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INEQUALITY AVERSION AND THE OPTIMAL COMPOSITION OF GOVERNMENT EXPENDITURE

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This paper examines the choice between government expenditure on public goods and transfer payments, in the form of a pension, in an overlapping-generations model. Government expenditure is tax-financed on a pay-as-you-go basis. A utilitarian judge chooses expenditures to maximize a social welfare function. The nonlinear solution is found to involve the ratio of a welfare-weighted average income, which depends on the inequality aversion of the judge, to arithmetic mean income. An approximation for this ratio is found that produces explicit solutions for the optimal composition. The result is used to obtain an indication of "implicit" inequality aversion for a range of countries.

Keywords: Overlapping Generations, Composition of Government Expenditure, Utilitarian Social Welfare Function

1. INTRODUCTION

A number of recent studies emphasize the importance of unobserved cultural factors in shaping tax and expenditure policies. Alesina and Glaeser (2004), on the basis of cross-national survey data, conjecture that different preferences for a more equal income distribution in Europe and the United States may play an important role in accounting for the considerable differences in their expenditure policies. Corneo and Grüner (2002), using a large international survey, present empirical evidence of systematic differences across countries in the preference for redistributive policies. To understand the way in which different views regarding inequality may influence the composition of expenditure, this paper carries out a normative exploration of the expenditure mix that would be chosen by a utilitarian judge having a degree of inequality aversion.

The framework of analysis is that of the choice of expenditure on nonrival and nonexcludable public goods and transfer payments, in the form of an unconditional

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pension, in a growing economy with overlapping generations. Expenditure is tax-financed on a pay-as-you-go basis. The utilitarian judge is regarded as maximizing a social welfare function expressed in terms of the utilities of heterogeneous individuals, subject to the pay-as-you-go financing constraint. By taking a growth context and focusing on the expenditure composition for a given tax rate,[1] the model allows for a range of factors other than the judge's inequality aversion that may contribute to the division of expenditure, such as consumers' preferences, tax and interest rates, and the growth rates of income and population. The transfer payment in the model is referred to as a pension, because it is received in the second period of life. However, it may be thought of more broadly as a standard type of income transfer because it involves a decision regarding income shifting between periods within the life cycle as well as intra- and intergenerational redistribution.

The optimal expenditure ratio, defined as the ratio of transfer payments (the pension) to public goods expenditure, is shown to depend on a welfare-weighted mean income in relation to arithmetic mean income, along with individuals' preferences for public goods, the discount rate, population and income growth rates, and the tax rate. However, a closed-form solution for the expenditure ratio is not available, as the welfare-weighted mean income in general depends on the choice of pension expenditure.[2] A useful approximation is therefore considered for the welfare-weighted mean income, using an explicit form for the welfare function. This produces an approximate solution for the optimal expenditure ratio, and simulations show that the approximation performs well. Comparative statics based on the approximate solution are shown to accord with expectations. In particular, a higher inequality aversion gives rise, ceteris paribus, to the transfer payment forming a larger proportion of total government expenditure.

It is of course unreasonable to suggest that, in practice, expenditure decisions arise from the optimizing plans of a single omniscient and omnipotent judge. However, it is of interest to consider the hypothetical question of what value judgments could be considered to be implicit in the observed expenditure policies of countries. The approximate closed-form solution for the optimal expenditure ratio allows an implied value of inequality aversion to be obtained for a range of countries, given data for other determinants of the expenditure ratio for each country. The results show that different degrees of inequality aversion are evident among the countries examined. The average implied inequality aversion for Scandinavian countries is consistently higher than that of Commonwealth countries, whereas the United States has a lower degree of inequality aversion than average Scandinavian and Commonwealth countries. These results are consistent with those of Lambert et al. (2003), who measure the degree of inequality aversion for a large sample of countries from income inequality indexes. Germany has the highest implied inequality aversion among all countries. Significant inequality aversion in Germany was also found in Lambert et al. (2003) and Schwarze and Härpfer (2007), where the latter use regional life-satisfaction data to estimate inequality aversion in Germany. These results suggest that variations in expenditure patterns could possibly reflect different social preferences regarding inequality, which may result

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from cultural differences across countries. This study thus provides a structural way to investigate the conjecture of Alesina and Glaeser (2004).

In view of the complexity of decision making in practice and the frequent need for compromises, it is typically far from straightforward to "reconstruct" the values that may be implied by actual policies. The evaluation of implicit values can thus help to judge the consistency of actual policies with certain stated objectives of policymakers and commentators. This study therefore relates to a large literature that attempts to recover implicit value judgments in government tax and expenditure policies. For example, early attempts to impute a value of inequality aversion implicit in government tax decisions include Stern (1977) and Christiansen and Jansen (1978).[3] More recent studies include Madden (1992, 1995) and Cragg (1991) in the context of indirect tax and Oliver and Spadaro (2004) and Spadaro (2007) in the context of direct taxation.[4] Lambert et al. (2003) and Schwarze and Härpfer (2007), as mentioned above, provide empirical examinations of attitudes toward inequality. By using a structural model to infer inequality aversion for a range of countries, the present study obtains results that are consistent with these earlier studies.

The paper is arranged as follows. Section 2 describes the economic environment. Section 3 examines the optimal choice of expenditure by the utilitarian judge. Section 4 describes the approximation that produces the closed-form solution for the expenditure ratio and discusses the comparative statics. Implicit value judgments for a range of countries are investigated in Section 5. Brief conclusions are presented in Section 6.

