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# Low-frequency electrostatic waves in a bounded dusty magnetoplasma

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**Abstract.** A rigorous theoretical investigation is made of obliquely propagating low-frequency electrostatic waves in a cylindrically bounded magnetized dusty plasma. A number of different modes, such as modified convective cells, coupled ion-cyclotron and dust-ion-acoustic waves, modified lower-hybrid waves, coupled dust-cyclotron and dust-acoustic waves, etc., are investigated. It is shown that the effects of the cylindrical boundary of the dusty plasma system, the external magnetic field, and the obliqueness (of the propagating modes) significantly modify the dispersion properties of these different low-frequency electrostatic waves. The implications of our results for laboratory dusty magnetoplasmas are briefly pointed out.

## 1. Introduction

During the last decade, there has been a great deal of interest in understanding different types of collective processes in dusty plasmas (plasmas with extremely massive and negatively charged dust grains), not only because of the ubiquity of such plasmas in space and astrophysical environments, such as asteroid zones, planetary atmospheres, the interstellar medium, circumstellar disks, dark molecular clouds, cometary tails, nebulae, and the Earth's environment, (Horanyi and Mendis 1985, 1986; Goertz 1989; Northrop 1992; Mendis and Rosenberg 1992, 1994), but also because of their vital role in laboratory experiments (Barkan et al. 1995a,b, 1996; Pieper and Goree 1996; Prabhakara and Tana 1996; Thompson et al. 1997; Merlino et al. 1998; Nakamura et al. 1999; Molotkov et al. 1999; Thomas and Watson 1999).

The presence of static charged dust grains can alter the equilibrium quasineutrality condition of an electron–ion plasma. Thus, the existing plasma-wave spectra of an electron–ion plasma are modified even when stationary dust grains are added. Rao et al. (1990) and Shukla and Silin (1992) have shown the existence of dust-acoustic and dust-ion-acoustic waves, which are now the two new normal modes of an unmagnetized dusty plasma. These dust-acoustic and dust-ion-acoustic modes have been observed in several laboratory dusty plasma devices (Barkan et al. 1995a,b, 1996; Pieper and Goree 1996; Prabhakara and Tana 1996; Thompson et al.

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1997; Merlino et al. 1998; Nakamura et al. 1999; Molotkov et al. 1999; Thomas and Watson 1999).

Since most of the dusty plasmas in laboratory and space environments are confined in an external magnetic field, it is of practical interest to examine the properties of dusty plasma waves in a magnetoplasma (Shukla 1992, 1996, 1999; Rosenberg 1993, 1996; Chow and Rosenberg 1995, 1996; Bharuthram and Pather 1996; Shukla and Rahman 1998; D'Angelo 1990, 1998). Shukla (1992) and Shukla and Rahman (1998) have shown the existence of lower-hybrid and dust-cyclotron modes in a magnetized dusty plasma, whereas Rosenberg (1993, 1996), Chow and Rosenberg (1995, 1996), D'Angelo (1998), and Bharuthram and Pather (1996) have studied the instabilities of electrostatic ion-cyclotron waves (Chow and Rosenberg 1995, 1996), dust-cyclotron waves (D'Angelo 1998) and dust-acoustic waves (Rosenberg 1996; Bharuthram and Pather 1996) in magnetized dusty plasmas. Shukla (1996, 1999) has also rigorously investigated the different possible modes that have so far been found to exist in such a magnetized dusty plasma.

