Answers to problems

(Unless otherwise stated, the mass of a nucleus of mass number A is approximated as A amu.)

Chapter 1

1.1 Ratio $= Gm_e^2(4\pi\varepsilon_0/e^2) = 2.4 \times 10^{-43}$.

1.2(*b*) (*i*) $+1$, (*ii*) -1, (*iii*) -1.

1.3(*a*) Wavelength
$\lambda = 2\pi c/\omega = 2\pi(\hbar c)/(\hbar\omega) = 2\pi(197 \text{ MeV fm})/(1 \text{ MeV}) = 1240 \text{ fm}$.

Chapter 2

2.1 Group velocity $= \mathrm{d}\omega/\mathrm{d}k = c^2k/\omega = c^2\,\hbar k/\hbar\omega = c^2p/E$.
For a particle of velocity v, $E = \gamma mc^2$ and $p = \gamma mv$.
Hence group velocity $=$ particle velocity.

2.2 From equation (2.8) the electrostatic energy is of order of magnitude $e^2/4\pi\varepsilon_0 a_0$.
From equation (2.13) and after, the weak interaction energy is of order of magnitude $\varepsilon_0^{-1}(\hbar/M_Zc)^2e^2/a_0^3$.
Ratio $= 4\pi(\hbar/a_0M_Zc)^2 \sim 10^{-15}$.

2.3 By momentum conservation the momenta of the two photons must be equal in magnitude (and opposite in direction). They will therefore have equal energy.

2.4 There is a frame of reference (the centre-of-mass frame) in which the total momentum of the e^+e^- pair is zero. The photon would therefore

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have zero momentum and hence zero energy: energy conservation would be violated.

2.5 Since $E = 1$ MeV is much less than the rest energy of the muon we may use non-relativistic mechanics, and the velocity $v = c\sqrt{(2E/m_\mu c^2)} = 4.1 \times 10^7\ \mathrm{ms}^{-1}$. In time τ_μ the muon travels $v\tau_\mu = 90$ m.

2.6(a) Not allowed. Such a process need not violate the conservation laws of energy, momentum, angular momentum or electric charge, but it would violate the conservation laws of electron number and muon number. Although searched for, this decay has never been seen. (b) and (c) can occur.

2.8 Consider a point charge e at position $\mathbf{R}$; then

$$\mathbf{E}(\mathbf{r}) = \frac{e}{4\pi\varepsilon_0}\frac{\mathbf{r}-\mathbf{R}}{|\mathbf{r}-\mathbf{R}|^3}.$$

Under reflection in the origin, $\mathbf{r} \to \mathbf{r}' = -\mathbf{r}$, $\mathbf{R} \to \mathbf{R}' = -\mathbf{R}$ and

$$\mathbf{E}'(\mathbf{r}') = \frac{e}{4\pi\varepsilon_0}\frac{\mathbf{r}'-\mathbf{R}'}{|\mathbf{r}'-\mathbf{R}'|^3} = -\mathbf{E}(\mathbf{r}).$$

The magnetic field due to a current I in a loop is

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0 I}{4\pi}\int\frac{\mathrm{d}\mathbf{R}\times(\mathbf{r}-\mathbf{R})}{|\mathbf{r}-\mathbf{R}|^3}.$$

Under reflection the vector product does not change sign. Hence $\mathbf{B}'(\mathbf{r}') = +\mathbf{B}(\mathbf{r})$; $\mathbf{B}(\mathbf{r})$ is an axial vector field.

2.9 The reduced masses are

$m_\mu m_\mathrm{d}/(m_\mu + m_\mathrm{d}) = 100.025$ MeV for d–μ system,

$m_\mu m_\mathrm{t}/(m_\mu + m_\mathrm{t}) = 101.829$ MeV for t–μ system.

The difference in binding energies is

$$\frac{1}{2}(\Delta m)c^2(e^2/4\pi\varepsilon_0\hbar c)^2 = 48\ \mathrm{eV}.$$

Chapter 3

3.1 The nucleon magnetic dipole moments are vectors aligned with the nucleon spin. In the deuteron the spins are parallel and the moments add to give a net magnitude $\mu_\mathrm{p} + \mu_\mathrm{n} = (2.792\,84 - 1.913\,04)e\hbar/2m_\mathrm{p} =$

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$0.8798e\hbar/2m_p$ (from equation (3.2)).
The measured magnitude is

$$\mu_d = 0.8574e\hbar/2m_p = (1 - 0.026)(\mu_p + \mu_n).$$

The discrepancy (0.026) could be due to a contribution to the magnetic moment from the orbital motion of the nucleons, associated with the small d-wave component of the deuteron wave-function. (Appendix C, §C.4.)

3.2(*a*) The magnetic field at distance $\mathbf{r}$ from dipole (1) is

$$\mathbf{B} = -\left(\frac{\mu_0}{4\pi}\right)\nabla\left(\frac{\mu\boldsymbol{\sigma}_1 \cdot \mathbf{r}}{r^3}\right) = \left(\frac{\mu_0}{4\pi}\right)\left(-\frac{\mu\boldsymbol{\sigma}_1}{r^3} + \frac{3(\mu\boldsymbol{\sigma}_1 \cdot \mathbf{r})\mathbf{r}}{r^5}\right).$$

The energy of dipole (2) at $\mathbf{r}$ in this field is

$$-\mu\boldsymbol{\sigma}_2 \cdot \mathbf{B} = -\left(\frac{\mu_0}{4\pi}\right)\frac{\mu^2}{r^3}\Omega_T.$$

3.3 Subtracting 1 MeV from the rest energies of the charged particles gives

(udd) 940, (uud) 937; 3 MeV for the extra d quark;
(dds) 1196, (uds) 1192, (uus) 1188; 4 MeV for the extra d quark;
(d$\bar{\text{s}}$) 498, (u$\bar{\text{s}}$) 493; 5 MeV for the extra d quark.

Interchanging a d quark for a u quark always increases the rest energy, in this sample by an average of 4 MeV.

3.4(*a*) Not allowed. Does not conserve electric charge.

(*b*) Not allowed. Does not conserve baryon number or electron number.

(*c*) Not allowed. Does not conserve baryon number.

(*d*) Allowed.

3.5

$\Xi^{*-} \to K^- + \Sigma^0$	Strong. Does not require the weak or the electromagnetic interaction.
$\Sigma^0 \to \Lambda^0 + \gamma$	Electromagnetic.
$\Lambda^0 \to p + e^- + \bar{\nu}_e$	Involves an anti-neutrino, therefore weak.
$K^- \to \pi^- + \pi^0$	An s quark changes to a d quark, therefore weak.
$\pi^0 \to \gamma + \gamma$	Electromagnetic (Fig. 3.6).
$\pi^- \to \mu^- + \bar{\nu}_\mu$	Weak (cf. Fig. 3.5).
$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$	Weak (Fig. 2.2).