2. THE ECONOMIC ENVIRONMENT

This section describes an overlapping-generations model in which a pure public good and a transfer payment, in the form of a pension, are tax-financed on a pay-as-you-go basis.

Each individual is assumed to live for two periods, a working and a retirement period, so that the economy is populated by two overlapping cohorts in any given period. Let N_t denote the number of individuals born in period t. Individuals have identical preferences but are heterogeneous with respect to incomes. A young individual i, born at time t, works in the first period and receives an exogenously fixed income, $y_{i,t}$. Income is taxed at rate τ, which is the same for all individuals and is assumed to be exogenously determined. In period t, young individual i allocates disposable income between current consumption, $c_{1i,t}$, and savings, $s_{i,t}$. In the second period of life, the individual finances consumption of private goods, $c_{2i,t+1}$, using the unconditional and untaxed pension from the government, b_{t+1}, and the return on savings, $(1+r)s_{i,t}$, where r is the constant interest rate at which individuals can borrow or lend. The price of the private consumption goods is normalized to unity, so that c also denotes private consumption expenditure.

The quantity of the pure public good in period t is denoted as Q_t, which can be consumed by all individuals in period t. Assume that the production of public

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goods involves a constant unit cost, p. Then government expenditure on public goods in t is simply $G_t \equiv pQ_t$.

Suppose all individuals have Cobb–Douglas preferences, expressed for person i as

$$U_{i,t} = c_{1i,t} c_{2i,t+1}^{\beta} Q_t^{\gamma} Q_{t+1}^{\gamma\beta}, \tag{1}$$

where $0 < \beta = 1/(1+\rho) < 1$ is the discount factor, ρ is the time preference rate, and γ is the utility weight attached to consumption of public goods, reflecting the preference for public goods relative to private goods.

In view of the intergenerational transfers, in addition to the income shifting and intra-generational redistribution, it is desirable to allow for population and income growth. Suppose the average income of young individuals grows at a constant rate of ω over time, and there is constant growth, at a rate n, in the population. It is assumed that income growth involves an equal proportional change at all income levels and that population growth involves an equal proportional change in population frequencies at each income level, so that ω and n are independent.

3. THE OPTIMAL CHOICE OF A UTILITARIAN JUDGE

This section investigates the optimal choice of expenditure by a utilitarian judge who aims to maximize a social welfare function. It begins by describing each individual's consumption and saving choice conditional on government expenditures, which yields indirect utility as a function of those expenditures. It then defines the social welfare function in terms of individual utilities and characterizes the optimal expenditure ratio, defined as the ratio of total expenditure on pensions to that on public goods, that maximizes the social welfare function.

From Section 2, the lifetime budget constraint of individual i is given by

$$c_{1i,t} + \frac{c_{2i,t+1}}{(1+r)} = (1-\tau) y_{i,t} + \frac{b_{t+1}}{(1+r)} \equiv M_{i,t}. \tag{2}$$

This form allows for the fact that tax-financed public goods are nonexcludable, so that individuals are not charged at the point of consumption. The consumption plans, conditional on the values of public expenditure and the pension, are given, using the standard properties of Cobb–Douglas utility functions, as

$$c_{1i,t} = M_{i,t}/(1+\beta), \quad c_{2i,t+1} = \beta\,(1+r)\,M_{i,t}/(1+\beta). \tag{3}$$

Thus the indirect utility function, $V_{i,t}$, is

$$V_{i,t} = \left(\frac{M_{i,t}}{1+\beta}\right)\left[\frac{\beta\,(1+r)\,M_{i,t}}{(1+\beta)}\right]^{\beta} Q_t^{\gamma} Q_{t+1}^{\gamma\beta}. \tag{4}$$

The utilitarian judge, recognizing that individuals' consumption and saving decisions are conditional on its expenditure choices, chooses a sequence of expenditures on public goods and pensions, $\{b_{t+1}, G_{t+1}\}_{t=0}^{\infty}$ (b_0 and G_0 are given), to

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maximize a social welfare function, subject to the government budget constraint in every period.

In the pay-as-you-go financing structure, income tax revenue in each period must be sufficient to finance the transfer payments to those currently retired along with the public good. Hence, at time t, the government budget constraint, where $\bar{y}_t$ denotes arithmetic mean income, is given by

$$G_t + N_{t-1} b_t = \tau N_t \bar{y}_t. \tag{5}$$

The social welfare function is defined as the following discounted sum of all generations' utilities:

$$\mathrm{SW} = \sum_{t=-1}^{\infty} \delta^t W_t, \tag{6}$$

where δ is the judge's discount factor, which could be different from private individuals' discount factor, β. The social welfare associated with generation t, W_t, is defined as a function of utilities of all individuals born in period t,

$$W_t = W(V_{1,t}, \ldots, V_{i,t}, \ldots V_{N_t,t}), \tag{7}$$

where $V_{i,t}$ is defined in (4) and the function W is time-invariant, individualistic, and Paretian. To ensure that the problem is well defined, it is also assumed that W is additively separable and is strictly increasing and (weakly) concave in each of its arguments.[5]

A simplification arises from the fact that the imposition of pay-as-you-go financing, as well as the two-period overlapping-generations nature of the model, ensures that maximization of the social welfare function in (6) is equivalent to the maximization of W_t in every period t. This property also arises because there is no income shifting between other "disconnected" generations, only those overlapping at any time.

