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Collective dynamics in strongly coupled dusty plasma medium

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A simplified description of dynamical response of strongly coupled medium is desirable in many contexts of physics. The dusty plasma medium can play an important role in this regard due to its uniqueness, as its dynamical response typically falls within the perceptible grasp of human senses. Furthermore, even at room temperature and normal densities it can be easily prepared to be in a strongly coupled regime. A simplified phenomenological fluid model based on the visco - elastic behaviour of the medium is often invoked to represent the collective dynamical response of a strongly coupled dusty plasma medium. The manuscript reviews the role of this particular Generalized Hydrodynamic (GHD) fluid model in capturing the collective properties exhibited by the medium. In addition the paper also provides new insights on the collective behaviour predicted by the model for the medium, in terms of coherent structures, instabilities, transport and mixing properties.

1. Introduction

It is well known that the dusty plasma medium can be prepared in the strongly coupled state rather easily. This is because the coupling parameter $\Gamma = \frac{(Z_d e)^2}{4\pi\epsilon_0 a_d k_B T_d}$ (where Z_d, a_d and T_d are the quantum of charge, inter-dust separation and temperature of dust species respectively) defining the ratio of inter dust potential energy to the average thermal kinetic energy can easily exceed unity due to the large charge of the dust particles. Thus even at room temperature and normal densities the dusty plasma medium can be in a strong coupling regime. Additionally, the response time and length scales of the medium are typically within the perceptible grasp of human senses. Thus, one does not require involved diagnostics and can directly view the phenomena as they occur. It is thus a unique medium in which one can directly view simple interactions at micro levels leading to complexity at macro scales.

The entire physics community is currently grappling with questions and appropriate simplified descriptions of strongly coupled state of matter. In this regard the understanding gleamed from the dusty plasma medium may have significant relevance to other fields of physics. This opportunity needs to be exploited in a best possible fashion. This expectation may not be so far fetched as a recent work (Bannur 2006) on obtaining the equation of state for the strongly coupled QGP (Quark Gluon Plasma) matter developed from an understanding of strongly coupled state of plasmas has found good acceptance in the high energy physics community (Cho and Zahed 2009). Studies concerning dynamics of strongly coupled state of matter are also important

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and timely, given the fact that the Alice detector at CERN is poised to provide results on the strongly coupled state of matter.

A simplified descriptions of the dynamical response of the medium in the strongly coupled limit is, therefore, necessary. The inputs from experiments, Molecular Dynamics simulations, etc., are important in this regard. Theoretical understanding gleamed from such inputs and the natural symmetries of nature can then be employed for the development of simplified phenomenological models. A Generalized Hydrodynamic (GHD) fluid model capturing the visco - elastic behaviour of the dusty plasma system, coupled with the Poisson equation has been found to be quite suitable for the description of electrostatic phenomena below the crystallization limit in this regard (Kaw and Sen 1998).

The dynamical response of any medium is best understood in terms of its collective behaviour. We, therefore, focus on the influence of strong coupling on certain collective properties, namely (i) Coherent structures (ii) instabilities and (iii) turbulent transport and mixing within the framework of the GHD model. The coherent structures like solitons, vortex structures has been reported in dusty plasma experiments (Morfill et al. 1999; Schwabe et al. 2011; Sharma et al. 2014). Hydrodynamic instabilities like Taylor instability has been observed in dusty plasma experiments by Pacha et al while Kelvin-Helmholtz instability has been studied in experiments/observations and simulations related to astrophysical scenarios (Otto and Fairfield 2000; Wiechen 2006). A brief introduction of the model equations are provided in the next section. In the subsequent Secs. 3–5 we discuss the role of strong coupling on (i) certain coherent solutions, (ii) instabilities and (iii) the behaviour of 2-D turbulence and mixing respectively, predicted by this model.