Chapter 4

4.2(*a*) Since $\mathbf{q} = \mathbf{k}_f - \mathbf{k}_i$, where $\mathbf{k}_i$ and $\mathbf{k}_f$ are the initial and final wave vectors,

$$q^2 = k_f^2 + k_i^2 - 2k_f k_i \cos\theta.$$

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Neglecting the electron mass, $k_i = p_i/\hbar = E/\hbar c$, and in scattering from a fixed target there is no energy loss. Hence $k_f = k_i = E/\hbar c$ and

$$q^2 = 2E^2(1 - \cos\theta)/\hbar^2 c^2, \quad q = (2E/\hbar c)\sin(\theta/2).$$

4.3 By the uncertainty principle, the mean magnitude of the lepton momentum $p \sim \hbar/a$. But $p = mv$, so that $(v/c) \sim \hbar/amc = e^2/4\pi\varepsilon_0 \hbar c = \frac{1}{137}$. The characteristic time $t = a/v = 137a/c = 1.2 \times 10^{-19}$ s, and $(\tau_\mu/t) \sim 2 \times 10^{13}$.

4.4(b) This follows from perturbation theory in quantum mechanics. Since the integral is over nuclear dimensions $r \leqslant R$, it is reasonable to approximate $\psi(r)$ by $\psi(0) = \pi^{-\frac{1}{2}}(Z/a)^{\frac{3}{2}}$, which with Problem 4.4(a) gives the result.

4.5 The total binding energy of two α-particles is 56.60 MeV, 0.1 MeV greater than the binding energy of $^8_4$Be. $^8_4$Be decays to two α-particles and to conserve energy the 0.1 MeV of nuclear energy is converted into the kinetic energy of their motion. $^{12}_6$C is more strongly bound than three α-particles by 7.26 MeV. The binding energy of $^6_3$Li is 31.99 MeV, 1.47 MeV greater than the total binding energy of $^2_1$H and $^4_2$He. Energy is conserved overall, and the nuclear energy released is taken by the γ-ray and the kinetic energy of the $^6_3$Li.

4.6 Treating A as a continuous variable, the maximum is where $\mathrm{d}(B/A)/\mathrm{d}A = 0$, i.e. at

$$\frac{A}{2} = Z = \frac{b}{d} = 25.7$$

The nearest integer is $Z = 26$, which gives the maximum of (B/A).

4.7 For $A = 100$, the formula gives $Z = 43$. $^{100}_{43}$Tc is an odd–odd nucleus and unstable. Both $^{100}_{42}$Mo and $^{100}_{44}$Ru are stable. For $A = 200$, the formula gives $Z = 80$, and $^{200}_{80}$Hg is stable.

4.8 Suppose the sample contains N ^{14}C nuclei. Then the mean number of decays per second is $N/\tau = 15.3/60\ \mathrm{s}^{-1}$, and hence $N = 6.7 \times 10^{10}$. The atomic mass of natural carbon is 12.01 amu $= 2 \times 10^{-23}$ g. Therefore 1 g of carbon contains 5×10^{22} atoms, and the proportion of ^{14}C in the sample is 1.3×10^{-12}.

Assuming (i) that the proportion of ^{14}C in the atmosphere remains constant, (ii) that the hut had been built from new timber, and (iii) that the carbon in the timber had all been bound in at the time of its growth, the average number of decays per minute would be reduced by a factor $\mathrm{e}^{-t/\tau}$, where τ is the mean life of ^{14}C and t the age of the specimen. Thus the expected rate would be 9.4 decays per minute. Conversely, an average of 9.4 decays per minute would suggest an age of 4000 years.

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In practice, carbon dating is far more complicated than this problem suggests. In the past there have been small variations in the proportion of ^{14}C in the atmosphere, as appears when radiocarbon dating can be calibrated against dating obtained by other methods, such as dendrochronology (counting tree rings). These variations can lead to ambiguities in the inversion of the calibration curve.

4.9 The number of atoms of ^{82}Se in the sample is $N = (0.97 \times 14\ \text{g})/(82 \times 1.66 \times 10^{-24}\ \text{g}) \approx 1.00 \times 10^{23}$. If τ is the mean life, in a time $t \ll \tau$ there will be Nt/τ decays, giving

$$\begin{aligned}\tau &= Nt/(\text{number of events}) \\ &= 10^{23} \times 7960\ \text{hr}/(35/0.062) \\ &= 1.6 \times 10^{20}\ \text{yr}.\end{aligned}$$

(The condition $t \ll \tau$ is well satisfied!)

4.10 In the absence of neutrinos to share the energy released in the decay, the sum of the energies of the two electrons emitted would be sharply peaked at the decay energy. (The recoil energy of the nucleus would be small.)

4.11 As a rough rule, for A odd there is only one β-stable nucleus and for A even, two. Up to and including $A = 209$ there are 105 odd-A nuclei and 104 even-A nuclei, and so about 310 β-stable nuclei. All these have $Z \leqslant 83$ which implies an average of about 3.7 stable isotopes per element.

Chapter 5

5.2(*a*) From equations (5.4) and (5.5), for neutrons with kinetic energy E,

$$\mathcal{N}(E) = N(E/E_n^F)^{\frac{3}{2}}.$$

The density of states is $n(E) = d\mathcal{N}/dE = \frac{3}{2} N E^{\frac{1}{2}} (E_n^F)^{-\frac{3}{2}}$.
Hence the total neutron kinetic energy is

$$\int_0^{E_n^F} En(E)dE = \frac{3}{5} N E_n^F.$$

A similar result holds for the protons, noting that proton kinetic energies are given by $(E_p - \bar{U})$.

(*b*) Again consider the neutrons; since E_n^F is proportional to $N^{\frac{2}{3}}$ we can write

$$E_n^F = E_0^F (N/N_0)^{\frac{2}{3}} = E_0^F (1 + \Delta N/N_0)^{\frac{2}{3}}.$$

The result follows on expansion.
Similarly $E_p^F - \bar{U} = E_0^F (1 - \Delta N/N_0)^{\frac{2}{3}}$.

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(*c*) $\frac{3}{5}NE_{\mathrm{n}}^{\mathrm{F}} = \frac{3}{5}N_0E_0^{\mathrm{F}}(1+\Delta N/N_0)^{\frac{5}{3}}$

$$\approx \tfrac{3}{5}N_0E_0^{\mathrm{F}}\left(1+\frac{5}{3}\left(\frac{\Delta N}{N_0}\right)+\frac{5}{9}\left(\frac{\Delta N}{N_0}\right)^2\right).$$

Adding the proton kinetic energy, one obtains the total kinetic energy

$$\tfrac{3}{5}E_0^{\mathrm{F}}A+\tfrac{1}{3}E_0^{\mathrm{F}}\frac{(N-Z)^2}{A}.$$

5.3 $$\bar{U}_{\mathrm{c}} = \frac{(Z-1)e^2}{4\pi\varepsilon_0 R}\left(\frac{4\pi R^3}{3}\right)^{-1}\int_0^R 4\pi r^2\left(\frac{3}{2}-\frac{r^2}{2R^2}\right)\mathrm{d}r$$

$$= \frac{6}{5}\frac{(Z-1)e^2}{4\pi\varepsilon_0 R} \approx 21.5\ \mathrm{MeV} \quad \text{for}\ {}^{208}_{82}\mathrm{Pb}.$$

5.4 Since $Z = N$, the energy due to the strong nucleon–nucleon interaction should be the same for a neutron as for a proton in a similar state; thus $\bar{U}$ should be given by the Coulomb contribution, $\bar{U}_{\mathrm{c}}$ of Problem 5.3(*a*). This yields $\bar{U} = 8.7$ MeV. The separation energy is the binding energy at the Fermi level. Assuming $E_{\mathrm{p}}^{\mathrm{F}} \approx E_{\mathrm{n}}^{\mathrm{F}} + \bar{U}$, it follows that $S_{\mathrm{p}} = (15.6 - 8.7)$ MeV.