Thus the first-order conditions for choice of b_{t+1}, for $t = 0, 1, \ldots$, are given by

$$\sum_{i=1}^{N_t} \frac{\partial W_t}{\partial V_{i,t}} \left(\frac{\partial V_{i,t}}{\partial b_{t+1}} + \frac{\partial V_{i,t}}{\partial G_{t+1}} \frac{dG_{t+1}}{db_{t+1}} \right) = 0. \tag{8}$$

Define $v_{i,t} = (\partial W_t / \partial V_{i,t})(\partial V_{i,t} / \partial b_{t+1})$ as the welfare weight attached to an increase in i's pension income. Then

$$\sum_{i=1}^{N_t} \frac{\partial W_t}{\partial V_{i,t}} \frac{\partial V_{i,t}}{\partial G_{t+1}} = \sum_{i=1}^{N_t} v_{i,t} \left(\frac{\partial V_{i,t} / \partial G_{t+1}}{\partial V_{i,t} / \partial b_{t+1}} \right). \tag{9}$$

Hence, writing $\tilde{v}_{i,t} = v_{i,t} / \sum_{i=1}^{N_t} v_{i,t}$, where $\sum_{i=1}^{N_t} \tilde{v}_{i,t} = 1$, and noting that from the government budget constraint in (5), $dG_{t+1}/db_{t+1} = -N_t$, substitution in (8)

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for each t gives

$$\sum_{i=1}^{N_t} \tilde{v}_{i,t} \left(\frac{\partial V_{i,t}/\partial G_{t+1}}{\partial V_{i,t}/\partial b_{t+1}} \right) = \frac{1}{N_t}. \tag{10}$$

This result does not depend on the precise form of V or W, but further progress can be made using (4) above, which gives

$$\frac{\partial V_{i,t}}{\partial G_{t+1}} = \frac{\beta\gamma V_{i,t}}{G_{t+1}}, \tag{11}$$

$$\frac{\partial V_{i,t}}{\partial b_{t+1}} = \frac{(1+\beta)V_{i,t}}{(1+r)(1-\tau)y_{i,t} + b_{t+1}}. \tag{12}$$

Substituting into (10) and writing $\tilde{y}_t = \sum_i^{N_t} \tilde{v}_{i,t} y_{i,t}$ gives

$$\frac{\beta\gamma\{(1+r)(1-\tau)\tilde{y}_t + b_{t+1}\}}{(1+\beta)G_{t+1}} = \frac{1}{N_t}. \tag{13}$$

The term $\tilde{y}_t$ is a welfare-weighted average of individual incomes with weights $\tilde{v}_{i,t}$. Substituting the government budget constraint into (13) gives the optimal per capita expenditure on the pension, b_{t+1}. Therefore the ratio of expenditure on pensions to total expenditure, $B_{t+1} \equiv N_t b_{t+1}/(\tau N_{t+1}\bar{y}_{t+1})$, is given by

$$B_{t+1} = \frac{1}{(1+\beta+\beta\gamma)} \left\{ (1+\beta) - \beta\gamma \frac{(1+r)(1-\tau)}{\tau\,(1+n)\,(1+\omega)} \frac{\tilde{y}_t}{\bar{y}_t} \right\}. \tag{14}$$

The share of total expenditure on public goods, $G_{t+1}/\tau N_{t+1}\bar{y}_{t+1}$, is thus simply obtained as $1 - B_{t+1}$. Therefore the ratio of total expenditure on pensions to that on public goods, R_{t+1}, is

$$R_{t+1} = \frac{[(1+\beta)/\beta\gamma]\,(1+n)(1+\omega) - (1+r)\left(\frac{1-\tau}{\tau}\right)(\tilde{y}_t/\bar{y}_t)}{(1+n)(1+\omega) + (1+r)\left(\frac{1-\tau}{\tau}\right)(\tilde{y}_t/\bar{y}_t)}. \tag{15}$$

This result shows how the optimal expenditure ratio, R_{t+1}, depends, inter alia, on the ratio of the welfare-weighted mean income to the arithmetic mean income at time t as well as parameters regarding population growth, income growth, the tax rate, and preferences.[6] This optimal choice is characterized by a balanced growth path, along which all aggregate endogenous variables grow at the same rate as aggregate income, and per capita variables grow at the same rate as average income.

The expression in (15) cannot, of course, be regarded as providing a closed-form solution for the expenditure ratio, as the welfare-weighted mean income, in general, depends on the value of b. However, a useful approximation is considered in the following section, using an explicit form of the welfare function.

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4. APPROXIMATING THE OPTIMAL EXPENDITURE RATIO

This section shows how an approximation to the optimal expenditure ratio can be obtained by approximating the welfare-weighted average income, $\tilde{y}_t$, using a widely adopted social welfare function that takes the isoelastic form[7]

$$W_t = \frac{1}{1-\varepsilon}\sum_{i=1}^{N_t} V_{i,t}^{1-\varepsilon}, \tag{16}$$

where $\varepsilon \geq 0$ denotes the relative inequality aversion of the judge. The case of $\varepsilon = 0$ corresponds to the absence of inequality aversion and gives the "classical utilitarian" welfare function.

