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Effect of spin-induced magnetization and Hall current on self-gravitational instability of magnetized viscous quantum plasma

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The Jeans self-gravitational instability is studied for dense quantum viscous plasma with Hall term and intrinsic magnetization generated by collective electron spin. The quantum magnetohydrodynamic model is employed to formulate the basic equations of the problem. The dispersion relation is obtained using the normal mode analysis, and further reduced for both transverse and longitudinal modes of propagation. The transverse mode of propagation is found to be unaffected by the Hall term but affected by quantum effect, viscosity, and magnetization parameters. The Jeans criterion of instability in the transverse direction is modified by Alfven velocity, magnetization parameter, and quantum effect. The non-gravitating magnetized mode is obtained in the longitudinal direction, which is modified by Hall parameter and is not affected by quantum term, whereas the gravitational mode is unaffected by the magnetization parameter but affected by viscosity and quantum parameters. It is observed that the Jeans condition of instability is affected by the quantum term. The growth rate of Jeans instability is plotted for various values of magnetization, quantum, and viscosity parameters of the quantum plasma medium.

1. Introduction

There has been a growing interest in quantum plasmas (Haas 2007, and the references therein). The understanding of dynamics of quantum plasma is an important issue for various physical, astrophysical, and laboratory situations such as semiconductor industries (Haas et al. 2003), astrophysical matters (Jung 2001; Opher et al. 2001), laser-produced plasmas (Kremp et al. 1999), and Fermi gas (Manfredi and Haas 2001). Looking at the importance of quantum plasma in different environments, studies in quantum plasmas are gaining momentum in both experimental and theoretical aspects (Haas et al. 2000; Manfredi 2005; Fletcher et al. 2006; Glenzer et al. 2007) of various problems. In dense plasma regions, plasma electrons degenerate and the dynamics of charged particles needs new pressure laws associated with the Fermi–Dirac distribution functions. The new forces associated with the quantum Bohm potential (Shukla 2006) and the Bohr magnetization involving electron spin (Brodin and Marklund 2007) start governing the quantum system. These forces are dominant and the system considerably affects wave propagations and corresponding instabilities in the dense quantum plasmas.

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In addition, the dynamics of particles has been influenced by spin-induced magnetization as a probe of quantum physical phenomena (Brodin and Marklund 2007; Modestov et al. 2009; Misra et al. 2010). It is thus expected that the collective spin effects can influence the propagation characteristics of waves in magnetized quantum plasma (Marklund and Brodin 2007; Zamanian et al. 2009), and the spin-induced magnetization due to electrons' spin contributes to collective effects in quantum magnetoplasmas (Zamanian et al. 2009). Harding and Lai (2006) have discussed that in the physics of pulsars and magnetars there is strong magnetic field, and spin-induced magnetization may be important. It is established that different contributions to the dynamics of particles appeared when the electron spin-induced magnetization effect is considered. Consequently, spin-induced magnetization has been incorporated by various researchers in quantum plasma. The spin current-generated magnetization has shown to change the frequency of electromagnetic wave in the study done by Shukla et al. (2010). Brodin et al. (2008) have discussed the presence of a new wave mode due to wave-particle resonances that depend directly on the anomalous magnetic moment of electrons with spin-induced magnetization. The influence of the spin of electrons on the propagation of circularly polarized waves in magnetized plasma is considered by Misra et al. (2010). A complete set of multi-fluid spin plasma equations has been presented by Brodin and Marklund (2007). The effect of spin-induced magnetization has been shown on the growth of the Rayleigh–Taylor instability in quantum plasma by Modestov et al. (2009). Markulund and Brodin (2007) have described the dynamics of plasma, including spin-induced magnetization. A Hall MHD type of model generalized to account for the spin-induced magnetization is presented by Zamanian et al. (2009). Yang et al. (2011) have derived nonlinear equation for a quantum magnetoplasma with spin force and the Bohm potential. Asenjo (2012) has discussed the quantum effects of spin and the Bohm potential in the oblique propagation of magnetosonic modes in warm plasma with ions and electrons. Bychkov et al. (2010) have discussed the magnetohydrodynamic instability in plasma with intrinsic magnetization. Lundin et al. (2008) have derived the modified Jeans instability criterion for magnetized plasma with intrinsic magnetization due to collective electron spin effects. Iqbal and Shukla investigated spin and ion effects in quantum plasma and derived the dispersion relation for wave propagation in homogeneous, strongly coupled magnetized plasma. Sharma and Chhajlani (2014a) have discussed the Jeans instability considering the effects of spin-induced magnetization, viscosity, and finite electrical resistivity in quantum plasma. Sharma and Chhajlani (2014b) have also studied the effect of spin-induced magnetization on the self-gravitational Jeans instability in presence of viscosity, resistivity, and Hall current in quantum plasma. Thus, we find that the effect of spin-produced magnetization is also important along with quantum effect in quantum plasma.