To the best of our knowledge, most of these studies on waves in dusty plasmas for both magnetized and unmagnetized cases, are concerned with a dusty plasma of infinite extent. It is obvious that dusty plasmas in laboratory devices are of finite extent (Barkan et al. 1995a,b, 1996; Pieper and Goree 1996; Prabhakara and Tana 1996; Thompson et al. 1997; Merlino et al. 1998; Nakamura et al. 1999; Molotkov et al. 1999; Thomas and Watson 1999). Recently, Shukla and Rosenberg (1999) have considered the boundary effects on dust-ion-acoustic and dust-acoustic waves in an unmagnetized dusty plasma. However, for understanding the physics of laboratory magnetized dusty plasmas, which should be suitable for future experiments on different waves and instabilities in a magnetized dusty plasma, one must consider a bounded magnetized dusty plasma system and obliqueness of the associated modes (which are not always restricted to propagate along or perpendicular to the external magnetic field). Thus, in the present work, we have considered obliquely propagating low-frequency electrostatic waves in a cylindrically bounded magnetized dusty plasma, and have systematically investigated the effects of obliqueness, the external magnetic field, and the cylindrical boundary of the plasma system on different low-frequency electrostatic modes, such as modified convective cells, coupled ion-cyclotron (IC) and dust-ion-acoustic (DIA) waves, modified lower-hybrid waves, coupled dust-cyclotron and dust-acoustic waves, etc. It has been found that the effects of a boundary of the dusty plasma system, the external magnetic field, and the obliqueness of (the propagating modes) significantly modify the dispersion properties of these different low-frequency electrostatic waves.

The paper is organized as follows. We present our basic equations in Sec. 2. We derive the dispersion relation and investigate the dispersion properties of different electrostatic modes in Sec. 3. Section 3.1 is concerned with these modes modified by the presence of static charged dust grains, whereas Sec. 3.2 deals with the dust modes associated with the dynamics of the dust particles. Finally, a brief discussion is presented in Sec. 4.

## 2. Governing equations

We consider a bounded three-component magnetized dusty plasma consisting of negatively charged dust particles and electrons, and positively charged ions. Thus, at equilibrium, we have $n_{i0} = Z_d n_{d0} + n_{e0}$, where $n_{i0}\,(n_{d0})$ is the equilibrium ion

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(dust) number density, $Z_d$ is the number of electrons residing on the surface of negatively charged dust grains, and $n_{e0}$ is the equilibrium electron number density. This plasma system is assumed to be immersed in an external static magnetic field. The macroscopic state of our magnetized dusty plasma system is described by

$$\frac{\partial N_s}{\partial t} + \boldsymbol{\nabla} \cdot (N_s \mathbf{U}_s) = 0, \tag{1}$$

$$N_s \left( \frac{\partial}{\partial t} + \mathbf{U}_s \cdot \boldsymbol{\nabla} \right) \mathbf{U}_s = \frac{N_s q_s}{m_s} \left[ -\boldsymbol{\nabla}\Phi + \frac{1}{c}(\mathbf{U}_s \times \mathbf{B}_0) \right] - \frac{\gamma_s T_s}{m_s} \boldsymbol{\nabla} N_s, \tag{2}$$

$$\nabla^2 \Phi = -4\pi \sum_s q_s N_s, \tag{3}$$

where $m_s$, $q_s$, and $N_s$ are respectively the mass, the charge, and the number density of species $s$ (the dust grains, the ions, and the electrons), $\mathbf{U}_s$ is the hydrodynamic velocity, $\mathbf{T}_s$ is the thermal energy, $\gamma_s$ is the adiabatic index, $\Phi$ is the electrostatic wave potential, and $c$ is the speed of light in vacuum. We are interested in looking at different low-frequency electrostatic modes propagating obliquely with an external magnetic field $\hat{\mathbf{z}} B_0$, where $\hat{\mathbf{z}}$ is the unit vector along the $z$ axis and $B_0$ is the strength of the external magnetic field.

## 3. Dispersion properties

To study obliquely propagating electrostatic modes in a bounded magnetized dusty plasma system, we shall carry out a normal-mode analysis. We first express our dependent variables $N_s$, $U_s$, and $\Phi$ in terms of their equilibrium and perturbed parts as

$$N_s = n_{s0} + n_s, \tag{4a}$$

$$\mathbf{U}_s = 0 + \mathbf{u}_s, \tag{4b}$$

$$\Phi = 0 + \phi. \tag{4c}$$

By using (4*a–c*) we can linearize (1)–(3) to a first order approximation as

$$\frac{\partial n_s}{\partial t} + \boldsymbol{\nabla} \cdot (n_{s0} \mathbf{u}_s) = 0, \tag{5}$$