Thus $\partial W_t/\partial V_{i,t} = V_{i,t}^{-\varepsilon}$, and obtaining $\partial V_{i,t}/\partial b_{t+1}$ from (4), the welfare weights, $v_{i,t} = (\partial W_t/\partial V_{i,t})(\partial V_{i,t}/\partial b_{t+1})$, are[8]

$$v_{i,t} = \frac{(1+\beta)\left[\dfrac{\beta^\beta(1+r)^\beta Q_t^\gamma Q_{t+1}^{\beta\gamma}}{(1+\beta)^{(\beta+1)}}\right]^{1-\varepsilon}}{(1+r)}\left[(1-\tau)y_{i,t} + \frac{b_{t+1}}{(1+r)}\right]^{\beta-\varepsilon(1+\beta)}. \tag{17}$$

This form is clearly intractable, but it is reasonable to suppose that b_{t+1} is small relative to $y_{i,t}$.[9] In this case an approximation, $\tilde{y}_{A,t}$, for the welfare-weighted mean is obtained as the ratio of two fractional moments:

$$\tilde{y}_{A,t} = \frac{\frac{1}{N_t}\sum y_{i,t}^{1+\beta-\varepsilon(1+\beta)}}{\frac{1}{N_t}\sum y_{i,t}^{\beta-\varepsilon(1+\beta)}}. \tag{18}$$

Suppose further that $y_{i,t}$ is lognormally distributed as $\Lambda(y_{i,t}|\mu_t, \sigma_t^2)$, with mean and variance of logarithms of μ_t and σ_t^2, respectively. Using properties of the lognormal moment generating function, it can be found that[10]

$$\frac{\tilde{y}_{A,t}}{\bar{y}_t} = \exp\left[\{2\beta - 2\varepsilon(1+\beta)\}\frac{\sigma_t^2}{2}\right]. \tag{19}$$

Higher ε therefore reduces $\tilde{y}_{A,t}$ relative to $\bar{y}_t$, by giving a lower weight to higher incomes. The classical utilitarian case of $\varepsilon = 0$ gives rise to $\tilde{y}_t/\bar{y}_t = \exp(2\beta)$. Using (19) to approximate $\tilde{y}_t/\bar{y}_t$ in (14) and (15) gives a closed-form solution for the transfer as a proportion of total expenditure and the expenditure ratio. Appendix A conducts a simulation exercise that shows that this approximation works well. Note from (14) that the transfer B could be negative if γ were sufficiently large while ε was sufficiently low, implying a poll tax.

It is now possible to discuss the comparative static properties of the model. Examination of (15), using (19) to approximate $\tilde{y}_t/\bar{y}_t$, shows that a higher tax rate is associated with a greater proportion of expenditure being devoted to transfer

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payments, whereby R_{t+1} is increasing in τ at a decreasing rate. An increase in the tax rate gives the government more income to spend on both types of expenditure, but the increase in pension expenditure is relatively higher than the increase in public goods expenditure.[11] Population and income growth have similar effects, as they appear in (15) together in the form $(1 + n)(1 + \omega)$: they are perfect substitutes for each other. Increases in ω and n increase the share of expenditure on pensions and decrease the share of expenditure on public goods, and also their ratio. With income growth or population growth, tax revenues of the government are increased so that the government is able to spend more on both types of expenditure. However, the increase in the expenditure on pensions is higher than the expenditure on public goods.

An increase in the interest rate has a negative effect on the share of expenditure on pensions and on the ratio of pensions to public goods, but it increases the total expenditure on public goods. A higher interest rate leads to more private savings by individuals at a young age. This results in a higher public goods expenditure and a lower ratio of pensions to public goods expenditure.

An increase in income inequality, as measured by σ^2, reduces $\tilde{y}_A/\bar{y}$, from (19), and thus increases the proportion of expenditure devoted to the pension. Hence more basic inequality implies a more redistributive expenditure policy. This result is consistent with models examining majority voting over the tax rate, with an unconditional transfer payment, in which a uniform result is that more basic inequality leads to the choice of a more redistributive tax and transfer structure; for example, see Meltzer and Richard (1981). Similarly, an increase in inequality aversion, ε, also reduces $\tilde{y}_A/\bar{y}$ and therefore raises the share of expenditure on redistribution.

The comparative static results with respect to the weight attached to public goods, γ, suggest that an increase in the preference for public goods unambiguously increases the share of expenditure on public goods, but decreases the share of expenditure on pensions and the ratio of pensions to public goods expenditure. Analytic results relating to β were found to be equivocal, but the numerical results reported below demonstrates that both B and R fall as β increases.

A numerical exercise is implemented to illustrate the comparative static properties and further examine the sensitivity of the optimal expenditure ratio to changes in parameters. The baseline values for parameters τ, r, β, ω, n, and σ^2 were set to the average of the sample reported in Appendix B. The baseline value for γ is 1. The baseline value for ε is the average implied ε across the sample countries for $\gamma = 1$. Section 5 describes how ε is calculated. The parameter variations considered are 10 and 20% changes around the baseline values. Table 1 reports the values of B and R as parameters vary around their baseline values. In each case only one parameter at a time is varied, with all other parameters kept at their baseline values.

