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Ion temperature gradient-driven dipolar vortices in dusty plasmas

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Abstract. The nonlinear dynamics of the ion temperature gradient (ITG) mode is investigated in a dusty plasma assuming the dust grains to be tiny and highly charged. The electrostatic drift waves cannot be ignored in the linear dispersion relation if polarization drifts of ions and dust are taken into account. The dipolar vortices are predicted to be formed in the nonlinear regime. The relevance of this study to space and astrophysical plasmas is pointed out.

1. Introduction

A great deal of research work has been done on the ion temperature gradient (ITG) mode for inhomogeneous electron–ion plasmas [1–3]. The ITG mode is most commonly known as the eta-i (η_i) mode. This can play an effective role in particle and energy transport in plasmas. Several researchers have studied the ion temperature gradient-driven instabilities in the presence of magnetic field inhomogeneity and magnetic shear [4, 5]. Both the linear and nonlinear solutions have been discussed. The chaotic behavior of this mode has also been investigated under certain assumptions [6].

A lot of research work is being carried out in the area of dusty plasmas due to its applications in space, astrophysics and laboratory plasmas. The dust dynamics or even the presence of a stationary dust in electron–ion plasmas can introduce new modes or modify the standard plasma modes [7–9].

In general, the dust particles are very heavy compared with the ions and $\Omega_d \ll \Omega_i$ (where $\Omega_i = eB_0/m_i$ is the ion gyrofrequency and $\Omega_d = ez_dB_0/m_d$ is the dust gyrofrequency). The plasmas of space and astrophysical origin may contain several different sizes, masses and charges of dust grains. To deal with real situations, one must consider the charge and mass distribution of dust grains. However, for simplicity many investigations assume a constant mass and charge for dust particles. We also adopt this assumption but consider a situation of tiny dust particles heavily charged so that $(\Omega_i/\Omega_d) \sim O(1)$. So the dust dynamics becomes equally important as that of ions. Recently, Vranjes et al. [10] studied the η_i mode in this limit by including the parallel motion of dust. They explored the distinct features of the dusty plasma by investigating the linear instability in the limit $|\partial_t| \ll \Omega_i, \Omega_d$. In this work the nonlinear regime has not been studied. Since the system is linearly unstable, therefore it seems natural to investigate the nonlinear stage of the instability.

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In this paper, we study the nonlinear dynamics of such a system including the effects of polarization drifts of ions and dust grains. The nonlinear solution of the partial differential equations in the form of dipolar vortices is found. The polarization drifts of ions and dust particles also modify the linear behavior of the modes. The electron drift and ion-acoustic waves are retained in the dispersion relation of the $\eta_{\rm i}$-mode since the smaller term (polarization drift of ions) is also taken into account. In this case several low-frequency modes can couple with the $\eta_{\rm i}$-mode. In the next section, the set of governing equations is presented. In Sec. 3 the linear dispersion relation is found. The nonlinear stationary solution is investigated in Sec. 4, while Sec. 5 is devoted to a summary.

2. Set of governing equations

Let us consider the dusty plasma embedded in the external magnetic field ($\mathbf{B}_0 = B_0\hat{z}$) with the density gradients along the x-axis, i.e. $\nabla n_{j0} = (dn_{j0}/dx)\hat{x}$, where j stands for e (electron), i (ion) and d (dust). The equilibrium ion temperature also has a gradient along the x-axis. For simplicity, we consider a uniform magnetic field and a collisionless plasma. The dusty plasma in the steady state satisfies the following condition:

$$n_{\rm i0} = n_{\rm e0} + z_{\rm d} n_{\rm d0}, \tag{1}$$

where n_{j0} is the unperturbed number density of the particle species. Here $z_{\rm d}$ is the constant number of negative charges residing on the dust grain surface. We assume the perturbation to be low frequency, i.e. $|\partial_t| \ll \Omega_{\rm d}, \Omega_{\rm i}$ therefore the inertia of the electrons is ignored. Thus, electrons follow the Boltzmann distribution,

$$n_{\rm e} = n_{\rm e0} \exp(e\phi/T_{\rm e}), \tag{2}$$

where $\mathbf{E} = -\nabla\phi$ has been assumed. The equation of motion for the jth species in the collisionless case can be written as