$$\frac{\partial \mathbf{u}_s}{\partial t} = -\frac{q_s}{m_s} \boldsymbol{\nabla}\phi + \frac{q_s B_0}{m_s c}(\mathbf{u}_s \times \hat{\mathbf{z}}) - \frac{\gamma_s T_s}{n_{s0} m_s} \boldsymbol{\nabla} n_s, \tag{6}$$

$$\nabla^2 \phi = -4\pi \sum_s q_s n_s. \tag{7}$$

To derive the dispersion relations for different electrostatic modes, we first express (5) and (6) in terms of $\mathbf{u}_{s\perp}$ (the velocity perpendicular to $\hat{\mathbf{z}}$) and $u_{sz}$ (the velocity parallel to $\hat{\mathbf{z}}$) as

$$\frac{\partial n_s}{\partial t} + n_{s0} \boldsymbol{\nabla} \cdot (\mathbf{u}_{s\perp} + \hat{\mathbf{z}} u_{sz}) = 0, \tag{8}$$

$$\left( \frac{\partial^2}{\partial t^2} + \omega_{cs}^2 \right) \mathbf{u}_{s\perp} = \frac{c}{B_0} \omega_{cs}^2 (\hat{\mathbf{z}} \times \boldsymbol{\nabla}_\perp) \left( \phi + \frac{\gamma_s T_s}{q_s n_{s0}} n_s \right) - \frac{q_s}{m_s} \frac{\partial}{\partial t} \boldsymbol{\nabla}_\perp \left( \phi + \frac{\gamma_s T_s}{q_s n_{s0}} n_s \right), \tag{9}$$

$$\frac{\partial u_{sz}}{\partial t} = -\frac{q_s}{m_s} \frac{\partial \phi}{\partial z} - \frac{\gamma_s T_s}{m_s n_{s0}} \frac{\partial n_s}{\partial z}, \tag{10}$$

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where $\omega_{cs} = |q_s|B_0/m_s c$ is the gyrofrequency. We now study different possible low-frequency electrostatic dust-associated modes in the bounded magnetized dusty plasma in two situations: one for static charged dust particles and other for mobile charged dust grains.

### *3.1. Static charged dust particles*

Here, we consider the presence of static charged dust particles (i.e. $u_d = 0$ and $n_d = 0$) and investigate the dispersion properties of modified convective cells, coupled ion-cyclotron (IC) and dust-ion-acoustic (DIA) waves, and modified lower-hybrid waves in the bounded dusty plasma under consideration.

*3.1.1. Modified convective cells.* To study convective cells in the bounded dusty plasma, we first consider the quasineutrality condition (i.e. $n_e = n_i$) and appropriate approximations for convective cells, which are characterized by $k_z v_{te,ti} \ll \omega \ll \omega_{ce,ci}$. Thus, we deduce from (8)–(10)

$$\boldsymbol{\nabla} \cdot (n_{i0}\mathbf{u}_{i\perp} - n_{e0}\mathbf{u}_{e\perp}) + \frac{\partial}{\partial z}(n_{i0}u_{iz} - n_{e0}u_{ez}) = 0, \tag{11}$$

$$\mathbf{u}_{i\perp} = \frac{c}{B_0}\hat{\mathbf{z}} \times \boldsymbol{\nabla}_\perp \phi - \frac{e}{m_i \omega_{ci}^2}\frac{\partial}{\partial t}\boldsymbol{\nabla}_\perp \phi, \tag{12}$$

$$\mathbf{u}_{e\perp} = \frac{c}{B_0}\hat{\mathbf{z}} \times \boldsymbol{\nabla}_\perp \phi + \frac{e}{m_e \omega_{ce}^2}\frac{\partial}{\partial t}\boldsymbol{\nabla}_\perp \phi, \tag{13}$$

$$\frac{\partial u_{iz}}{\partial t} = -\frac{e}{m_i}\frac{\partial \phi}{\partial z}, \tag{14}$$

$$\frac{\partial u_{ez}}{\partial t} = \frac{e}{m_e}\frac{\partial \phi}{\partial z}. \tag{15}$$

