

Substituting the parameters in Eq. (36) into Eq. (35) yields

$$-Y^T\sigma + \frac{1}{2}\dot{\Gamma}[\hat{\pi} - \pi - \hat{\rho}(t)] + \Gamma[\dot{\hat{\pi}} - \dot{\hat{\rho}}(t)] = 0 \tag{37}$$

Then:

$$\begin{gathered} -Y^T\sigma + \frac{1}{2}\dot{\Gamma}(\hat{\pi} - \pi) + \Gamma\dot{\hat{\pi}} \\ -[\frac{1}{2}\dot{\Gamma}\hat{\rho}(t) + \Gamma\dot{\hat{\rho}}(t)] = 0 \end{gathered} \tag{38}$$

Eq. (38) can be divided into two groups as:

$$-Y^T\sigma + \frac{1}{2}\dot{\Gamma}(\hat{\pi} - \pi) + \Gamma\dot{\hat{\pi}} = 0 \tag{39}$$

$$-(\frac{1}{2}\dot{\Gamma}\hat{\rho}(t) + \Gamma\dot{\hat{\rho}}(t)) = 0 \tag{40}$$

If Eqs. (39) and (40) are equal to zero, the system will be stable. Equation (39) can also be written as:

$$\frac{1}{2}\dot{\Gamma}(\hat{\pi} - \pi) + \Gamma\dot{\hat{\pi}} = Y^T(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\sigma \tag{41}$$

In order to solve the differential Eq. (41), Γ is chosen as a positive definite function such that:

$$\Gamma = e^{\int Y^T\sigma dt}, \quad \dot{\Gamma} = Y^T\sigma e^{\int Y^T\sigma dt} \tag{42}$$

Substitution of Eq. (42) into Eq. (41) yields

$$e^{\int Y^T\sigma dt}\dot{\hat{\pi}} + \frac{1}{2}Y^T\sigma e^{\int Y^T\sigma dt}(\hat{\pi} - \pi) = Y^T\sigma \tag{43}$$

Dividing Eq. (43) by $e^{\int Y^T\sigma dt}$

$$\begin{gathered} \dot{\hat{\pi}} + \frac{1}{2}Y^T\sigma\hat{\pi} - \frac{1}{2}Y^T\sigma\pi = e^{-\int Y^T\sigma dt}Y^T\sigma \\ \Rightarrow \dot{\hat{\pi}} + \frac{1}{2}Y^T\sigma\hat{\pi} = e^{-\int Y^T\sigma dt}Y^T\sigma + \frac{1}{2}Y^T\sigma\pi \end{gathered} \tag{44}$$

Multiplying Eq. (44) by the factor $e^{1/2\int Y^T\sigma dt}$

$$\begin{gathered} e^{1/2\int Y^T\sigma dt}\dot{\hat{\pi}} + \frac{1}{2}Y^T\sigma e^{1/2\int Y^T\sigma dt}\hat{\pi} = e^{1/2\int Y^T\sigma dt} \\ e^{-\int Y^T\sigma dtt}Y^T\sigma + \frac{1}{2}Y^T\sigma e^{1/2\int Y^T\sigma dt}\pi \end{gathered} \tag{45}$$

Eq. (45) can be arranged as:

$$\begin{gathered} \frac{d}{dt}(e^{1/2\int Y^T\sigma dt}\hat{\pi}) = e^{-1/2\int Y^T\sigma dt}Y^T\sigma \\ + \frac{1}{2}Y^T\sigma e^{1/2\int Y^T\sigma dt}\pi \end{gathered} \tag{46}$$

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Integrating both sides of Eq. (46) yields:

$$e^{1/2\int Y^T\sigma dt}\hat{\pi} = \int e^{-1/2\int Y^T\sigma dt} Y^T\sigma dt + \frac{1}{2}\int \pi Y^T\sigma e^{1/2\int Y^T\sigma dt} dt \tag{47}$$

$$e^{1/2\int Y^T\sigma dt}\hat{\pi} = -2e^{-1/2\int Y^T\sigma dt} + \pi e^{1/2\int Y^T\sigma dt} + C \tag{48}$$

Dividing Eq. (48) by $e^{-1/2\int Y^T\sigma dt}$

$$\hat{\pi} = -2e^{-\int Y^T\sigma dt} + \pi + Ce^{-1/2\int Y^T\sigma dt} \tag{49}$$

