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Dynamic Factors and Asset Pricing

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Abstract

This study develops an econometric model that incorporates features of price dynamics across assets as well as through time. With the dynamic factors extracted via the Kalman filter, we formulate an asset pricing model, termed the dynamic factor pricing model (DFPM). We then conduct asset pricing tests in the in-sample and out-of-sample contexts. Our analyses show that the ex ante factors are a key component in asset pricing and forecasting. By using the ex ante factors, the DFPM improves upon the explanatory and predictive power of other competing models, including unconditional and conditional versions of the Fama and French (1993) 3-factor model. In particular, the DFPM can explain and better forecast the momentum portfolio returns, which are mostly missed by alternative models.

I. Introduction

Since the preeminence of the capital asset pricing model (CAPM) was challenged by many studies, conditional versions of the CAPM with time-varying parameters or the arbitrage pricing theory (APT) were proposed as alternative models.[1] Given the fact that the explanatory power of these models was not satisfactory, yet another series of alternative models was proposed, and two of the most prominent are Fama and French's (FF) (1993) 3-factor model and Carhart's (1997) 4-factor model. Since the publication of these studies, the 2 models have

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[1]For example, see Ross (1976), Harvey (1989), Ferson and Harvey (1991), Ferson and Korajczyk (1995), Jagannathan and Wang (1996), and Brennan, Wang, and Xia (2004).

been the most popular benchmarks in the modern empirical asset pricing literature. While these studies have contributed considerably to our understanding of asset pricing, we also recognize that the 2 models are essentially static and thus lack dynamic features. In this sense, we believe that it is meaningful to extend these models by considering a dynamic-factor representation.

In this study we propose a new approach to identifying factors that can incorporate features of price dynamics across assets as well as through time. We start with the same data sets as the FF 3-factor (FF3) model uses, developing an econometric method that can extract more informative risk factors. In the spirit of Stock and Watson (1989), (1991), we employ the Kalman filter to estimate unobservable state variables (i.e., 3 ex ante factors and 3 ex post factors). After extracting the latent factors via the Kalman filter, we formulate an asset pricing model. Because the Kalman filter repeats a sequence of forecasting and updating as the information set develops, we specifically refer to this model as the dynamic factor pricing model (DFPM). We then compare the performance of the DFPM with that of other existing asset pricing models using the same test assets. Our empirical analyses show that the dynamic-factor approach outperforms other existing models in the in-sample and out-of-sample contexts. These results are by and large consistent with the findings reported by Ferson and Harvey (1999) and Simin (2008).

Ferson and Harvey (1999) test the empirical performance of the FF3 model, concluding that the FF3 is rejected as a pricing model. Moreover, Simin (2008) shows that existing conditional and unconditional asset pricing models produce very poor 1-step-ahead forecasts compared to a simple benchmark. His result is intriguing because it implies that existing asset pricing models provide little guidance to investment practitioners, to whom acquiring more accurate forecasts in future asset returns is of primary concern. Keeping these studies in mind, we ensure that our model has both cross-sectional and time-series features that distinguish our model from the alternative models. Cross-sectionally, the factors are identified through a prespecified factor-loading matrix, where parameter restrictions are imposed across assets. Through time, the dynamic factors are formed as linear combinations of both contemporaneous and past returns so that they can capture the entire history of the information set. These features of our dynamic factors appear to play an important role in enhancing the performance. In particular, our finding suggests that the ex ante size factor is a good proxy for information uncertainty, effectively capturing the momentum effect missed by the FF3.

Using the 25 portfolios, we first conduct asset pricing tests in the in-sample context through the 2-pass regression procedure along the lines of Cochrane (2005) and Brennan et al. (2004). The result shows that with the residual explanatory power of the ex ante factors, the DFPM can better explain returns than the FF3 model. To alleviate the possible data-snooping biases described in Lo and MacKinlay (1990), we then extend the scope of test assets by adding 10 momentum portfolios to the above 25 portfolios. The 2-pass procedure with the augmented 35 portfolios exhibits that the DFPM further improves upon the performance of both unconditional and conditional versions of the FF3 model. In particular, the cross-sectional regressions show that, owing to the role of the ex ante size factor, the DFPM explains the momentum portfolio returns well: None of the

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pricing errors is statistically significant, and none has a pricing error greater than 0.20% per month. This presents a sharp contrast to the large pricing errors and their high significance levels in the FF3 model. Considering that the FF3 model or its conditional versions cannot explain the returns on portfolios formed on short-term past returns (FF (1996)), our results are quite encouraging.

Following Simin (2008), we also perform out-of-sample tests for several competing models, presenting pair-wise comparisons of the accuracy in 1-step-ahead forecasts. The out-of-sample tests reassure that by using the ex ante factors, the DFPM provides more accurate forecasts than the naive benchmark (9.35%), the FF3 model, the Carhart (1997) 4-factor model, and the conditional versions of the FF3 model. The better predictive power of the DFPM over the other competing models occurs mainly in the extreme portfolios such as the loser/winner deciles and the small stock portfolios, which are exactly the ones for which traditional asset pricing models have difficulties in pricing or forecasting.

Our motivation for using the Kalman filter is to utilize an econometric technique that helps us extract more efficient state variables and demonstrate that the FF3s may miss some important intertemporal information that is vital to explain the momentum effect. While the momentum effect remains a puzzle for the FF3s, it can be explained well by our ex ante dynamic factors. We try to figure out what causes the better explanatory power of the dynamic factors. One reason may stem from the fact that our dynamic factors are weakly correlated with each other but highly serially correlated.[2] In contrast, the FF3s are correlated with each other but not serially correlated. Given that the FF3s, like any factors, are subject to some noise in their measurement, our study shows that the Kalman filter provides us with a recursive process in extracting more efficient unobserved factors.

The remainder of this paper is organized as follows. Section II presents the model. Section III discusses data, estimation of model parameters, descriptive statistics of the factors, and their relations to other variables. Section IV formulates the null and alternative hypotheses associated with the models. In Section V we conduct in-sample performance tests of the DFPM and compare the results with those of other alternative models. In Section VI we perform out-of-sample tests, presenting pair-wise comparisons of the accuracy in 1-step-ahead forecasts. Section VII concludes.

II. The Model

A. Model Specification

Stock and Watson (1989), (1991) form a composite index of coincident economic indicators based on the notion that comovements in observed macroeconomic time series have a common component that can be captured by a single time-varying latent variable. Along the lines of Stock and Watson (1989), (1991),

[2]However, the (absolute) levels of serial correlation in our dynamic factors are much lower than the value (95% or higher), which may induce the spurious regression biases described in Ferson, Sarkissian, and Simin (2003). Therefore, the high serial correlation in the dynamic factors does not cause any concern for such spurious results.

we develop a multifactor model of stock returns, with each of the factors being identified within a prespecified factor structure.

Our model may be viewed as a conditional APT with time-varying expected factor premiums but with constant factor variances. Stambaugh (1983), Gibbons and Ferson (1985), and Connor and Korajczyk (1989) follow a similar approach of time-varying factor premiums and constant betas. A different approach is considered by Harvey and Kirby (1996), Ferson and Harvey (1999), and others in order to express the betas as linear functions of conditioning variables. Ferson and Harvey (1991), Evans (1994), and Ferson and Korajczyk (1995) argue that predictability is primarily driven by changes in risk premiums through time, whereas the impact of variation in conditional betas is marginal. Moreover, researchers have raised a concern that conditional beta models are subject to model misspecification (Ghysels (1998)) and variable selection biases (Wang (2004)). Our model can largely circumvent these empirical difficulties, because a model specification test is conducted to ensure that the estimated factors are unbiased.

Following FF (1993), we begin our discussion on the model specification by looking at the returns of the 6 portfolios formed on size and book-to-market equity (BTM). Given the fact that most commonly used risk factors in the finance literature are the FF3s (MKT, SMB, and HML)[3] and these factors originate from the 6 size- and BTM-sorted portfolios, we conjecture that presumably useful latent variables may be extracted from the 6 portfolios formed in this way. Our goal in this study is to develop an econometric method that enables us to extract risk factors more efficiently using the same data sets that the FF3 model employs. This is why we use the 6 size- and BTM-sorted portfolios as our starting point.

We notate the 6 size- and BTM-sorted portfolios as SL, SM, SH, BL, BM, and BH.[4] Now let $\underline{R}_t \equiv [\underline{R}_{\mathrm{SL},t}\ \underline{R}_{\mathrm{SM},t}\ \underline{R}_{\mathrm{SH},t}\ \underline{R}_{\mathrm{BL},t}\ \underline{R}_{\mathrm{BM},t}\ \underline{R}_{\mathrm{BH},t}]'$ denote a vector of demeaned excess returns on the 6 portfolios at month t.[5] Also let $D_t \equiv [D_{\mathrm{MKT},t}\ D_{\mathrm{SIZE},t}\ D_{\mathrm{BTM},t}]'$ denote a vector of 0-mean unobserved state variables (termed as "dynamic" factors: specifically, market (MKT), size (SIZE), and BTM factors, respectively) at month t. The model is then specified as follows: The 6 observation or measurement equations are given (in the form of deviations from the means) by

$$(1)\qquad \begin{bmatrix} \underline{R}_{\mathrm{SL},t} \\ \underline{R}_{\mathrm{SM},t} \\ \underline{R}_{\mathrm{SH},t} \\ \underline{R}_{\mathrm{BL},t} \\ \underline{R}_{\mathrm{BM},t} \\ \underline{R}_{\mathrm{BH},t} \end{bmatrix} = \begin{bmatrix} \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \\ \beta_{4,1} & \beta_{4,2} & \beta_{4,3} \\ \beta_{5,1} & \beta_{5,2} & \beta_{5,3} \\ \beta_{6,1} & \beta_{6,2} & \beta_{6,3} \end{bmatrix} \begin{bmatrix} D_{\mathrm{MKT},t} \\ D_{\mathrm{SIZE},t} \\ D_{\mathrm{BTM},t} \end{bmatrix} + \begin{bmatrix} w_{\mathrm{SL},t} \\ w_{\mathrm{SM},t} \\ w_{\mathrm{SH},t} \\ w_{\mathrm{BL},t} \\ w_{\mathrm{BM},t} \\ w_{\mathrm{BH},t} \end{bmatrix},$$

[3]MKT is the excess return on the market portfolio, SMB is the return on a 0 net investment portfolio that is long in small firms and short in large firms, and HML is the return on a 0 net investment portfolio that is long in high BTM firms and short in low BTM firms.

[4]As usual, S and B denote "small" and "big" in firm size, respectively. Similarly, L, M, and H denote "low," "medium," and "high" in the BTM ratio, respectively.

[5]Throughout the paper, we denote $R_{i,t}$ as the excess return (over the 1-month T-bill rate) on asset i at time t, $\overline{R}_i$ as the sample mean of $R_{i,t}$, and $\underline{R}_{i,t} \equiv R_{i,t} - \overline{R}_i$ as the demeaned excess return.

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and the 3 transition or state equations are expressed as

$$(2)\qquad \begin{bmatrix} D_{\mathrm{MKT},t} \\ D_{\mathrm{SIZE},t} \\ D_{\mathrm{BTM},t} \end{bmatrix} = \begin{bmatrix} \phi_{\mathrm{MKT}} & 0 & 0 \\ 0 & \phi_{\mathrm{SIZE}} & 0 \\ 0 & 0 & \phi_{\mathrm{BTM}} \end{bmatrix} \begin{bmatrix} D_{\mathrm{MKT},t-1} \\ D_{\mathrm{SIZE},t-1} \\ D_{\mathrm{BTM},t-1} \end{bmatrix} + \begin{bmatrix} v_{\mathrm{MKT},t} \\ v_{\mathrm{SIZE},t} \\ v_{\mathrm{BTM},t} \end{bmatrix},$$

where the βs in equation (1) are the loadings on the state variables and the ϕs in equation (2) are the AR(1) coefficients of the factors. The state-space representation of the dynamics of D_t is also given in matrix form by the following system of equations:

$$(3)\qquad \underline{R}_t = BD_t + w_t,$$

$$(4)\qquad D_t = \Phi D_{t-1} + v_t,$$

where B is a prespecified constant factor-loading matrix whose columns have identifying restrictions on the MKT, SIZE, and BTM factors; w_t is a vector of idiosyncratic returns on the 6 portfolios; Φ is a constant matrix whose diagonal elements consist of the AR(1) coefficients that capture the time-series predictability of the 3 factors; and v_t is a vector of factor disturbances. Our model is *dynamic* in the sense that the state vector D_t follows a stationary vector autoregressive process of order 1 (VAR(1)). That is, the 3 equations in equation (2) or equation (4) describe the dynamics of the state variables, $D_t \equiv [D_{\mathrm{MKT},t}\ D_{\mathrm{SIZE},t}\ D_{\mathrm{BTM},t}]'$.