Substituting (12)–(15) into (11), we obtain

$$\frac{\partial^2}{\partial t^2}\nabla_\perp^2 \phi + \varepsilon \omega_{gm}^2 \frac{\partial^2 \phi}{\partial z^2} = 0, \tag{16}$$

where

$$\varepsilon = \frac{n_{e0}}{n_{i0}}\left(1 + \frac{n_{i0}}{n_{e0}}\frac{m_e}{m_i}\right) \qquad \omega_{gm} = (\omega_{ce}\omega_{ci})^{1/2}.$$

Expressing $\nabla_\perp^2 \phi$ in terms of cylindrical coordinates $(r, \theta, z)$ as

$$\nabla_\perp^2 \phi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2} \tag{17}$$

and assuming $\phi$ as

$$\phi = \phi(r)e^{-i\omega t + ik_z z + il\theta}, \tag{18}$$

we can rewrite (16) in the form

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi}{\partial r}\right) - \frac{l^2}{r^2}\phi + \beta_c \phi = 0, \tag{19}$$

where $\beta_c = \varepsilon k_z^2 \omega_{gm}^2/\omega^2$. Equation (19) is an $l$th-order Bessel equation whose solution is

$$\varphi = J_l(\beta_c^{1/2} r), \tag{20}$$

where $\varphi = \phi/\phi_0$ is the electrostatic wave potential normalized by the unperturbed electrostatic potential $\phi_0$. On the surface of a cylindrical waveguide with radius $R$,

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we have $J_l(\beta_c^{1/2} r) = 0$. Thus, if $\mu_{lc}$ is a root of $J_l$, $\beta_c^{1/2} R = \mu_{lc}$ gives

$$\omega = \left(\frac{n_{e0}}{n_{i0}}\right)^{1/2} \omega_{gm} \frac{k_z R}{\mu_{lc}}, \tag{21}$$

which is the dispersion relation for the convective cells in a magnetized dusty plasma waveguide. The finite cylindrical boundary leads to an effective wavenumber given by $(k_z^2 + k_\perp^2)^{1/2}$, where the effective radial wavenumber $k_\perp = \mu_{lc}/R$ is quantized.

*3.1.2. Coupled IC and DIA waves.* To study coupled IC and DIA waves in a bounded dusty plasma system, we first reduce (8)–(10) to the form

$$\frac{\partial}{\partial t}\left(\frac{\partial^2}{\partial t^2} + \omega_{ci}^2\right) n_i - \frac{n_{i0} e}{m_i} \frac{\partial}{\partial t} \nabla_\perp^2 \left(\phi + \frac{3T_i}{n_{i0} e} n_i\right) + n_{i0} \left(\frac{\partial^2}{\partial t^2} + \omega_{ci}^2\right) \frac{\partial u_{sz}}{\partial z} = 0, \tag{22}$$

$$\frac{\partial u_{iz}}{\partial t} = -\frac{e}{m_i} \frac{\partial}{\partial z} \left(\phi + \frac{3T_i}{n_{i0} e} n_i\right). \tag{23}$$

We now assume the quasineutrality condition (i.e. $n_e = n_i$), and appropriate approximations for the ion-cyclotron or ion-acoustic waves, namely $\omega \ll k_z v_{te}, \omega_{ce} k_z / k_\perp$. These considerations yield $n_i = n_e \simeq n_{e0} e\phi / T_e$. Thus, we can combine (22) and (23) to obtain

$$\left(\frac{\partial^2}{\partial t^2} - C_s^2 \frac{\partial^2}{\partial z^2}\right)\left(\frac{\partial^2}{\partial t^2} + \omega_{ci}^2\right)\phi - C_s^2 \frac{\partial^2}{\partial t^2} \nabla_\perp^2 \phi = 0, \tag{24}$$

where

$$C_s^2 = \frac{T_e}{m_i} \frac{n_{i0}}{n_{e0}} \left(1 + 3 \frac{T_i}{T_e} \frac{n_{e0}}{n_{i0}}\right).$$