Apart from this, it is a well-established fact that in understanding the process of formation of astrophysical objects the self-gravitation plays a key role. After the pioneer work by Chandrasekhar (1961), the study of Jeans instability has occupied a wide range of research area with a number of physical parameters in which the Hall current is of much interest. The Hall parameter plays an important role in the structure formation of astrophysical dense plasma, magnetic reconnection process, plasma confinement problem, and plasma channel flow. Kalra and Talwar (1964) have investigated the influence of Hall effect on gravitational instability in unbound plasma. Hans (1966) has studied gravitational instability in the presence of Hall current and the finite Larmor radius effect. Gravitational instability of

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self-gravitating rotating plasma has been discussed, including finite Larmor radius effects by Sharma (1974). Prajapati et al. (2008) have examined the Jeans instability of magnetized plasma using generalized polytrope laws considering the effects of Hall current, electron inertia, rotation, and electrical resistivity. Shaikh et al. (2008) investigated the Jeans instability of thermally conducting partially ionized gravitating plasma considering the effects of Hall current, viscosity, resistivity, and collisions. Mangesha and Tessema (2013) have carried out investigation of effect of viscosity on the propagation of magnetohydrodynamic waves in astrophysical plasma. The self-gravitational instability of rotating viscous plasma with Hall term and radiative effects has been shown by Prajapati et al. (2010). In all these studies, the importance of gravitational instability with Hall effect is established in astrophysical and interstellar situations.

Furthermore, the response of Fermi plasma to wave propagations is different, and this has made the investigation of waves and instability in the dense Fermi degenerate quantum plasma a fascinating research area. In this direction, Hass (2005) has presented a model for magnetized quantum plasmas and provided the quantum magnetohydrodynamic (QMHD) set of equations. In recent years, many researchers have discussed the Jeans instability of quantum plasma with QMHD equations. The Jeans instability of quantum magnetoplasma with Hall current and resistivity effects has been studied by Wu et al. (2010). Shukla and Stenflo (2008) have tried to explore the effect of Hall and quantum effects for a warm dense magnetoplasma with an anisotropic pressure. Effect of Hall current and viscosity on the Jeans instability of quantum magnetoplasma has been investigated by Prajapati and Chhajlani (2010). The problem of Jeans instability of quantum plasma in the presence of magnetic field and resistive effects is discussed by Ren et al. (2009). The Jeans instability criterion for magnetized systems, including spin-induced magnetization, has been investigated by Lundin et al. (2008). In all these studies (Lundin et al. 2008; Shukla and Stenflo 2008; Ren et al. 2009; Prajapati and Chhajlani 2010; Wu et al. 2010), the QMHD model (Hass 2005) is used for the investigations of various problems in quantum plasma.