2. Simplified GHD description

The continuity and Poisson equation coupled with a two parameter (τ_m and η) representation of a generalized momentum equation expresses the GHD fluid model for the dusty plasma medium.

$$\left[1+\tau_m\left(\frac{\partial}{\partial t}+\vec{v}\cdot\nabla\right)\right]\left[\rho\left(\frac{\partial}{\partial t}+\vec{v}\cdot\nabla\right)\vec{v}+\nabla P+\rho_c\nabla\phi\right] = \eta\nabla^2\vec{v}+(\eta/3+\zeta)\nabla(\nabla\cdot\vec{v}) \tag{2.1}$$

Here, ρ, ρ_c, η and ζ are the mass density, charge density, longitudinal and shear viscosity of dust fluid respectively. Here τ_m represents the memory relaxation parameter responsible for the elasticity in the medium. Thus the GHD model is a continuum simplified description with visco - elastic effects in terms of two parameters η and τ_m. There are other visco elastic models for which an addition of solvent in a solute leads to elastic stresses. For instance, the visco - elastic behaviour of polymeric fluids wherein the elastic stresses arise due to the polymer additives in a background fluid (Berti and Boffetta 2010) belongs to this class. In the dusty plasma medium discussed here, the dust plays a dual role of solvent as well as the solute. Here, essentially, the underlying fluid itself is responsible for providing the elastic stresses in the system.

The predictions concerning the normal modes of this model have been verified experimentally. In particular, the transverse shear wave mode, premised on this model has been observed experimentally (Pramanik et al. 2002). Also, the change in the dispersion characteristics of longitudinal Dust acoustic waves predicted by

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the model, namely the turning over at higher wavenumbers, has been observed in experiments (Bandyopadhyay et al. 2007).

The results presented in Secs. 4 and 5 have been obtained by numerically simulating the following coupled representation of the GHD model in incompressible limit.

$$\left(\frac{\partial}{\partial t} + \vec{v}\cdot\nabla\right)\vec{v} + \frac{\nabla P}{\rho} + \frac{\rho_c}{\rho}\nabla\phi = \vec{\psi}$$
$$\left(\frac{\partial}{\partial t} + \vec{v}\cdot\nabla\right)\vec{\psi} = -\frac{\vec{\psi}}{\tau_m} + \frac{\eta}{\tau_m}\nabla^2\vec{v} \tag{2.2}$$

Here, η is now termed as the kinematic viscosity. The above equation is used in its normalized form with length, time and density normalized with characteristic length L_0, time T_0 and equilibrium density N_0 of the system respectively. These normalizations are conveniently chosen so that one has to primarily deal with numbers of the order of unity in simulation and analysis for any system. The relaxation parameter τ_m is also normalized with T_0. The normalized kinematic viscosity $\eta_N = \eta/mnU_0L_0$ and the normalized pressure $P_N = P/mnU_0^2$ with $U_0 = L_0/T_0$. The numerical simulations are carried out using time splitting technique in two dimensions with the help of flux corrected finite difference method Boris et al. (1993). The code was validated by reproducing the linear propagation of transverse modes as shown in (Tiwari et al. 2014*a*).

3. Coherent structures

The possibility of the existence of exact coherent nonlinear solutions and their stability characteristics are other features of interest in the context of any medium. For weakly nonlinear cases a reductive perturbative analysis of the GHD model equations shows that the set of equations reduce to the regular KdV form only in the limit of weak coupling. In the strong coupling regime such an analysis results into an entirely different form of Hunter Saxton (HS) equation

$$(u_\tau + uu_\xi)_\xi = \frac{1}{2}u_\xi^2 \tag{3.1}$$

Here $\tau = t$ and $\xi = x - \beta t$. While the KdV equation permits localized solitonic solutions, the HS equation can not permit any smooth localized or periodic solutions due to the positive definite integral appearing on the right hand side when integrated with respect to ξ. It is in fact well known that this equation permits shock solutions which are both of conservative as well as dissipative varieties (Hunter and Ralph 1991). A simple physical interpretation of such solutions have been provided by us by considering an example of an elastic spring (Das et al. 2011). In the context of dusty plasmas the conservative shock corresponds to a case wherein the kinetic energy of the particles get stored as elastic energy in the shock region, which can again bounce back as the kinetic energy of the medium. The dissipative shock, on the other hand, would mean that the energy in the shock is lost in the thermal kinetic energy of the particles. The resultant increase in temperature may then result in the loss of the strong coupling properties. Thus, a material breakdown kind of scenario (shown in the context of spring in Das et al. (2011)) prevails here as well.