5.5 In the simple shell model, ${}^{31}_{15}\mathrm{P}$ has an odd proton; Table 5.1 suggests this is in the $2s_{\frac{1}{2}}$ shell, giving nuclear spin and parity $j^P = \frac{1}{2}^+$.

${}^{67}_{30}\mathrm{Zn}$ has an odd neutron. Suggested shell $f_{\frac{5}{2}}$; $j^P = \frac{5}{2}^-$.
${}^{115}_{49}\mathrm{In}$ has an odd proton. Suggested shell $1g_{\frac{9}{2}}$; $j^P = \frac{9}{2}^+$.

These suggestions are all in agreement with experiment.

5.6 The spins and parities of all but ${}^{26}_{13}\mathrm{Al}$ are in accord with pairing and shell filling as in Table 5.1. ${}^{26}_{13}\mathrm{Al}$ is odd–odd; the model suggests the odd neutron and proton both to be in $\frac{5}{2}^+$ states. Such a configuration would have the measured parity of ${}^{26}_{13}\mathrm{Al}$; and the measured spin of 5 suggests that the spins are paired parallel.
For the magnetic moments, equation (5.26) gives:

${}^{43}_{20}\mathrm{Ca}$, odd neutron, $l = 3$, $j = l + \frac{1}{2}$; $\mu = -1.92\ \mu_{\mathrm{N}}$
${}^{93}_{41}\mathrm{Nb}$, odd proton, $l = 4$, $j = l + \frac{1}{2}$; $\mu = 6.80\ \mu_{\mathrm{N}}$
${}^{137}_{56}\mathrm{Ba}$, odd neutron, $l = 2$, $j = l - \frac{1}{2}$; $\mu = 1.15\ \mu_{\mathrm{N}}$
${}^{197}_{79}\mathrm{Au}$, odd proton, $l = 2$, $j = l - \frac{1}{2}$; $\mu = 0.12\ \mu_{\mathrm{N}}$.

${}^{26}_{13}\mathrm{Al}$. The fact that the two angular momenta appear to be aligned suggests that we can simply add the Schmidt values to obtain the estimate $\mu = 2.9\ \mu_{\mathrm{N}}$.

5.7 For a proton, $j = \frac{1}{2}$, $\mu = 2.80\ \mu_{\mathrm{N}}$.
For ${}^{43}_{20}\mathrm{Ca}$, $j = \frac{7}{2}$, $\mu = -1.32\ \mu_{\mathrm{N}}$.

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Taking the nuclear magneton $\mu_N = 3.15 \times 10^{-14}$ MeV T^{-1} and using equation (5.21) gives

$\nu = \omega/2\pi = 43$ MHz for protons,
$\nu = \omega/2\pi = 2.9$ MHz for $^{43}_{20}$Ca.

5.8 The volume of the ellipsoid is $(4\pi/3)a^2b$ and hence the charge density is $3Ze/4\pi a^2 b$.

$$Q_{zz} = \frac{3Z}{4\pi a^2 b}\iiint (2z^2 - x^2 - y^2)\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z,$$

where the integral is through the ellipsoid.
Make a change of scale: $x = ax'$, $y = ay'$, $z = bz'$; then

$$Q_{zz} = \frac{3Z}{4\pi}\iiint (2b^2 z'^2 - a^2 x'^2 - a^2 y'^2)\,\mathrm{d}x'\,\mathrm{d}y'\,\mathrm{d}z',$$

where the integral is now through the unit sphere. Also

$$\iiint x'^2\,\mathrm{d}x'\,\mathrm{d}y'\,\mathrm{d}z' = \frac{1}{3}\int_0^1 r'^2 4\pi r'^2\,\mathrm{d}r' = \frac{4\pi}{15}, \quad \text{etc.},$$

giving

$$Q_{zz} = \frac{2Z}{5}(b^2 - a^2).$$

Taking the density of nuclear matter to be $\rho_0 = 0.17$ nucleons fm^{-3} and

$$\frac{4\pi}{3}a^2 b\rho_0 = A,$$

these equations lead to $b = 7.7$ fm, $a = 5.6$ fm.

Chapter 6

6.1 $Q = 0.094$ MeV, $r_s \approx 2 \times 4^{\frac{1}{3}} \times 1.1$ fm $= 3.5$ fm,

$r_c = 4e^2/4\pi\varepsilon_0 Q = 61$ fm.
$r_s/r_c = 0.057$ and Fig. 6.3 or equation (6.16) gives $\mathscr{G} = 0.70$.

The reduced mass is $m_\alpha/2$. Hence $G = 13$.

Taking $\tau_0 \approx 7 \times 10^{-23}$ s, as in other α-decays, leads to the estimate $\tau = 3 \times 10^{-17}$ s, though this excellent agreement is fortuitous.

6.2(*a*) To apply equations (6.2) and (6.15) to a positron, $2Z_d \to Z_d$ and m becomes the positron mass.
Thus $r_c = 112$ fm. Values of r_s between $r_s = 0$ and $r_s = 1.1A^{\frac{1}{3}}$ fm $=$

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6.37 fm are reasonable, giving $\mathscr{G} = 1, 0.70$; $G = 1.81, 1.27$, and a suppression factor e^{-G} in the range 0.16 to 0.28.

(*b*) $Q = 1.8$ Mev, $r_c = 123$ fm, $r_s \approx 8.1$ fm, $\mathscr{G} = 0.68$, $G = 154$, which with $\tau_0 = 7 \times 10^{-23}$s gives a partial mean life for α-decay $\tau_\alpha \sim 10^{37}$ years.
(Age of Solar System $\sim 10^9$ years.)

6.3 In α-decay in material, the kinetic energy is largely converted into heat and N atoms of ^{238}Pu would on average produce $N(5.49\ \text{MeV})/\tau$ of power.
For 1 kW $= 6.24 \times 10^{15}$ MeV s^{-1} we need $N = 4.6 \times 10^{24}$, or 1.8 kg of ^{238}Pu.
The decay rate of the by-product ^{234}U is so low that the heat from its decay is negligible.
For the remaining mass of plutonium to be 1.8 kg after 50 years requires 2.7 kg initially.

6.4 Suppose that when the sample of rock was formed, say T years ago, it contained no lead but N_1 atoms of ^{238}U (mean life τ_1) and N_2 atoms of ^{235}U (mean life τ_2). Then it would now contain $N_1 e^{-T/\tau_1}$ atoms of ^{238}U and, since each decayed uranium atom becomes a lead atom, $N_1(1 - e^{-T/\tau_1})$ atoms of ^{206}Pb. Setting $(1 - e^{-T/\tau_1})/e^{-T/\tau_1} = 0.0797$ suggests $T = 497 \times 10^6$ years. Similarly for ^{235}U and ^{207}Pb, $(1 - e^{-T/\tau_2})/e^{-T/\tau_2} = 0.675$ suggests $T = 531 \times 10^6$ years. (The discrepancy could be due to the effect of water on the rock, for example.)