The results show that the share of expenditure on pensions, public goods, and their ratio are quite sensitive to variations in the tax rate, the weight attached to public goods, the variance of the distribution of income, and the discount factor. They are less sensitive to changes in the interest rate and the growth rates of

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TABLE 1. Share of pension expenditure and expenditure ratio for alternative parameter values

		τ	r	β	ω	n	σ^2	ε	γ
	−20%	0.433	0.502	0.526	0.480	0.491	0.461	0.416	0.569
	−10%	0.468	0.499	0.509	0.489	0.493	0.479	0.459	0.531
B	Baseline	0.496	0.496	0.496	0.496	0.496	0.496	0.496	0.496
	+10%	0.519	0.493	0.485	0.502	0.498	0.511	0.526	0.463
	+20%	0.538	0.490	0.476	0.509	0.500	0.525	0.551	0.432
	−20%	0.764	1.007	1.108	0.924	0.965	0.856	0.711	1.319
	−10%	0.879	0.995	1.038	0.957	0.974	0.920	0.850	1.133
R	Baseline	0.984	0.984	0.984	0.984	0.984	0.984	0.984	0.984
	+10%	1.079	0.972	0.941	1.009	0.993	1.045	1.111	0.862
	+20%	1.165	0.960	0.908	1.037	1.002	1.105	1.229	0.760

population and income. The expenditure ratio is most sensitive to variations in ε around the baseline value. For the classical utilitarian welfare function, that is, for $\varepsilon = 0$, and other parameters at their baseline values, the resulting expenditure on transfers as a proportion of total expenditure is 0.056, and the expenditure ratio is 0.060. An absence of inequality aversion therefore implies some expenditure on redistributive transfer payments. This arises because the social welfare function is a simple sum of indirect utilities, which are in turn concave functions of transfer income. Hence there is a benefit from income shifting.[12]

5. AN APPLICATION OF THE MODEL: IMPLICIT INEQUALITY AVERSION

This section considers the following hypothetical question. Supposing that actual expenditure policies were determined by a utilitarian judge of the type considered above, what degree of inequality aversion is implied? This type of question has previously been considered in the context of static models of taxation. Actual policymaking is of course far too complicated to be captured by such a simple model, and in practice value judgments are extremely difficult to articulate precisely and thus are never expressed explicitly by policymakers. But information about implicit value judgments can provide a useful input into rational policy analysis.

The present framework allows an implied value, $\hat{\varepsilon}$, of inequality aversion to be obtained. Using the approximation for $\tilde{y}_A/\bar{y}$ in (19) above, $\hat{\varepsilon}$ is, after some rearrangement,

$$\hat{\varepsilon} = \frac{1}{(1+\beta)}\left[\beta + \frac{1}{2}\frac{\log(\tilde{y}_A/\bar{y})}{\left(-\sigma_t^2/2\right)}\right]. \tag{20}$$

For the lognormal distribution, median income is $y_m = e^{\mu}$, so the ratio of median to mean income is given by $\log(y_m/\bar{y}_t) = -\sigma_t^2/2$. Hence (20) can be

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seen to involve a ratio of two terms consisting of logarithms of income ratios, one containing the welfare-weighted mean and the other involving the median income.

From equation (14), it is possible to express $\tilde{y}/\bar{y}$, and hence $\tilde{y}_A/\bar{y}$, for the relevant period in terms of β, γ, τ, n, ω, r, and the share of pension expenditure, B. However, a problem is raised by the difficulty of obtaining detailed comparable data on the composition of government expenditure for a wide range of countries. A major difficulty in the present context is that inadequate information about expenditure components is available. Ideally, separate details regarding pension expenditure are required, but only aggregate expenditure on transfer payments can be obtained: the use of the aggregates effectively requires the assumption that the composition of transfers is similar across countries.[13] Hence the results presented here should be treated with much caution. Information about σ^2 can be obtained from the Gini coefficient, again using properties of the lognormal distribution (as described in Appendix B).

The data for relevant variables needed to infer $\hat{\varepsilon}$ are described in detail in Appendix B. Data for the Gini measure are not available for each year and for each country, so the average of 2000–2006 was used. The growth rates of income and population, as well as the real interest rate, were obtained from the average annual growth rates of GDP per capita and population, as well as the average annual real deposit interest rate between 2000 and 2006 for each country. The value of β is obtained, for each country, by assuming that the time preference rate is equal to the real interest rate. It is necessary to adjust the various rates, so that instead of being annual rates, they correspond to the overlapping-generations framework used here; this is discussed in Appendix B, assuming that a time period in the model is 20 years.

Information can be obtained from various data sources about all these variables, except for the value of γ, the relative weight attached to public goods in the utility function. Independent evidence on preferences for public goods is notoriously difficult to obtain, in view of the well-known problems of preference revelation with nonrival and nonexcludable goods. It is possible only to obtain, for a given country at any time, a value of $\tilde{y}/\bar{y}$ conditional on an assumed value of γ. Hence, the sensitivity of results to variations in γ is examined. In considering sensible values for γ, it was decided to use a "benchmark case" of $\gamma = 1$; that is, consumers have the same preference for public goods as for private goods. Then variations of 10 and 20% around this value were considered. The results reported here are for the benchmark case where individuals' preferences are assumed to be the same in all countries.[14]

Table 2 shows the implied values of inequality aversion, ε, for each country in the sample.[15] For each value of γ the countries are ranked in increasing order, that is, from low implied inequality aversion to high implied aversion. Furthermore, the countries are divided into various groups. The Scandinavian countries include Denmark, Finland, and Norway. The group of Commonwealth countries includes the United Kingdom, Australia, Canada, and New Zealand.