If $\hat{\pi}(0) = \pi$ is taken as an initial condition, the constant C is equivalent to 2. Hence, the parameter adaptation law is derived as:

$$\begin{aligned}\hat{\pi} &= -2e^{-\int Y^T\sigma dt} + \pi + 2e^{-1/2\int Y^T\sigma dt} \\ &= 2(e^{-1/2\int Y^T\sigma dt} - e^{-\int Y^T\sigma dt}) + \pi\end{aligned} \tag{50}$$

The adaptive parameter estimation law is obtained as a solution of Eq. (39). As a result of Eq. (40), the robust parameter estimation law $\hat{\rho}(t)$ can also be obtained. Substitution of Eq. (42) into Eq. (40) yields:

$$-(e^{1/2\int Y^T\sigma dt}\dot{\hat{\rho}}(t) + \frac{1}{2}Y^T\sigma e^{\int Y^T\sigma dt}\hat{\rho}(t)) = 0 \tag{51}$$

By dividing Eq. (51) by the factor $e^{1/2\int Y^T_{\sigma dt}}$, the following expression is found.

$$-(e^{1/2\int Y^T\sigma dt}\dot{\hat{\rho}}(t) + \frac{1}{2}Y^T\sigma e^{1/2\int Y^T\sigma dt}\hat{\rho}(t)) = 0 \tag{52}$$

If Eq. (52) is arranged according to $\hat{\rho}(t)$

$$-(e^{1/2\int Y^T\sigma dt}\hat{\rho}(t))\frac{d}{dt} = 0 \tag{53}$$

Integrating both sides of Eq. (53) yields:

$$e^{1/2\int Y^T\sigma dt}\hat{\rho}(t) = -C \Rightarrow \hat{\rho}(t) = -Ce^{-1/2\int Y^T\sigma dt} \tag{54}$$

If $\hat{\rho}(0) = \rho$ is taken as an initial condition, the constant C is equivalent to ρ. Hence the robust parameter estimation algorithm is derived as:

$$\hat{\rho}(t) = -\rho e^{-1/2\int Y^T\sigma dt} \tag{55}$$

Since $\delta(t) = -\hat{\rho}(t)$, the control vector can be written as:

$$\begin{aligned}u = Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)[2(e^{-1/2\int Y^T\sigma dt} - e^{-\int Y^T\sigma dt}) \\ + \pi + \rho e^{-1/2\int Y^T\sigma dt}] + K\sigma\end{aligned} \tag{56}$$

If π_i, denote the ith component of the π, $\hat{\pi}_i$ is the ith component of $\hat{\pi}$, ρ_i denote the ith component of ρ, v_i denote the ith component of the $Y^T\sigma$, the parameter estimation and the additional control input for each parameter are defined, respectively, as:

$$\begin{aligned}\hat{\pi}_i &= 2(e^{-1/2\int v_i dt} - e^{-\int v_i dt}) + \pi_i \quad \text{and} \\ \delta_i(t) &= \rho_i e^{-1/2\int v_i dt}\end{aligned} \tag{57}$$

5. DESIGN EXAMPLE

As an illustration of the adaptive robust control, the above algorithms (57) are applied to a two-link robot arm manipulators shown in Figure 1. The robot link parameters are:

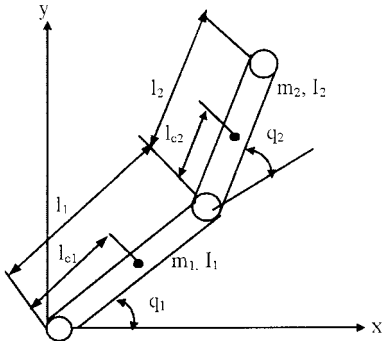


Fig. 1. Two-link planar robot.[16]

$$\begin{aligned}\pi_1 &= m_1 l_{c1}^2 + m_2 l_1^2 + I_1 \qquad \pi_2 = m_2 l_{c2}^2 + I_2 \\ \pi_3 &= m_2 l_1 l_{c2} \qquad \pi_4 = m_1 l_{c1} \\ \pi_5 &= m_2 l_1 \qquad \pi_6 = m_2 l_{c2}\end{aligned} \tag{58}$$