6.5 Neglecting the excitation energy, the kinetic energy of the fragments can be estimated using equation (6.18), which gives $\Delta B = 178$ MeV. Each fragment would then have velocity 12×10^6 m s^{-1}. In the frame in which the fragment is at rest, a 2 MeV neutron has velocity 20×10^6 m s^{-1}. In the laboratory frame, the distribution of emitted neutrons is peaked in the direction of the moving fragment.

Chapter 7

7.3(*a*) $E_i + E_0 = E_f +$ (excitation energy) + (^{17}O* recoil energy).
Hence the recoil energy is 0.26 MeV and the recoil velocity is $v/c = 5.7 \times 10^{-3}$ (approximating the mass of ^{17}O* by 17 amu).

(*b*) If the photon has energy E_γ it has momentum E_γ/c, and to conserve momentum this must be the recoil momentum of the ^{17}O. Hence the ^{17}O recoil energy E_R is $E_R = (E_\gamma/c)^2/(34\ \text{amu})$.
To conserve energy, $E_\gamma + E_R = 0.87$ MeV.
We could solve these equations for E_γ, but clearly E_R is small, and to two significant figures it is sufficient to take $E_\gamma = 0.87$ MeV in the first equation to give $E_R = 24$ eV.

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(*c*) The photon energy will be a maximum if it is emitted parallel to the motion of the ^{17}O*. By a Lorentz transformation to the laboratory frame

$$E_\gamma^{\text{lab}} = (1 + v/c)E_\gamma/(1 - v^2/c^2)^{\frac{1}{2}}$$

and hence $E_\gamma^{\text{lab}} - E_\gamma = 5$ keV.
Similarly the photon energy will be a minimum if it is emitted anti-parallel, in which case $E_\gamma^{\text{lab}} - E_\gamma = -5$ keV.

7.4 $^{11}_{6}$C is less bound than $^{11}_{5}$B by 2.762 MeV. The difference of Coulomb energies of uniformly charged spheres of net charge $6e$ and $5e$ and radius $R = 1.1 \times 11^{\frac{1}{3}}$ fm is

$$\frac{3}{5}\frac{e^2}{4\pi\varepsilon_0 R}(6^2 - 5^2) = 4 \text{ MeV}.$$

This is a 50% over-estimate of the observed energy difference, and we would need to take $R_C = 1.45\ R$ to obtain agreement. The approximation of a uniform charge distribution is inadequate for precise calculations, especially for light nuclei. In reality some charge is displaced to larger distances (see Fig. 4.3) thereby reducing the energy. Calculations using the more realistic distributions are in better accord with the data.

7.5 The decay by neutron emission with a kinetic energy release of 0.41 MeV need involve only the strong interaction. There is no Coulomb barrier, and only a small angular momentum barrier: to conserve angular momentum and parity the angular momentum of the ^{16}O–n pair must be $l = 1$.
The mean life is still quite long on the nuclear time scale of $\sim 10^{-22}$ s. $\Gamma = \hbar c/\tau c = 0.04$ MeV.

7.6 The nuclear transition is $\frac{1}{2}^- \rightarrow \frac{3}{2}^-$, so the photon will have positive parity and angular momentum quantum number $2 \geqslant j \geqslant 1$. The most likely transition is with $j = 1$, which would be magnetic dipole. The photon energy is about 2.13 MeV. From Fig. 7.6, a rough estimate of the mean life is $\tau \sim 10^{-17} \times (100/A)^{\frac{2}{3}} \times 20A^{\frac{2}{3}}$ s $\sim 4 \times 10^{-15}$ s. (The experimental mean life is 5.2×10^{-15} s.) An electric quadrupole transition with $j = 2$ is also possible, but Fig. 7.6 suggests its partial decay rate to be much slower than the magnetic dipole rate.

7.7 The lowest six energy levels (comprising 26 states) all have positive parity. $^{10}_{5}$B has three protons in the $p_{\frac{3}{2}}$ shell and three neutrons in the $p_{\frac{3}{2}}$ shell. There are many combinations of the single nucleon p-states and they all have positive parity, $(-1)^6$. The lowest observed states can be considered to be constructed from these.
The 1.74 MeV level can decay to the 0.72 MeV level by a magnetic

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dipole ($0^+ \rightarrow 1^+$) transition with $E_\gamma \approx 1.02$ MeV. This level can in turn decay to ground by an electric quadrupole ($1^+ \rightarrow 3^+$) transition with $E_\gamma \approx 0.72$ MeV. Neglecting internal conversion the ratio of photons emitted is clearly one-to-one.

The 1.74 MeV level can also decay directly to ground with $E_\gamma \approx 1.74$ MeV but this ($0^+ \rightarrow 3^+$) transition is magnetic octapole and very slow. Using Fig. 7.6 the number of photons emitted with energies 1.02 MeV, 0.72 MeV and 1.74 MeV should be in proportion $1{:}1{:}10^{-8}$.

Chapter 8

8.2 A neutron with kinetic energy 0.1 eV has $v/c = 1.46 \times 10^{-5}$ giving $\sigma = 2670$ b, $l \approx 1.56$ cm. The probability of a neutron penetrating a distance x into the gas without interaction is $e^{-x/l}$. For this probability to be 0.1, we require $x = 3.6$ cm. The active region of the detector should be at least of this thickness.

8.3(*a*) $\tau = \hbar/\Gamma = \hbar c/\Gamma c = 1.3 \times 10^{-21}$ s.

(*b*) In this example the elastic width equals the total width to a good approximation, since there is not enough energy to induce other nuclear reactions. The spin of the neutron is $s_1 = \frac{1}{2}$ and the spin of ${}^4_2\mathrm{H}$ is $s_2 = 0$. Hence the statistical factor in the Breit–Wigner formula is $(2j+1)/(2s_1+1)(2s_2+1) = 2$, and the cross-section at energy E is

$$\sigma(E) = \frac{2\pi}{k^2}\frac{\Gamma^2}{(E-E_0)^2 + \Gamma^2/4} \quad \text{(equation (D.11))}.$$

At energy $E = E_0$, $\sigma = 8\pi/k^2 = 4\pi\hbar^2/mE_0$, where m is the reduced mass. Hence $\sigma \approx 3.2$ b.

8.4 The coefficient of the $(1/v)$ term is large if the incident neutron can easily induce a nuclear reaction (as in the case of ${}^{235}\mathrm{U}$ fission), or if there is an excited state close to zero incident neutron energy. Neither of these conditions is apparently satisfied in the case of ${}^{238}\mathrm{U}$. However, one would expect to see a small $(1/v)$ contribution at even lower neutron energies due for example to residual radiative capture.

