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TOTAL CHORD LENGTH WHEN A RANDOM DISC INTERSECTS EQUISPACED PARALLEL LINES

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Abstract

Construct the lines $x = k$ for integers k. Centre a disc of radius r on $(S, 0)$, where S has a uniform distribution on $[0, 1]$. Let L be the total chord length from the intersection of the disc with the lines. This article obtains the cumulative distribution function for L.

2020 *Mathematics subject classification*: primary 60D05; secondary 51M04.

Keywords and phrases: stereology, geometric probability.

1. Introduction

In the Euclidean plane $\mathbb{R}^2$, construct the lines $x = k$ for integers k. Centre a disc of radius r on $(S, 0)$, where S has a uniform distribution on $[0, 1]$. Let $L \equiv L(r)$ be the total chord length from the intersection of the disc with the lines. Let $F(\ell) \equiv \mathbb{P}\{L \le \ell\}$ be the cumulative distribution function (cdf) for L. This article obtains $F(\ell)$.

THEOREM 1.1. *For $x \in \mathbb{R}$, let*

$$u(x) = \sum_{k=\lceil x-r \rceil}^{\lfloor x+r \rfloor} u_k(x), \quad u_k(x) = 2\sqrt{r^2 - (k-x)^2}. \tag{1.1}$$

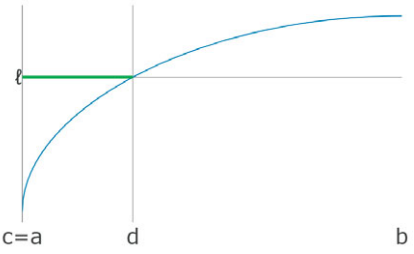
Write $r = n + s$ where n is an integer and $s \in [0, 1)$. Set $a = \lfloor -2s \rfloor/2$, $b = a + 1/2$, and $u_{\min} = u(-s)$, $u_{\max} = \max\{u(a), u(b)\}$. Then

$$F(\ell) = \begin{cases} 0 & \text{if } \ell < u_{\min}, \\ 2(d-c) & \text{if } u_{\min} \le \ell \le u_{\max}, \\ 1 & \text{if } \ell > u_{\max}, \end{cases} \tag{1.2}$$

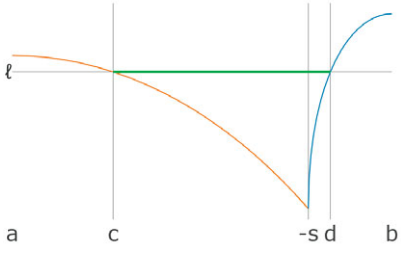
where c and d are specified in the following table (see also Figure 1).

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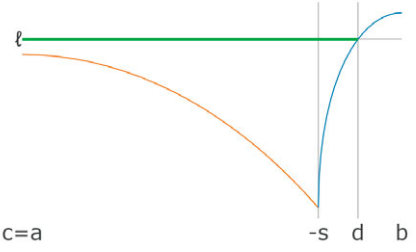
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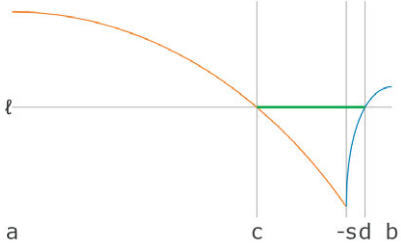
Case 1: $s = 0$ or $s = 1/2$.



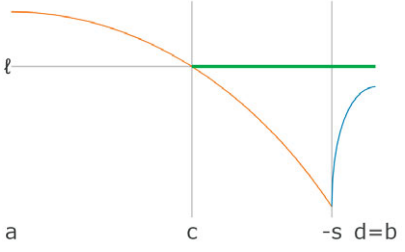
Case 2: $\ell \le u(a) \le u(b)$.



Case 3: $u(a) \le \ell \le u(b)$.



Case 4: $\ell \le u(b) \le u(a)$.



Case 5: $u(b) \le \ell \le u(a)$.