With this parameterisation, y_{ij} of $Y(q,\dot{q},\ddot{q})$ in Eq. (2) are given as:

$$\begin{aligned}&y_{11} = \ddot{q}_1; \quad y_{12} = \ddot{q}_1 + \ddot{q}_2; \quad y_{13} = \cos(q_2)(2\ddot{q}_1 + \ddot{q}_2) \\ &- \sin(q_2)(\dot{q}_2^2 + 2\dot{q}_1\dot{q}_2); \quad y_{14} = g_c\cos(q_1); \\ &y_{15} = g_c\cos(q_1); \quad y_{16} = g_c\cos(q_1+q_2); \quad y_{21} = 0; \\ &y_{22} = \ddot{q}_1 + \ddot{q}_2; \quad y_{23} = \cos(q_2)\ddot{q}_1 + \sin(q_2)(\dot{q}_1^2); \\ &y_{24} = 0; \quad y_{25} = 0; \quad y_{26} = g_c\cos(q_1+q_2)\end{aligned} \tag{59}$$

$Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)$ in (18) has the component:

$$\begin{aligned}&y_{11} = \ddot{q}_{r1}; \quad y_{12} = \ddot{q}_{r1} + \ddot{q}_{r2}; \quad y_{13} = \cos(q_2) \\ &(2\ddot{q}_{r1} + \ddot{q}_{r2}) - \sin(q_2)((\dot{q}_1\dot{q}_{r2} + \dot{q}_1\dot{q}_{r2} + \dot{q}_2\dot{q}_{r2}); \\ &y_{14} = g_c\cos(q_1; \quad y_{15} = g_c\cos(q_1); \\ &y_{14} = g_c\cos(q_1+q_2); \quad y_{21} = 0; \quad y_{22} = \ddot{q}_{r1} + \ddot{q}_{r2}; \\ &y_{23} = \cos(q_2)\ddot{q}_2\ddot{q}_{r1} + \sin(q_2)(\dot{q}_1\dot{q}_{r1}); \\ &y_{24} = 0; \quad y_{25} = 0; \quad y_{26} = g_c\cos(q_1+q_2)\end{aligned} \tag{60}$$

For illustrated purposes, let us assume that the parameters of the unloaded manipulator are known and the chosen values of the link parameters are given by Table I. Using these values in Table I, the ith components of π obtained by means of Eq. (58) are given in Table II. These parametric values also show lower and unloaded robot parameters.

If an unknown load carried by the robot is regarded as part of the second link, then the parameters m_2, l_{c2}, and I_2

Table I. Inertia values for the unloaded arm.[16]

m_1	m_2	l_1	l_2	l_{c1}	l_{c2}	I_1	I_2
10	5	1	1	0.5	0.5	10/12	5/12

Table II. π_i for the unloaded arm.[16]

π_1	π_2	π_3	π_4	π_5	π_6
8.33	1.67	2.5	5	5	2.5

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Table III. Nominal inertia parameter vector, π_0.[16]

π_{01}	π_{02}	π_{03}	π_{04}	π_{05}	π_{06}
13.33	8.96	8.75	5	10	8.75

Table IV. Uncertainty bound.[16]

ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6
5	7.29	6.25	0	5	6.25

will change $m_2 + \Delta m_2$, $l_{c2} + \Delta l_{c2}$ and $I_2 + \Delta I_2$, respectively. A controller will be designed that provides robustness in the intervals:

$$0 \le \Delta m_2 \le 10; \quad 0 \le \Delta l_{c2} \le 0.5; \quad 0 \le I_2 \le \frac{15}{12} \qquad (61)$$

π_0 is chosen as a vector of nominal parameters and it also has loaded arm parameters and their upper bounds. The computed values for ith component of π_0 are given in Table III. The difference between values given in Table III and Table II describe the uncertainty bound ρ, and the relevant values are given in Table IV.

6. DISCUSSION

In this scheme, a new adaptive-robust control law based on the Lyapunov theory is derived. The basic structure is similar to that of reference [19]. ln the reference study,[19] K_π is chosen as a positive definite constant gain matrix and the adaptation algorithm is a function of $q, \dot{q}, \dot{q}_r, \ddot{q}_r$, and σ. If $Y^T\sigma$ is positive, then there is a deviation from the desired motion and the adaptation algorithm increases the driving torque. The updating algorithm stops when it reaches its known bounds, and resumes updating as soon as the corresponding derivative changes signs. Eqs. (18) and (21) represent a robust control law derived by Spong.[16] The nominal parameters π_0, do not change or are not updated in time as they would be in an adaptive control strategy. However, in the adaptive robust control law, both parameter and additional control input are updated in time as an exponential function of $Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)$ and trajectory tracking error. In this scheme, Γ is chosen as a time varying function and taken as $\Gamma = e^{\int Y^T \sigma dt}$. In order to determine the appropriate value of Γ, it is also possible to derive various parameter estimation laws for this system.