We also make the following conventional assumptions for equations (3) and (4): w_t and v_t follow joint normal distributions, with their covariance matrices given by

$$(5)\qquad \mathrm{E}[w_t w_\tau'] = \begin{cases} \Delta, & \text{for } t = \tau \\ 0, & \text{for } t \neq \tau \end{cases},$$

$$(6)\qquad \mathrm{E}[v_t v_\tau'] = \begin{cases} \Omega, & \text{for } t = \tau \\ 0, & \text{for } t \neq \tau \end{cases}, \quad \text{and}$$

$$(7)\qquad \mathrm{E}[w_t v_\tau'] = 0, \quad \text{for all } t \text{ and } \tau,$$

where $\Delta \equiv \mathrm{diag}[\sigma^2_{\mathrm{SL}}, \sigma^2_{\mathrm{SM}}, \sigma^2_{\mathrm{SH}}, \sigma^2_{\mathrm{BL}}, \sigma^2_{\mathrm{BM}}, \sigma^2_{\mathrm{BH}}]$ is a (6×6) diagonal covariance matrix of idiosyncratic risk for the 6 portfolios, and $\Omega \equiv \mathrm{diag}[\sigma^2_{\mathrm{MKT}}, \sigma^2_{\mathrm{SIZE}}, \sigma^2_{\mathrm{BTM}}]$ is a (3×3) diagonal covariance matrix of factor disturbances.

The model has both cross-sectional and time-series features that distinguish this model from the FF3 model or other statistical factor models such as Roll and Ross (1980), Lehmann and Modest (1988), and Connor and Korajczyk (1988). Cross-sectionally, the 3 factors are identified through a prespecified factor-loading matrix, in which parameter restrictions are imposed across assets. As a result, the 3 factors are not only readily interpretable, but they also preserve some desirable properties (e.g., mutually uncorrelated, minimum variances) of purely statistical factors. Through time, the 3 factors are formed as linear combinations of both contemporaneous and past returns, thereby incorporating the entire history of the information set.

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B. Factor Identification

The first factor, D_{MKT}, in equation (1) is identified from the 1st column of the coefficient matrix, B, with no restrictions on the 6 factor loadings. Using the firm characteristic notations employed above for the 6 portfolios, we can denote the parameters in the 1st column of B as follows:

$$\begin{aligned} \beta_{1,1} &= \beta_{\text{SL}}, \quad \beta_{2,1} = \beta_{\text{SM}}, \\ \beta_{3,1} &= \beta_{\text{SH}}, \quad \beta_{4,1} = \beta_{\text{BL}}, \\ \beta_{5,1} &= \beta_{\text{BM}}, \quad \beta_{6,1} = \beta_{\text{BH}}. \end{aligned} \tag{8}$$

Given that the 6 portfolios consist of market-wide assets, this factor (D_{MKT}) can capture price dynamics common across the 6 types of assets or portfolios formed on firm characteristics. That is why we term this factor as the *market* factor. In this sense, the market factor may be interpreted as a composite index.

The size factor, D_{SIZE}, is identified from the 2nd column of matrix B by imposing the restrictions that βs should be the same within each of the small- and big-stock groups. That is, the 1st set of parameter restrictions are

$$\begin{aligned} \beta_{1,2} &= \beta_{2,2} = \beta_{3,2} = \beta_{\text{S}}, \\ \beta_{4,2} &= \beta_{5,2} = \beta_{6,2} = \beta_{\text{B}}. \end{aligned} \tag{9}$$

These restrictions are reasonable, because assets within the same size group must have the same sensitivity, while assets belonging to different size groups should have different sensitivities to the size-related factor. The restriction ensures that if any systematic variation in returns exists beyond that captured by the market factor, this factor will pick up the size-related variation.

Similarly, the BTM factor, D_{BTM}, is identified from the 3rd column of B by imposing another set of restrictions that βs should be the same within each of the low, medium, and high BTM groups, that is,

$$\begin{aligned} \beta_{1,3} &= \beta_{4,3} = \beta_{\text{L}}, \\ \beta_{2,3} &= \beta_{5,3} = \beta_{\text{M}}, \quad \text{and} \\ \beta_{3,3} &= \beta_{6,3} = \beta_{\text{H}}. \end{aligned} \tag{10}$$

These restrictions ensure that D_{BTM} captures the variation that is related to the BTM effect beyond that captured by the market factor.

Finally, we normalize the variances so that each of the factors has a unit variance (i.e., $\sigma^2_{\text{MKT}} = \sigma^2_{\text{SIZE}} = \sigma^2_{\text{BTM}} = 1$) as in Stock and Watson (1991). In addition, because the 3 factors are, by construction, mutually uncorrelated (i.e., diagonal Φ and Ω), the model uniquely identifies the 3 common factors with 0 means and unit residual variances.[6] Our factors may be viewed as the intertemporal CAPM

[6]The restrictions on the factor-loading matrix B are both necessary and sufficient to identify the 3 factors. If no restriction is imposed, then the total number of free parameters (18 factor loadings + 6 variances) exceeds the number of independent variance-covariance terms of returns (21). In addition, the factor-loading restrictions make the 3 factors free from arbitrary rotations, thereby keeping the factor identities unique. To see this, consider a (3×3) orthogonal transformation matrix M ($\text{MM}' = I$), so that equation (3) becomes $\underline{R}_t = (\text{BM})(\text{M}'D_t) + w_t$. Any nonzero off-diagonal elements in the 2nd or 3rd row of M destroy the factor-loading pattern and violate the definition of the size or BTM factor.

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(ICAPM) state variables (Merton (1973)) that contemporaneously span investors' investment opportunity sets, or as the coincident financial indicators (Chauvet and Potter (2000)) that capture investors' contemporaneous beliefs about the cycles of financial markets.

III. Estimation of the Model via the Kalman Filter

Under the assumptions in equations (5)–(7), the Kalman filter is a statistically optimal procedure to extract the unobserved factors from a finite set of observed returns. Using the Kalman filter, we first estimate the parameters in matrices B, Δ, and Φ specified by equations (1)–(7), with the state vector being initialized by the unconditional means and variances of D_t.[7] The Kalman filter is then iterated for $t = 1, \ldots, T$ to recursively extract conditional expectations of the dynamic factors, $D_{t|t-1}$ and $D_{t|t}$, as well as their mean squared error (MSE) matrices, $P_{t|t-1}$ and $P_{t|t}$. The procedure for extracting the factors via the Kalman filter is briefly described in the Appendix.

A. Data and Estimation Methods

We obtain monthly returns on the 6 portfolios formed by sorting on size and BTM from Kenneth French's Web site (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/). To compute excess returns on the 6 portfolios, we subtract the 1-month T-bill rate. The sample used in this study ranges from January 1955 to December 2004, covering the past 600 months (50 years) in the U.S. stock market.

With the excess returns on the 6 portfolios, we first estimate the 20 model parameters in matrices B, Δ, and Φ using the maximum likelihood method. For this purpose, we run the Kalman filter as described in Kim and Nelson (1999) with iterations until all the parameters converge to a high precision.[8] Given these parameter estimates, we run the Kalman filter again to recursively extract the ex ante (forecast at month $t-1$) factors $D_{t|t-1} \equiv (D_{\mathrm{MKT},t|t-1}\ D_{\mathrm{SIZE},t|t-1}\ D_{\mathrm{BTM},t|t-1})'$ as well as the ex post (updated at month t) factors $D_{t|t} \equiv (D_{\mathrm{MKT},t|t}\ D_{\mathrm{SIZE},t|t}\ D_{\mathrm{BTM},t|t})'$ for $t =$ 1955:01 to 2004:12.

B. Descriptive Statistics and Estimated Parameters

Means and variances of excess returns (over the 1-month T-bill rate) for the 6 size- and BTM-sorted portfolios are given in Panel A of Table 1. It indicates that mean returns increase monotonically with BTM within each of the 2 size groups (small (S) and big (B)), while volatilities of returns are highest for growth stocks (L) within each of the 2 size groups. The former fact is consistent with the well-known BTM effect. Considering that growth portfolios are likely to consist of small, tech-oriented stocks, the latter is also a reasonable feature.

[7] See, for example, Kim and Nelson (1999) and Hamilton (1994) for details.

[8] The maximum likelihood estimates converge quickly to a preset 10^{-5} convergence criterion and are robust to different sets of reasonably chosen starting values of parameters.

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TABLE 1

Maximum Likelihood Estimates for the Model

Panel A of Table 1 reports the means and variances of excess returns (over the 1-month T-bill rate, in %) on the 6 size and book-to-market (BTM) sorted portfolios over the past 600 months (50 years: 1955:01–2004:12). Panel B reports the maximum likelihood estimates of the parameters for the model obtained using the Kalman filter. The 6 size- and BTM-sorted portfolios are notated as SL, SM, SH, BL, BM, and BH, where S and B denote "small" and "big" in firm size, respectively, and L, M, and H denote "low," "medium," and "high" in the BTM ratio, respectively. $\underline{R}_t \equiv [\underline{R}_{SL,t}\ \underline{R}_{SM,t}\ \underline{R}_{SH,t}\ \underline{R}_{BL,t}\ \underline{R}_{BM,t}\ \underline{R}_{BH,t}]'$ is a (6×1) vector of demeaned excess returns on the 6 portfolios at month t; and $D_t \equiv [D_{MKT,t}\ D_{SIZE,t}\ D_{BTM,t}]'$ is a (3×1) vector of 0-mean unobserved state vector at month t. The model is specified as

i) observation equations:

$$\begin{bmatrix} \underline{R}_{SL,t} \\ \underline{R}_{SM,t} \\ \underline{R}_{SH,t} \\ \underline{R}_{BL,t} \\ \underline{R}_{BM,t} \\ \underline{R}_{BH,t} \end{bmatrix} = \begin{bmatrix} \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \\ \beta_{4,1} & \beta_{4,2} & \beta_{4,3} \\ \beta_{5,1} & \beta_{5,2} & \beta_{5,3} \\ \beta_{6,1} & \beta_{6,2} & \beta_{6,3} \end{bmatrix} \begin{bmatrix} D_{MKT,t} \\ D_{SIZE,t} \\ D_{BTM,t} \end{bmatrix} + \begin{bmatrix} w_{SL,t} \\ w_{SM,t} \\ w_{SH,t} \\ w_{BL,t} \\ w_{BM,t} \\ w_{BH,t} \end{bmatrix} \quad \text{and}$$

ii) state equations:

$$\begin{bmatrix} D_{MKT,t} \\ D_{SIZE,t} \\ D_{BTM,t} \end{bmatrix} = \begin{bmatrix} \phi_{MKT} & 0 & 0 \\ 0 & \phi_{SIZE} & 0 \\ 0 & 0 & \phi_{BTM} \end{bmatrix} \begin{bmatrix} D_{MKT,t-1} \\ D_{SIZE,t-1} \\ D_{BTM,t-1} \end{bmatrix} + \begin{bmatrix} v_{MKT,t} \\ v_{SIZE,t} \\ v_{BTM,t} \end{bmatrix},$$

where $\Delta \equiv \mathrm{E}[w_t w_t'] = \mathrm{diag}[\sigma^2_{SL}, \sigma^2_{SM}, \sigma^2_{SH}, \sigma^2_{BL}, \sigma^2_{BM}, \sigma^2_{BH}]$, $\Omega \equiv \mathrm{E}[v_t v_t'] = \mathrm{diag}[\sigma^2_{MKT}, \sigma^2_{SIZE}, \sigma^2_{BTM}]$ with $\sigma^2_{MKT} = \sigma^2_{SIZE} = \sigma^2_{BTM} = 1$, and $\{w_t, v_t\}_{t=1}^{T}$ follows multivariate normal distributions, with w_t and v_t uncorrelated at all leads and lags. Using the firm characteristic notations employed above for the 6 portfolios, we can denote βs in the first column of the parameter matrix in the observation equation as follows: $\beta_{1,1} = \beta_{SL}$, $\beta_{2,1} = \beta_{SM}$, $\beta_{3,1} = \beta_{SH}$, $\beta_{4,1} = \beta_{BL}$, $\beta_{5,1} = \beta_{BM}$, and $\beta_{6,1} = \beta_{BH}$. In the same context, the restrictions that βs should be the same within each of the small- and big-stock groups can be expressed as $\beta_{1,2} = \beta_{2,2} = \beta_{3,2} = \beta_S$ and $\beta_{4,2} = \beta_{5,2} = \beta_{6,2} = \beta_B$. The restrictions that βs should be the same within each of the low, medium, and high BTM groups can be expressed as: $\beta_{1,3} = \beta_{4,3} = \beta_L$, $\beta_{2,3} = \beta_{5,3} = \beta_M$, and $\beta_{3,3} = \beta_{6,3} = \beta_H$. * and ** indicate coefficients significantly different from 0 at the 5% and 1% levels, respectively.

Panel A. Descriptive Statistics about the 6 Portfolios Formed on Size and BTM

Item	SL	SM	SH	BL	BM	BH
Mean	0.52	0.88	1.06	0.49	0.60	0.74
Var	45.02	26.22	26.95	21.55	16.93	19.41

Panel B. Maximum Likelihood Estimates of the Parameters for the Model

Parameter	Estimated Value	Parameter	Estimated Value	Parameter	Estimated Value
β_{SL}	6.26**	β_S	1.32**	σ^2_{SL}	1.51**
β_{SM}	4.79**	β_B	2.86**	σ^2_{SM}	0.38**
β_{SH}	4.72**			σ^2_{SH}	0.60**
β_{BL}	3.14**	β_L	–1.09**	σ^2_{BL}	1.91**
β_{BM}	2.59**	β_M	0.47*	σ^2_{BM}	1.42**
β_{BH}	2.81**	β_H	1.21**	σ^2_{BH}	1.62**
ϕ_{MKT}	0.18**	ϕ_{SIZE}	–0.14**	ϕ_{BTM}	0.21**

Panel B of Table 1 presents the maximum likelihood parameter estimates for the model specified in equations (1)–(7). The parameter estimates are all statistically significant at the 5% level: The unreported t-values (absolute) for βs are all greater than 1.96 and are mostly larger by a factor of 10–30; those for ϕs are 2.8–4.5; and those for σ^2s are larger than 5. These results, including the highly significant factor loadings, suggest that excess returns on the 6 portfolios conform to our 3-dimensional factor structure.