8.5 The total width Γ is given approximately by $\Gamma = \Gamma_\gamma + \Gamma_\mathrm{n}$, and the relative probability of neutron radiative capture is

$$\Gamma_\gamma/\Gamma = 1/(1 + \Gamma_\mathrm{n}/\Gamma_\gamma).$$

In this application of the Breit–Wigner formula the neutron spin $s_1 = \frac{1}{2}$ and the spin of the even–even nucleus ${}^{238}\mathrm{U}$ is $s_2 = 0$. Also $j = \frac{1}{2}$. Hence at resonance

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$$\sigma_\gamma = \frac{4\pi}{k^2}\frac{\Gamma_n\Gamma_\gamma}{(\Gamma_n+\Gamma_\gamma)^2},$$

where $k^2 = 2mE_0/\hbar^2$.
From Fig. 8.5, $E_0 = 6.7$ eV and $\sigma_\gamma = 2\times10^4$ b. Hence

$$\frac{\Gamma_n\Gamma_\gamma}{(\Gamma_n+\Gamma_\gamma)^2} = \frac{\Gamma_n/\Gamma_\gamma}{(1+\Gamma_n/\Gamma_\gamma)^2} = 0.052.$$

Since we are told that Γ_n/Γ_γ is small, we take the solution $\Gamma_n/\Gamma_\gamma = 0.058$. Hence capture is 95% probable.

Chapter 9

9.1 The molecules $CH_4 + 2O_2$ have a mass ≈ 80 amu and release 9 eV in chemical reaction, i.e. 0.11 eV per amu. ^{235}U has a mass ≈ 235 amu and releases about 200×10^6 eV on fission (Table 9.1 and discussion), giving 0.85×10^6 eV per amu.
Ratio $\approx 8\times10^6$.

9.3 $S_n = B(N,Z) - B(N-1,Z)$. The quoted difference comes from the pairing energy terms. All other terms in the mass formula give contributions to S_n which for a heavy nucleus vary only slowly with A.

9.4 The probability of the neutron inducing fission at the nth collision is $p(1-p)^{n-1}$.
By definition the mean number of collisions is

$$\bar{n} = \sum_1^\infty np(1-p)^{n-1} = -p\frac{d}{dp}\sum_1^\infty(1-p)^n.$$

Summing the geometric series

$$\bar{n} = -p\frac{d}{dp}[1-(1-p)]^{-1} = \frac{1}{p}.$$

9.5(a) If v is the neutron velocity in the laboratory frame, its velocity in the centre-of-mass frame is

$$v - \frac{m_n v}{M+m_n} = \frac{Mv}{M+m_n}.$$

In the centre-of-mass frame it loses no energy on scattering, but suppose it is deflected through an angle θ. In the laboratory frame it will then have a component of velocity $v(m_n + M\cos\theta)/(M+m_n)$ in the original direction and a perpendicular component $Mv\sin\theta/(M+m_n)$. The result follows on averaging over all angles θ.

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(*b*) On average, after N collisions a neutron with initial energy E_0 will have energy $E_N = \alpha^N E_0$, where

$$\alpha = \frac{M^2 + m_n^2}{(M + m_n)^2} = 0.86.$$

For $E_0 = 2$ MeV, $E_N = 0.1$ eV, the number of collisions required is $N \sim 110$.

The mean time between collisions for a neutron of energy E is $\Delta t = 1/\sigma\rho v = 1/\sigma\rho\sqrt{(2E/m_n)}$, and it loses energy $\Delta E = (1-\alpha)E$. Approximating the mean rate of change of energy by

$$\frac{dE}{dt} \approx -\frac{\Delta E}{\Delta t} = -(1-\alpha)\sigma\rho(2/m_n)^{\frac{1}{2}}E^{\frac{3}{2}}$$

gives the time to 'cool' to $E_N = 0.1$ eV:

$$\text{time} = \frac{1}{(1-\alpha)\sigma\rho c}\sqrt{\left(\frac{m_n c^2}{2}\right)}\int_{E_N}^{E_0}\frac{dE}{E^{\frac{3}{2}}} \approx \frac{1}{(1-\alpha)\sigma\rho c}\left(\frac{2m_n c^2}{E_N}\right)^{\frac{1}{2}} = 8 \times 10^{-5}\text{s}.$$

9.6 From the text $l = 3$ cm, $l/v = 1.5 \times 10^{-9}$s, $\nu = 2.5$ and $t_p = 9 \times 10^{-9}$ s. Substituting the form $\rho(r, t) = f(r)e^{\lambda t}$ into the equation yields

$$\lambda f(r) = \frac{(\nu - 1)}{t_p}f(r) + \frac{D}{r}\frac{d^2}{dr^2}(rf(r)),$$

which has solutions of the form $f(r) = (1/r)\sin(kr)$ provided

$$\lambda = \frac{(\nu - 1)}{t_p} - Dk^2.$$

To satisfy the boundary condition,

$$k(R + 0.71l) = n\pi, \quad n = 1, 2, \ldots$$

To avoid an exponential increase in density $\lambda \leqslant 0$ for all n and therefore

$$(R + 0.71l)^2 \leqslant \frac{t_p}{(\nu - 1)}D\pi^2,$$

i.e.

$$R \leqslant \left[\pi\sqrt{\left[\frac{t_p v}{3(\nu - 1)l}\right]} - 0.71\right]l,$$

and the critical radius in this approximation is 8.8 cm.

9.7(*a*) Suppose that the probability of a neutron induced fission to result in a fragment which produces a delayed neutron is ν_d, and that the number of such fragments at any time is $N(t)$, then

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$$\frac{\mathrm{d}N}{\mathrm{d}t} = -\frac{N(t)}{\tau_\beta} + \frac{q\nu_\mathrm{d}}{t_\mathrm{p}} n(t).$$

This equation has the solution

$$N(t) = \frac{q\nu_\mathrm{d}}{t_\mathrm{p}} \int_{-\infty}^{t} \mathrm{e}^{-(t-t')/\tau_\beta} n(t')\,\mathrm{d}t'.$$

Including delayed neutrons in equation (9.1) gives

$$\frac{\mathrm{d}n}{\mathrm{d}t} = \frac{(\nu_\mathrm{q} - 1)}{t_\mathrm{p}} n(t) + \frac{N(t)}{\tau_\beta},$$

which is the equation quoted.

(*b*) $$\lambda = \frac{(\nu q - 1)}{t_\mathrm{p}} + \frac{\nu_\mathrm{d} q}{t_\mathrm{p}(1 + \lambda\tau_\beta)}$$

or

$$(\lambda\tau_\beta)^2 + (\lambda\tau_\beta)[1 - (\nu q - 1)\tau_\beta/t_\mathrm{p}] - (\tau_\beta/t_\mathrm{p})[(\nu + \nu_\mathrm{d})q - 1] = 0.$$

(*c*) Clearly $\lambda = (\nu q - 1)/t_\mathrm{p} = 1$ s when $\nu_\mathrm{d} = 0$.

(*d*) Substituting the given values in the quadratic equation gives

$$(\lambda\tau_\beta)^2 + 781(\lambda\tau_\beta) - 10 = 0.$$

The positive solution for $(\lambda\tau_\beta)$ corresponds to an exponentially increasing $n(t)$, with time scale $1/\lambda = 13$ min.