The new adaptive robust control law is similar to adaptive[19] and robust[16] control laws, but in this new scheme, both adaptive and robust control law are implemented simultaneously. Parameter estimation is utilized by the adaptive control law, while robust control input compensates for disturbances due to unmodelled dynamics and increase robustness to uncertainty. The new parameter estimation law works like a control law (11). The updating algorithm stops when it reaches its known bounds and resumes updating when $2(e^{-1/2\int Y^T\sigma dt} - e^{-\int Y^T\sigma dt})$ changes sign. But in this new scheme the parameter convergence depends on the exponential function of $\int Y^T\sigma dt$. If $\int Y^T\sigma dt < 0$ and $\int Y^T\sigma dt$ approaches zero or infinity, the first term of the developed parameter estimation (26) approaches zero, The first exponential term increases the value of $\hat{\pi}$ while the second one reduces it. If $\int Y^T\sigma dt$ is negative, $\hat{\pi}$ decreases up to the lower bound of the parameter.

For computer simulation, a fifth order polynomial is selected as a reference trajectory for both Joints. Joint angles change from 0 to 2.5 rad in 3 seconds. Sampling time is selected as 0.01 sec. For the comparison of the new control law with the known controller (11), each control algorithm with the same controller K and Λ is applied to the model system for the same trajectory in order to analyse performance of each control law. For that purpose, the matrix K and Λ are chosen to be identical for the two controllers with K = diag (80 80), Λ = diag (80 80) and K = diag (100 100) and Λ = diag (100 100). The obtained results are plotted in Figures 2 and 3 for the pure adaptive controller (11), in Figure 4 for the pure robust controller (21) and in Figures 5 and 6 for the adaptive-robust controller. As seen from Figure 2 the imaximum tracking error in transient state for the first joint is about 0.013 rad, for the second joint it is 0.007 rad with the control parametes K = diag (80 80) and Λ = diag (80 80). Tracking performance can be increased by choosing control parameters as K = diag (100 100), Λ = diag (100 100), thus the maximum tracking error in transient state is reduced to a value of 0.009 and 0.005 for first and second joints, respectively. For the pure robust control law, the maximum tracking error in the transient state is 0.005 for first joint, 0.009 for second joint, and there is chattering in the error response. Adaptive approaches are feasible for a wider variation range of parameters, but for unmodelled dynamics and disturbances. Another disadvantage of adaptive controllers is to exhibit poor transient behaviour. The adaptive-robust control law eliminates the disadvantage of the adaptive controller, improves transient performance and decreases the steady-state-error.[8] As seen from Figures 5 and 6, the transient and steady state behaviour of the adaptive-robust controller is better than that of a pure adaptive and robust control law. The maximum tracking error is about 0.00025 for first joint; it is 0.00022 for second joint at transient state with the control parametes K = diag (80 80) and Λ = diag (80 80). It is also possible to increase tracking performance of the controller (26) by choosing appropriate values of K and Λ as K = diag (100 100), Λ = diag (100 100). The maximum tracking error can be reduced the values of 0.00015 for the first joint, and 0.00011 for the second joint. These results show that the proposed control law can achieve fast and accurate convergence of robot parameters to their asymptotic

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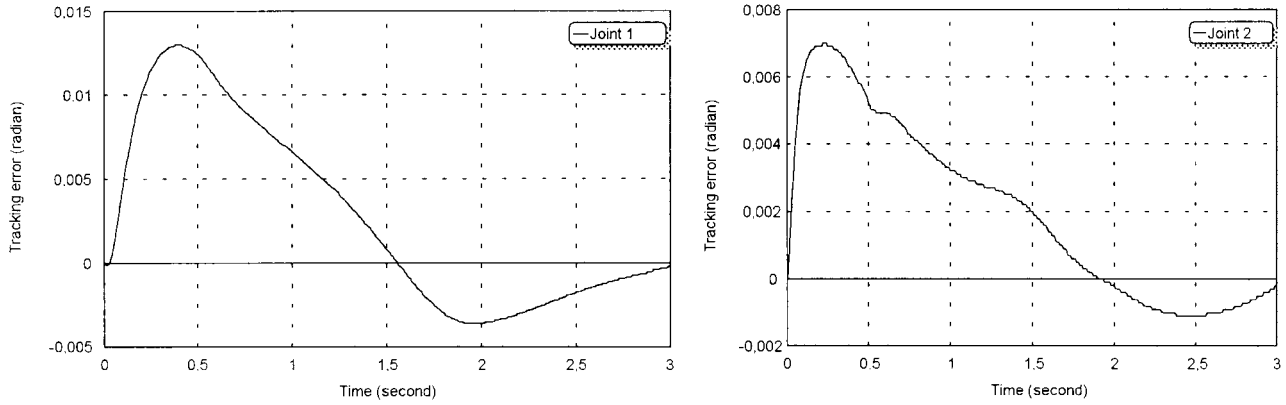