FIGURE 1. Calculating $\mathbb{P}\{L \le \ell\}$. The orange and blue curves plot $u(x)$ on $[a, b]$. The thick green line marks the interval $[c, d]$. If X follows a uniform distribution on $[a, b]$ then $\{X \in [c, d]\} \equiv \{L \le \ell\}$.

	Case	*c*	*d*
(1)	$s = 0$ *or* $s = 1/2$	a	$u^{-1}(\ell)$ *on* $[a, b]$
(2)	$\ell \le u(a) \le u(b)$	$u^{-1}(\ell)$ *on* $[a, -s]$	$u^{-1}(\ell)$ *on* $[-s, b]$
(3)	$u(a) \le \ell \le u(b)$	a	$u^{-1}(\ell)$ *on* $[-s, b]$
(4)	$\ell \le u(b) \le u(a)$	$u^{-1}(\ell)$ *on* $[a, -s]$	$u^{-1}(\ell)$ *on* $[-s, b]$
(5)	$u(b) \le \ell \le u(a)$	$u^{-1}(\ell)$ *on* $[a, -s]$	b

Theorem 1.1 adds to the literature of geometric probability, and may have applications in stereology. It will be shown that when $r \le 1/2$, Theorem 1.1 agrees with earlier findings on the length of a single random chord [1, Equation (1.66)].

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2. Proof of Theorem 1.1

PROOF. The probability function $F(\ell)$ is constructed by inverse transform sampling: Proposition 2.1 establishes $L = u(X)$ where X is uniformly distributed on $[a, b]$. To invert $u(x)$, Propositions 2.4 and 2.5 prove that $u(x)$ is strictly decreasing on $[a, -s]$ and strictly increasing on $[-s, b]$. So $\{X \in [c, d]\} \equiv \{L \le \ell\}$, whereby $\mathbb{P}\{L \le \ell\} = (d - c)/(b - a)$. □

To prove Propositions 2.1, 2.4 and 2.5, first note that $u_k(x)$ is the length of the chord made by the line $x = k$ when it crosses a disc of radius r centred on $(x, 0)$.

PROPOSITION 2.1. *The total chord length L can be realised as $L = u(X)$ where X is uniformly distributed on $[a, b]$.*

PROOF. Observe that $u(x)$ is constructed as the total chord length formed by the lines $x = k$ for integers k, for a disc of radius r centred on $(x, 0)$: the sum is over $k = \lceil x - r \rceil, \ldots, \lfloor x + r \rfloor$ because lines outside that range will not intersect the disc. Now the lines are at unit spacing, which creates a translation symmetry, whereby $u(x + k) = u(x)$ for all integers k. Moreover, a disc is symmetric in reflection, so $u(-x) = u(x)$ for all real numbers x. Thus, L is fully characterised by $u(X)$ where X is drawn uniformly from an interval that is of width $1/2$ and has an integer as one of its boundaries. The interval $[a, b]$ is one such interval. □

To prove monotonicity of $u(x)$, take the chords in pairs. Within each pair, it will be shown that the shorter chord grows faster than the longer chord shrinks. The following lemma locates the shorter chord within a pair.

LEMMA 2.2. *Centre a disc of radius r on $(x, 0)$. Let $k \ge 0$ and $j \in \mathbb{R}$. Then the chord formed by the line $x = k$ is strictly shorter (longer) than the chord formed by the line $x = -k + j$ if and only if $(2k - j)(2x - j) < 0$* (respectively, > 0).