$$m_j n_j (\partial_t + \mathbf{v}_j \cdot \nabla)\mathbf{v}_j = q_j n_j (\mathbf{E} + \mathbf{v}_j \wedge \mathbf{B}_0) - \nabla p_j, \tag{3}$$

which gives for ions

$$\mathbf{v}_{\rm i\perp} \simeq \frac{1}{B_0}\mathbf{E}_\perp \wedge \hat{z} - \frac{1}{en_{\rm i}B_0}\nabla p_{\rm i} \wedge \hat{z} - \frac{1}{B_0\Omega_{\rm i}}(\partial_t + \mathbf{v}_{\rm i} \cdot \nabla)\nabla_\perp\phi \tag{4}$$

or

$$\mathbf{v}_{\rm i\perp} = \mathbf{v}_E + \mathbf{v}_{\rm Di} + \mathbf{v}_{\rm pi}$$

where

$$\mathbf{v}_E = \frac{1}{B_0}\mathbf{E}_\perp \wedge \hat{z}, \quad \mathbf{v}_{\rm Di} = -\frac{1}{en_{\rm i}B_0}\nabla p_{\rm i} \wedge \hat{z}$$

and

$$\mathbf{v}_{\rm pi} = -\frac{1}{B_0\Omega_{\rm i}}(\partial_t + \mathbf{v}_{\rm i} \cdot \nabla)\nabla_\perp\phi$$

are the electric drift, ion diamagnetic drift and ion polarization drift, respectively. For dust we have,

$$\mathbf{v}_{\rm d\perp} \simeq \frac{1}{B_0}\mathbf{E}_\perp \wedge \hat{z} + \frac{1}{B_0\Omega_{\rm d}}(\partial_t + \mathbf{v}_{\rm d} \cdot \nabla)\nabla_\perp\phi \tag{5}$$

or

$$\mathbf{v}_{\rm d\perp} = \mathbf{v}_E + \mathbf{v}_{\rm pd}$$

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where

$$\mathbf{v}_{\rm pd} = \frac{1}{B_0\Omega_{\rm d}}(\partial_t + \mathbf{v}_{\rm d}\cdot\nabla)\nabla_\perp\phi$$

is the dust polarization drift and we ignore the dust temperature effects. Let us assume that $\mathbf{v}_E$ and $\mathbf{v}_{\rm Di}$ are of the same order of magnitude and $|v_{{\rm p}j}/v_E| \sim O(\epsilon)$ where $\epsilon \ll 1$. Furthermore, $|v_z\partial_z| \ll |\mathbf{v}_E\cdot\nabla| \sim |\partial_t|$ is also assumed and we define $d_t = \partial_t + \mathbf{v}_E\cdot\nabla$. The contribution of diamagnetic drift to the convective derivative in the polarization drift is treated separately (not included in d_t) and hence we may write

$$\mathbf{v}_{\rm pi} = -\frac{1}{B_0\Omega_j}(d_t + \mathbf{v}_{\rm Di}\cdot\nabla)\nabla_\perp\phi.$$

The parallel equations of motion for ions and dust become, respectively,

$$d_t v_{{\rm i}z} = -\frac{e}{m_{\rm i}}\partial_z\phi - \frac{1}{m_{\rm i}n_{\rm i}}\partial_z p_{\rm i} \tag{6}$$

and

$$d_t v_{{\rm d}z} = \frac{ez_{\rm d}}{m_{\rm d}}\partial_z\phi. \tag{7}$$

In deriving (6) we have used the relation

$$(\mathbf{v}_{\rm Di}\cdot\nabla)v_{{\rm i}z} = -\frac{1}{m_{\rm i}n_{\rm i}}(\nabla\cdot\mathbf{\Pi}_{\rm i})_z,$$

where $\mathbf{\Pi}_{\rm i}$ is the ion stress tensor. The ion continuity equation can be written as

$$d_t\frac{n_{\rm i1}}{n_{\rm i0}} + \frac{1}{B_0}\frac{1}{n_{\rm i0}}(\nabla n_{\rm i0}\wedge\hat{z})\cdot\nabla_\perp\phi - \frac{1}{B_0\Omega_{\rm i}}[d_t + \mathbf{v}_{\rm Di}\cdot\nabla]\nabla_\perp^2\phi + \partial_z(n_{\rm i}v_{{\rm i}z}) = 0, \tag{8}$$