Singular structures in the context of dusty plasmas have been observed experimentally. Theoretically it appears that, they could either correspond to the localized solitonic solutions for the fully nonlinear system in the weak coupling regime at the wave breaking limit or the singular solutions predicted by GHD model

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in the strong coupling regime. We have shown that the former is stable and accessible by carrying fluid simulations of the dusty plasma equations in the weak coupling regime (Tiwari et al. 2012*c*). The GHD simulations of complete set with arbitrary initial conditions have also indicated formation of singular shock like structures from arbitrary initial conditions (Tiwari et al. 2012*b*).

Interestingly, it is worth noting that the MD simulations carried out with Yukawa potential for the inter-dust interaction may not be very accurate in capturing such singular behaviour. This is so because the linearized expansion of the Boltzmann expression for ions and electrons in the close neighborhood of dust will not be valid. The singular solutions correspond to having a very high value of dust particle density at the singular point.

4. Instabilities

The GHD model being a fluid description it is worthwhile to examine the role of strong correlations for some of the well known fluid instabilities. The shear flow and the stratification against gravity being some of routine scenarios, the Kelvin Helmholtz (KH) and the Rayleigh Taylor (RT) modes are particularly important in this regard.

It has been demonstrated that the KH mode gets stabilized and the unstable domain of the wavenumber also shrinks in the presence of strong coupling effects (Tiwari et al. 2012*a*). There exists an interplay between the KH mode and the transverse shear wave emission for any sheared flow configuration. At lower flow speeds the transverse shear emission and propagation is important. A detailed discussion on this interplay can be found in Ref (Tiwari et al. 2012*a*). In the strong coupling case an additional local instability driven by the first derivative of velocity flow profile is also permitted. This is quite unlike the KH mode for which the finite second derivative or a inflection point in the flow profile is a must. The 2-D nonlinear simulations of the sheared flow profile show emission of compressional as well as transverse shear mode and the development of the instabilities. An interesting recurrence phenomena is observed wherein the vortices formed as a result of KH destabilization, repeatedly stretch and coalesce and form shear layers, which is destabilized again. This happens for several cycles, until the formed shear layer gets weak enough to forbid any further destabilization. The nonlinear stage in 2-D constitutes of a significant number of short scale structures in this case, which is quite unlike what happens in simple 2-D fluid which shows preferential cascade of power spectrum towards longer scales materializing through coalescence of the vortices.

A gravitationally stratified dusty plasma suffers the usual Rayleigh Taylor destabilization. However, in this case too the strong coupling effects are seen to stabilize the instability. Das et al have recently studied the effect of strong coupling on Rayleigh Taylor instability using GHD description and have shown the stabilization of this mode due to the strong coupling effects. The implication of this in the context of inertial fusion have been shown by them (Das and Kaw 2014). We have also carried out the numerical simulations of the GHD set of equations in the presence of gravity. In Fig. 1 we provide a comparison of the development of the RT instability for simple fluid and GHD fluid with increasing values of η/τ_m (measuring the square of the phase velocity of the transverse shear wave and hence the increasing strong coupling). It is clear from the figure that the instability gets weaker with increasing strong coupling.