In this direction, considering the importance of the problem of Jeans instability in quantum systems with various parameters, we incorporated the spin-generated magnetization effect and Hall current effect with viscosity in the study of Jeans instability in quantum plasma. The present study is motivated by Lundin et al. (2008), Ren et al. (2009), and Prajapati and Chhajlani (2010). Lundin et al. (2008) studied the Jeans instability of quantum plasma with spin-induced magnetization; Ren et al. (2009) studied the Jeans instability with resistive effects; and Prajapati and Chhajlani (2010) studied the same with viscosity and Hall effects using the QMHD set of equations. However, none of the works has considered the combined effect of viscosity, spin-induced magnetization, and Hall current on the study of Jeans instability in quantum plasma. Thus, considering the importance of present problem in the fragmentation process of dense astrophysical system, where viscosity is of importance, we investigate the Jeans instability of infinitely conducting quantum plasma incorporating the simultaneous effects of Hall current, viscosity, and spin-induced magnetization. The paper is organized as follows. Section 2 is devoted to the formulation of the model using basic equations from the QMHD theory, including Hall current, magnetization, and viscosity in quantum plasma. In Section 3, linearized perturbation equations and the modified dispersion relation are derived and discussed analytically and numerically. In the last section, some conclusive comments and results are presented.

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2. Basic set of equations

The QMHD theory with spin-induced magnetization in the presence of Hall current and viscosity is considered in quantum plasma. The quantum effects are introduced through the Bohm potential term in momentum transfer equation, which describes the quantum forces. We follow the QMHD set of equations derived by Hass (2005) in which the magnetic field is treated as classical. In the description of the QMHD picture of plasma, the electrons are assumed to behave as quantum mechanically and the ions are often treated classically. Therefore, we neglected the quantum magnetization of plasma, and treated magnetic field and generated effects as classical. The influence of viscous force, Hall current, and spin-induced magnetization is also incorporated in the equations. This will be a modification in the QMHD model given by Hass (2005), Lundin et al. (2008), and Ren et al. (2009). The governing equations for the self-gravitating ideal QMHD plasma with spin effects are given as the momentum conservation equation for viscous magnetized quantum fluid with spin-induced magnetization,

$$\rho\left(\frac{\partial}{\partial t}+\boldsymbol{v}.\nabla\right)\boldsymbol{v}=-\nabla p+\rho\nabla\phi-\left(\nabla\times\frac{\boldsymbol{H}}{4\pi}\right)\times\boldsymbol{H}-(\nabla\times\boldsymbol{M})\times\boldsymbol{H}+\nu\rho\nabla^2\boldsymbol{v}+\frac{\hbar^2\rho}{2m_em_i}\nabla\left(\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}\right). \tag{1}$$

The mass density conservation law is

$$\frac{\partial\rho}{\partial t}+\nabla.(\rho\,\boldsymbol{v})=0. \tag{2}$$

The equation for idealized Ohm's law is

$$\frac{\partial\mathbf{H}}{\partial t}=\nabla\times(\boldsymbol{v}\times\mathbf{H})-\frac{c}{4\pi Ne}\left\{\nabla\times[(\nabla\times\boldsymbol{H})\times\boldsymbol{H}]\right\}, \tag{3}$$

and the Poisson's equation for the self-gravitational potential is

$$\nabla^2\phi=-4\pi G\rho. \tag{4}$$

The equation of state for pressure in quantum plasma is

$$\nabla p=c_s^2\nabla\rho, \tag{5}$$

where $c_s=(dp/d\rho)^{1/2}$, the modified ion-acoustic velocity due to quantum effects, can be given as

$$c_s^2=v_{\mathrm{ti}}^2+\frac{m_i}{m_e}v_{\mathrm{te}}^2+\frac{3}{5}\frac{m_i}{m_e}v_{\mathrm{Fe}}^2, \tag{6}$$

where v_{ti} and v_{te} are the ion and electron thermal velocities, $v_{\mathrm{Fe}}=\hbar(3\pi^2n_e)/m_e$ is the Fermi velocity (Hass 2005; Lundin et al. 2008; Shukla and Stenflo 2008; Ren et al. 2009; Prajapati and Chhajlani 2010; Wu et al. 2010), and $\boldsymbol{v}$ $(v_x,\ v_y,\ v_z)$, $p, \phi, G, \nu, N, \rho, c, e, m_e, m_i$, and $\hbar$ denote the fluid velocity, fluid pressure, gravitational potential, gravitational constant, fluid viscosity, electron number density, fluid density, speed of light, electronic charge, electron mass, ion mass, and Planck's constant divided by 2π respectively. Further, the last term (Bohm potential) in the momentum equation gives the correction due to quantum diffraction effects because of density correlations. It is important to mention that we have not incorporated the effect due to spin-induced magnetization in the Hall term of (3). The