9.8(*a*) Take the energy release per fission to be (178 + 15) MeV (Table 9.1). The number of fissions in time $\mathrm{d}t'$ is then $P\mathrm{d}t'/(193\ \mathrm{MeV})$. For steady power output, the rate of release of ionising energy at time t after shutdown at time $t = 0$ is, from equation (9.2),

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{P}{(193\ \mathrm{MeV})} \int_{-T}^{0} 2.66 \left(\frac{1\ \mathrm{s}}{t - t'}\right)^{1.2} \mathrm{d}t'\ \mathrm{MeV\ s^{-1}}$$

$$= 0.07P \left[\left(\frac{1\ \mathrm{s}}{t}\right)^{0.2} - \left(\frac{1\ \mathrm{s}}{t + T}\right)^{0.2}\right].$$

(*b*) 7.8 MW, 4.3 MW, 0.8 MW.

9.9 The mean free path l of a fission neutron is given by $l = 1/\rho_\mathrm{nuc}\sigma$, where ρ_nuc is the number density of ^{239}Pu nuclei, and σ is the total neutron cross-section. If R_c is the radius of the critical mass M at atmospheric pressure,

$$\rho_\mathrm{nuc} = (M/m_\mathrm{nuc})/(4\pi/3)R_\mathrm{c}^3.$$

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Hence if $R_c = Kl$, where K is a constant,

$$R_c = (4\pi/3)KR_c^3 m_{nuc}/\sigma M.$$

A mass m and its critical radius r_c are related by a similar equation, so that

$$\left(\frac{r_c}{R_c}\right) = \left(\frac{m}{M}\right)^{1/2}.$$

If r is the radius of m at atmospheric pressure,

$$\left(\frac{R_c}{r}\right) = \left(\frac{M}{m}\right)^{1/3}.$$

Eliminating R_c,

$$\left(\frac{r_c}{r}\right) = \left(\frac{m}{M}\right)^{1/6}.$$

If $m = 0.8\,M$, $r_c/r = 0.96$.

Chapter 10

10.1(*a*) 6.5×10^{14} m^{-2} s^{-1}.

(*b*) Neutrino mean free path $l = 1/\sigma\rho_{nuc}$ and $\rho_{nuc} \leqslant (1\ \text{Å})^{-3} = 10^{30}$ m^{-3}. Hence $l \sim 10^{15}$ km $\sim 10^{11}$ Earth diameters.

10.2 Thermal velocities in the gaseous state of hydrogen exceed the escape velocity in the Earth's gravitational field. Only hydrogen that is chemically bound remains.

10.3 4.86 MeV.

10.4(*a*) $\rho_p = 3.4 \times 10^{31}$ m^{-3}, $\quad \tau^2 e^{-\tau} = 2.5 \times 10^{-4}$,

$$\lambda_{pp} = \overline{v\sigma}_{pp} = 1.4 \times 10^{-49}\ \text{m}^3\ \text{s}^{-1}.$$

The p–p reaction rate $= \frac{1}{2}\lambda_{pp}\rho_p^2 = 8.1 \times 10^{13}$ m^{-3} s^{-1}.
Each p–p reaction produces 13.1 MeV and hence the contribution to $\varepsilon = 170$ W m^{-3}.

(*b*) The p-^{12}C reaction rate $= \rho_p\rho_c\lambda_{pc}$, and hence the mean time for one carbon nucleus to react is $1/\rho_p\lambda_{pc} \approx 10^6$ years.

10.5(*a*) The reaction rate per unit volume is $\rho_d^2\overline{\sigma v}$ and each reaction reduces ρ_d by one. Since this is the dominant reaction (Fig. 10.4)

$$\frac{d\rho_d}{dt} = -\rho_d^2\overline{\sigma v}.$$

(*b*) This equation can be integrated to give

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$$\overline{\sigma v}t = \frac{1}{\rho_d} - \frac{1}{\rho_0},$$

and hence the proportion 'burned' is

$$\frac{\rho_0 - \rho_d}{\rho_d} = \frac{\rho_0 t_c \overline{\sigma v}}{1 + \rho_0 t_c \overline{\sigma v}}.$$

(*c*) 10^{20} m^{-3} s.

10.6(*a*) $\dfrac{dV}{dr} = 0$ at $r = r_0$ where $r_0^3 = \dfrac{(e^2/4\pi\varepsilon_0)}{2K}$.

(*b*) A Taylor expansion of $V(r)$ about $r = r_0$ gives

$$V(r) \approx V(r_0) + \frac{1}{2} M\omega^2 (r - r_0)^2,$$

where $M\omega^2 = 3(e^2/4\pi\varepsilon_0)/r_0^3$.
Classically a particle of mass M in this potential undergoes simple harmonic motion about $r = r_0$ at angular frequency ω.

(*c*) The lowest quantum state in this potential has angular momentum $L = 0$ and energy

$$E_0 = V(r_0) + \frac{1}{2}\hbar\omega.$$

Taking $r_0 = 500$ fm gives

$$\omega = 1.66 \times 10^{18}\ \text{s}^{-1},\ E_0 = 4.87\ \text{keV}.$$

(*d*) Neglecting the Kr^2 term in the potential, we may estimate the tunnelling probability to be $\exp[-G(E_0)]$ where

$$G(E_0) = \frac{\pi}{\hbar c}\frac{e^2}{4\pi\varepsilon_0}\sqrt{\frac{2Mc^2}{E_0}}G(r_s/r_c)$$

(see equation (6.15)).
Since $r_s \sim 3$ fm and $r_c \sim 300$ fm, $G(r_s/r_c) \approx 1$ and $G(E_0) \approx 15.6$.
Tunnelling probability $= \exp[-G(E_0)] = 1.68 \times 10^{-7}$.

(*e*) In a semi-classical picture, the deuteron and triton approach each other at intervals of $2\pi/\omega = 3.8 \times 10^{-18}$ s.
The mean number of approaches $\bar{n}$ before tunnelling (and presumably fusion) takes place is (cf. §2.3)

$$\bar{n} = \exp[G(E)].$$

The corresponding time is $(2\pi/\omega)\bar{n} = 2.3 \times 10^{-11}$ s.

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Chapter 11

11.1 All states are occupied up to $k = k_F$ (equation (B.5)). Using equation (B.4) the mean value of k is

$$\bar{k} = \int_0^{k_F} k^3\,\mathrm{d}k \Big/ \int_0^{k_F} k^2\,\mathrm{d}k = (3/4)k_F.$$

For extreme relativistic electrons, $E \approx pc = \hbar ck$; hence the mean energy is $\hbar c\bar{k} = (3/4)\,\hbar ck_F$.

11.3 The reaction is endothermic and requires an energy input of 0.78 MeV $= Q$. The reaction will proceed if $\varepsilon_F > Q$. (At $T = 0$ K the proton Fermi energy is less than the electron Fermi energy E_e^F by a factor $\sim m_e/m_p$.) Using the non-relativistic formula $\varepsilon_F = \hbar^2 k_F^2/2m_e$ for a rough estimate, and equation (B.5), the number density of electrons ρ_e must satisfy

$$\rho_e \geqslant \frac{1}{3\pi^2}\left(\frac{2m_e c^2 Q}{(\hbar c)^2}\right)^{\frac{3}{2}} = 3 \times 10^{-9}\ \mathrm{fm}^{-3}.$$

The corresponding hydrogen density is $\sim 5 \times 10^9$ kg m^{-3}.