Specifically, the loadings on the market factor (D_{MKT}) in the small-stock group (β_{SL}, β_{SM}, and β_{SH}) are significantly larger than those in the big-stock group (β_{BL}, β_{BM}, and β_{BH}). This suggests that the market factor captures well the notion that small stocks are relatively riskier than big stocks. As for the loadings

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on the size factor (D_{SIZE}), the magnitude of loadings in the big-stock group (β_{B}) is more than twice as large as that in the small-stock group (β_{S}), indicating that returns are more sensitive to the size factor in the big-stock group. We also find that low (L) BTM stocks have a significantly negative loading (–1.09) on the BTM factor (D_{BTM}), while high (H) BTM stocks have a significantly positive loading (1.21), providing evidence of the growth and value effects. The loading on the factor in the medium BTM group (β_{M}) is much smaller. This is a desirable feature because it implies that in a portfolio with a medium level of BTM, stock returns are not very sensitive to the BTM factor.

As for the predictability of our dynamic factors, we find in Panel B of Table 1 that the AR(1) coefficients on the 3 factors specified in equation (2) are all statistically significant at any conventional level. A change in the factor realizations by 1 standard deviation in the current month will affect the unobserved factor realizations by 14%–21% in the following month.[9] To visualize the level of time variation in investors' optimal forecasts about the reward/risk ratio, we draw a graph of the maximum ex ante squared Sharpe ratio ($\text{SSR}_{t|t-1}$) based on equation (A-1) over the sample period (1955:01–2004:12). Figure 1 shows that substantial variation exists in investors' forecasts about their risk-return tradeoff. The average value of the ex ante SSR is 0.09, which is comparable to that of the FF3s (0.06). The ratio is highest (1.76) in March 2000, when the S&P 500 index reached its peak in our sample period.

C. Descriptive Statistics and Correlations of the Factors

As we have examined that our model is appropriately specified, we now report in Table 2 the descriptive statistics and correlations of the dynamic factors after estimating the ex post factors ($D_{\text{MKT},t|t}$, $D_{\text{SIZE},t|t}$, and $D_{\text{BTM},t|t}$) as well as the ex ante factors ($D_{\text{MKT},t|t-1}$, $D_{\text{SIZE},t|t-1}$, and $D_{\text{BTM},t|t-1}$) using the Kalman filter. For notational simplicity, we denote (D^1_{MKT}, D^1_{SIZE}, D^1_{BTM}) and (D^0_{MKT}, D^0_{SIZE}, D^0_{BTM}) as the time-series collection of $D_{t|t}$ and $D_{t|t-1}$, respectively. In general, we let superscript "1" denote ex post and "0" ex ante. For comparison purposes, we also report the statistics about the FF3s and the excess return on the equal-weighted market (EWM) index.

[9]We conduct a specification test to ensure that our dynamic factors are unbiased. The hypothesis of the model specification is that forecast errors of portfolio returns, $e_{t|t-1}$, shown in equation (A-2) in the Appendix should be unpredictable based on I_{t-1}. Following Stock and Watson (1989), (1991), we test the predictability by regressing 1-step-ahead forecast errors of portfolio returns on 6 lagged returns and forecast errors as in the equation

$$e_{i,t} \quad = \quad a + \sum_{s=1}^{6} \gamma_{i,t-s} \underline{R}_{i,t-s} + \sum_{s=1}^{6} \theta_{i,t-s} e_{i,t-s} + \varsigma_{i,t},$$

where $e_{i,t} = \underline{R}_{i,t} - \underline{R}_{i,t|t-1}$ denotes forecast errors obtained from the Kalman filter, and $\underline{R}_{i,t-s}$ denotes lagged returns on portfolio i (i= SL, SM, SH, BL, BM, BH). The null hypothesis is that the 6 coefficients (γs) of $\underline{R}_{i,t-s}$ ($s = 1, \ldots, 6$) or the 6 coefficients (θs) of $e_{i,t-s}$ ($s = 1, \ldots, 6$) for each portfolio i are jointly 0. The p-values (unreported to save space) of the joint F-test show that the hypothesis is rejected at the 5% significance level only for 4 cases out of the total 72 tests. This suggests that our model is reasonably well specified.

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FIGURE 1

Maximum Ex Ante Squared Sharpe Ratio over the Past 50 Years

Figure 1 plots the maximum ex ante squared Sharpe ratio ($SSR_{t|t-1}$) over the past 600 months (50 years: 1955:01–2004:12). The maximum ex ante squared Sharpe ratio ($SSR_{t|t-1}$) is defined as $SSR_{t|t-1} \equiv D'_{t|t-1} P^{-1}_{t|t-1} D_{t|t-1}$, where $D_{t|t-1} \equiv [D_{MKT,t|t-1}\ D_{SIZE,t|t-1}\ D_{BTM,t|t-1}]'$ is a vector of the ex ante expectations of the 3 dynamic factors estimated using the Kalman filter. $P_{t|t-1}$ is a (3 × 3) matrix of the mean squared error (MSE) of the ex ante expected factors, $D_{t|t-1}$.

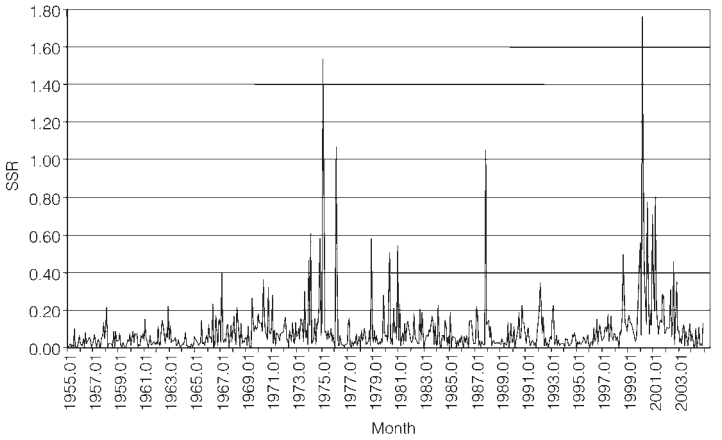


TABLE 2

Descriptive Statistics and Correlations of the Estimated Dynamic Factors and Other Related Variables

Table 2 reports the descriptive statistics and correlations of the estimates of the dynamic factors and other related variables over the past 600 months (50 years: 1955:01–2004:12). The variables are defined as follows: Mean, the mean of the variable in %; STD, the standard deviation of the variable; $D_{t|t} = (D^1_{MKT}\ D^1_{SIZE}\ D^1_{BTM})'$, a vector of the ex post (updated at month t) dynamic factors; $D_{t|t-1} = (D^0_{MKT}\ D^0_{SIZE}\ D^0_{BTM})'$, a vector of ex ante (forecast at month $t-1$) dynamic factors; (MKT SMB HML)′, a vector of the FF3; EWM, the excess return on the equal-weighted market index; and ρ_k, an autocorrelation coefficient of order k.

			Cross Correlations										Autocorrelations		
Item	Mean	STD	D^1_{MKT}	D^1_{SIZE}	D^1_{BTM}	D^0_{MKT}	D^0_{SIZE}	D^0_{BTM}	MKT	SMB	HML	EWM	ρ_1	ρ_2	ρ_6
D^1_{MKT}	0.00	1.01	1.00										0.18	0.00	0.04
D^1_{SIZE}	0.00	0.95	0.03	1.00									–0.14	0.02	–0.03
D^1_{BTM}	0.00	0.93	0.02	–0.01	1.00								0.21	0.10	0.08
D^0_{MKT}	0.00	0.18	0.18	–0.07	0.08	1.00							0.18	0.00	0.04
D^0_{SIZE}	0.00	0.13	–0.12	0.14	–0.08	–0.03	1.00						–0.14	0.02	–0.03
D^0_{BTM}	0.00	0.20	–0.01	–0.04	0.21	0.02	0.01	1.00					0.21	0.10	0.08
MKT	0.52	4.33	0.77	0.64	–0.12	0.09	0.00	–0.04	1.00				0.07	–0.04	–0.03
SMB	0.21	3.05	0.82	–0.55	0.02	0.18	–0.16	0.02	0.27	1.00			0.06	0.04	0.06
HML	0.40	2.79	–0.33	–0.02	0.92	0.00	–0.03	0.14	–0.39	–0.27	1.00		0.13	0.04	0.06
EWM	0.80	5.48	0.95	0.23	0.05	0.24	–0.08	–0.03	0.85	0.66	–0.29	1.00	0.21	–0.01	0.00

The first part of Table 2 shows that the means of the ex post and ex ante estimates of the dynamic factors are all 0 by construction. The volatility of the ex ante factor estimates is about 14%–22% of the ex post factor volatility. The middle part of the table presents the correlations between the 2 types of dynamic factors and the FF3s. Most notable is that the FF3s are highly correlated with each other: The absolute values of the correlation coefficients range from 27% to 39%.

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However, both the ex post and the ex ante factors are very weakly correlated with each other within each of the 2 groups: The absolute values of the correlation coefficients are only 1%–3%. This implies that each of D_{MKT}, D_{SIZE}, and D_{BTM} independently captures specific aspects in return variations, whereas MKT, SMB, and HML tend to jointly explain return variations.

Looking at the cross-correlations between the dynamic factors and the FF3s, the ex post BTM factor (D^1_{BTM}) is strongly correlated with HML (92%). The ex post market factor (D^1_{MKT}) is also highly correlated with MKT (77%), and especially with SMB (82%). It is interesting to see that the ex post size factor (D^1_{SIZE}) is negatively correlated with SMB (−55%), while it is positively correlated with MKT (64%). The ex ante factors are relatively weakly correlated with the FF3s in general, but there are some statistically significant relations: positive correlation of D^0_{MKT} with SMB, negative correlation of D^0_{SIZE} with SMB, and positive correlation of D^0_{BTM} with HML. Another noteworthy aspect is that the correlations of the EWM with D^1_{MKT} and D^0_{MKT} are 95% and 24%, respectively.

Lastly, both ex post and ex ante factors are highly autocorrelated, compared to the FF3s. For example, the 1st- and 2nd-order autocorrelation coefficients of D^0_{BTM} are 21% and 10%, respectively. This indicates that our 2 types of dynamic factors are more predictable than the FF3s. Particularly interesting is that D^0_{SIZE} has a negative and statistically significant 1st-order autocorrelation (−14%), while SMB has a small positive serial correlation (6%). These features may have some implications for the better performance of our dynamic factors.

IV. Formulating the Asset Pricing Tests

Assume that the excess return on a given asset or portfolio, R_i ($i = 1, \ldots, N$), follows a conditional factor structure with the true factors identified as $D_t \equiv (D_{\text{MKT},t}\ D_{\text{SIZE},t}\ D_{\text{BTM},t})'$ whose prior (ex ante) expectations are extracted as $D_{t|t-1} \equiv (D_{\text{MKT},t|t-1}\ D_{\text{SIZE},t|t-1}\ D_{\text{BTM},t|t-1})'$, and posterior (ex post) expectations are extracted as $D_{t|t} \equiv (D_{\text{MKT},t|t}\ D_{\text{SIZE},t|t}\ D_{\text{BTM},t|t})'$ using the Kalman filter. Both $D_{t|t-1}$ and $D_{t|t}$ have 0 unconditional means. Also assume that the information set (I_{t-1}) contains only the excess returns on the 6 portfolios formed on size and BTM up to time $t-1$.

Now consider the following return-generating process for asset i conditional on I_{t-1}:

$$R_{i,t} = R_{i,t|t-1} + B^1_i (D_t - D_{t|t-1}) + \epsilon_{i,t}, \tag{11}$$

where $R_{i,t|t-1} \equiv \text{E}[R_{i,t} \mid I_{t-1}]$, $D_{t|t-1} \equiv \text{E}[D_t \mid I_{t-1}]$, and $B^1_i \equiv [\beta^1_{i,\text{MKT}}\ \beta^1_{i,\text{SIZE}}\ \beta^1_{i,\text{BTM}}]$ is a (1×3) row vector of constant factor loadings on the factor innovations for asset i. Let the error term $\epsilon_{i,t}$ be $\text{E}[\epsilon_{i,t}|I_{t-1}] = 0$ and $\text{E}[\epsilon_{i,t}D_t|I_{t-1}] = 0$. Furthermore, conditional asset pricing assumes that $R_{i,t|t-1}$ is linearly related to $D_{t|t-1}$ so that

$$R_{i,t|t-1} = \overline{R}_i + B^0_i D_{t|t-1}, \tag{12}$$

where $\overline{R}_i$ is the unconditional mean of $R_{i,t}$, and $B^0_i \equiv [\beta^0_{i,\text{MKT}}\ \beta^0_{i,\text{SIZE}}\ \beta^0_{i,\text{BTM}}]$ is a (1×3) row vector of constant factor loadings on $D_{t|t-1}$. For convenience, equation (12) can be expressed in a demeaned form as $\underline{R}_{i,t|t-1} \equiv R_{i,t|t-1} - \overline{R}_i = B^0_i D_{t|t-1}$.