Fig. 2. Response for the adaptive control law (11). K_{π}^{-1} = diag ([1 1 1 1 1 1]), Λ = diag ([80 80]), K = diag ([80 80]).

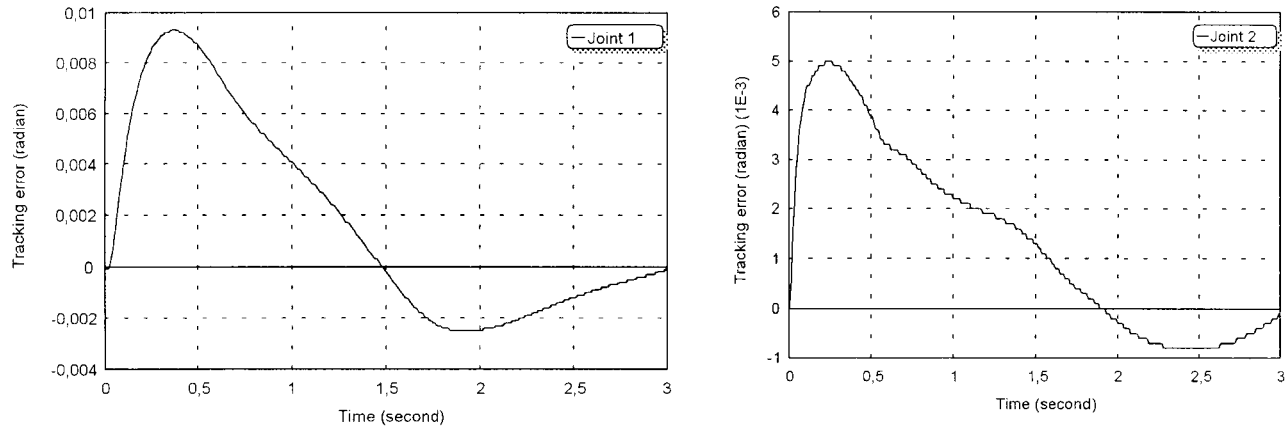


Fig. 3. Response for the new adaptive control law (11). K_{π}^{-1} = diag ([1 1 1 1 1 1]), Λ = diag ([100 100]), K = diag ([100 100]).

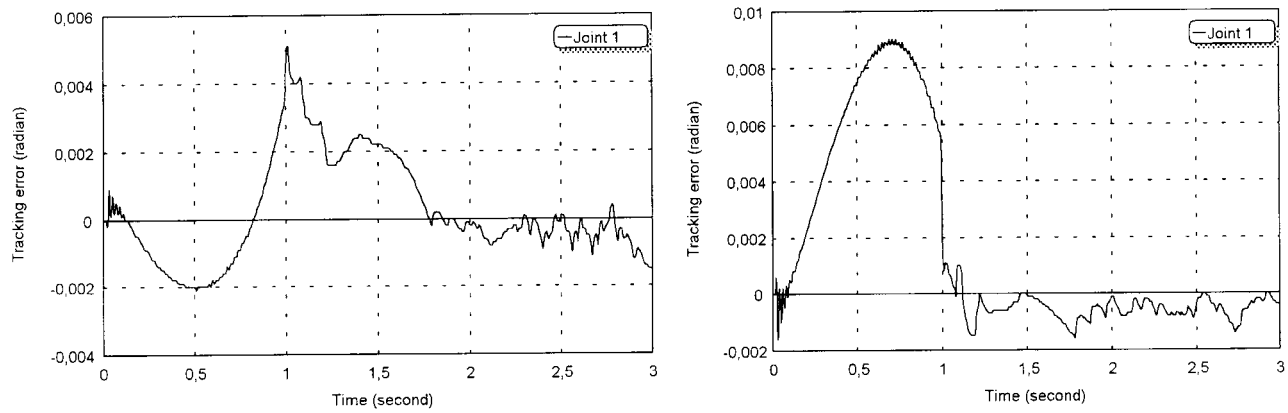


Fig. 4. Tracking errors of joints 1 and 2 for the robust control law. Λ = diag ([20 20]) K = diag ([55 55]).

values, by using while the robust input also compensates for the disturbances due to uncertainties in the dynamic model.