where

$$\mathbf{v}_{\rm Di}\cdot\nabla = \left\{v_{\rm i0}\partial_y - \left[\frac{1}{eB_0}(\nabla T_{\rm i1}\wedge\hat{z})\cdot\nabla + \frac{T_{\rm i0}}{eB_0n_{\rm i0}}(\nabla n_{\rm i1}\wedge\hat{z})\cdot\nabla\right]\right\},$$

$$v_{\rm i0} = v_{{\rm i}*}(1+\eta_{\rm i}), \quad v_{{\rm i}*} = -\frac{T_{\rm i0}}{eB_0}\kappa_{\rm ni}, \quad \eta_{\rm i} = \kappa_{\rm Ti}/\kappa_{\rm ni}, \kappa_{\rm ni} = -\partial_x\ln[n_{\rm i0}(x)]$$

and

$$\kappa_{\rm Ti} = -\partial_x\ln[T_{\rm i0}(x)].$$

Here it seems important to mention that in the continuity equation the convective diamagnetic drift contribution to $\nabla\cdot(n\mathbf{v}_p)$ is exactly cancelled by the $\nabla\cdot(n\mathbf{v}_\Pi)$-term (where $\mathbf{v}_\Pi = -(1/eB_0n_{\rm i})\hat{z}\wedge(\nabla\cdot\mathbf{\Pi}_{\rm i})$) if collisionless gyroviscosity is taken into account in the limit $\nabla\cdot\mathbf{v}_\perp \simeq \nabla\cdot\mathbf{v}_E = 0$. However if $\nabla\cdot\mathbf{v} = \nabla\cdot\mathbf{v}_p + \partial_z v_{{\rm i}z} \neq 0$ is also considered as a small correction then this cancellation does not occur. In this case in the polarization drift $\nabla\cdot(n\mathbf{v}_{\rm pi})$, a term like $\nabla\cdot[(\mathbf{v}_{\rm Di}\cdot\nabla)\nabla_\perp\phi]$ (for $T_{\rm i0} \neq 0$) appears as in (5) of [4]. However, they have ignored the stress tensor contribution. Similarly, for simplicity we ignore the stress tensor which can give only minor correction in this case, while we retain $v_{\rm i0}\partial_y$-term and corresponding nonlinear terms in (8). Note that in the linear limit we shall assume $v_{\rm i0}\partial_y \sim |\partial_t|$.

The dust continuity equation yields

$$d_t n_{\rm d1} + \frac{1}{B_0}(\nabla n_{\rm d0}\wedge\hat{z})\nabla_\perp\phi + \frac{n_{\rm d0}}{B_0\Omega_{\rm d}}d_t\nabla_\perp^2\phi + \partial_z(n_{\rm d}v_{{\rm d}z}) = 0. \tag{9}$$

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The energy equation for ions can be written as

$$d_t\left(\frac{T_{\mathrm{i}1}}{T_{\mathrm{i}0}}\right)+\left(\eta_{\mathrm{i}}-\frac{2}{3}\right)\frac{1}{B_0}\kappa_{\mathrm{n}i}\partial_y\phi-\frac{2}{3}d_t\left(\frac{n_{\mathrm{i}1}}{n_{\mathrm{i}0}}\right)=0. \tag{10}$$

Here the term $-\frac{2}{3}d_t(n_{\mathrm{i}1}/n_{\mathrm{i}0})$ appears because $\boldsymbol{\nabla}\cdot\mathbf{v}_\perp+\partial_z v_{\mathrm{i}z}\neq 0$ is taken into account.

If we normalize the ϕ as $\Phi=e\phi/T_{\mathrm{e}}$, then the charge conservation equation $\boldsymbol{\nabla}\cdot\mathbf{J}=0$ gives

$$d_t\Phi+v_{\mathrm{e}*}\partial_y\Phi-\rho_{\mathrm{s}}^2 d_t\nabla_\perp^2\Phi-\delta_{\mathrm{i}e}\rho_{\mathrm{s}i}^2 v_{\mathrm{i}0}\partial_y\nabla_\perp^2\Phi+\frac{1}{eB_0}\delta_{\mathrm{i}e}\rho_{\mathrm{s}i}^2\{(\boldsymbol{\nabla}T_{\mathrm{i}1}\wedge\hat{z})\cdot\boldsymbol{\nabla}\}\nabla_\perp^2\Phi$$
$$+\frac{T_{\mathrm{i}0}}{en_{\mathrm{e}0}B_0}\rho_{\mathrm{s}i}^2\{(\boldsymbol{\nabla}n_{\mathrm{i}1}\wedge\hat{z})\cdot\boldsymbol{\nabla}\}\nabla_\perp^2\Phi+\delta_{\mathrm{i}e}\partial_z(v_{\mathrm{i}z})-\delta_{\mathrm{d}e}\partial_z(v_{\mathrm{d}z})=0, \tag{11}$$