Combining equations (11) and (12), we have the following asset pricing test equation:

$$R_{i,t} = \overline{R}_i + B_i^* D_{t|t-1} + B_i^1 D_t + \varepsilon_{i,t}, \tag{13}$$

where $B_i^* \equiv B_i^0 - B_i^1 = [(\beta_{i,\text{MKT}}^0 - \beta_{i,\text{MKT}}^1) \ (\beta_{i,\text{SIZE}}^0 - \beta_{i,\text{SIZE}}^1) \ (\beta_{i,\text{BTM}}^0 - \beta_{i,\text{BTM}}^1)] \equiv [\beta_{i,\text{MKT}}^* \ \beta_{i,\text{SIZE}}^* \ \beta_{i,\text{BTM}}^*]$ denotes a (1×3) row vector of the differences between the loadings on $D_{t|t-1}$ and those on D_t for asset i. What B_i^* captures is the portion of factor loadings B_i^0 that is left over by B_i^1. Henceforth, B_i^* is referred to as the *residual* factor loadings, and $B_i^* D_{t|t-1}$ is referred to as asset i's residual predictability. Eliminating $B_i^* D_{t|t-1}$ leads to a bias because it is part of the predictable return but is not incorporated into the asset's ex post expected value. We call this model specified in equation (13) the DFPM.

Then equation (13) involves a testable restriction that distinguishes between the null and alternative hypotheses. If we set the null hypothesis as H_0: $B_i^* = 0$, this in turn implies

$$R_{i,t} = \overline{R}_i + B_i^1 D_t + \varepsilon_{i,t}. \tag{14}$$

Given that the vector of true factors (D_t) is not observable to an econometrician in a finite sample of assets, D_t in equations (13) and (14) is proxied by its factor-mimicking estimate, $D_{t|t}$, for our empirical testing purposes (e.g., see equation (17)). Under the null hypothesis ($B_i^* = 0$), equation (13) reduces to equation (14) as B_i^1 fully incorporates all the past and present information, thereby making $D_{t|t-1}$ irrelevant in pricing asset i. Under the alternative hypothesis of H_a: $B_i^* \neq 0$, however, D_t (and hence $D_{t|t}$) alone does not fully explain conditional expected returns, and thus $D_{t|t-1}$ has explanatory power beyond what $D_{t|t}$ captures in equation (13).

For equation (13), asset pricing imposes the following restriction on the unconditional mean of excess returns:

$$\overline{R}_i = B_i^* \lambda^0 + B_i^1 \lambda^1, \tag{15}$$

where $\lambda^0 \equiv [\lambda_{\text{MKT}}^0 \ \lambda_{\text{SIZE}}^0 \ \lambda_{\text{BTM}}^0]'$ is a 3-vector of ex ante factor risk premiums (corresponding to $D_{t|t-1}$), and $\lambda^1 \equiv [\lambda_{\text{MKT}}^1 \ \lambda_{\text{SIZE}}^1 \ \lambda_{\text{BTM}}^1]'$ is a 3-vector of ex post factor risk premiums (corresponding to D_t and hence $D_{t|t}$).

Equation (15) decomposes the mean excess return into a risk-adjusted return attributed to risk factors, $B_i^1 \lambda^1$, plus a portion of predictable return that is missed by risk adjustments, $B_i^* \lambda^0$. Under the null hypothesis ($B_i^* = 0$), equation (15) reduces to

$$\overline{R}_i = B_i^1 \lambda^1. \tag{16}$$

Thus, testing equation (15) is virtually equivalent to gauging the economic significance of $B_i^* \lambda^0$.

V. Testing the Dynamic Factor Pricing Model

As we will see below, the high explanatory power of the residual loadings on the ex ante factors results in a strong rejection of the null hypothesis (H_0: $B_i^* = 0$)

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in favor of the unrestricted model (i.e., the DFPM specified in equation (13)). In this section, therefore, we focus on testing the DFPM and compare it with other alternative models. For expositional convenience, we first conduct in-sample tests in this section and then perform out-of-sample tests in the next section.

A. With 25 Portfolios as Test Assets

Following Cochrane ((2005), chap. 12) and Brennan et al. (2004), we run the 2-pass regressions in order to test our DFPM using the 25 size- and BTM-sorted portfolios as test assets.

1. Time-Series Regressions

To test the DFPM, we first estimate the factor loadings by regressing the excess return on the ex ante dynamic factors as well as the ex post dynamic factors, as in the following equation:

$$R_{i,t} = \alpha_i + \sum_{j=1}^{3} \beta^1_{i,j} D_{j,t|t} + \sum_{j=1}^{3} \beta^*_{i,j} D_{j,t|t-1} + u_{i,t}, \tag{17}$$

where $R_{i,t}$ is the excess return on portfolio i ($i = 1, \ldots, 25$), and $D_{j,t|t}$ and $D_{j,t|t-1}$ are ex post and ex ante factors j ($j =$ MKT, SIZE, BTM), respectively.

Panel A of Table 3 contains the estimates of the residual factor loadings ($\beta^*_{i,j}$s) ($i = 1, \ldots, 25$) from the time-series regressions for the 25 portfolios.[10] For brevity, we do not tabulate all the corresponding results from the FF3 model, but some parts or averaged values are reported in Panel B for comparison purposes. Note that many of the residual loadings for the DFPM are statistically significant at the 5% level, especially in the extreme size or BTM groups. Given the statistical significance levels of the loadings on the ex post and ex ante factors, it is not likely that our 2nd-step results are subject to the "useless" factor problem described in Kan and Zhang (1999).[11] In particular, the magnitude of the residual loadings and their statistical significance are often much higher in the corner portfolios. For instance, the loadings on the market and the BTM factors in the portfolio with the smallest size and lowest BTM are 2.14 and –1.45, respectively. Also, the average of adjusted R^2 (Avg R^2) is higher than that of the FF3 model, while the average of standard errors (Avg $s(u)$) is lower. These results suggest that the ex post dynamic factors alone cannot fully explain the return variation over time.

[10] We do not report the estimates of the loadings on the ex post factors ($\beta^1_{i,j}$s), because they are mostly statistically significant at any conventional level.

[11] Kan and Zhang (1999) suggest testing the significance of factor loadings as a diagnostic tool to detect useless factors. To formally test the null hypothesis of jointly 0 coefficients on the dynamic factors, we conduct the F-test in a seemingly unrelated regression (SUR) system for the 25 portfolios. The test strongly rejects the null hypothesis.

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TABLE 3

Two-Pass Regression Results to Test the Dynamic Factor Pricing Model

Table 3 reports the 2-pass regression results to test the dynamic factor pricing model (DFPM). Panel A contains the 1st-step results that include the estimates of the residual factor loadings (β^*s) for each of the 25 portfolios, and the averaged values of adjusted R^2 (Avg R^2) and standard errors (Avg $s(u)$) after running the time-series regression

$$R_{i,t} = \alpha_i + \sum_{j=1}^{3} \beta^1_{i,j} D_{j,t|t} + \sum_{j=1}^{3} \beta^*_{i,j} D_{j,t|t-1} + u_{i,t},$$

where $R_{i,t}$ is the excess return on portfolio $i (i = 1, \ldots, 25)$ formed on size and book-to-market equity (BTM), $D_{j,t|t}$ and $D_{j,t|t-1}$ are ex post and ex ante dynamic factors j (j = MKT, SIZE, BTM) extracted from the Kalman filter. Panel B reports the summary of the time-series regression results from the Fama and French 3-factor (FF3) (MKT, SMB, HML) model for comparison purposes. Panel C contains the results from the 2nd step, which involves cross-sectionally regressing the sample mean of monthly excess returns on the estimated factor loadings as in the equation

$$\bar{R}_i = \sum_{j=1}^{3} \lambda^1_j \hat{\beta}^1_{i,j} + \sum_{j=1}^{3} \lambda^0_j \hat{\beta}^*_{i,j} + e_i,$$

where $\bar{R}_i$ is the sample mean of monthly excess returns on portfolio $i (i = 1, \ldots, 25)$, $\hat{\beta}^1_{i,j}$ is the estimated loading on ex post factor j (j = MKT, SIZE, BTM), $\hat{\beta}^*_{i,j}$ is the estimated loading on ex ante factor j, λ^1_j and λ^0_j denote the coefficient to be estimated as factor premiums, and e_i is the residual term that measures the pricing error of the DFPM in portfolio i. From the regression, we report the estimated factor premiums (PREM) and pricing errors (es) in %. The (unreported) t-statistics for the factor premiums, the (unreported) t-statistics for the pricing errors, and the covariance matrix of the pricing errors (Σ) are computed with Shanken's (1992) correction. AAE stands for the average absolute error, and SSE = $e'e$ is the sum of squared errors. $e'\Sigma^{-1}e$ is the quadratic test statistic, which follows a $\chi^2(19)$ distribution under the null hypothesis that pricing errors are jointly 0. For comparison purposes, the cross-sectional regression results from the FF3 (MKT, SMB, HML) model are also reported in Panel D. The sample range is the past 600 months (50 years: 1955:01–2004:12) in the U.S. stock market. * and ** indicate coefficients and pricing errors significantly different from 0 at the 5% and 1% levels, respectively.

Panel A. Time-Series Regression Results for DFPM

	Book-to-Market					Book-to-Market				
	1 (low)	2	3	4	5 (high)	1 (low)	2	3	4	5 (high)
Size	β^*_{MKT}					β^*_{SIZE}				
1 (small)	2.14**	1.02*	0.91**	1.29**	2.24**	0.92	1.69**	1.20**	0.83*	0.47
2	–0.34	–0.10	–0.65*	–0.67**	–0.91**	0.26	0.43	–0.74*	–0.12	–0.58
3	–2.09**	–0.56	–0.05	–0.80**	–0.23	–1.17**	–0.87	–0.08	–0.84*	0.48
4	–0.91**	0.31	–0.23	–1.25**	–0.56	0.15	0.02	0.17	0.04	0.46
5 (big)	0.79*	0.55	–0.39	–0.50	0.51	–0.73	–0.86*	–0.36	0.37	–1.40*

	Book-to-Market				
	1 (low)	2	3	4	5 (high)
Size	β^*_{BTM}				
1 (small)	–1.45**	–0.09	–0.10	–0.69*	–0.14
2	0.18	0.55*	–0.05	0.25	0.29
3	1.01**	0.10	0.01	0.01	–0.40
4	0.45	0.30	–0.10	–0.54	–0.35
5 (big)	0.79**	0.68*	0.39	–0.54	–1.52**

Averages of R^2 and $s(u)$	
Avg R^2:	0.92
Avg $s(u)$:	1.50

Panel B. Summary of Time-Series Regression Results for FF3

Avg R^2: 0.90 Avg $s(u)$: 1.61

Panel C. Cross-Sectional Regression Results for DFPM

	λ^1_{MKT}	λ^1_{SIZE}	λ^1_{BTM}	λ^0_{MKT}	λ^0_{SIZE}	λ^0_{BTM}
PREM (in %)	0.14**	0.05	0.24**	0.04	0.00	0.17**

	Book-to-Market				
	1 (low)	2	3	4	5 (high)
Size	e (in %)				
1 (small)	–0.14*	0.04	0.00	0.26**	0.06
2	–0.12	–0.13	0.07	0.00	–0.04
3	–0.02	0.04	–0.09	0.00	–0.01
4	0.20**	–0.15*	0.04	0.13	–0.10
5 (big)	0.14	–0.08	0.02	–0.04	–0.01

Other Statistics	
AAE:	0.076
SSE:	0.255
$e'\Sigma^{-1}e$:	39.68
p-value:	0.004

(continued on next page)

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TABLE 3 (continued)

Two-Pass Regression Results to Test the Dynamic Factor Pricing Model

Panel D. Cross-Sectional Regression Results for FF3

	λ_{MKT}	λ_{SMB}	λ_{HML}
PREM (in %)	0.48**	0.21	0.44**

	Book-to-Market						
	1 (low)	2	3	4	5 (high)		
Size	e (in %)					Other Statistics	
1 (small)	–0.37**	0.04	0.04	0.23**	0.17**	AAE:	0.105
2	–0.17**	–0.03	0.09	0.08	0.06	SSE:	0.463
3	0.02	0.06	–0.05	0.04	–0.02		
4	0.16**	–0.09	0.04	0.03	–0.14	$e'\Sigma^{-1}e$:	74.52
5 (big)	0.22**	0.02	0.06	–0.14*	–0.24*	p-value:	0.000

2. Cross-Sectional Regressions

Based on equation (15), we cross-sectionally regress the sample mean of monthly excess returns on the ex post and *residual* factor loadings (estimated in the 1st step above) as in the equation

$$\overline{R}_i = \sum_{j=1}^{3} \lambda_j^1 \widehat{\beta}_{i,j}^1 + \sum_{j=1}^{3} \lambda_j^0 \widehat{\beta}_{i,j}^* + e_i, \tag{18}$$

where $\overline{R}_i$ is the mean excess return on portfolio i ($i = 1, \ldots, 25$), $\widehat{\beta}_{i,j}^1$ is the loading on ex post factor j (j = MKT, SIZE, BTM), $\widehat{\beta}_{i,j}^*$ is the *residual* factor loading on ex ante factor j, λ_j^1 and λ_j^0 denote the coefficients to be estimated as factor risk premiums, and e_i is the residual term that measures the pricing error of the DFPM for portfolio i.