PROOF. Since $u_k(x) = 2\sqrt{r^2 - (k - x)^2}$ and $u_{-k+j}(x) = 2\sqrt{r^2 - (-k + j - x)^2}$, the inequality $u_k(x) < u_{-k+j}(x)$ holds if and only if $(k - x)^2 > (-k + j - x)^2$. Observe that $(k - x)^2 - ((-k + j) - x)^2 = \cancel{k^2} - 2kx + \cancel{x^2} - (\cancel{k^2} - 2jk + j^2) + 2(-k + j)x - \cancel{x^2} = -2(2k - j)\,x + (2k - j)j = -(2k - j)(2x - j)$. The result follows. □

Henceforth, for any $m \in \mathbb{R}$, let $y_m \equiv y_m(x) = \sqrt{r^2 - (m - x)^2}$. Note $y'_m \equiv y'_m(x) = (m - x)/y_m$. The following lemma will establish whether the net rate of growth for a pair of chords is positive or negative.

LEMMA 2.3. *Let $k, j \in \mathbb{R}$. If $x < (>) j/2$ and $(k - x)(y_{-k+j} - y_k) > (<) 0$ then $y'_k + y'_{-k+j} > (<) 0$.*

PROOF. Observe that

$$y'_k + y'_{-k+j} = \frac{k - x}{y_k} + \frac{-k + j - x}{y_{-k+j}} = \frac{y_{-k+j}(k - x) + y_k(-k + j - x)}{y_k y_{-k+j}}.$$

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Since $y_k, y_{-k+j} > 0$, it is sufficient to consider the numerator

$$\Delta \equiv y_{-k+j}(k-x) + y_k(-k+j-x) = k(y_{-k+j} - y_k) - y_{-k+j}x + y_k(j-x).$$

Now $x < j/2$ if and only if $j - x > j - j/2 = j/2 > x$, whereby

$$\Delta > k(y_{-k+j} - y_k) - y_{-k+j}x + y_k x = (k-x)(y_{-k+j} - y_k).$$

Conversely, if $x > j/2$ then the inequalities are reversed. The result follows. □

Monotonicity can now be established, in either direction from $-s$.

PROPOSITION 2.4. *The function $u(x)$ is strictly increasing on $[-s, b]$.*

PROOF. It is sufficient to show that $u'(x) > 0$ for $x \in (-s, b)$, as the result will follow from $u(x)$ being continuous on $[-s, b]$. For $0 \le s < 1$, if $x \in (-s, b)$ then we have $x + r > -s + n + s = n$ and $x + r < b + n + s \le n + 1/2$, therefore $\lfloor x + r \rfloor = n$. We consider three cases.

(1) $s = 0$. If $x \in (-s, b)$, then $x - r > -s - n - s = -n$ and $x - r < b - n - s = -n + 1/2$. Therefore, $\lceil x - r \rceil = -n + 1$. Thus,

$$u(x) = \sum_{k=-n+1}^{n} u_k(x) = \sum_{k=1}^{n} (u_k(x) + u_{-k+1}(x)).$$

Set $j = +1$.

(2) $0 < s \le 1/2$. If $x \in (-s, b)$, then $x - r > -s - n - s > -n - 1$ and so we have $x - r < b - n - s < -n$. Therefore, $\lceil x - r \rceil = -n$. Thus,

$$u(x) = \sum_{k=-n}^{n} u_k(x) = u_0(x) + \sum_{k=1}^{n} (u_k(x) + u_{-k}(x)).$$

Set $j = 0$.

(3) $1/2 < s < 1$. If $x \in (-s, b)$, then $x - r > -s - n - s > -n - 2$ and so we have $x - r < b - n - s < -n - 1$. Therefore, $\lceil x - r \rceil = -n - 1$. Thus,

$$u(x) = \sum_{k=-n-1}^{n} u_k(x) = \sum_{k=0}^{n} (u_k(x) + u_{-k-1}(x)).$$

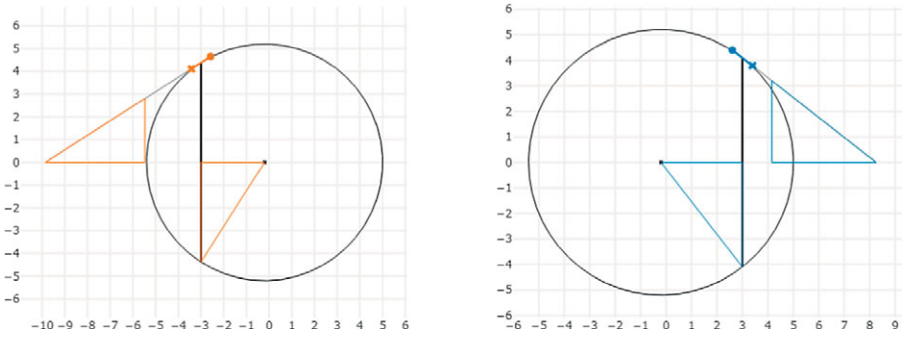
Set $j = -1$.