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TABLE 2. Implied inequality aversion for alternative values of γ

$\gamma = 0.8$		$\gamma = 0.9$		$\gamma = 1$		$\gamma = 1.1$		$\gamma = 1.2$	
Country	ε	Country	ε	Country	ε	Country	ε	Country	ε
Ireland	0.30	Ireland	0.49	Ireland	0.68	Ireland	0.87	Ireland	1.09
Greece	0.35	Greece	0.57	Greece	0.79	Greece	0.99	Greece	1.20
Hungary	0.44	Hungary	0.79	Hungary	1.13	Hungary	1.45	Hungary	1.75
Czech Rep.	0.49	Czech Rep.	0.90	Czech Rep.	1.30	Czech Rep.	1.68	U.S.	1.98
Iceland	0.61	Iceland	1.08	Iceland	1.50	U.S.	1.85	Czech Rep.	2.07
Belgium	0.99	Belgium	1.30	Belgium	1.60	Poland	1.86	Poland	2.09
Netherlands	1.01	Poland	1.36	Poland	1.61	Belgium	1.90	Italy	2.16
Poland	1.08	U.K.	1.42	Italy	1.67	Iceland	1.90	Belgium	2.20
U.K.	1.11	Netherlands	1.42	U.S.	1.71	Italy	1.91	N.Z.	2.21
Finland	1.16	Italy	1.44	U.K.	1.72	N.Z.	1.98	Iceland	2.27
N.Z.	1.19	N.Z.	1.47	N.Z.	1.73	U.K.	2.01	U.K.	2.29
Italy	1.20	U.S.	1.56	Netherlands	1.81	Netherlands	2.19	Netherlands	2.55
U.S.	1.41	Finland	1.60	Spain	2.01	Spain	2.31	Canada	2.61
Spain	1.43	Spain	1.72	Finland	2.05	Canada	2.38	Spain	2.64
Norway	1.51	Canada	1.93	Canada	2.16	Finland	2.50	Finland	2.97
France	1.54	Norway	2.02	Australia	2.42	Australia	2.74	Australia	3.07
Austria	1.68	France	2.02	Austria	2.49	Austria	2.89	Austria	3.29
Canada	1.69	Austria	2.09	Norway	2.50	Norway	2.96	Norway	3.41
Australia	1.77	Australia	2.10	France	2.50	France	2.99	France	3.50
Denmark	2.30	Denmark	2.86	Switzerland	3.33	Switzerland	3.66	Switzerland	4.01
Switzerland	2.69	Switzerland	3.01	Denmark	3.41	Denmark	3.94	Denmark	4.48
Germany	3.24	Germany	3.73	Germany	4.21	Germany	4.71	Germany	5.24
				Scandinavian countries					
Average	1.66		2.16		2.65		3.14		3.62
Coeff var	0.35		0.30		0.26		0.24		0.21
				Commonwealth countries					
Average	1.44		1.73		2.01		2.28		2.54
Coeff var	0.24		0.20		0.17		0.16		0.15
				All countries					
Average	1.33		1.68		2.01		2.35		2.69
Coeff var	0.56		0.47		0.42		0.39		0.37

From the generally high values of implied aversion reported in Table 2, it seems likely that the inability to measure a more restricted concept of transfer payments has imparted a significant upward bias. It is therefore perhaps more useful to concentrate on the relative orders of magnitude for different countries. There is much consistency in the rankings as γ is varied. The average implied inequality aversion for Scandinavian countries is consistently higher than that of Commonwealth countries, and the former are more homogeneous, as shown by the lower coefficient of variation. The relative positions of the two groups of countries accord with expectations. Countries within each group become more homogeneous with regard to the implicit inequality aversion as γ is increased. This can be seen by the reduction in the coefficient of variation within each group.

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A notable change in ranking by implied inequality aversion, as γ is increased, is for the United States. This country moves from around the middle to around the bottom quartile, which is perhaps more consistent with a priori expectations. Germany has the highest implied inequality aversion for all values of γ, which may perhaps be regarded as consistent with its "social-market model." Significant inequality aversion in Germany was also found by Schwarze and Härpfer (2007). Furthermore, Lambert et al. (2003) suggested that countries that have lower population growth tend to have higher inequality aversion, and indeed Germany has a much lower population growth rate than the other countries examined. Similarly, Scandinavian countries have relatively low population growth rates. Exceptions to this result are for the former communist countries of Hungary, Poland, and the Czech Republic, which have low population growth rates combined with relatively low implicit inequality aversion. The low implied inequality aversion for Ireland is consistent with the findings of Madden (1992, 1995).[16]

6. CONCLUSIONS

This paper has examined the optimal composition of government expenditure, as chosen by an independent utilitarian judge, in a two-period overlapping-generations model. The main focus was on public goods and a transfer payment, in the form of a pension, which are financed by income tax on a pay-as-you-go basis. The choice of expenditure by the judge, who maximizes a social welfare function, is complicated by the fact that the pension involves a combination of income-shifting between phases of the life cycle with both intergenerational and intragenerational transfers. The latter arise because the basic pension is unrelated to income, whereas the tax is proportional to income. The former arises from the pay-as-you-go feature of financing, whereby each generation can benefit from productivity and thus income growth accruing to the following generation. This modeling framework, despite its simplicity, therefore offers useful insights into the nature of the policy choices involved.

The analysis is a normative exercise in welfare economics rather than a positive attempt to explain differences in the composition of government expenditure in different countries. There is no suggestion that policies are actually based on the "simple" maximization of a clearly stated social welfare function. The same point of view is of course taken in all models that examine optimal policies, however defined. Nevertheless it is of interest to consider what values are implied by actual policies, using a "what if" approach. Thus, if outcomes are considered as arising from maximisation of a welfare function, what extent of inequality aversion do they reveal?