Panel C of Table 3 reports the estimated factor risk premiums (PREM) and pricing errors (es). The levels of the risk premiums for the ex post dynamic factors are 0.05%–0.24%, with the premiums for the market and BTM factors (λ_{MKT}^1 and λ_{BTM}^1) being statistically significant at any conventional level. But the premium for the size factor is not significant. The monthly risk premiums for the FF3s in Panel D are of a similar pattern. Notice here that the premium for the ex ante BTM factor, λ_{BTM}^0, is positive and statistically significant at the 1% level (i.e., the beta risk of the ex ante BTM factor is priced). The strong residual predictability of the ex ante BTM factor ($D_{\text{BTM},t|t-1}$) is consistent with Lewellen (1999).

Panel C of Table 3 also indicates that, in general, the pricing errors (es) of the DFPM become much lower (especially in the extreme portfolios) compared to those of the FF3 model reported in Panel D. It follows that the average absolute error (AAE) is down by 27.6% from 10.5 basis points (bp) to 7.6 bp per month, and the sum of squared errors (SSE) decreases from 0.46 to 0.26. In particular, the decrease in pricing errors mainly occurs in the corner portfolios. For instance, the pricing error in the smallest growth portfolio for the DFPM is only −0.14% per month, while its counterpart is −0.37% in the FF3 model. The reduction in pricing errors is mainly because of the residual power of the loading on the ex

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ante BTM factor. Given the incremental role of the ex ante factors, it is obvious that the DFPM performs better than the FF model.

Next, we examine the joint test statistics $e'\Sigma^{-1}e$. This quadratic term follows a $\chi^2(19)$ distribution under the null hypothesis. For the DFPM, $e'\Sigma^{-1}e$ is 39.68, which is much smaller than that for the FF3 model. However, the null hypothesis of a jointly 0 pricing error is still rejected at the 5% level for this model, as the p-value indicates. However, as FF (1993), (1996) suggest, the rejection of the DFPM appears to reflect the high explanatory power of the test. The time-series regressions for the 25 portfolios produce an average R^2 of 92%, leading to a small covariance matrix, Σ. Thus, even small pricing errors suffice to generate rejectable test statistics. In the same context, the reason that some recent ICAPM studies cannot reject the null hypothesis may be that the explanatory power of their tests is low (Daniel and Titman (2005)).[12] As FF (1993), (1996) suggest, therefore, it may be more meaningful to assess the *economic* significance of pricing errors. Judging by this criterion, we see that the DFPM performs better than the FF3 model, particularly in the extreme size or BTM quintiles. For the DFPM, only 2 portfolios have pricing errors larger than 2% per year, and the largest error is about 3% per year.

Overall, our analyses demonstrate that the ex ante factors are a key component in asset pricing. Ferson and Harvey (1999) find that loadings on lagged macroeconomic variables have significant premiums that explain variation in expected returns not captured by the FF model. However, they do not propose an alternative model that can explain expected returns better than the FF3 model. Our asset pricing tests present that the residual power of the DFPM stems from the fact that loadings on the ex post factors alone do not fully incorporate all the past and current information. Once the bias is adjusted by adding the ex ante factors, the DFPM can better explain expected returns than the FF3 model.

B. With Extended Sets of Portfolios

Lo and MacKinlay (1990) raise an issue of possible data-snooping biases when drawing inferences from samples of characteristic-sorted data. To reduce the potential problem, we now extend the scope of test assets by adding 10 momentum portfolios formed on past returns to the 25 size- and BTM-sorted portfolios (total 35 portfolios). Following FF (1996), we form 10 momentum portfolios by splitting New York Stock Exchange (NYSE) common stocks into deciles based on continuously compounded returns over the past 11 months (from month $t-1$ to $t-11$) in ascending order, and then computing equal-weighted excess returns (in excess of the 1-month T-bill rate) for each decile over the sample period (1955:01–2004:12). We then apply the 2-pass regression procedure in the same way as described in the previous subsection. As shown in Panel A of Table 4, the 10 portfolio monthly returns, by construction, monotonically increase from

[12]For example, the ICAPM of Brennan et al. (2004) produces comparable pricing errors to those of the FF3 model, but the ICAPM cannot be rejected due to a larger residual covariance matrix (p. 1764). In addition, Petkova (2006) cannot reject an ICAPM for a similar size of pricing errors because the model leaves a large portion of portfolio variance unexplained (compare the R^2 values in Tables III and IV of her paper).

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0.11% (decile 1) to 1.38% (decile 10). Note also that the portfolios show U-shaped volatility, with the returns of extreme portfolios (e.g., decile 1 and decile 10) being more volatile.

We apply the 2-pass procedure to the DFPM and the FF3 model with the augmented set of portfolios and check whether the DFPM has incremental explanatory power beyond what the FF3 model can capture in pricing assets. Panel B of Table 4 presents the 1st-step (time-series regression) results for the 10 momentum portfolios. The loading on the ex post market factor, β^1_{MKT}, shows a U-shaped pattern, while the loadings on the ex post size and BTM factors exhibit little variation across the deciles. The statistical significance of the loadings on the 3 ex post factors is very strong. The features of the loadings on the ex ante factors, β^*s, may be of more interest, however. While β^*_{MKT} shows a U-shaped pattern, this and β^*_{BTM} are both statistically insignificant. On the contrary, the residual loading on the ex ante size factor, β^*_{SIZE}, increases monotonically with momentum, and it is significant at the 5% level in 8 of the 10 portfolios. This indicates that the ex ante size factor plays an important role in describing momentum portfolio returns.

Panel C of Table 4 presents the results from the cross-sectional regressions using the 35 portfolios. The risk premiums for the ex post factors, λ^1s, are comparable to those reported in Panel C of Table 3. The size and statistical significance of the premium for the ex ante BTM factor (λ^0_{BTM}) are also similar to those in

TABLE 4

Two-Pass Regression Results to Test the DFPM Using an Extended Set of 35 Portfolios

Table 4 reports the 2-pass regression results to test the dynamic factor pricing model (DFPM) using the 25 size and book-to-market (BTM) sorted portfolios augmented by 10 momentum portfolios (total 35 portfolios). The 10 momentum portfolios are formed by splitting NYSE common stocks into deciles after sorting in ascending order by the continuously compounded return over the past 11 months (from month $t-1$ to $t-11$), and then the equal-weighted excess return (in excess of the 1-month T-bill rate) is computed for each decile over the sample period. Decile 1 (10) is a portfolio with the lowest (highest) continuously compounded past returns. The 2-pass regressions are performed in the same way as described in Table 3 with the 35 portfolios. Panel A contains the means (Mean) and standard deviations (STD) of the excess returns for each decile. Panel B contains the time-series regression results for the 10 momentum portfolios (factor loadings) from the DFPM. Panel C reports the cross-sectional regression results across the 35 portfolios (factor premiums, PREM, for the ex post and ex ante dynamic factors, and the pricing errors (es)) from the DFPM. AAE stands for average absolute error of the 35 portfolios, SSE = $e'e$ stands for the sum of squared errors of the 35 portfolios, and $e'\Sigma^{-1}e$ is the quadratic test statistic, which follows a $\chi^2(29)$ distribution under the null hypothesis of jointly 0 pricing errors. The (unreported) t-statistics for the pricing errors and the covariance matrix of the pricing errors (Σ) are computed with Shanken's (1992) correction. For comparison purposes, the 2-pass regression results from the FF 3-factor model (FF3) are also reported in Panel D. The upper part of Panel D contains the adjusted R^2s (R^2) and residual standard errors ($s(u)$) from the time-series regressions for the 10 momentum portfolios. The pricing errors (es) from the cross-sectional regressions across the 35 portfolios are reported (for the 10 momentum portfolios only) in the middle part of Panel D. In the lower part of Panel D, the other statistics corresponding to those in Panel C are reported. The sample range is the past 600 months (50 years: 1955:01–2004:12) in the U.S. stock market. * and ** indicate coefficients and pricing errors significantly different from 0 at the 5% and 1% levels, respectively.

	Momentum Decile									
Item	1 (loser)	2	3	4	5	6	7	8	9	10 (winner)
Panel A. Descriptive Statistics for the 10 Momentum Portfolios										
Mean	0.11	0.53	0.55	0.64	0.74	0.77	0.89	0.97	1.08	1.38
STD	7.64	5.65	5.15	4.84	4.68	4.57	4.60	4.80	5.07	6.03
Panel B. Time-Series Regression Results for DFPM										
β^1_{MKT}	5.94**	4.50**	4.05**	3.90**	3.78**	3.68**	3.71**	3.93**	4.16**	5.03**
β^1_{SIZE}	1.93**	2.11**	2.25**	2.17**	2.21**	2.22**	2.25**	2.27**	2.26**	2.07**
β^1_{BTM}	1.01**	0.95**	1.04**	0.91**	0.89**	0.87**	0.72**	0.61**	0.50**	0.08
β^*_{MKT}	1.19	0.76	0.33	0.14	0.15	–0.16	–0.04	0.05	0.06	0.27
β^*_{SIZE}	–3.58**	–1.64*	–1.83**	–1.74**	–0.93*	–0.74*	–0.06	0.58	1.24*	1.99*
β^*_{BTM}	–1.16	–0.11	–0.27	0.28	0.14	0.12	0.04	0.04	0.04	0.24

(continued on next page)

TABLE 4 (continued)

Two-Pass Regression Results to Test the DFPM Using an Extended Set of 35 Portfolios

Panel C. Cross-Sectional Regression Results for DFPM

	Risk Premium					
	λ^1_{MKT}	λ^1_{SIZE}	λ^1_{BTM}	λ^0_{MKT}	λ^0_{SIZE}	λ^0_{BTM}
PREM (in %)	0.12**	0.09*	0.23**	–0.01	0.16**	0.21**

	Pricing Error for 35 Portfolios									
	Momentum Decile									
	1 (loser)	2	3	4	5	6	7	8	9	10 (winner)
e (in %)	–0.16	–0.12	–0.02	0.00	0.01	0.02	0.08	0.05	0.05	0.20

	e (in %) for the 25 Portfolios					
	Book-to-Market					
Size	1 (low)	2	3	4	5 (high)	Other Statistics
1 (small)	–0.05	–0.12	–0.08	0.27**	0.15	AAE: 0.108
2	–0.14	–0.19	0.20*	0.00	0.05	SSE: 0.599
3	0.06	0.15	–0.09	0.10	–0.09	
4	0.09	–0.21*	–0.05	0.05	–0.22	$e' \sum^{-1} e$: 36.02
5 (big)	0.16	–0.04	–0.05	–0.19	0.24	*p*-value: 0.173

Panel D. Two-Pass Regression Results for FF3

	Momentum Decile									
Item	1 (loser)	2	3	4	5	6	7	8	9	10 (winner)
Time-Series Regressions:										
R^2	0.71	0.81	0.84	0.87	0.90	0.90	0.90	0.90	0.88	0.82
$s(u)$	4.12	2.46	2.06	1.72	1.49	1.44	1.42	1.51	1.78	2.55
Cross-Sectional Regressions:										
e (in %)	–0.92**	–0.33**	–0.27**	–0.13**	–0.02	0.02	0.16**	0.23**	0.33**	0.61**

	Other Statistics			
AAE:	0.172		SSE:	2.153
$e' \sum^{-1} e$:	112.13		*p*-value:	0.000

Table 3. However, the premium for the ex ante size factor, λ^0_{SIZE}, is 0.16% and becomes statistically significant at the 1% level, which contrasts with its counterpart in Panel C of Table 3. This significant premium, coupled with the increasing pattern of the loadings described previously, suggests that the ex ante size factor ($D_{SIZE,t|t-1}$) plays a key role in explaining the momentum portfolio returns. To gauge the incremental impact of this factor, we see that monthly excess returns for the extreme loser (decile 1) and winner (decile 10) portfolios are -0.57% ($= -3.58 \times 0.16\%$) and 0.32% ($= 1.99 \times 0.16\%$), respectively. This means that a large portion of explainable returns is missed by the FF3 model.