In each case, $b = j/2$ and hence $x < j/2$. Moreover, $2k - j > 0$ for each index k in the sums. Thus, for each such k, $y_k < y_{-k-1}$ by Lemma 2.2, hence $y'_k + y'_{-k-1} > 0$ by Lemma 2.3. For case 2, note $y'_0 = -x/y_0 > 0$. The result follows from $u'_k(x) = 2y'_k(x)$ for all k. □

PROPOSITION 2.5. *The function $u(x)$ is strictly decreasing on $[a, -s]$.*

PROOF. It is sufficient to show that $u'(x) < 0$ for $x \in (a, -s)$, as the result will follow from $u(x)$ being continuous on $[a, -s]$. If $s = 0$ then $a = 0$, whereby $(a, -s)$ is a degenerate interval. Attention can thus be constrained to $0 < s < 1$. Thus, if $x \in (a, -s)$

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← disc motion disc motion →

FIGURE 2. Rate of change in chord length. The triangle inside the disc shows $y'_k(x) = (k-x)/y_k(x)$. The triangle outside the disc shows the tangent to $(k, y_k(x))$. The triangles are congruent, so $y'_k(x)$ matches the gradient of the tangent at $(k, y_k(x))$.

then $x + r > a + n + s > n - 1/2$ and $x + r < -s + n + s = n$, therefore $\lfloor x + r \rfloor = n - 1$. We consider two cases.

(1) $0 < s \le 1/2$. If $x \in (a, -s)$, then $x - r > a - n - s \ge -n - 1$ and so we have $x - r < -s - n - s < -n$. Therefore, $\lceil x - r \rceil = -n$. Thus,

$$u(x) = \sum_{k=-n}^{n-1} u_k(x) = \sum_{k=0}^{n-1} (u_k(x) + u_{-k-1}(x)).$$

Set $j = -1$.

(2) $1/2 < s < 1$. If $x \in (a, -s)$, then $x - r > a - n - s \ge -n - 2$ and so we have $x - r < -s - n - s < -n - 1$. Therefore, $\lceil x - r \rceil = -n - 1$. Thus,

$$u(x) = \sum_{k=-n-1}^{n-1} u_k(x) = u_{-1}(x) + \sum_{k=0}^{n-1} (u_k(x) + u_{-k-2}(x)).$$

Set $j = -2$.

In each case, $a = j/2$, hence $x > j/2$. Moreover, $2k - j > 0$ for each index k in the sums. Thus, for each such k, $y_k > y_{-k-1}$ by Lemma 2.2. Now the indices k also satisfy $x < k$, so $y'_k + y'_{-k-1} < 0$ by Lemma 2.3. For case 2, note $y'_{-1} = (-1 - x)/y_{-1} < 0$. The result follows from $u'_k(x) = 2y'_k(x)$ for all k. □

2.1. Visual proof. Figures 2 and 3 offer a visual proof. Construct $v_k^+(x) \equiv (-y_k(x), k - x)$ and $v_k^-(x) \equiv (y_k(x), x - k)$ as the *growth vectors* for the chord from the line $x = k$, when moving respectively in the positive x and negative x directions. Then the growth vectors match the gradient of the tangents to the disc at $(k, y_k(x))$ (see Figure 2). Note further that if $v_k^+(x)$, $v_k^-(x)$ are each considered in their components (v_x, v_y), then $v_y/v_x = -y'_k(x)$. Hence, each growth vector is congruent with the rate of