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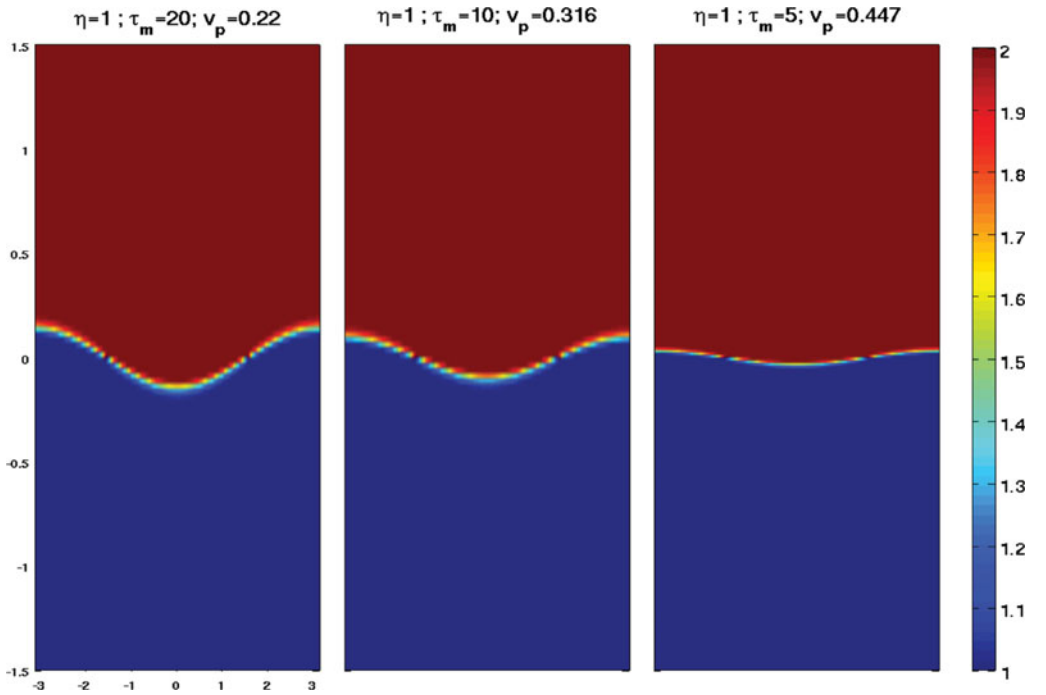


FIGURE 1. (Colour online) Growth of Rayleigh Taylor instability in strongly coupled dusty plasma using GH model. Here, $v_p = \sqrt{\eta/\tau_m}$ is the phase velocity of transverse shear wave in strongly coupled dusty plasma medium. All the three snapshots have been taken at $t = 10$ in normalized time units.

5. Turbulence and mixing

It has been observed that the 2-D GHD system of equations are such that the short scales seem to persist even in two dimensional case. This is quite in contrast to the well known behaviour of inverse cascade observed in the context of 2-D hydrodynamic fluids. This feature would have interesting implications in turbulence, transport and mixing properties of the medium.

The vortex patterns being the typical structures observed in any kind of fluid systems, we chose to investigate through numerical simulations, the mixing of fluids entrapped in such patterns with the outside fluid in the two cases of neutral fluid and the GHD system. The system length is chosen as 12π in both x and y direction and system is centered at 0. The velocity profile is chosen such that $v_x = -y, v_y = x$ for $x^2 + y^2 = r \leqslant 6$ and $v_x = v_y = 0$ elsewhere in the simulation box. Various values of η and τ_m were considered and it was noted that the mixing was better as the value of η/τ_m was increased. The initial profile and the evolved sharp vortex patch at $t = 50$ in normalized units for hydrodynamic fluid and the GHD fluid have been shown in Fig. 2. It clearly illustrates that in a hydrodynamic fluid the KH destabilization is primarily responsible for mixing. However, such a mixing is not very strong, Severe inhomogeneous and non-uniform patches continue to persist. In contrast a vortex flow in a GHD system in turn emits both radially inward and outward propagating transverse shear waves along with the KH destabilization. The figure clearly shows that the mixing is much better and uniform in this case.