Introducing (18) into (24), we obtain

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \phi}{\partial r}\right) - \frac{l^2}{r^2} \phi + \beta_s \phi = 0, \tag{25}$$

where

$$\beta_s = \frac{(\omega^2 - \omega_{ci}^2)(\omega^2 - k_z^2 C_s^2)}{\omega^2 C_s^2}.$$

Equation (25) is again the $l$th-order Bessel equation. Thus, as before, the dispersion relation $\beta_s^{1/2} R = \mu_{lc}$ gives

$$(\omega^2 - \omega_{ci}^2)(\omega^2 - k_z^2 C_s^2) = \omega^2 C_s^2 \frac{\mu_{lc}^2}{R^2}. \tag{26}$$

Equation (26) is the dispersion relation for the coupled IC and DIA waves in a magnetized dusty plasma waveguide. We can now consider different frequency limits, namely $\omega \ll \omega_{ci}$, $\omega \gg \omega_{ci}$, and $\omega \gg k_z C_s$, and find the corresponding dispersion relations

$$\omega = k_z C_s \left(1 + \frac{C_s^2 \mu_{lc}^2}{\omega_{ci}^2 R^2}\right)^{-1/2} \quad \text{for } \omega \ll \omega_{ci}, \tag{27}$$

$$\omega = k_z C_s \left(1 + \frac{\mu_{lc}^2}{k_z^2 R^2}\right)^{1/2} \quad \text{for } \omega \gg \omega_{ci}, \tag{28}$$

$$\omega = \omega_{ci} \left(1 + \frac{C_s^2 \mu_{lc}^2}{\omega_{ci}^2 R^2}\right)^{1/2} \quad \text{for } \omega \gg k_z C_s. \tag{29}$$

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It is obvious from (27)–(29) that the boundary effects and the effects of obliqueness and external magnetic field significantly modify the dispersion properties of both the dust-ion-acoustic and the ion-cyclotron modes.

*3.1.3. Modified lower-hybrid waves.* To derive the dispersion relation for the lower-hybrid waves in a bounded dusty plasma system, we first consider some appropriate approximations for lower-hybrid waves, namely $k_z v_{te,ti}, \omega_{ci} \ll \omega \gg \omega_{ce}$. These considerations allow us to write (9) and (10) as

$$\frac{\partial \mathbf{u}_{i\perp}}{\partial t} = -\frac{e}{m_i}\boldsymbol{\nabla}_\perp \phi, \tag{30}$$

$$\mathbf{u}_{e\perp} = \frac{c}{B_0}\hat{\mathbf{z}} \times \boldsymbol{\nabla}_\perp \phi + \frac{e}{m_e \omega_{ce}^2}\frac{\partial}{\partial t}\boldsymbol{\nabla}_\perp \phi, \tag{31}$$

$$\frac{\partial u_{iz}}{\partial t} = -\frac{e}{m_i}\frac{\partial \phi}{\partial z}, \tag{32}$$

$$\frac{\partial u_{ez}}{\partial t} = \frac{e}{m_e}\frac{\partial \phi}{\partial z}. \tag{33}$$

Thus, using (7), (8), and (30) in (33), we obtain

$$\frac{\partial^2}{\partial t^2}\left(\nabla^2 + \frac{\omega_{pe}^2}{\omega_{ce}^2}\nabla_\perp^2\right)\phi + \omega_{pe}^2\frac{\partial^2 \phi}{\partial z^2} + \omega_{pi}^2 \nabla^2 \phi = 0. \tag{34}$$

Expressing $\nabla^2\phi$ in terms of cylindrical coordinates $(r, \theta, z)$ and introducing (18) into (34), we obtain

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi}{\partial r}\right) - \frac{l^2}{r^2}\phi + \beta_L \phi = 0, \tag{35}$$

where

$$\beta_L = \frac{\omega_{pe}^2}{(\omega^2 - \omega_{LH}^2)}\frac{k_z^2}{(1 + \omega_{pe}^2/\omega_{ce}^2)}, \tag{36}$$

$$\omega_{LH} = \frac{\omega_{pi}}{(1 + \omega_{pe}^2/\omega_{ce}^2)^{1/2}}. \tag{37}$$