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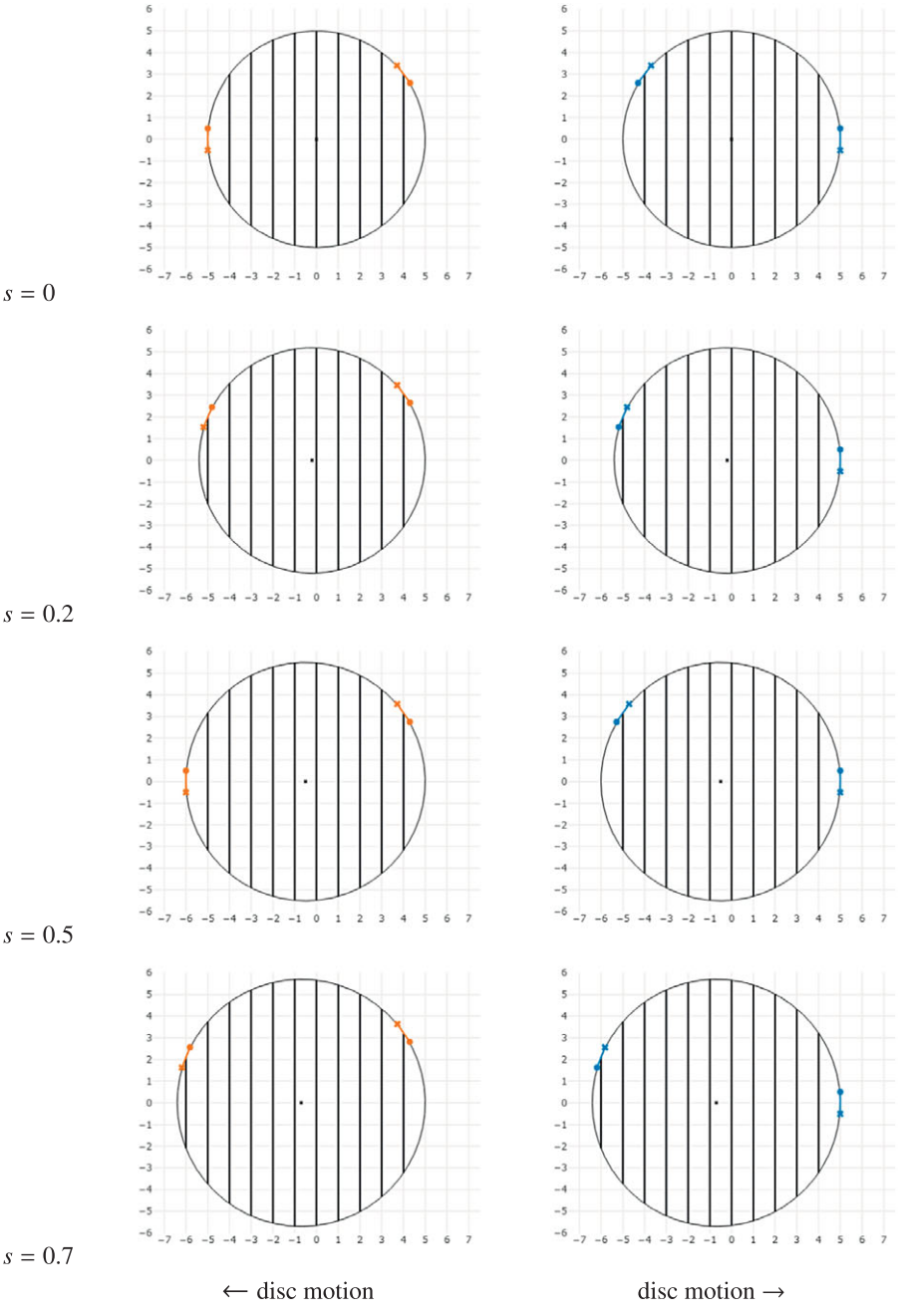


FIGURE 3. Pairing the chords. The arrows show the rate of change in length for one pair, namely the chord that is farthest along the direction of movement, and its counterpart under the pairing. Within each pair, the chord that is further along the direction of movement grows faster than its counterpart shrinks.

