Math. Proc. Camb. Phil. Soc. (1990), **107**, 273
Printed in Great Britain

A note on Morita context functors

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(*Received* 8 *April* 1989; *revised* 1 *August* 1989)

It is claimed in [3], proposition 4 that if a Morita context (R, V, W, S) is such that $WVW = W$ and S/WV is flat as a right S-module, then $Vw = 0$, $w \in W$, implies $w = 0$. The proof given for this uses proposition 5 of Azumaya [1] in which rings have unity and so is only applicable when the ring S has a unity. Thus proposition 4 in [3] should be amended so that the additional hypothesis that S has a unity is imposed for statement (4) to be equivalent to the first three statements. It is the purpose of this note to give an example to show that the claim in [3] is false in general.

Let G be a group with unity e, and A a ring with unity 1. Then A is said to be G-graded if $A = \bigoplus_{g \in G} A_g$ (as additive groups) such that $A_g A_h \leqslant A_{gh}$ for all $g, h \in G$.

In [4] we introduce the ring $A^2 = A \otimes_{A_e} A$ with multiplication defined as follows:

$$(a \otimes b)(x \otimes y) = a(bx)_e \otimes y = a \otimes (bx)_e y.$$

The definition of multiplication then extends to finite sums as usual.

Since A_e is a unitary subring of A, A is a left and right unitary A_e-module. The left and right A^2-module structure of A are given by

$$(x \otimes y)a = x(ya)_e \quad \text{and} \quad a(x \otimes y) = (ax)_e y.$$

It is clear that A is both an $A^2 - A_e$-bimodule and $A_e - A^2$-bimodule, and

$$\{A_e, {}_{A_e}A_{A^2}, {}_{A^2}A_{A_e}, A^2, (\ ,\ ), [\ ,\ ]\}$$

is a Morita context, where $(\ ,\ ): A \otimes_{A^2} A \to A_e$ is given by $(x, y) = (xy)_e$; and

$$[\ ,\ ]: A \otimes_{A_e} A \to A^2$$

is given by $[x, y] = x \otimes y$.

Obviously $[A, A] = A^2$ and $A^2 A = A$ so the cited hypotheses from [3], proposition 4 hold and the assertion there would imply that $(A, a) = 0$, $a \in A$, implies $a = 0$, from which it would follow that every group grading on any associative ring is non-degenerate, i.e. if $0 \neq a_g \in A_g$ then $A_{g^{-1}} a_g \neq 0$ (see [2], lemma 2·5(1)).

This result is false because we have a counter-example as follows. Let G be a group with presentation $\langle a \mid a^2 = e \rangle$, and $A = \mathbb{C}x/(x^2) = \mathbb{C} + \mathbb{C}\bar{x}$, where x is an indeterminate over the complex number field $\mathbb{C}$. Let $A_e = \mathbb{C}, A_a = \mathbb{C}\bar{x}$; then $A = A_e \oplus A_a$ is G-graded.

Obviously $0 \neq \bar{x} \in A_a$, but $\bar{x} A_{a^{-1}} = 0$. It follows that the grading is not non-degenerate.

The author is indebted to the referee of the paper for some useful comments.

https://doi.org/10.1017/S0305004100068547 Published online by Cambridge University Press

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https://doi.org/10.1017/S0305004100068547 Published online by Cambridge University Press