Equation (35) is again the $l$th-order Bessel equation. Thus, as before, the dispersion relation $\beta_L^{1/2} R = \mu_{lc}$ becomes

$$\omega = \omega_{LH}\left(1 + \frac{n_{e0}}{n_{i0}}\frac{m_i}{m_e}\frac{k_z^2 R^2}{\mu_{lc}^2}\right)^{1/2}, \tag{38}$$

which, for $\omega_{pe} \gg \omega_{ce}$, reduces to

$$\omega = \omega_{gm}\left(\frac{n_{i0}}{n_{e0}}\right)^{1/2}\left(1 + \frac{n_{e0}}{n_{i0}}\frac{m_i}{m_e}\frac{k_z^2 R^2}{\mu_{lc}^2}\right)^{1/2}. \tag{39}$$

Equation (39) is the dispersion relation for the modified lower-hybrid waves in a magnetized dusty plasma of finite extent. It can be seen that the lower-hybrid mode is significantly modified by the obliqueness and boundary of the plasma.

### *3.2. Dynamics of charged dust particles*

Here, we consider the dynamics of charged dust particles, and derive the dispersion relations for the coupled dust-cyclotron and dust-acoustic waves in a bounded dusty

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plasma system. We first express (8)–(10), for the cold dust fluid ($T_d = 0$), as

$$\frac{\partial n_d}{\partial t} + n_{d0}\boldsymbol{\nabla}\cdot(\mathbf{u}_{d\perp} + \hat{\mathbf{z}}u_{dz}) = 0, \tag{40}$$

$$\left(\frac{\partial^2}{\partial t^2} + \omega_{cd}^2\right)\mathbf{u}_{d\perp} = \frac{c}{B_0}\omega_{cd}^2(\hat{\mathbf{z}}\times\boldsymbol{\nabla}_\perp\phi) + \frac{Z_d e}{m_d}\frac{\partial}{\partial t}\boldsymbol{\nabla}_\perp\phi, \tag{41}$$

$$\frac{\partial u_{dz}}{\partial t} = \frac{Z_d e}{m_d}\frac{\partial\phi}{\partial z}, \tag{42}$$

where $\omega_{cd} = Z_d eB_0/m_d c$. We introduce the quasineutrality condition ($\sum_s q_s n_s = 0$) and appropriate approximations for the dust-cyclotron or dust-acoustic waves, namely $\omega \ll \omega_{ce}$, $\omega_{ce,ci}k_z/k_\perp$, and $k_\perp\rho_i \gg 1$, where $\rho_i$ is the ion Larmor radius. Hence, we have $n_e - n_i = Z_d n_d$, $n_e \simeq n_{e0}e\phi/T_e$, and $n_i \simeq -n_{i0}e\phi/T_i$. Thus, we can combine (40)–(42) to obtain

$$\left(\frac{\partial^2}{\partial t^2} - C_{DA}^2\frac{\partial^2}{\partial z^2}\right)\left(\frac{\partial^2}{\partial t^2} + \omega_{cd}^2\right)\phi - C_{DA}^2\frac{\partial^2}{\partial t^2}\nabla_\perp^2\phi = 0, \tag{43}$$

where

$$C_{DA}^2 = \frac{T_i}{m_d}\frac{Z_d n_{d0}}{n_{i0}}\left(1 + \frac{T_i}{T_e}\frac{n_{e0}}{n_{i0}}\right).$$

Introducing (18) into (43), we obtain

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right) - \frac{l^2}{r^2}\phi + \beta_d\phi = 0, \tag{44}$$

where

$$\beta_d = \frac{(\omega^2 - \omega_{cd}^2)(\omega^2 - k_z^2 C_{DA}^2)}{\omega^2 C_{DA}^2}.$$