where $\rho_{\mathrm{s}}^2=(\delta_{\mathrm{i}e}\rho_{\mathrm{s}i}^2+\delta_{\mathrm{d}e}\rho_{\mathrm{s}d}^2)$, $\rho_{\mathrm{s}i}^2=(c_{\mathrm{s}i}^2/\Omega_{\mathrm{i}}^2)$, $\rho_{\mathrm{s}d}^2=(c_{\mathrm{s}d}^2/\Omega_{\mathrm{d}}^2)$, $c_{\mathrm{s}i}^2=(T_{\mathrm{e}}/m_{\mathrm{i}})$, $c_{\mathrm{s}d}^2=(z_{\mathrm{d}}T_{\mathrm{e}}/m_{\mathrm{d}})$, $v_{\mathrm{e}*}=(T_{\mathrm{e}0}/eB_0)\kappa_{n\mathrm{e}}$, $\kappa_{n\mathrm{e}}=-\partial_x\ln[n_{\mathrm{e}0}(x)]$, $\delta_{\mathrm{i}e}=n_{\mathrm{i}0}/n_{\mathrm{e}0}$, $\delta_{\mathrm{d}e}=z_{\mathrm{d}}n_{\mathrm{d}0}/n_{\mathrm{e}0}$ and $n_j=n_{j0}+n_{j1}$, $T_{\mathrm{i}}=T_{\mathrm{i}0}+T_{\mathrm{i}1}$ have been used.

3. Linear dispersion relation

Here we assume that the perturbed quantities are proportional to $\exp(ik_y y+ik_z z-i\omega t)$. After linearizing the coupled equations (2), (6), (7) and (9)–(11), we obtain the dispersion relation,

$$H_0\omega^4+H_1\omega^3+H_2\omega^2+H_3\omega+H_4=0, \tag{12}$$

where

$$H_0=1+\rho_{\mathrm{s}}^2k_y^2,\quad H_1=-\big[\omega_{\mathrm{e}*}+\delta_{\mathrm{i}e}\rho_{\mathrm{s}i}^2k_y^2(1+\eta_{\mathrm{i}})\omega_{\mathrm{i}*}\big],$$
$$H_2=-\big[\delta_{\mathrm{i}e}c_{\mathrm{s}i}^2k_z^2\big\{\big(1+\tfrac{5}{3}\delta_{\mathrm{e}i}\tau\big)+\tfrac{5}{3}\tau\delta_{\mathrm{d}i}\rho_{\mathrm{s}d}^2k_y^2\big\}+\delta_{\mathrm{d}e}c_{\mathrm{s}d}^2k_z^2\big],$$
$$H_3=-\delta_{\mathrm{i}e}c_{\mathrm{s}i}^2k_z^2\big[\big(\tfrac{2}{3}-\eta_{\mathrm{i}}\big)\omega_{\mathrm{i}*}+\tfrac{5}{3}\{-(\omega_{\mathrm{i}*}+\tau\delta_{\mathrm{e}i}\omega_{\mathrm{e}*})\}\big],$$
$$H_4=\tfrac{5}{3}\tau\delta_{\mathrm{d}e}c_{\mathrm{s}i}^2c_{\mathrm{s}d}^2k_z^4,$$
$$\omega_{\mathrm{e}*}=v_{\mathrm{e}*}k_y,\omega_{\mathrm{i}*}=v_{\mathrm{i}*}k_y\quad\text{and}\quad\tau=T_{\mathrm{i}0}/T_{\mathrm{e}0}.$$

Equation (12) is the dispersion relation of the coupled drift waves, η_{i}-mode, ion-acoustic and dust-acoustic waves in the limit $\Omega_{\mathrm{i}}\Omega_{\mathrm{d}}$.