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change in chord length, with a minus sign because moving the disc in the positive (negative) x direction is equivalent to moving the lines in the negative (positive) x direction.

Figure 3 plots the growth vector for the chord that is farthest along the direction of movement (positive or negative x axis), and the negative growth vector for the chord to which it is paired. By extrapolation, it can be seen that each pair consists of a chord on the leading side of the disc, and one on the trailing side. Moreover, within each pair, the chord on the leading side grows faster than its counterpart shrinks. Indeed, when moving in the positive x axis, the vector $-v^+_{k+j}(x)$ is clockwise from the vector $v^+_k(x)$ (clockwise with respect to the x–y axes under the right-hand rule). When moving in the negative x axis, the relationship is anti-clockwise. This corresponds to the identity $\Delta = v^+_k(x) \times -v^+_{k+j}(x)$.

2.2. Experiment. Figure 4 presents the results of experiments to check Theorem 1.1 for disc radii $r = 0.2, 0.3, \ldots, 0.9, 1$ and $r = 3.2, 3.3, \ldots, 3.9, 4$.

(1) Stochastic simulation was used to sample the total chord lengths ($n = 100\,000$).
(2) The distribution predicted by Theorem 1.1 was compared with the empirical distribution for the sample, via a quantile–quantile plot. The predicted quantiles were obtained by numeric inversion of equation (1.2).

The intent was to check for discrepancies between the empirical and predicted distributions, as this would have indicated a problem in the analysis. However, over the range of disc radii that were tested, no discrepancies appeared and the quantile–quantile plots show good correlation. The software is available from the author upon request.

3. Remarks

3.1. Intersection with a single line. If $r \le 1/2$ then the disc will intersect at most one line. This case has a simple exact solution.

COROLLARY 3.1. *If $r \le 1/2$, then*

$$F(\ell) = 1 - \sqrt{(2r)^2 - \ell^2} \tag{3.1}$$

for $0 \le \ell \le 2r$.

PROOF. If $r \le 1/2$, then $u(x) = 2\sqrt{r^2 - x^2}$ for $x \in [-r, r]$ (and 0 otherwise), whereby $u^{-1}(\ell) = \sqrt{r^2 - (\ell/2)^2} = \frac{1}{2}\sqrt{(2r)^2 - \ell^2}$. Hence, $F(\ell) = 2(\frac{1}{2}\sqrt{(2r)^2 - \ell^2} - (-1/2)) = 1 - \sqrt{(2r)^2 - \ell^2}$ as required. □