11.4 At low temperature, only particles at the top of the Fermi distributions can take part in the reactions

$$\mathrm{n} \rightarrow \mathrm{p} + \mathrm{e}^- + \bar{\nu}_e, \qquad \mathrm{p} + \mathrm{e}^- \rightarrow \mathrm{n} + \nu_e.$$

In thermal equilibrium

$$\varepsilon_F(\mathrm{n}) = \varepsilon_F(\mathrm{p}) + \varepsilon_F(\mathrm{e}),$$

where

$$\varepsilon_F(\mathrm{n}) \approx m_n c^2 + \frac{\hbar^2}{2m_n}k_F(\mathrm{n})^2, \qquad \varepsilon_F(\mathrm{p}) \approx m_p c^2 + \frac{\hbar^2}{2m_n}k_F(\mathrm{p})^2,$$

and $\varepsilon_F(\mathrm{e}) \approx \hbar ck_F(\mathrm{e})$, since we expect the electrons to be highly relativistic.

For electrical neutrality, the number density of electrons must equal the number density of protons. Hence

$$k_F(\mathrm{e})^3 = k_F(\mathrm{p})^3 = 3\pi^2\rho_p, \qquad k_F(\mathrm{n})^3 = 3\pi^2\rho_n.$$

The equilibrium condition becomes

$$m_n c^2 + \frac{(\hbar c)^2}{2m_n c^2}[3\pi^2\rho_n]^{2/3} = m_p c^2 + \frac{(\hbar c)^2}{2m_p c^2}[3\pi^2\rho_p]^{2/3} + \hbar c[3\pi^2\rho_p]^{1/3}.$$

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Taking $\rho_n = 0.17$ nucleons fm^{-3} and solving for ρ_p gives $\rho_p = \rho_e = 8.44 \times 10^{-4}$ fm^{-3}, and $\rho_p/\rho_n = 5.0 \times 10^{-3}$.

11.5 If $E > E_0 = 8$ MeV, $k_B T = 0.5$ MeV, then $E/k_B T > 16$. Hence

$$n \approx \frac{1}{\pi^2(\hbar c)^3}\int_{E_0}^{\infty} E^2 e^{-E/k_B T}\,dE \approx \frac{1}{\pi^2(\hbar c)^3} E_0^2 k_B T e^{-E_0/k_B T},$$

giving $n \approx 5 \times 10^{31}$ m^{-3}.

11.6(*a*) From §10.3,

$$\overline{v\sigma} = \left(\frac{2}{\pi}\right)^{\frac{1}{2}}\left(\frac{m_\alpha}{2k_B T}\right)^{\frac{3}{2}}\int_0^{\infty} v\sigma e^{-E/k_B T} v^2\,dv$$

where $E = \frac{1}{2}(m_\alpha/2)v^2$, i.e.

$$\overline{v\sigma} = \left(\frac{1}{m_\alpha k_B T}\right)^{\frac{3}{2}} 8\pi^{\frac{1}{2}}\hbar^2\Gamma^2 \int \frac{e^{-E/k_B T}}{(E-E_0)^2 + \Gamma^2/4}\,dE.$$

Integrating over the narrow resonance peak gives the result.

(*b*) In the plasma, the rate of production of ^{8}Be is $\frac{1}{2}\rho_\alpha^2\overline{v\sigma}$ (equation (10.6)). The rate of decay per unit volume is $\rho_{Be}/\tau = \rho_{Be}\Gamma/\hbar$. In equilibrium these rates are equal.

(*c*) 2.3×10^{-10}.

Chapter 12

12.1 $fT_{\frac{1}{2}} = 4760$ s. For a free neutron $fT_{\frac{1}{2}} = 1015$ s, from §12.8. In the simple shell model the 1s neutron and proton spatial wave-functions would be the same if Coulomb distortions were neglected, and the spin states similar to those of a free neutron and free proton. Thus the predicted $fT_{\frac{1}{2}}$ value would be the same. However, since $Z = 15$, Coulomb distortions are not insignificant. Also shell model predictions for the Gamow–Teller matrix elements, like those for the similar magnetic moment matrix elements (§5.6), are not accurate.

12.2 Note there is no Coulomb factor in the matrix element.

$E_e = 0.71$ MeV; $\quad \sigma = 3.3 \times 10^{-20}$ b.

12.3 ^{11}Be decay: $|\mathbf{R}_{f0}| = 0.7$ fm, a nuclear size.
Atomic decay: $|\mathbf{R}_{f0}| = 0.4$ Å, an atomic size.

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Chapter 13

13.1 Replacing $S_0(E_e)$ in equation (12.5) by $(E_0 - E_e)^2 E_e^2$, the mean electron energy is clearly $E_0/2$ by symmetry. The mean life is inversely proportional to

$$f(Z_d, E_0) \approx \left(\frac{1}{m_e c^2}\right)^5 \int_0^{E_0} (E_0 - E_e)^2 E_e^2 \, dE_e = \frac{1}{30}\left(\frac{E_0}{m_e c^2}\right)^5.$$

The proportion of decays within ΔE of the end-point is

$$\frac{1}{f(Z_d, E_0)}\left(\frac{1}{m_e c^2}\right)^5 \int_{E_0-\Delta E}^{E_0} (E_0 - E_e)^2 E_e^2 \, dE_e \approx \frac{1}{f}\frac{E_0^2 \Delta E^3}{3(m_e c^2)^5}.$$

Substituting for f, the result follows.

13.2 By momentum conservation

$$p_\nu^2 c^2 = p_{Li}^2 c^2 = 2 \times 6536 \text{ MeV} \times (55.9 \pm 1.0) \text{ eV}$$
$$= (0.7307 \pm 0.0131) \text{ MeV}^2,$$

and by energy conservation

$$m_\nu^2 c^4 = [(0.862)^2 - 0.7307 \pm 0.0131] \text{ MeV},$$

giving $0 \leq m_\nu c^2 \leqslant 160$ keV.

13.3(*a*) From equation (A.4), $l = 1/(\rho_{nuc}\sigma)$.

(*b*) Number of steps $= (R/l)^2$.
Time for each step $= l/c$.
Total time $= R^2/(lc)$.

13.4 Measuring $k_B T$ in MeV,

$$a_\nu = \frac{3 \times 7}{8}\frac{\pi^2 c}{60(\hbar c)^3} = 1.6 \times 10^{46} \text{ MeV}^{-3} \text{ s}^{-1} \text{ m}^{-2}.$$

Since $\int_0^\infty e^{-t/\tau} dt = \tau$, the total energy loss is

$\sim a_\nu (k_B T_0)^4 4\pi R^2 \tau = 3 \times 10^{58}$ MeV.

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Chapter 14

14.1 $$R_{\mathrm{p}}(T) = m_{\mathrm{p}}c^2 \int_0^{T/m_{\mathrm{p}}c^2} \mathrm{d}u/F(u).$$

$$\begin{aligned} R_M(T_M) &= (Mc^2/z^2) \int_0^{T_M/Mc^2} \mathrm{d}u/F(u) \\ &= (M/z^2 m_{\mathrm{p}}) m_{\mathrm{p}}c^2 \int_0^{T_M/Mc^2} \mathrm{d}u/F(u) \\ &= (M/z^2 m_{\mathrm{p}}) R_{\mathrm{p}}(m_{\mathrm{p}} T_M/M). \end{aligned}$$

14.2 $T = [Mc^2/\sqrt{(1 - v^2/c^2)}] - Mc^2$

gives

$$\frac{v^2}{c^2} = 1 - \frac{1}{(1+u)^2}, \quad \text{where } u = T/Mc^2.$$

Taking a constant $\bar{L}$, the integration is straightforward.