Here we discuss some limiting cases of interest. If the Larmor radius terms are dropped, we can obtain the dispersion relation (14) of [10] with some modifications. The above coefficients in this case become, as follows:

$$H_0=1,\quad H_1=-\omega_{\mathrm{e}*}$$
$$H_2=-\big[c_{\mathrm{s}i}^2k_z^2\big\{\delta_{\mathrm{i}e}+\big(1+\tfrac{2}{3}\big)\tau\big\}+\delta_{\mathrm{d}e}c_{\mathrm{s}d}^2k_z^2\big],$$
$$H_3=c_{\mathrm{s}i}^2k_z^2\big[\delta_{\mathrm{i}e}\big(\eta_{\mathrm{i}}-\tfrac{2}{3}\big)\omega_{\mathrm{i}*}+\big(1+\tfrac{2}{3}\big)(\delta_{\mathrm{i}e}\omega_{\mathrm{i}*}+\tau\omega_{\mathrm{e}*})\big],$$
$$H_4=\big(1+\tfrac{2}{3}\big)\tau\delta_{\mathrm{d}e}c_{\mathrm{s}i}^2c_{\mathrm{s}d}^2k_z^4.$$

It may be noted here that the terms including the factor $\frac{2}{3}$ in H_2, H_3 and H_4 appear because we have not dropped the $\boldsymbol{\nabla}\cdot\mathbf{v}=\partial_z v_{\mathrm{i}z}$-term in the ion temperature

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equation unlike [2, 10]. If we drop the ion temperature, i.e. η_i-mode, then we have the drift wave coupling with acoustic waves as in the dispersion relation (17) of [9], which is

$$\left(1+\rho_s^2 k_y^2\right)\omega^2 - \omega_{e*}\omega - \left(\delta_{ie}c_{si}^2 + \delta_{de}c_{sd}^2\right)k_z^2 = 0. \tag{13}$$

If we drop the dust dynamics then we can find from (12), the general linear dispersion relation for a collisionless, non-dissipative electron–ion plasma system as

$$\left(1+\rho_{si}^2 k_y^2\right)\omega^3 - \left[\omega_{e*} + \rho_{si}^2 k_y^2(1+\eta_i)\omega_{i*}\right]\omega^2 - c_{si}^2 k_z^2\left(1+\tfrac{5}{3}\tau\right)\omega - \omega_{i*}\left(\tfrac{2}{3}-\eta_i\right)c_{si}^2 k_z^2 = 0, \tag{14}$$

where $\nabla_\perp \cdot \mathbf{v}_{pi} \neq 0$ and $\partial_z v_{iz} \neq 0$ have been used.

4. Nonlinear solution

Now we look for the vortex solution of the nonlinear equations (2), (6), (7) and (9)–(11). Let us define a new variable $\xi = y + \alpha z - ut$, where u is the translational speed of the vortex along the y-axis and α is a constant angle between the wave front normal and the (x, y) plane.

The dust parallel momentum equation, i.e. (7), can be written as

$$D_\Phi\left(v_{dz} + \frac{\alpha c_{sd}^2}{u}\Phi\right) = 0, \tag{15}$$

where

$$d_t = -uD_\Phi, \quad D_\Phi = \left[\partial_\xi - \frac{c_{si}^2}{u\Omega_i}(\partial_x\Phi\partial_\xi - \partial_\xi\Phi\partial_x)\right].$$

The trivial solution of (15) is

$$v_{dz} = -\frac{\alpha c_{sd}^2}{u}\Phi. \tag{16}$$

The ion parallel equation (6) can be written as

$$D_\Phi v_{iz} = \frac{\alpha c_{si}^2}{u}(\partial_\xi\Phi + \tau\partial_\xi N_{i1} + \tau\partial_\xi T_1), \tag{17}$$

where $N_{i1} = n_{i1}/n_{i0}$ and $T_1 = T_{i1}/T_{i0}$. The energy equation (10) can be written as

$$D_\Phi\left[T_1 + \frac{1}{\tau}\left(\eta_i - \frac{2}{3}\right)\frac{v_{i*}}{u}\Phi - \frac{2}{3}N_{i1}\right] = 0. \tag{18}$$