Panel C of Table 4 also reports the pricing errors of the DFPM for the 35 portfolios (10 momentum portfolios plus 25 size- and BTM-sorted portfolios). We find that once biases are adjusted with the ex ante factors, the DFPM describes the momentum portfolio returns reasonably well: None of the pricing errors is statistically significant, and economically, none has a pricing error greater than 0.20% per month. This presents a sharp contrast to the large pricing errors and their high significance levels in the FF3 model reported in Panel D. For instance,

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compare −0.16% of decile 1 in the middle part of Panel C with −0.92% in the cross-sectional regression part of Panel D, and 0.20% of decile 10 in Panel C with 0.61% in Panel D. For the 25 size and BTM portfolios, the largest pricing error is 0.27% per month for the DFPM. Moreover, the AAE of the 35 portfolios is less than 0.11% in Panel C, but its counterpart of the FF3 model in Panel D is 0.17%. Finally, given the magnitude of the *p*-value, we cannot reject the null hypothesis that pricing errors in the 35 portfolios are jointly 0.[13]

For a robustness check, we also reestimate the model specified in equations (3) and (4) to extract another set of ex ante and ex post dynamic factors over the period from July 1963 to December 1993, which is the same sample range studied by FF (1996). The parameter estimation shows that the magnitude of the AR(1) coefficients is larger for this sample period. The DFPM test results with the new set of factors using the 35 portfolios again confirm the additional explanatory power left out by the FF3 model. The DFPM explains momentum in all the deciles, except for decile 1, whose pricing error is −0.45% per month. Although this level is still economically significant, it is a substantial improvement upon the corresponding error of −1.15% reported in FF (1996). The DFPM also explains excess returns on the 25 size- and BTM-sorted portfolios, generating the largest pricing error of 0.33% in the smallest growth portfolio, which is again much lower than that of −0.45% reported in the above study.[14]

Grundy and Martin (2001) and Jegadeesh and Titman (2002) document that risk adjustments have little to do with the momentum effect. With bias adjustments, however, the DFPM explains the momentum portfolio returns quite well. This result is very encouraging, given that the FF3 model misses the continuation of returns for portfolios formed by sorting on short-term past returns (FF (1996)). What captures this effect is the residual loading on the ex ante size factor. To our knowledge, this is the first study that documents this feature in the asset pricing literature.[15]

C. With More Competing Models

One concern in the above analyses is that the better performance of the DFPM over the FF3 model may be primarily driven by the additional factors used

[13]However, we do not place much significance upon the nonrejection of the joint test. The joint test with the 35 portfolios may not have the same power as that with the 25 portfolios because of the different residual covariance matrices between the 2 sets of portfolios. A more meaningful criterion would be the economic significance of the pricing errors and the relative performance between the different models. See Cochrane ((2005), chap. 12) for similar discussion.

[14]In addition to the above checks, we conduct a number of other tests. For example, we again extend the scope of assets by forming 10 reversal portfolios based on the continuously compounded returns for the past 13–60 months. We then follow the 2-pass procedure with the new augmented set of the 35 portfolios. Furthermore, we conduct a more comprehensive test with the 45 portfolios after pooling the 25 size- and BTM-sorted, 10 momentum, and 10 reversal portfolios all together. We find that the DFPM explains both momentum and reversal portfolios, with the largest (absolute) pricing error across the 45 portfolios being below 0.30%, and the AAE below 0.11% per month.

[15]Avramov and Chordia (2006) show that the conditional FF3 model provides the best result. In their study, it is "best" in the sense that the conditional FF3 model captures the impact of size and BTM on stock returns. However, the model cannot capture the predictive ability of past returns (momentum variables) even when the momentum factor is included in the 1st-pass regressions.

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in the model, because the FF model uses only 3 factors. To address this issue, we consider 2 competing models that have more than 3 factors.

One is Carhart's (1997) 4-factor model (denoted as CARHART), which includes a momentum portfolio return, UMD,[16] in addition to the FF3s. The other is a conditional version of the FF (1993) 3-factor model (conditional FF3 (CFF3)). Let us consider a 3-factor model:

$$R_t = \alpha_0 + a_{t-1}\text{MKT}_t + b_{t-1}\text{SMB}_t + c_{t-1}\text{HML}_t + \varepsilon_t. \tag{19}$$

Here, we assume that each of the 3-factor loadings on the FF (1993) factors is a linear function of an instrumental variable, Z. That is,

$$\begin{aligned} a_{t-1} &= a_0 + a_1 Z_{t-1}, \\ b_{t-1} &= b_0 + b_1 Z_{t-1}, \quad \text{and} \\ c_{t-1} &= c_0 + c_1 Z_{t-1}. \end{aligned} \tag{20}$$

Then, the CFF3 model can be written as:

$$\begin{aligned} R_t = {} & \alpha_0 + a_0\text{MKT}_t + b_0\text{SMB}_t + c_0\text{HML}_t \\ & + a_1 Z_{t-1}\text{MKT}_t + b_1 Z_{t-1}\text{SMB}_t + c_1 Z_{t-1}\text{HML}_t + \varepsilon_t. \end{aligned} \tag{21}$$

Note that equation (21) now has 6 terms ("factors"), similar to the DFPM, which make the CFF3 more comparable to the DFPM.

In line with Ferson and Harvey (1999) and Simin (2008), as a candidate for an instrument (Z_{t-1}) in equations (20) and (21) we employ the following 5 variables: i) TB1m: the 1-month T-bill rate; ii) TB3m1m: the difference between 3- and 1-month T-bill rates; iii) TERM: the term spread between 10- and 1-year Treasury bond yields; iv) DEF: the default spread between Moody's Baa and Aaa corporate bond yields; and v) DIV: the dividend yield of the S&P 500 index.

From this point on, we use the 35 extended portfolios as test assets, because we believe that it is more relevant to the asset pricing tests. The in-sample pricing errors from the cross-sectional regressions using the previous models are reported in Table 5. The Carhart (1997) model is known to price size, BTM, and momentum portfolios. Thus, it is not surprising to see in Panel A of Table 5 that most pricing errors from the CARHART model are smaller than those from the DFPM (Panel C of Table 4). But some of them are larger in the "corner" portfolios.[17] Overall, the in-sample test results indicate that the DFPM is comparable to the CARHART model. To avoid a data-snooping bias, however, it would be prudent to compare the 2 models in the out-of-sample context, which is discussed in the next section.

We next look at the pricing errors from a CFF3 model in Panel B of Table 5. For brevity, we report only the result from the CFF3 that uses the 1-month T-bill rate as a conditioning variable out of the 5 choices for equation (21) (in this

[16]UMD is defined as the average return on the high (highest 30%) prior (months $t-2$ to $t-13$) return portfolios minus the average return on the low (lowest 30%) prior return portfolios. It is available from Ken French's Web site (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/).

[17]The rejection of the null hypothesis for the CARHART model (p-value = 0.001) occurs because the 4 factors generate a very small covariance matrix (Σ). So even small pricing errors can generate large $e'\Sigma^{-1}e$, resulting in a statistical rejection.

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TABLE 5

Two-Pass Regression Results from the Carhart (1997) 4-Factor Model and the Conditional Fama-French 3-Factor Model Using the 35 Portfolios

Table 5 reports in-sample pricing errors, *e*s, from the cross-sectional regressions for the Carhart (1997) 4-factor model (CARHART) (Panel A) and the conditional Fama-French (1993) 3-factor (FF3) model (Panel B) using the 25 size- and BTM-sorted portfolios augmented by 10 momentum portfolios (total 35 portfolios). For the conditional FF3 model (CFF3(TB1m)), each of the loadings on the FF3s is modeled as a linear function of the (lagged) 1-month T-bill rate. The 10 momentum portfolios are formed by splitting NYSE common stocks into deciles after sorting in ascending order by the continuously compounded return over the past 11 months (from month $t - 1$ to $t - 11$), and then the equal-weighted excess return (in excess of the 1-month T-bill rate) is computed for each decile over the sample period. Decile 1 (10) is a portfolio with the lowest (highest) continuously compounded past returns. To obtain the pricing errors, the 2-pass regressions are performed in the same way as described in Table 3 with the 35 portfolios. AAE stands for average absolute error of the 35 portfolios, SSE = $e'e$ stands for the sum of squared errors of the 35 portfolios, and $e'\Sigma^{-1}e$ is the quadratic test statistic, which follows a $\chi^2(29)$ distribution under the null hypothesis of jointly 0 pricing errors. The (unreported) *t*-statistics for the pricing errors and the covariance matrix of the pricing errors (Σ) are computed with Shanken's (1992) correction. The sample range is the past 600 months (50 years: 1955:01–2004:12) in the U.S. stock market. * and ** indicate pricing errors significantly different from 0 at the 5% and 1% levels, respectively.

Panel A. Pricing Errors from the Carhart (1997) 4-Factor Model (CARHART)

	Momentum Decile									
	1 (loser)	2	3	4	5	6	7	8	9	10 (winner)
e (in %)	–0.10	0.19**	0.08	0.07	0.07	–0.01	0.03	0.00	–0.02	0.07

	e (in %) for the 25 Portfolios						
	Book-to-Market						
Size	1 (low)	2	3	4	5 (high)	Other Statistics	
1 (small)	–0.32**	0.00	0.00	0.18**	0.15**	AAE:	0.087
2	–0.14*	–0.02	0.07	0.03	0.01	SSE:	0.428
3	0.04	0.04	–0.05	0.01	–0.01		
4	0.12*	–0.08	0.03	0.02	–0.13	$e'\sum^{-1}e$:	62.28
5 (big)	0.19**	–0.01	–0.01	–0.15*	–0.24*	*p*-value:	0.001

Panel B. Pricing Errors from the Conditional FF 3-Factor Model (CFF3(TB1m))

	Momentum Decile									
	1 (loser)	2	3	4	5	6	7	8	9	10 (winner)
e (in %)	–0.28*	–0.04	–0.18	–0.14	–0.08	–0.09	0.05	0.05	0.14	0.28*

	e (in %) for the 25 Portfolios						
	Book-to-Market						
Size	1 (low)	2	3	4	5 (high)	Other Statistics	
1 (small)	–0.25*	0.21	0.04	0.32**	0.30**	AAE:	0.129
2	–0.21*	–0.05	–0.01	–0.10	0.09	SSE:	0.968
3	–0.06	–0.02	0.00	–0.03	0.11		
4	0.13	–0.27**	0.02	–0.07	–0.08	$e'\sum^{-1}e$:	38.51
5 (big)	0.41**	0.00	–0.18	–0.05	0.18	*p*-value:	0.111

case, the CFF3 is specifically denoted as CFF3(TB1m)).[18] Comparing the pricing errors in the 10 momentum portfolios from the CFF3(TB1m) with those of the (unconditional) FF3 (1993) model reported in Panel D of Table 4, we find that the CFF3(TB1m) performs much better than the unconditional FF model in the in-sample test. This is consistent with Simin (2008). When the errors are compared with those from the DFPM (Panel C in Table 4), however, the CFF3(TB1m) tends to underperform the DFPM, especially in momentum deciles 1, 3, 4, 9, and 10 as well as in the lowest-BTM portfolios. Other unreported results also indicate

[18]The in-sample pricing errors from the other 4 CFF3 models that use 1 of the 4 different instruments (denoted as CFF3(TB3m1m), CFF3(TERM), CFF3(DEF), and CFF3(DIV), respectively) are all qualitatively similar to those of the CFF3(TB1m). The results are available from the authors.

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that the CFF3 models do not explain the returns of the momentum portfolios very well, no matter what instrument is used as a conditioning variable. This is consistent with the findings in Avramov and Chordia (2006). However, the better performance of the DFPM is found to be more pronounced in the out-of-sample tests, which are discussed in Section VI.

To assess the incremental role of the ex ante factors, we now examine the extent to which our dynamic factors can explain the momentum effect in asset pricing. For this purpose, we obtain the time series of the momentum factor, UMD, from the Web site of Kenneth French (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/). Because our goal is to investigate the incremental explanatory power of the dynamic factors after accounting for the effects of the FF3s, we first estimate the risk-adjusted (against FF3s) momentum factor. Then we test if the dynamic factors can explain the risk-adjusted momentum factor by running a time-series regression,

$$\text{UMD}_t^* = a + \sum_j \delta_j^1 D_{j,t}^1 + \sum_j \delta_j^0 D_{j,t}^0 + \zeta_t, \tag{22}$$

where $j =$ MKT, SIZE, and BTM while UMD* is the residual from the regression of UMD on the FF3s (1993). As we see in Specification 1 of Table 6, none of the ex post factors can explain the momentum effect. However, D_{SIZE}^0 has strong explanatory power, as shown in Specification 2 in the table. When we include both groups in equation (22), the result is again similar: The ex ante size factor plays an important role in describing the momentum effect.

TABLE 6

Regression Results for FF3-Adjusted Momentum Portfolio Returns Using the Dynamic Factors

Table 6 reports the time-series regression results using the risk-adjusted momentum portfolio return over the past 600 months (50 years: 1955:01–2004:12) from the equation

$$\text{UMD}_t^* = a + \sum_{j=1} \delta_j^1 D_{j,t}^1 + \sum_{j=1} \delta_j^0 D_{j,t}^0 + \zeta_t, \quad j = \text{MKT, SIZE, BTM,}$$

where the dependent variable, UMD*, is the residual from the regression of the momentum portfolio return (UMD) on Fama-French's (1993) 3 factors (FF3); UMD is the winner portfolio (decile 10) return less the loser portfolio (decile 1) return (after the CSRP stocks are sorted into deciles based on the past returns (from month $t-2$ to month $t-12$)), which is obtained from Kenneth French's Web site; $D_j^1 = (D_{\text{MKT}}^1\ D_{\text{SIZE}}^1\ D_{\text{BTM}}^1)'$ is a vector of the ex post (updated at month t) dynamic factors; $D_j^0 = (D_{\text{MKT}}^0\ D_{\text{SIZE}}^0\ D_{\text{BTM}}^0)'$ is a vector of ex ante (forecast at month $t-1$) dynamic factors. The values in the row for each explanatory variable are the coefficients of the variable. Adj. R^2 is the adjusted R^2 in the regressions, and N is the number of observations in the regressions. * and ** indicate coefficients significantly different from 0 at the 5% and 1% levels, respectively.