We have also investigated the nature of turbulence for the GHD system by carrying out several simulations for distinct initial choices of randomly phased Fourier mode

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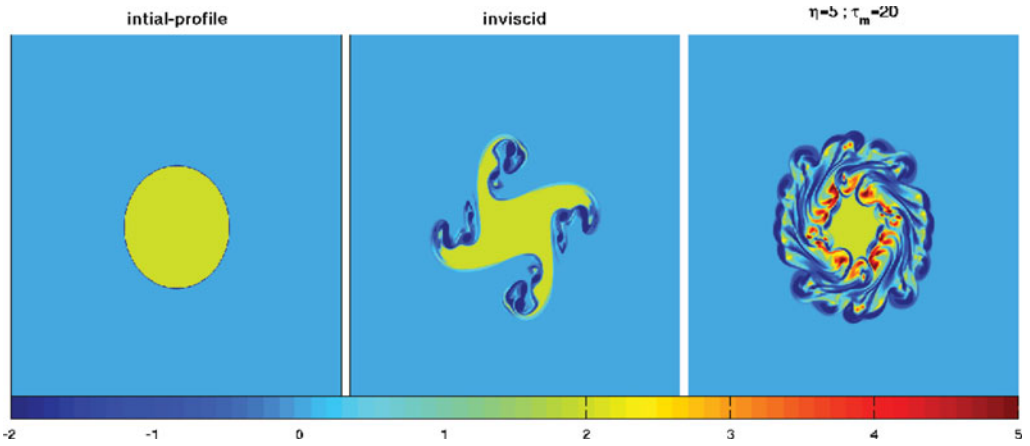


FIGURE 2. (Colour online) Vortex evolution in strongly coupled dusty plasma medium with subplots (a) initial vortex flow profile with sharp edge (b) evolution in hydrodynamic fluid (c) evolution in visco-elastic fluid.

spectrum. This includes both a peaked spectrum at certain wavenumber location and also a fairly broad initial spectrum. The evolved spectra is broad irrespective of initial choice and the fact that the Reynold's number was much lower than the threshold value for turbulence excitation, this is suggestive of elastic turbulence, for which both Reynold's number Re as well as the Weissenberg number Wi are important. The Weissenberg number provides the ratio of relaxation time to the typical stretching time scale of the system. For our system of normalized equations $Re = \eta^{-1}$ and $Wi = \tau_m$.

In all cases the spectrum was seen to acquire an asymptotic form, however, no specific power law dependence was observed. There also appeared to be a piling up of power at specific wave-numbers whose location and sharpness was dependent on the initial choice of spectrum. When initial power was chosen to lie largely at shorter scales the piling was more prominent. A perceptible shift of the location of the piling wave number towards longer scales was observed with time as well as with increasing value of η/τ_m. The details of these results have been presented in (Tiwari et al. 2014*b*).

The piling up is indicative of the inhibition of the inverse spectral cascade. The shift of piling wave number towards longer scales can be understood by realizing that an inherent hydrodynamics character for time scales slower than τ_m resides in the GHD set of equations. The GHD dynamics with an intrinsic special time scale τ_m seems to provide a barrier to the intrinsic inverse cascade process involved in hydrodynamics. Barriers in the inverse spectral cascade have been observed earlier in the context of Hasegawa Mima equation (at ion gyro-radius) (Hasegawa and Mima 1978); EMHD (at electron skin depth). In fact that led to the formation of quasi crystalline structures formation observed by (Kukharkin et al. 1995). In the context of GHD system, however, it appears that the barrier does not remain fixed at any specific scale but 'keeps yielding' (e.g. shifting towards longer scales). Neither does it vanish completely as the presence of waves in the context of the HM system have been shown to do.

6. Summary

The dusty plasma system has an important role to play in the context of understanding the physics of strongly coupled systems. This is because of the

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simplicity with which the dusty plasma medium can be made to lie in a strong coupling regime and the ease with which its properties and behaviour can be diagnosed. Phenomenological simplified models based on experimental inputs, extensive simulations and rules of general symmetry and constraints posed by nature are most welcome in this regard. The visco - elastic GHD fluid model appears a suitable description. Further improvisations of this model would be extremely fruitful.

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