Using quasineutrality, this equation becomes

$$D_\Phi\left[T_1 + \frac{1}{\tau}\left(\eta_i - \frac{2}{3}\right)\frac{v_{i*}}{u}\Phi - \frac{2}{3}\delta_{ei}\Phi - \frac{2}{3}\delta_{di}N_{d1}\right] = 0, \tag{19}$$

where $N_{d1} = n_{d1}/n_{d0}$, $\delta_{di} = z_d n_{d0}/n_{i0}$, $\delta_{ei} = n_{e0}/n_{i0}$. Equation (9) gives

$$D_\Phi\left[N_{d1} - \frac{1}{u}(\delta_{id}v_{i*} + \delta_{ed}v_{e*})\Phi + \rho_{sd}^2\nabla_\perp^2\Phi + \frac{\alpha^2 c_{sd}^2}{u^2}\Phi\right] = 0. \tag{20}$$

The trivial solution of (20) yields

$$N_{d1} = \frac{1}{u}(\delta_{id}v_{i*} + \delta_{ed}v_{e*})\Phi - \rho_{sd}^2\nabla_\perp^2\Phi - \frac{\alpha^2 c_{sd}^2}{u^2}\Phi, \tag{21}$$

where $\delta_{id} = n_{i0}/z_d n_{d0}$ and $\delta_{ed} = n_{e0}/z_d n_{d0}$. Using (21) in (19) we obtain,

$$D_\Phi\left[T_1 + \frac{1}{\tau}\left(\eta_i - \frac{2}{3}\right)\frac{v_{i*}}{u}\Phi - \frac{2}{3}\delta_{ei}\Phi - \frac{2}{3}\delta_{di}\left\{\frac{1}{u}(\delta_{id}v_{i*} + \delta_{ed}v_{e*})\Phi - \rho_{sd}^2\nabla_\perp^2\Phi - \frac{\alpha^2 c_{sd}^2}{u^2}\Phi\right\}\right] = 0. \tag{22}$$

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Equation (22) implies

$$T_1 = Q_1\Phi + Q_2\nabla_\perp^2\Phi, \tag{23}$$

where

$$Q_1 = \left[\frac{1}{\tau}\left\{\frac{2}{3}(1+\tau) - \eta_{\rm i}\right\}\frac{v_{\rm i*}}{u} + \frac{2}{3}\delta_{\rm ei}\left(1+\frac{v_{\rm e*}}{u}\right) - \frac{2}{3}\delta_{\rm di}\frac{\alpha^2 c_{\rm sd}^2}{u^2}\right]$$

and

$$Q_2 = -\frac{2}{3}\delta_{\rm di}\rho_{\rm sd}^2.$$

Using (21) and (23) in (17), we obtain

$$D_\Phi v_{{\rm i}z} = \frac{\alpha c_{\rm si}^2}{u}D_\Phi\left[\{1+\tau\delta_{\rm ei}+\tau\delta_{\rm di}Q_3+\tau Q_1\}\Phi - \frac{5}{3}\tau\delta_{\rm di}\rho_{\rm sd}^2\nabla_\perp^2\Phi\right]. \tag{24}$$

Its solution implies

$$v_{{\rm i}z} = \frac{\alpha c_{\rm si}^2}{u}[Q_4\Phi - Q_5\nabla_\perp^2\Phi], \tag{25}$$

where

$$Q_3 = \left\{\frac{1}{u}(\delta_{\rm id}v_{\rm i*}+\delta_{\rm ed}v_{\rm e*}) - \frac{\alpha^2 c_{\rm sd}^2}{u^2}\right\}, \quad Q_4 = \{1+\tau\delta_{\rm ei}+\tau\delta_{\rm di}Q_3+\tau Q_1\}$$

and $Q_5 = \frac{5}{3}\tau\delta_{\rm di}\rho_{\rm sd}^2$. In the moving frame, (11) can be written as

$$\partial_\xi\Phi - \frac{v_{\rm e*}}{u}\partial_\xi\Phi - \rho_{\rm s}^2\partial_\xi\nabla_\perp^2\Phi + \rho_{\rm s}^2\left(\frac{c_{\rm si}^2}{u\Omega_{\rm i}}\right)\{\Phi,\nabla_\perp^2\Phi\} + \delta_{\rm ie}\rho_{\rm si}^2\frac{v_{\rm i0}}{u}\partial_\xi\nabla_\perp^2\Phi$$
$$+\delta_{\rm ie}\rho_{\rm si}^2\frac{T_{\rm i0}}{ueB_0}\{T_1,\nabla_\perp^2\Phi\} + \delta_{\rm ie}\rho_{\rm si}^2\frac{T_{\rm i0}}{ueB_0}\{N_{\rm i1},\nabla_\perp^2\Phi\} - \delta_{\rm ie}\frac{\alpha}{u}\partial_\xi v_{{\rm i}z} + \delta_{\rm de}\frac{\alpha}{u}\partial_\xi v_{{\rm d}z}, \tag{26}$$