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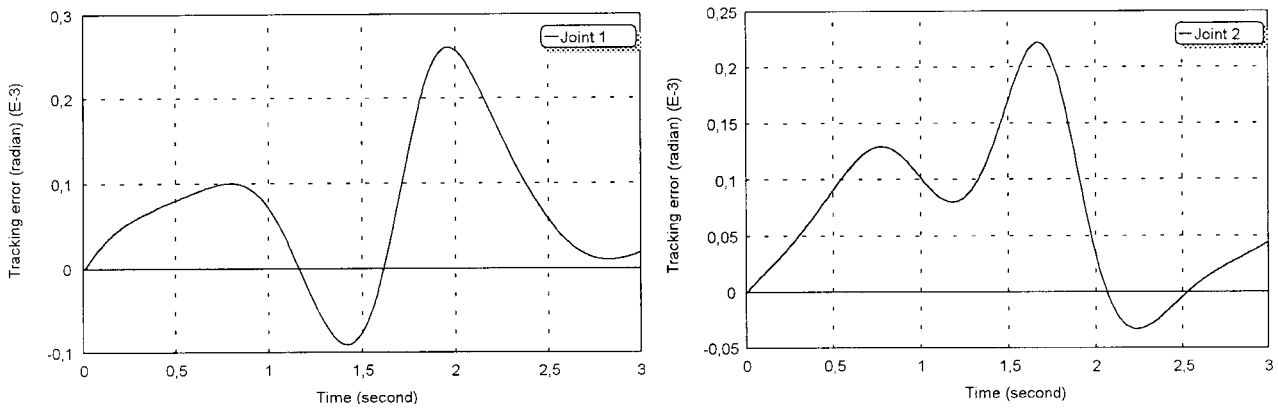


Fig. 5. Tracking errors of joints 1 and 2 for the adaptive-robust control law. Λ = diag ([80 80]) K = diag ([80 80]).

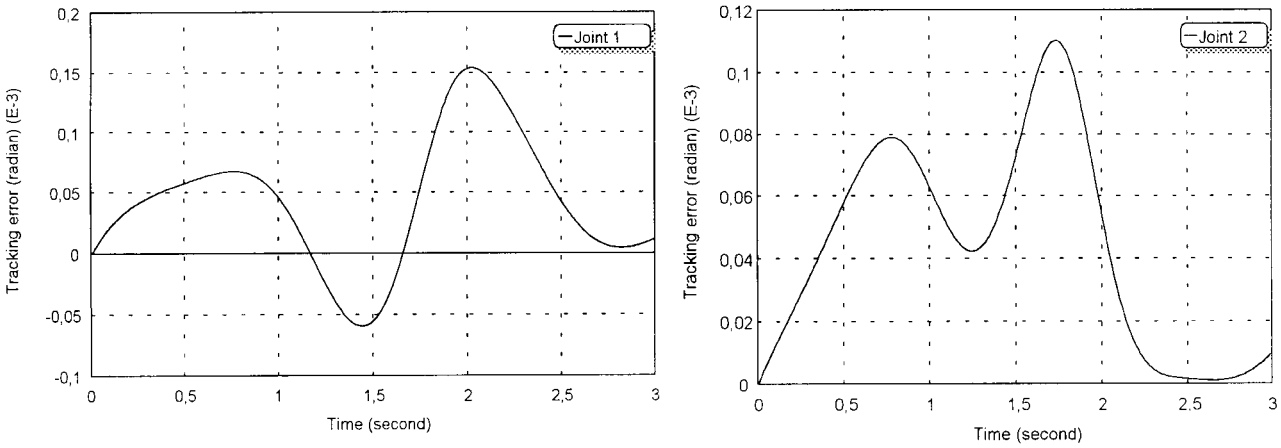


Fig. 6. Tracking errors of joints 1 and 2 for the adaptive-robust control law. Λ = diag ([100 100]) K = diag ([100 100]).

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