Expressions for the choice of expenditure levels and their ratio were found to depend on the ratio of a welfare-weighted average income to arithmetic mean income in each generation. This welfare-weighted arithmetic mean income level was shown to depend on the degree of relative inequality aversion of the judge. Using an approximation for this welfare-weighted mean income, it was thus

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possible to produce, for a range of countries, implicit values of the inequality aversion parameter, conditional on assumed preferences of individuals for public goods relative to private goods. The implicit values of inequality aversion, revealed by the policies of different countries, were generally found to accord with expectations and also consistent with previous studies. However, their absolute values are likely to be biased upward by the difficulty of measuring transfer payments, corresponding to the more limited concept specified in the overlapping-generations model constructed.

NOTES

1. Because the emphasis of the present study is on the composition of expenditure, rather than its total, the income tax rate is assumed to be exogenously fixed. In practice, tax and expenditure policies are debated separately.

2. It is known from the optimal tax literature that explicit solutions cannot be obtained, even in the simple case where a proportional tax finances only an unconditional transfer in a static framework.

3. See also Mera (1969), Moreh (1981), and Brent (1984). Ravallion (1988) examined inequality aversion implicit in regional disbursement policies.

4. By examining the shape of the implicit welfare function, Spadaro (2007) found that it was concave, thereby supporting the idea that tax policy could be regarded as the outcome of an optimizing process. Implicit values in the choice of adult equivalent scales were examined by van de Ven and Creedy (2005).

5. The social welfare function defined this way is consistent with requirements for time-consistent policy. See Calvo and Obstfeld (1988) and Ambler (2000) for further discussion of this issue in overlapping-generations models. A simple form of W_t adopted by many studies, such as Ghiglino (2000), defines W_t as the weighted sum of lifetime utilities of individuals in cohort t.

6. The expenditure ratio does not depend on the constant cost of producing the public good, p. This property arises from the Cobb–Douglas form of utility functions. The distinction between G and Q is nevertheless made here for transparency.

7. For $\varepsilon = 1$ this takes the form $W_t = \sum_{i=1}^{N_t} \log V_{i,t}$.

8. The results clearly depend on the precise cardinalisation of individuals' utility functions.

9. Data on pension expenditure are not available. However, the total expenditure on transfers, including pensions, accounts for about 20% of GDP on average for the countries examined below. Furthermore, the simulations in Appendix A show that the approximation works well even with $b/\bar{y}$ around 0.25.

10. The vth moment is given by $\mu_v = \exp(v\mu + v^2\sigma^2/2)$, and thus $\bar{y} = \mu_1 = \exp(\mu + \sigma^2/2)$; see Aitchison and Brown (1957).

11. However, in a static model with labor supply, transfers, and a public good, Creedy and Moslehi (2009) find a concave relationship between the share of transfers and the tax rate. This arises from adverse incentive effects of the tax and transfer system.

12. Similarly, in standard static optimal tax models the optimal tax, and thus transfer, is positive even when $\varepsilon = 0$.

13. Also, not all nontransfer expenditure is on pure public goods. Furthermore, transfer expenditure is in practice endogenous, because governments only set levels and qualifying conditions for the receipt of benefits, not total expenditure.

14. However, comparisons can be made across columns.

15. The values may be considered as conditional estimates based on a single year, and therefore having no degrees of freedom. As mentioned above, information for each country is not available for all years. Furthermore, there is relatively little variation in the income distribution over time.

16. Madden's results were, however, confined to implications of indirect tax structures.

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17. This implies that $y_m/\bar{y} = 0.85$, which is the average for the sample of countries described in Appendix B.

18. Aggregated data are described in the interpretative guide (see in particular Section 3.3, Annex 1, and Annex 4), which is available at http://stats.oecd.org/OECDStatDownloadFiles/_OECDSOCX2007InterpretativeGuide_En.pdf.

19. Data for interest rates in the United States, the United Kingdom, and Austria were obtained, respectively, from Web sites of the Federal Reserve Bank of the United States, the Bank of England, and the Central Bank of Austria.

20. Unfortunately, the income concept varies across countries. In some countries gross income is considered and in others it is net income.

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APPENDIX A: TESTING THE APPROXIMATION

It is important to test the value of the approximation derived in Section 4. Hence values of the expenditure components using the approximation $\tilde{y}_A$ were compared with those obtained using a simulated population of size 15,000 drawn at random from a lognormal distribution with $\mu = 9.0$ and $\sigma^2 = 0.32$.[17] Using the simulated distribution, a range of values of b_{t+1} were investigated. For each b_{t+1}, the government budget constraint was used to obtain G_{t+1} and the resulting values were used to calculate each individual's level of utility. These were then used to obtain social welfare, using the isoelastic function with a specified inequality aversion parameter, ε. Finally, given a number of W_t measures for different b_{t+1}, the maximum was determined, giving the optimal composition. Table A.1 gives the results for a range of degrees of inequality aversion, with other parameters set at the average of the sample reported in Appendix B. These results show that the approximation gives values of expenditure levels and ratios which are close to those obtained using a large simulated population.