It follows that if $r \le 1/2$ then the probability density function for L is

$$f(\ell) = \frac{\ell}{\sqrt{(2r)^2 - \ell^2}}. \tag{3.2}$$

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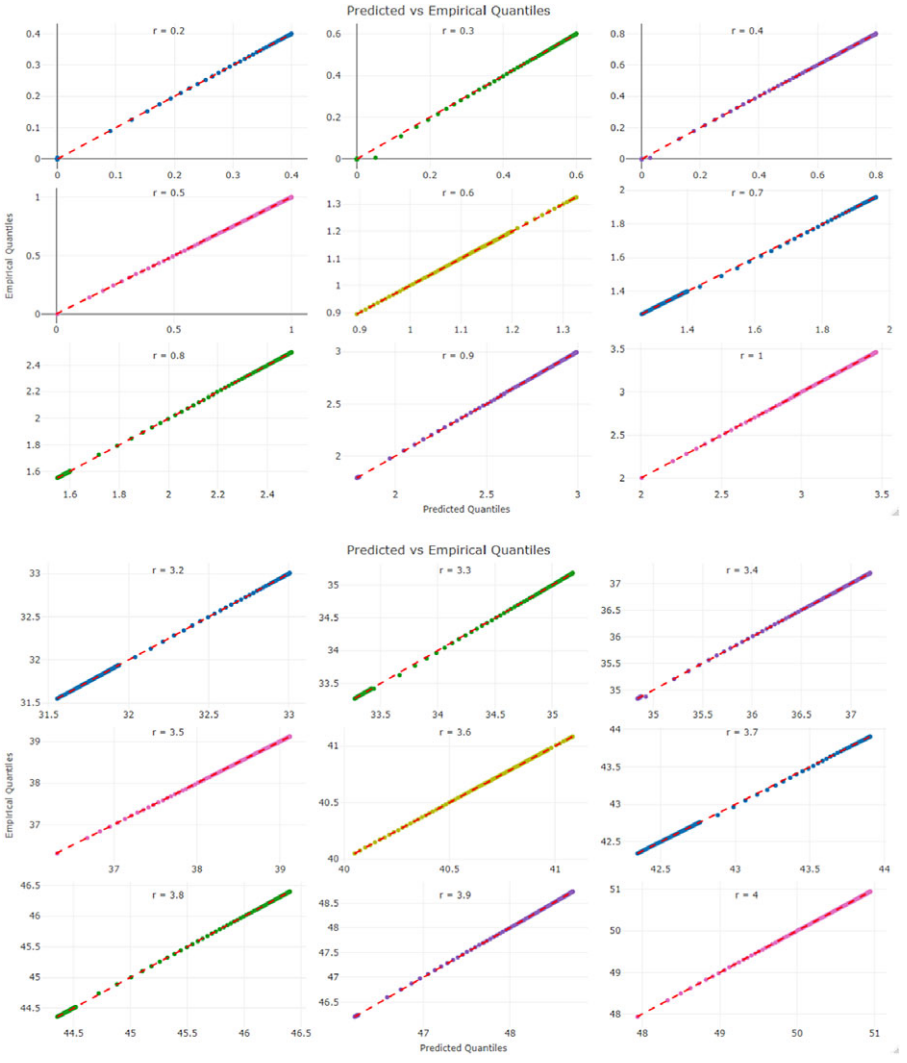


FIGURE 4. Quantiles as predicted by numerical inversion of equation (1.2), versus those obtained empirically from a random sample of total chord lengths ($n = 100\,000$).

Suppose that a line is described randomly across a disc of diameter r. In this case, the chord length has probability density function [1, Equation (1.66)]

$$h(\ell) = \frac{\ell}{2r\sqrt{(2r)^2 - \ell^2}} \tag{3.3}$$

(where $0 \le \ell \le 2r$). Equations (3.2) and (3.3) are consistent: $f(\ell) \equiv \mathbb{P}\{L \in [\ell, \ell + d\ell]\}$ and $h(\ell) \equiv \mathbb{P}\{L \in [\ell, \ell + d\ell] | L > 0\}$ when $d\ell \to 0$, and $\mathbb{P}\{L > 0\} = 2r/1$ is the

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probability of having an intersection between parallel lines at unit spacing, and a disc of radius $r \le 1/2$ centred on $(X, 0)$ where X is uniformly distributed on $[0, 1]$.

3.2. Quantiles. In some cases, the quantiles have an exact solution.

COROLLARY 3.2. *For $p \in [0, 1]$:*

(1) *if $s = 0$, then $F^{-1}(p) = u(p/2)$;*
(2) *if $s = 1/2$, then $F^{-1}(p) = u((p-1)/2)$;*
(3) *if $u(a) \le u(b)$ and $p \ge 2(-s-a)$, then $F^{-1}(p) = u(a + p/2)$;*
(4) *if $u(b) \le u(a)$ and $p \ge 2(b+s)$, then $F^{-1}(p) = u(b - p/2)$.*

PROOF. Write $F(\ell) \equiv 2(d-c) = p$. For clarity in the following, '$\equiv$' denotes that a value is assigned in the statement of Theorem 1.1.