where $\{\Phi,\nabla_\perp^2\Phi\}$, $\{T_1,\nabla_\perp^2\Phi\}$ and $\{N_{\rm i1},\nabla_\perp^2\Phi\}$ are the Poisson brackets defined as $\{\Phi,\nabla_\perp^2\Phi\} = \partial_x\Phi\partial_\xi\nabla_\perp^2\Phi - \partial_\xi\Phi\partial_x\nabla_\perp^2\Phi$. Using (16), (21), (23) and (25), equation (26) can be written as

$$h_1\partial_\xi\Phi + h_2\partial_\xi\nabla_\perp^2\Phi + h_3\{\Phi,\nabla_\perp^2\Phi\} = 0, \tag{27}$$

where $h_1 = (1-v_{\rm e*}/u-\delta_{\rm ie}(\alpha^2c_{\rm si}^2/u^2)Q_4-\delta_{\rm de}(\alpha^2c_{\rm sd}^2/u^2))$, $h_2 = -(\rho_{\rm s}^2-\delta_{\rm ie}\rho_{\rm si}^2(v_{\rm i0}/u)-\delta_{\rm ie}(\alpha^2c_{\rm si}^2/u^2)Q_5)$ and $h_3 = [\rho_{\rm s}^2(c_{\rm si}^2/u\Omega_{\rm i}) + \delta_{\rm ie}\rho_{\rm si}^2(T_{\rm i0}/ueB_0)Q_1 + \rho_{\rm si}^2(T_{\rm i0}/ueB_0)(1+\delta_{\rm de}Q_3)]$.

To solve (27) we use the ansatz,

$$\nabla_\perp^2\Phi = C_1\Phi + C_2x, \tag{28}$$

where C_1 and C_2 are arbitrary constants. Following the standard procedure as given in [12] we define new coordinates $x = r\cos\theta$ and $\xi = r\sin\theta$. Then a circle of radius r_0 is considered with outside region $r > r_0$ and inside region $r < r_0$. For the outside region we assume $C_2 = 0$ so that it does not have a x dependence for the solution to be bounded.

The solution in this region can be written as

$$\Phi_{\rm o}(r,\theta) = [A_1K_1(\lambda_1 r)]\cos\theta \tag{29}$$

and the inside ($r < r_0$) solution for $C_2 \neq 0$ can be written as

$$\Phi_{\rm i}(r,\theta) = \left[A_2J_1(\lambda_2 r) + \frac{C_{2i}}{\lambda_2^2}r\right]\cos\theta \tag{30}$$

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where $C_{1o} = \lambda_1^2 = -h_1/h_2$, $C_{1i} = -\lambda_2^2$ and $C_{2i} = -(h_2/h_3)(\lambda_1^2 + \lambda_2^2)$. It also should be mentioned here that the subscripts (o) and (i) denote the outside and inside regions, respectively. The constants A_1, A_2 and λ_2 can be determined from the continuity conditions of $\Phi, \partial_r \Phi$ and $\nabla_\perp^2 \Phi$ at the boundary of the circle.

5. Summary

To summarize, we have investigated the η_i-mode for dusty plasmas in the limit $(\Omega_i/\Omega_d) \sim O(1)$ including the effect of polarization drifts of ions and dust particles. We have found linear dispersion relation which shows coupling of η_i-mode with the drift and acoustic waves with the correction terms coming from ion and dust polarization drifts. The inclusion of dust dynamics affects the η_i instability as has been illustrated numerically in [10]. We have focused our attention on the nonlinear evolution of the system. To avoid repetition, the numerical study of the linear dispersion relation in this investigation has not been presented because it can be found in [10]. Since the system is linearly unstable, therefore a nonlinear investigation becomes important. In the nonlinear regime, we have found the stationary solution of the coupled equations in the form of dipolar vortices. The solution is bounded and well localized. This investigation predicts dipolar vortices in non-uniform dusty plasmas having $(\Omega_i/\Omega_d) \sim O(1)$ in the presence of an ion temperature gradient. In our opinion, the present investigation can be applicable to dusty plasmas of space, astrophysical and laboratory plasmas.

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