TABLE A.1. Composition of government expenditure: Alternative solutions

	Approximation			Simulation		
ε	b_{t+1}	B_{t+1}	R	b_{t+1}	B_{t+1}	R
			$\gamma = 0.9$			
1.81	2813.4	0.51	1.03	2660	0.48	0.93
2.01	2934.52	0.53	1.13	2730	0.49	0.97
2.21	3041.62	0.55	1.22	2790	0.50	1.01
			$\gamma = 1$			
1.81	2537.23	0.46	0.85	2420	0.44	0.78
2.01	2739.88	0.50	0.98	2530	0.46	0.84
2.21	2908.26	0.53	1.11	2630	0.48	0.91
			$\gamma = 1.10$			
1.81	2418.73	0.44	0.77	2280	0.41	0.70
2.01	2557.45	0.46	0.86	2350	0.42	0.74
2.21	2680.14	0.48	0.94	2420	0.43	0.77

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APPENDIX B: DATA SOURCES AND CONSTRUCTION OF VARIABLES

The OECD Social Expenditure Database covers 30 OECD countries for the period 1980–2005. The share of social expenditure as a percentage of total expenditure is obtained from this data set. Social expenditure includes old age, survivors, incapacity-related benefits, health, family, active labor market programs, unemployment, housing, and other social policy areas.[18] See Table A.2 for summary information.

The source of data for growth rates in income and population, and real interest rates, is *World Development Indicators* (2008).[19] Suppose the annual average growth of GDP

TABLE A.2. Summary information for each country and group of countries

Country	B	τ	r	β	ω	n	Gini	σ^2
Australia	0.50	0.25	0.11	0.90	0.46	0.30	30.56	0.31
Austria	0.53	0.36	0.27	0.79	0.39	0.10	29.15	0.28
Belgium	0.52	0.41	0.15	0.87	0.39	0.10	32.97	0.36
Canada	0.44	0.19	−0.12	1.14	0.47	0.21	32.56	0.35
Czech Republic	0.45	0.30	−0.14	1.16	1.37	0.01	25.33	0.21
Denmark	0.50	0.33	0.17	0.86	0.38	0.07	23.80	0.18
Finland	0.51	0.35	−0.03	1.03	0.83	0.06	26.88	0.24
France	0.54	0.40	0.18	0.85	0.33	0.14	27.50	0.25
Germany	0.57	0.28	0.36	0.74	0.32	0.01	28.31	0.26
Greece	0.44	0.34	−0.02	1.02	1.17	0.07	34.27	0.39
Hungary	0.44	0.34	0.41	0.71	1.27	−0.04	28.43	0.27
Iceland	0.38	0.29	0.88	0.53	0.74	0.33	25.00	0.20
Ireland	0.46	0.31	−0.55	2.24	1.23	0.45	34.28	0.39
Italy	0.51	0.35	−0.14	1.17	0.19	0.11	36.03	0.44
Netherlands	0.45	0.37	0.14	0.88	0.38	0.09	26.33	0.23
New Zealand	0.48	0.31	1.02	0.50	0.49	0.27	33.70	0.38
Norway	0.51	0.37	0.61	0.62	0.45	0.15	25.79	0.22
Poland	0.50	0.29	0.79	0.56	1.31	−0.03	34.49	0.40
Spain	0.53	0.26	−0.07	1.08	0.52	0.34	34.66	0.40
Switzerland	0.54	0.18	0.05	0.95	0.31	0.15	33.68	0.38
United Kingdom	0.51	0.36	0.39	0.72	0.56	0.10	33.00	0.35
United States	0.44	0.18	0.05	0.95	0.36	0.21	40.81	0.57
				Scandinavian countries				
Average	0.51	0.35	0.25	0.84	0.55	0.09	25.49	0.21
Coeff. variation	0.02	0.06	1.33	0.25	0.44	0.51	0.06	0.13
				Commonwealth countries				
Average	0.48	0.28	0.35	0.82	0.49	0.22	32.46	0.35
Coeff. variation	0.06	0.26	1.42	0.34	0.10	0.39	0.04	0.08
				All countries				
Average	0.49	0.31	0.20	0.92	0.63	0.15	30.80	0.32
Coeff. variation	0.09	0.21	1.83	0.39	0.60	0.89	0.14	0.30

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per capita is $x\%$ and the length of a time period in this overlapping-generations model is 20 years. The required adjusted growth rate is obtained as $(1+x)^{20}-1$. The same approach was used for population growth and the interest rate for each country. Also, by assuming that the time preference rate is equal to the real interest rate, $\rho = r$, the discount factor for each country, β, is obtained from $1/(1+r)$.[20]

The tax rate is obtained from *World Development Indicators* (2008), which includes tax revenue as well as social contributions. The ratio of tax revenue to GDP includes compulsory transfers to the central government for public purposes. The ratio of social contributions to GDP includes social security contributions by employees, employers, and self-employed individuals, and other contributions whose source cannot be determined. They also include actual or imputed contributions to social insurance schemes operated by governments.

Data on Gini inequality measures were obtained from *World Development Indicators* (2008) and the *World Income Inequality Database* (2007).[20] Suppose income follows the lognormal distribution with mean and variance of logarithms of income μ and σ^2. This variance can be obtained from the Gini index using $\sigma = \sqrt{2}\Phi^{-1}(\frac{\text{Gini}+1}{2})$, where Φ^{-1} is the inverse function of the standard normal cumulative distribution; see Aitchison and Brown (1957, p. 13). Because for most of the countries the Gini index is available only for selected years, the average of available years between 2000 and 2006 was used to calculate the ratio of median to mean income. Consequently, averages of growth of GDP per capita, population growth, the real interest rate, and government expenditures and revenues were computed over the same period.

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