Explanatory Variables	Dependent Variable = UMD*		
	1	2	3
Intercept	1.032**	1.032**	1.032**
D_{MKT}^1	–0.035		0.018
D_{SIZE}^1	–0.061		–0.138
D_{BTM}^1	0.032		0.083
D_{MKT}^0		0.434	0.338
D_{SIZE}^0		3.834**	4.033**
D_{BTM}^0		–0.177	–0.284
Adj. R^2	–0.003	0.015	0.012
N	600	600	600

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Previous studies find that information uncertainty plays an important role in explaining the momentum effect, and firm size is often used as a reasonable proxy for information uncertainty. For example, Daniel, Hirshleifer, and Subrahmanyam (1998), (2001) argue that return predictability should be stronger for firms with greater uncertainty because investors tend to be more overconfident when firms' businesses are hard to value. Zhang (2006) extends their idea and uses firm size as a proxy for information uncertainty, providing evidence that greater information uncertainty produces higher return momentum. In the same context, our finding suggests that the ex ante size factor (D^0_{SIZE}) is a good proxy for information uncertainty. What distinguishes D^0_{SIZE} from the FF3s (especially SMB) is that D^0_{SIZE} is essentially dynamic and more efficient in exploiting available information through the Kalman filter. Therefore, D^0_{SIZE} appears to effectively capture the momentum effect while the FF3s cannot.

VI. Out-of-Sample Tests

In the previous section we have shown that our DFPM generally performs better than the FF3 model as well as its conditional versions, and that the ex ante dynamic factors play key roles in explaining returns. However, these results are in the context of in-sample tests. Simin (2008) argues that while conditional asset pricing models outperform unconditional models in the in-sample tests, they perform poorly in the out-of-sample tests. He also documents that simply using the historical average return (6% annually, or 0.5% monthly) gives the best result in the 1-step-ahead forecasts for most of his test assets, compared to other sophisticated models (regardless of conditional or unconditional). From the perspective of practitioners, being able to obtain more accurate forecasts is very important in a variety of settings. In this section, therefore, we focus on the performance of our dynamic factors in the out-of-sample context. Specifically, we make pair-wise comparisons of the accuracy in 1-step-ahead forecasts.

In comparing the accuracy of the forecasts, we first use the following 4 competing models: i) CAPM; ii) 9.35%: a simple benchmark that uses a fixed annual rate of 9.35% (monthly 0.78%), which is the time-series average of the Center for Research in Security Prices (CRSP) value-weighted index returns over our 300-month training period (1955:01–1979:12); iii) FF3; and iv) DFPM. To see how our results compare to those of Simin (2008), we include the low cost forecast (historical average return of 9.35%) and the CAPM in the analysis.

Given the concern that the better performance of the DFPM may be achieved primarily by adding more factors, we also consider a 4-factor model and a conditional version of the FF3 model (described in Section V) as follows: v) CARHART: Carhart's (1997) 4-factor model, which includes an additional factor, UMD; and vi) CFF3(TB1m): a conditional beta version of the FF3 model, where each of the loadings on the FF3s is modeled as a linear function of the (lagged) 1-month T-bill rate.

We again use the 35 extended portfolios (10 momentum portfolios plus 25 size- and BTM-sorted portfolios) as test assets. For this purpose, the first 300 months (1955:01–1979:12) of our whole sample period (600 months: 1955:01–2004:12) are used as a training period in order to estimate the first set of model

parameters. The first 1-step-ahead forecast is then computed for January 1980 using the estimated model parameters. The 1-step-ahead forecasts for the remaining 299 months (1980:02–2004:12) are also computed based on the model parameters estimated using the relevant data from the 300-month rolling windows, resulting in the total 300 forecasts (1980:01–2004:12) for each model.[19]

For pair-wise comparisons between the competing models, the average of differentials in the mean squared forecast errors (MSFEs), $\bar{d}$, is computed as $\bar{d} = (1/T)\sum_{t=1}^{T}[u_{1,t}^2 - u_{2,t}^2]$, where $u_{i,t}$ is the time t forecast error of model i (i.e., $u_{i,t} = R_{i,t} - \widehat{R}_{i,t|t-1}$, where $\widehat{R}_{i,t|t-1}$ is the forecast of excess return $R_{i,t}$ at time t) and T is the total number of the forecasts (300). In a comparison pair, DFPM versus FF3, for instance, $u_{1,t}$ is the forecast error of the DFPM and $u_{2,t}$ is the forecast error of the FF3. Following Simin (2008), we compute DM-STAT based on Diebold and Mariano (1995) to test the null hypothesis of H_0: $\bar{d} = 0$. This test statistic is defined as

$$\text{DM-STAT} = \frac{\bar{d}}{\sqrt{\dfrac{2\pi \widehat{f}_d(0)}{T}}},$$

where $\widehat{f}_d(0)$ is a consistent estimator of the spectral density of $[u_{1,t}^2 - u_{2,t}^2]$ at frequency 0 and $2\pi/T$ is the length of time required for the process to repeat a full cycle. To examine if the average of $\bar{d}$s is 0, we also conduct the overall t-test for each of the pairs using $\bar{d}$s obtained from the 35 portfolios.

The results are reported in Table 7. Given the best performance of the simple benchmark (6%) reported in Simin (2008), we first compare the predictive ability of our fixed benchmark (9.35%) with that of the other 3 models. As Panels A and B indicate, the average MSFE differentials ($\bar{d}$s) of the first 2 pairs (CAPM vs. 9.35% and FF3 vs. 9.35%) are mostly positive (in both the 10 momentum portfolios and the 25 size- and BTM-sorted portfolios), and many of them are statistically significant at 5%. The overall t-test results in Panel C exhibit that $\bar{d}$s are positive on average and statistically significant at 1% for both pairs. This indicates that indeed the naive low cost forecast (9.35%) does a better job than the CAPM or the FF3 model, as Simin (2008) points out.

However, the 3rd comparison pair (DFPM vs. 9.35%) shows that $\bar{d}$s in many portfolios are negative (22 cases out of the 35 portfolios) and statistically significant at 5% in the extreme portfolios such as the loser and winner deciles as well as the small (MV1) stock portfolios. The overall t-test presents that $\bar{d}$s are negative on average and statistically different from 0 at the 1% level. This clearly demonstrates that the DFPM generally performs better than the naive benchmark in the 1-step-ahead forecasts.

We turn to the comparisons among the 3 models: CAPM, FF3, and DFPM. In examining the average MSFE differentials and the t-test for the 4th pair (FF3

[19]Note that, unlike our study, Simin (2008) uses the first 60 months (1926:01–1930:12) as the training period for estimating the first set of model parameters and repeats the estimation using the subsequent 60-month rolling windows. In his study, the 6% benchmark is the average of the CRSP value-weighted index returns during the initial 60-month training period (1926:01–1930:12).

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vs. CAPM), we see that the FF3 outperforms the CAPM, especially in the winner portfolios (deciles 6–10) and the high BTM (BTM3–BTM5) assets. Now we examine if the DFPM performs better than the FF3. As the last comparison pair (DFPM vs. FF3) exhibits, the average MSFE differentials are negative with no exception in the 10 momentum portfolios and significant at 5% in 2 cases. Panel B of Table 7 also indicates that the vast majority of $\bar{d}$s are negative except for some larger portfolios (MV4–MV5), and 6 cases are statistically significant at 5%. The overall t-test indicates that $\bar{d}$s are negative on average and significant at the 1%

TABLE 7

Out-of-Sample Tests: Comparison of Accuracy in One-Step-Ahead Forecasts

Table 7 presents the pair-wise comparison of the accuracy in 1-step-ahead forecasts from the competing models using the 35 portfolios (10 momentum portfolios and 25 size and book-to-market (BTM) sorted portfolios). As in Fama and French (1996), the 10 momentum portfolios in Panels A and D are formed by splitting NYSE common stocks into deciles after sorting in ascending order by the continuously compounded return over the past 11 months (from month $t-1$ to $t-11$). Thus, decile 1 in Panels A and D is the "loser" portfolio, while decile 10 is the "winner" portfolio. Similarly, the 25 portfolios in Panels B and E are formed by first splitting the component stocks into 5 portfolios based on firm size (market capitalization, MV), and then by splitting each of the 5 portfolios again into 5 portfolios based on BTM in ascending order. Thus, MV1BM5 in Panels B and E denotes the portfolio that includes stocks of size (MV) group 1 (smallest) and BTM group 5 (highest). The competing models are as follows: i) CAPM: the capital asset pricing model; ii) 9.35%: a simple benchmark that uses a fixed annual rate of 9.35% (monthly 0.78%), which is the time-series average of the CRSP value-weighted index returns over the 300-month training period (1955:01–1979:12); iii) FF3: the Fama-French (1993) 3-factor model; iv) DFPM: the dynamic-factor model defined in Section IV; v) CARHART: Carhart's (1997) 4-factor model, which includes an additional factor, "winner" portfolio return minus "loser" portfolio return (UMD); and vi) CFF3(TB1m): a conditional beta version of the FF3 model, where each of the loadings on the FF3s is modeled as a linear function of the (lagged) 1-month T-bill rate. The first 300 months (1955:01–1979:12) of our whole sample period (600 months: 1955:01–2004:12) are used as a training period in order to estimate the first set of model parameters. The first 1-step-ahead forecast is then computed for January 1980 using the estimated model parameters. The 1-step-ahead forecasts for the remaining 299 months (1980:02–2004:12) are also computed based on the model parameters estimated using the relevant data from the 300-month *rolling* windows, resulting in the total 300 forecasts (1980:01–2004:12) for each model. For pair-wise comparison between the competing models, the average of differentials in the mean squared forecast errors (MSFEs), $\bar{d}$, is computed as

$$\bar{d} = \frac{1}{T}\sum_{t=1}^{T}\left[u_{1,t}^2 - u_{2,t}^2\right],$$

where $u_{i,t}$ is the time t forecast error of model i (i.e., $u_{i,t} = R_{i,t} - \hat{R}_{i,t|t-1}$, where $\hat{R}_{i,t|t-1}$ is the forecast of excess return $R_{i,t}$ at time t) and T is the total number of the forecasts (300). In the comparison pair of DFPM versus FF3 in the table, for instance, $u_{1,t}$ is the forecast error of DFPM and $u_{2,t}$ is the forecast error of FF3. To test the null hypothesis of H_0: $\bar{d} = 0$, a test statistic, DM-STAT, is computed based on Diebold and Mariano (1995). The test statistic is defined as

$$\text{DM-STAT} = \frac{\bar{d}}{\sqrt{\frac{2\pi \hat{f}_d(0)}{T}}},$$

where $\hat{f}_d(0)$ is a consistent estimator of the spectral density of $[u_{1,t}^2 - u_{2,t}^2]$ at frequency 0 and $2\pi/T$ is the length of time required for the process to repeat a full cycle. Panels C and F contain the results for the overall t-test for each of the 12 pairs based on $\bar{d}$s obtained from the 35 portfolios. * and ** indicate $\bar{d}$s significantly different from 0 at the 5% and 1% levels, respectively.

	Average of Differentials in MSFEs ($\bar{d}$)				
Portfolio	CAPM vs. 9.35%	FF3 vs. 9.35%	DFPM vs. 9.35%	FF3 vs. CAPM	DFPM vs. FF3
Panel A. 10 Portfolios Formed on Past Return (Momentum)					
1 (loser)	–0.361*	0.529**	–2.391**	0.791*	–3.045**
2	0.019	0.078	–0.477	–0.043	–0.643
3	0.036	0.051	–0.208	–0.076	–0.300
4	0.155	0.042	–0.194	–0.201	–0.296
5	0.209	0.086	0.000	–0.196	–0.111
6	0.215	0.080	0.053	–0.213*	–0.051
7	0.278*	0.089	–0.056	–0.252*	–0.121
8	0.337**	0.134*	–0.147	–0.270**	–0.253
9	0.356*	0.126	–0.302	–0.295**	–0.401
10 (winner)	0.316*	0.142	–1.123*	–0.248**	–1.198*

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TABLE 7 (continued)