Equation (44) is the $l$th-order Bessel equation. The dispersion relation $\beta_d^{1/2}R = \mu_{lc}$ yields

$$(\omega^2 - \omega_{cd}^2)(\omega^2 - k_z^2 C_{DA}^2) = \omega^2 C_{DA}^2\frac{\mu_{lc}^2}{R^2}. \tag{45}$$

Equation (45) is the dispersion relation for the coupled dust-cyclotron and dust-acoustic waves in a magnetized dusty plasma waveguide. We can now consider different frequency regimes, namely $\omega \ll \omega_{cd}$, $\omega \gg \omega_{cd}$, and $\omega \gg k_z C_{DA}$, and obtain the corresponding dispersion relations

$$\omega = k_z C_{DA}\left(1 + \frac{C_{DA}^2\mu_{lc}^2}{\omega_{cd}^2 R^2}\right)^{-1/2} \quad \text{for } \omega \ll \omega_{cd}, \tag{46}$$

$$\omega = k_z C_{DA}\left(1 + \frac{\mu_{lc}^2}{k_z^2 R^2}\right)^{1/2} \quad \text{for } \omega \gg \omega_{cd}, \tag{47}$$

$$\omega = \omega_{cd}\left(1 + \frac{C_{DA}^2\mu_{lc}^2}{\omega_{cd}^2 R^2}\right)^{1/2} \quad \text{for } \omega \gg k_z C_{DA}. \tag{48}$$

Equations (46)–(48) clearly indicate that the dispersion properties of both the dust-acoustic and dust-cyclotron modes are significantly modified by the effects of the boundary, the obliqueness, and the external magnetic field.

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## 4. Discussion

A self-consistent and general description of linear low-frequency electrostatic waves (propagating obliquely with respect to an external magnetic field) in a bounded dusty magnetoplasma consisting of negatively charged dust particles, electrons, and positively charged ions has been presented. A number of different modes, such as the modified convective cell, coupled ion-cyclotron and dust-ion-acoustic waves, modified lower-hybrid waves, coupled dust-cyclotron and dust-acoustic waves, etc. have been investigated. It has been shown that the effects of a boundary of the dusty plasma system, the external magnetic field, and the obliqueness of the propagating modes significantly modify the dispersion properties of these different low-frequency electrostatic waves.

We have shown that the finite cylindrical boundary leads to an effective wavenumber (of all the wave modes considered) given by $(k_z^2 + k_\perp^2)^{1/2}$, where the effective radial wave number $k_\perp = \mu_{lc}/R$ is quantized. It has been observed that as we increase the quantity $k_\perp = \mu_{lc}/R$, the frequency of the ion-acoustic mode with $\omega \ll \omega_{ci}$ decreases, whereas that with $\omega \gg \omega_{ci}$ increases (cf. (27) and (28)). The same results are also found to hold for the case of dust-acoustic modes (cf. (46) and (47)). It has also been shown that the frequencies of the ion-cyclotron mode (where $\omega \gg k_z C_s$ holds), the lower-hybrid mode (where $k_z v_{te,ti}, \omega_{ci} \ll \omega \ll \omega_{ce}$ holds), and the dust-cyclotron mode (where $\omega \gg k_z C_{DA}$ holds) increase with $k_\perp = \mu_{lc}/R$ (cf. (29), (39), and (48)).

It should be mentioned that in our analysis we have neglected the variation of the dust-grain charge, which (for Maxwellian electrons and ions) causes damping of the dust-acoustic mode (Varma et al. 1993; Melandsø et al. 1993; Shukla 1996). To include this effect in our case, we have to calculate the perturbed distribution and charging cross-sections, which are functions of the external magnetic field and the impact parameter of the particles that approach the grain to within distances smaller than the particle size. This is, in fact, a rather complicated problem, and is beyond the scope of the present work. However, it has been shown by Rosenberg and Krall (1995) that such damping effects may need to be considered when the wave frequency $\omega$ is of the order of the charging frequency $\nu_{ch}$ of the dust grain. In conclusion, we stress that the results of our investigation should be useful for understanding the spectra of low-frequency electrostatic waves in magnetized dusty plasma laboratory devices, with which we plan to carry out our wave studies.

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