14.3 α-particle range ≈ 20 μm; electron range ~ 1 cm, a much greater distance.

14.4 $$\frac{\mathrm{d}}{\mathrm{d}x}(\tfrac{1}{2}Mv^2) = Mv\frac{\mathrm{d}v}{\mathrm{d}x} = M\frac{dx}{\mathrm{d}t}\frac{\mathrm{d}v}{\mathrm{d}x} = M\frac{\mathrm{d}v}{\mathrm{d}t}.$$

From equation (14.4), in the approximation $L = \bar{L}$,

$$\frac{\mathrm{d}v}{\mathrm{d}x} = \frac{\text{constant}}{v^3} \quad \text{and} \quad \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\text{constant}}{v^2}.$$

Hence

$$\text{(time to stop)/(range)} = \int_0^{v_0} v^2\,\mathrm{d}v \Big/ \int_0^{v_0} v^3\,\mathrm{d}v = 4/3v_0.$$

For the α-particle of question (13.3), time $\approx 1.7 \times 10^{-12}$ s.

14.5 The kinetic energy of ionising particles is 0.76 MeV. From the end of §13.1, the number of ion pairs produced is

~ 0.7 MeV/35 eV $\sim 2 \times 10^4$.

The proton will have the longest range. The proton energy is

$(m_{\mathrm{t}}/(m_{\mathrm{p}} + m_{\mathrm{t}})) \times 0.76$ MeV $= 0.57$ MeV.

To estimate its range take $I = 24$ eV (Problem 13.3) and estimate $\bar{L}(\sim 2.5)$, which gives a range ~ 0.5 cm.

14.6 From Fig. 13.4, at 50 keV the photon cross-section for lead is predominantly due to absorption and the linear attenuation coefficient

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$\mu \approx 93.2\ \text{cm}^{-1}$.
For the thickness x to be such that $e^{-\mu x} = 10^{-3}$, we require $x = 0.76$ mm.

Chapter 15

15.1 Since the mean life ^{137}Cs is 44 yr, we may neglect its radioactive depletion over 1 yr, and estimate the activity as

$$^{137}\text{Cs activity} = \frac{0.059 \times (3 \times 10^9\ \text{W}) \times (1\ \text{yr})}{(200\ \text{MeV}) \times (44\ \text{yr})} \approx 1.3 \times 10^{17}\ \text{Bq}.$$

If 13% of this activity were spread over 10^6 km^2, the activity per square metre $\approx 2 \times 10^4$ Bq. (Each ^{235}U fission releases ~ 200 MeV energy. See §9.3.)

15.2 Radon will be produced from the radium decay at a rate $R = 1$ Ci, but itself decays. Suppose $n(t)$ is the number of radon nuclei after time t. Then

$$\frac{dn}{dt} = R - \frac{n}{\tau}.$$

With the initial condition $n(0) = 0$, the solution of this equation is

$$\frac{n(t)}{\tau} = R(1 - e^{-t/\tau}) = (1 - e^{-t/\tau})\ \text{Ci}.$$

But $n(t)/\tau$ is the radon activity at time t.

After one month, the radon activity will be approximately constant at $R = 1$ Ci. Similarly the other decay products up to ^{210}Pb with their even shorter decay times will be in quasi-equilibrium, each with activity R, and the total activity will be 6 Ci.

15.3 4 GeV muons are highly relativistic. Hence in equation (14.4) we take $u \sim c$. Also $(Z/A) = 0.5$, $L = 14$, $\rho \approx 1\ \text{g cm}^{-3} = 10^3\ \text{kg m}^{-3}$. Then

$$-\frac{dE}{dx} = 0.307 \times 0.5 \times 14\ \text{MeV cm}^{-1} \sim 2\ \text{MeV cm}^{-1}.$$

Thus a muon passing through the body loses only a small fraction of its energy to ionisation, and

$$\frac{dE}{dt} = \frac{dE}{dx}\frac{dx}{dt} \sim c\frac{dE}{dx} \sim -10^{-2}\ \text{J s}^{-1}.$$

The number density n of muons in the body is

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$$n \sim \frac{\text{flux}}{c} \sim \frac{150}{3 \times 10^8}\ \text{m}^{-3} \sim 5 \times 10^{-7}\ \text{m}^{-3}.$$

Hence in 1 s, the received dose is

$$\sim \frac{(5 \times 10^{-7}) \times 10^{-2}}{10^3\ \text{kg}}\ \text{J} \sim 5 \times 10^{-12}\ \text{Sv}.$$

This suggests an annual dose of ~ 0.16 mSv from this process.

15.4 Taking a total body weight of 70 kg, the activity due to ^{14}C is ~ 3200 Bq. Assuming all the electron energy is deposited in the body (cf. Table 14.1), the dose per second is

$$3200 \times 0.052 \times 1.6 \times 10^{-13}\ \text{J}/70\ \text{kg} = 3.8 \times 10^{-13}\ \text{Sv}.$$

The annual dose is 12 μSv.

15.5 From the given data, a 70 kg body contains 2.47×10^{20} ^{40}K nuclei, which will yield 1.33×10^{11} decays per year. If we assume that all an electron energy, and 10% of a photon energy, is deposited in the body, the average energy per decay is

$$\sim [(0.89 \times 0.66) + (0.11 \times 0.145)]\ \text{MeV}$$
$$\sim 0.6\ \text{MeV}.$$

Annual dose $\sim 1.33 \times 10^{11} \times 0.6 \times 1.6 \times 10^{-13}/(70\ \text{kg})$
~ 0.18 mSv.

Appendices

B.1(b) The centre-of-mass coordinate $\mathbf{R} = (m_1\mathbf{r}_1 + m_2\mathbf{r}_2)/(m_1 + m_2)$, and the relative coordinate $\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$. The two particle wave-functions $\exp(\mathrm{i}\mathbf{k}_1 \cdot \mathbf{r}_1)\exp(\mathrm{i}\mathbf{k}_2 \cdot \mathbf{r}_2)$ and $\exp(\mathrm{i}\mathbf{k} \cdot \mathbf{R})\exp(\mathrm{i}\mathbf{k} \cdot \mathbf{r})$ must be identical. The result follows on equating coefficients for $\mathbf{r}_1$ and $\mathbf{r}_2$. The Jacobian of the transformation is unity. If the particles are identical, only one hemisphere of the angular integration of the $\mathbf{k}$-vector gives distinct states, since $\mathbf{k}$ and $-\mathbf{k}$ are equivalent.

C.2 The wave-functions of the state are linear combinations of spatial functions of fixed l, each of which has parity $(-1)^l$. The effect of the parity operator on the internal states $|+\tfrac{1}{2}\rangle$, $|-\tfrac{1}{2}\rangle$ of spin $\tfrac{1}{2}$ fermions (e.g. electrons, protons) is in fact a matter of convention, and they are taken to have positive parity. It is usually the relative parity of two states which is significant.

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