(1) This is Case 1 with $s = 0$: $c \equiv a = 0$, hence $d = p/2$. Further, $d \equiv u^{-1}(\ell)$.
(2) This is Case 1 with $s = 1/2$: $c \equiv a = -1/2$, hence $d = (p-1)/2$. Further, $d \equiv u^{-1}(\ell)$.
(3) This is Case 3: $c \equiv a$, hence $d = a + p/2$. Further, $d \equiv u^{-1}(\ell)$.
(4) This is Case 5: $d \equiv b$, hence $c = b - p/2$. Further, $c \equiv u^{-1}(\ell)$. □

In terms of Figure 1, Corollary 3.2 identifies the sub-domain inside $[a, b]$ in which $u(x)$ is a bijection. This fixes one end of the interval $[c, d]$, allowing the other end to be calculated from p.

3.3. Moments. Moments for L can be calculated from the quantile function.

COROLLARY 3.3. *For any $m \ge 0$,*

$$\mathbb{E}(L^m) = \int_0^1 F^{-1}(p)^m \, dp. \tag{3.4}$$

PROOF. Let X have a uniform distribution on $[0, 1]$. Then by the inverse probability integral transform, $F^{-1}(X)$ has the same distribution as L. Hence, $\mathbb{E}(L^m) = \mathbb{E}([F^{-1}(X)]^m) = \int_0^1 [F^{-1}(p)]^m \, dp$. □

COROLLARY 3.4. *If $s = 0$ or $s = 1/2$, then $\mathbb{E}(L) = \pi r^2$.*

PROOF. Let $A \equiv \int_{-1/2}^{1/2} u(x) \, dx$. Then $A = \pi r^2$. The integral is calculating the area of the circle, by partitioning it into strips of width δx and taking $\delta x \to 0$ [2]. Now a disc is symmetric in reflection, so $u(-x) = u(x)$ for all $x \in \mathbb{R}$. If $s = 0$, then $\mathbb{E}(L) = \int_0^1 u(p/2) \, dp = 2 \int_0^{1/2} u(x) \, dx = A$. Likewise if $s = 1/2$, then $\mathbb{E}(L) = \int_0^1 u((p-1)/2) \, dp = 2 \int_{-1/2}^0 u(x) \, dx = A$. □

REMARK 3.5. Reference [2] concluded that $\mathbb{E}(L) = \pi r^2$ for all radii r, on the geometric grounds of A being the infinitesimal sum of chord lengths.

3.4. Effect of the integer component n. The cdf $F(\ell)$ does not have an explicit dependence on n. To see why, for a disc of radius r centred on $(x, 0)$, let $u(x; r)$ be

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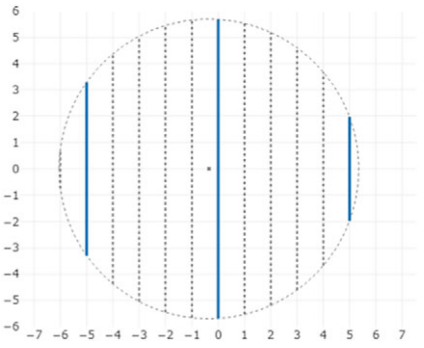


FIGURE 5. The disc of radius $r = 5.7$, highlighting the chords formed by the lines spaced by 1 unit, and the lines spaced by 5 units. The behaviour of $u(x)$ can be considered at either scale.

the total chord length and $u_k(x; r)$ be the length of the chord formed by the line $x = k$. By algebra, $u_n(x; r) = n \cdot u_1(x/n; 1 + s/n)$ (see also Figure 5). So $u(x; r)$ can be thought of as scaling $u(x/n; 1 + s/n)$ up by a factor of n, and then adding in the intervening chords. Thus, *in principle*, $u(\cdot; r)$ could be deduced from $u(\cdot; 1 + s/n)$.

Acknowledgements

The author thanks the reviewer for their feedback. Calculations were implemented in R [4] using RStudio [3] and `devtools` [6]. Figures were created using `plotly` [5].

References

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