Out-of-Sample Tests: Comparison of Accuracy in One-Step-Ahead Forecasts

	Average of Differentials in MSFEs ($\bar{d}$)				
Portfolio	CAPM vs. 9.35%	FF3 vs. 9.35%	DFPM vs. 9.35%	FF3 vs. CAPM	DFPM vs. FF3
Panel B. 25 Portfolios Formed on Size (MV) and Book-to-Market Equity (BTM)					
MV1BTM1	–0.250	–0.069	–4.608**	0.166	–4.477**
MV1BTM2	0.035	0.084	–2.462**	–0.029	–2.574**
MV1BTM3	0.194	0.049	–1.627**	–0.231	–1.708**
MV1BTM4	0.350*	0.010	–1.959**	–0.424*	–1.971**
MV1BTM5	0.381	–0.136	–2.494**	–0.608*	–2.437**
MV2BTM1	–0.035	0.032	–1.288	0.064	–1.295
MV2BTM2	0.122	0.113*	–0.474	–0.076	–0.592
MV2BTM3	0.331*	0.087	–0.337	–0.321*	–0.415
MV2BTM4	0.421**	0.025	–0.343	–0.464**	–0.379
MV2BTM5	0.358*	–0.115	–0.726*	–0.560*	–0.656*
MV3BTM1	0.088	0.121	–0.181	0.052	–0.376
MV3BTM2	0.216	0.156	0.145	–0.100	–0.029
MV3BTM3	0.214	0.099	–0.020	–0.200	–0.132
MV3BTM4	0.302	0.067	0.026	–0.312*	–0.054
MV3BTM5	0.442**	–0.164	–0.244	–0.677**	–0.137
MV4BTM1	0.074	0.184	0.087	0.148	–0.211
MV4BTM2	0.068	0.054	0.381	–0.054	0.247
MV4BTM3	0.194	0.068	–0.006	–0.209	–0.095
MV4BTM4	0.220	0.017	0.180	–0.288*	0.134
MV4BTM5	0.324*	–0.041	0.126	–0.432*	0.113
MV5BTM1	0.114	0.317	0.151	0.229	–0.259
MV5BTM2	0.165	0.185	0.423	0.020	0.111
MV5BTM3	0.093	0.041	0.404	–0.072*	0.239
MV5BTM4	0.142	0.049	0.278	–0.154	0.214
MV5BTM5	0.255	0.056	0.328	–0.281	0.255
Panel C. Overall t-Test					
Mean	0.182**	0.076**	–0.545**	–0.166**	–0.654**

	Average of Differentials in MSFEs ($\bar{d}$)		
Portfolio	DFPM vs. CARHART	FF3 vs. CFF3(TB1m)	DFPM vs. CFF3(TB1m)
Panel D. 10 Portfolios Formed on Past Return (Momentum)			
1 (loser)	–1.837**	–0.131**	–3.165**
2	–0.592	–0.021	–0.677*
3	–0.319	–0.010	–0.311
4	–0.375	–0.003	–0.301
5	–0.152	–0.008	–0.111
6	–0.066	–0.010	–0.057
7	–0.091	–0.021*	–0.139
8	–0.104	–0.018*	–0.270
9	–0.099	–0.041*	–0.443
10 (winner)	–0.722	–0.058**	–1.268*
Panel E. 25 Portfolios Formed on Size (MV) and Book-to-Market Equity (BTM)			
MV1BTM1	–4.550**	0.018	–4.477**
MV1BTM2	–2.588**	–0.002	–2.573**
MV1BTM3	–1.715**	–0.006	–1.710**
MV1BTM4	–1.982**	0.009	–1.961**
MV1BTM5	–2.452**	0.003	–2.433**
MV2BTM1	–1.326	0.001	–1.296
MV2BTM2	–0.624	0.009	–0.582
MV2BTM3	–0.442	–0.001	–0.418
MV2BTM4	–0.387	0.005	–0.376
MV2BTM5	–0.659*	0.013	–0.644*
MV3BTM1	–0.386	0.004	–0.365
MV3BTM2	–0.014	–0.004	–0.028
MV3BTM3	–0.147	–0.007	–0.139
MV3BTM4	–0.063	0.000	–0.052
MV3BTM5	–0.157	0.001	–0.136

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TABLE 7 (continued)

Out-of-Sample Tests: Comparison of Accuracy in One-Step-Ahead Forecasts

	Average of Differentials in MSFEs ($\bar{d}$)		
Portfolio	DFPM vs. CARHART	FF3 vs. CFF3(TB1m)	DFPM vs. CFF3(TB1m)
MV4BTM1	–0.224	0.009	–0.204
MV4BTM2	0.217	–0.009	0.242
MV4BTM3	–0.134	–0.004	–0.100
MV4BTM4	0.094	0.007	0.141
MV4BTM5	0.083	0.002	0.121
MV5BTM1	–0.318	0.047	–0.210
MV5BTM2	0.117	–0.002	0.107
MV5BTM3	0.283	–0.010	0.226
MV5BTM4	0.195	–0.001	0.214
MV5BTM5	0.263	–0.004	0.258
Panel F. Overall t-Test			
Mean	–0.608**	–0.007	–0.661**

level, demonstrating that the DFPM provides more accurate forecasts than the FF3. It is worth noting that the better predictive ability of the DFPM over the FF3 occurs mainly in the extreme portfolios such as the loser/winner deciles (deciles 1 and 10) and the small stock portfolios (MV1–MV2). These portfolios are exactly the ones for which traditional pricing models have difficulties in pricing or forecasting. On balance, there is solid evidence that the DFPM outperforms the naive benchmark (9.35%) and the FF3 in the 1-step-ahead forecasts.

Now we look into the issue of whether the better predictive power of the DFPM over the alternatives may be primarily driven by using additional factors in the model. For this purpose, the DFPM is again compared with the models that have more than 3 factors. The results are presented in Panels D–F of Table 7. We first see that most of the average MSFE differentials ($\bar{d}$s) in the first comparison pair (DFPM vs. CARHART) are negative. In particular, the DM-STATs are statistically significant at the 1%–5% levels in the loser portfolio and in the smaller-sized portfolios (MV1–MV2). The overall t-test shows that $\bar{d}$s are negative on average and statistically different from 0 at the 1% level, indicating that the DFPM generally outperforms the Carhart (1997) 4-factor model. Next, in the last pair we compare the DFPM with a time-varying beta version of the FF3 model (CFF3(TB1m)), which is a 6-factor equivalent. The results demonstrate that the DFPM again performs better than the conditional model, particularly in loser/winner deciles 1, 2, and 10 as well as in the smaller-sized portfolios (MV1–MV2).

As our last check, we examine how the CFF3 models compare to the unconditional version in the out-of-sample context. As can be seen in the middle pair (FF3 vs. CFF3(TB1m)), the negative $\bar{d}$s (with those in deciles 1 and 7–10 being significant) and the overall t-test suggest that the CFF3 model (CFF3(TB1m)) does not perform any better than the unconditional FF3 model. Although unreported, comparisons of the FF3 with the other 4 CFF3s (CFF3(TB3m1m), CFF3(TERM), CFF3(DEF), and CFF3(DIV)) show virtually the same patterns, confirming Simin's (2008) findings.

To sum up, the DFPM outperforms the competing models in the out-of-sample tests. We find that the ex ante dynamic factors, especially $D_{\mathrm{BTM},t|t-1}$

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and $D_{\text{SIZE},t|t-1}$, play key roles in describing and forecasting asset returns. We conjecture that the better explanatory and predictive power of our factors is related to their lower correlations within each group but higher serial correlations, which may stem from the dynamic features of the Kalman filter. As we have seen in Table 2, the FF3s are highly correlated with each other, while the ex ante and ex post factors are weakly correlated with each other within each of the 2 groups. However, autocorrelations of the ex post and ex ante factors are much higher than those of the FF3s. In particular, the 2nd-order autocorrelation of the ex ante BTM factor is relatively high (10%), compared with that of HML (4%). Moreover, the ex post and ex ante size factors have negative and substantially high 1st-order autocorrelations (−14%). These differences may induce the higher explanatory and predictive power of the dynamic factors relative to the FF3s. This is an interesting issue that deserves further investigation in future research.

VII. Conclusion

The Fama and French (1993) 3-factor (FF3) model has been subject to controversy because its empirical performance in describing stock returns is not unambiguously successful. Researchers also recognize that the forecast ability of some macroeconomic variables may be spurious, while predictive variables are not empirically observable, but their predictable elements may instead be reflected on some latent variables. In this study, we develop a model that incorporates features of price dynamics across assets as well as through time. These aspects of our factors extracted via the Kalman filter make the model distinctive from the FF3 model or other statistical factor models.

With the extracted factors, we formulate the dynamic factor pricing model (DFPM), which employs both the ex ante factors and the ex post factors. We then conduct in-sample asset pricing tests for the model through the 2-pass regression procedure using the 25 size- and BTM-sorted portfolios or more extensive sets of assets. In addition, we perform out-of-sample tests, thereby presenting pair-wise comparisons of the accuracy in 1-step-ahead forecasts.

We provide evidence that the ex ante factors, especially BTM and size factors, are a key component in asset pricing and prediction. The high explanatory and predictive power of the residual loadings on the ex ante factors leads the DFPM to outperform other competing models such as the FF3 model, the Carhart (1997) 4-factor model, and the conditional versions of the FF3 model, especially in the out-of-sample tests. In particular, owing to the role of the ex ante size factor, the DFPM can explain and better forecast the momentum portfolio returns, which are mostly missed by alternative models.

Appendix. The Kalman Filter as a Model of Conditional Expectations

At the beginning of month t, investors make prior assessments about the conditional means and variances of the unobserved factors (D_t) based on the information set I_{t-1}, which have the following characteristics:

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Property 1. Producing a Forecast of D_t Based on I_{t-1}. In a conditional distribution $D_t|I_{t-1} \sim N(D_{t|t-1}, P_{t|t-1})$ and from equation (4),

i) A vector of the ex ante (prior) expectations of the unobserved true factors (D_t) is given by $D_{t|t-1} = \Phi D_{t-1|t-1}$.

ii) $P_{t|t-1} = \Phi P_{t-1|t-1}\Phi' + \Omega$ is a function of the population parameters (hence a constant matrix).

iii) $D_{t|t-1}$ is the minimum mean squared error (MMSE) estimator of D_t with respect to $P_{t|t-1}$.

$D_{t|t-1}$ defines a vector of time-varying expected factors, which we term as the ex ante factors. The MMSE property of $D_{t|t-1}$ is a well-established result of the Kalman filter.[20] Constant $P_{t|t-1}$ follows from the stationary joint normal distributions of $\underline{R}_t$ and D_t. This characterizes the conditional structure of the model as having time-varying expected factors ($D_{t|t-1}$), constant conditional factor variances ($P_{t|t-1}$), and constant factor loadings (B), as in Stambaugh (1983). The constant and MMSE features of $P_{t|t-1}$ imply that we can define a time-varying maximum ex ante squared Sharpe ratio (SSR) as

$$\text{(A-1)} \qquad \text{SSR}_{t|t-1} \equiv D'_{t|t-1}P^{-1}_{t|t-1}D_{t|t-1}, \tag{A-1}$$

which reflects investors' optimal forecast of the anticipated reward-risk tradeoff in the factors. Due to the constant property of $P_{t|t-1}$, any intertemporal variation in $\text{SSR}_{t|t-1}$ is solely driven by the ex ante forecasts, $D_{t|t-1}$, that describe anticipated shifts in the investment opportunity set.

At the end of month t, investors make contemporaneous (real-time) assessments about the conditional means and variances of the unobserved factors as the data ($\underline{R}_t$) are observed, leading to the following characteristics:

Property 2. Updating the Inference about D_t Based on I_t. In a conditional distribution $D_t|I_t \sim N(D_{t|t}, P_{t|t})$,

i) A vector of the ex post (posterior) expectations of the true factors (D_t) is given by

$$\text{(A-2)} \qquad D_{t|t} = D_{t|t-1} + K_t e_{t|t-1}, \tag{A-2}$$

where $K_t \equiv P_{t|t-1}B'\Sigma^{-1}_{t|t-1}$ is the Kalman gain matrix (which is a function of the population parameters). In the Kalman gain matrix, $\Sigma_{t|t-1} \equiv (BP_{t|t-1}B' + \Delta)$ is the variance-covariance matrix of the forecast error, $e_{t|t-1} \equiv \underline{R}_t - \underline{R}_{t|t-1}$.

ii) $P_{t|t} = P_{t|t-1} - K_t B P_{t|t-1}$ is a function of the population parameters (hence a constant matrix).

iii) $D_{t|t}$ is the MMSE estimator of D_t with respect to $P_{t|t}$.

$D_{t|t}$ defines a set of ex post factors that capture investors' contemporaneous beliefs about the state of financial markets, and $P_{t|t}$ measures the constant real-time uncertainty when making new inferences about the unobserved factors, D_t. As we see in equation (A-2), the ex post factors ($D_{t|t}$) are formed as a linear

[20] Harvey (1989) and Hamilton (1994) provide details on the MMSE estimators obtained from the Kalman filter.

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combination of the factors predicted at time $t-1$ ($D_{t|t-1}$) and contemporaneous information surprises contained in the forecast error ($e_{t|t-1}$), with the weight being the constant (3×6) Kalman gain matrix, K_t. The MMSE property means that $D_{t|t}$ is extracted in a way that the variance of tracking error ($D_t - D_{t|t}$) is minimized. Thus, $D_{t|t}$ is said to mimic D_t in the MMSE sense, so that the ex post factors can replace the unobserved true factors within a finite sample of assets for exact arbitrage pricing.[21]

It is worth mentioning some advantages of our approach (that estimates the latent factors using the Kalman filter) relative to other statistical methods (e.g., principal component analysis). For instance, principal components are static and thus lack dynamic features. Moreover, they are purely statistical factors that cannot be intuitively understood or named. With the cross-sectional restrictions on the factor-loading matrix (i.e., equations (9) and (10)), however, our dynamic factors are intuitive and reasonably interpretable, which in turn enables us to call them (D_{MKT}, D_{SIZE}, D_{BTM}) market, size, and BTM factors. On balance, our employing the Kalman filter is justified, especially because the filter is a useful vehicle as an adaptive process with which a sequence of updating and forecasting becomes available in order to form rational expectations as the information set evolves over time.

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