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external magnetic field $\boldsymbol{H}(0, 0, H)$ is assumed in the z-direction. The magnetization $\boldsymbol{M}$ of the system on time scale shorter than the spin relaxation time scale is given by

$$\boldsymbol{M} = \frac{\mu_B \rho}{m_i} \tanh\left(\frac{\mu_B H}{k_B T}\right) \hat{H}, \tag{7}$$

where $\hat{H} = \boldsymbol{H}/H$ and $\mathrm{H} = |\mathbf{H}|$, and $\mu_B = e\hbar/2m_e$ is the Bohr magnetron and T is the temperature. We consider $\tanh(\mu_B H/k_B T)$ as constant for initially homogeneous plasma (Lundin et al. 2008) and this is also a function of temperature depending on a range of temperature with Fermi temperature.

3. Perturbation equations and dispersion relation

To accomplish the process of linearization, all variable quantities are assumed to be made up of equilibrium and perturbed parts as

$$\begin{aligned} &\boldsymbol{v} = \boldsymbol{v}_0 + \delta\boldsymbol{v}, \quad p = p_0 + \delta p, \quad \phi = \phi_0 + \delta\phi, \quad \boldsymbol{H} = \boldsymbol{H}_0 + \delta\boldsymbol{h}, \\ &\rho = \rho_0 + \delta\rho, \quad \boldsymbol{M} = \boldsymbol{M}_0 + \delta\boldsymbol{M}. \end{aligned} \tag{8}$$

The terms with subscript '0' are for equilibrium part, and the subscript 'δ' stands for perturbed part. Terms δp, $\delta\rho$, $\delta\boldsymbol{v}$, $\delta\phi$, $\delta\boldsymbol{h}$, and $\delta\boldsymbol{M}$ denote the perturbations in fluid pressure, fluid density, fluid velocity, gravitational potential, magnetic field, and magnetization respectively. We assume $\boldsymbol{v}_0 = 0$ and remove '0' from the equilibrium quantities for simplicity. We use (8) in (1)–(7), and obtain linearized perturbation equations of infinitely conducting, viscous, quantum plasma with Hall effect as follows:

$$\begin{aligned} \rho\frac{\partial\delta\boldsymbol{v}}{\partial t} = &-\nabla\left(\delta p - \rho\,\delta\phi + \frac{\hbar^2 k^2}{4\,m_e\,m_i}\delta\rho\right) - \nabla\boldsymbol{H}.\left(\frac{\delta\boldsymbol{h}}{4\pi} - \delta\boldsymbol{M}\right) + \nabla\boldsymbol{M}.\delta\boldsymbol{h} \\ &+ \boldsymbol{H}.\nabla\left(\frac{\delta\boldsymbol{h}}{4\pi} - \delta\boldsymbol{M}\right) - \nu\,\rho\,k^2\,\delta\,\boldsymbol{v}, \end{aligned} \tag{9}$$

$$\frac{\partial\delta\rho}{\partial t} = -\rho\nabla.\delta\boldsymbol{v}, \tag{10}$$

$$\frac{\partial\delta\boldsymbol{h}}{\partial t} = \nabla\times(\delta v\times\boldsymbol{H}) - \frac{c}{4\pi Ne}\left\{\nabla\times[(\nabla\times\delta\boldsymbol{h})\times\boldsymbol{H}]\right\}, \tag{11}$$

$$\nabla^2\delta\phi = -4\pi G\delta\rho, \tag{12}$$

$$\delta p = c_s^2\delta\rho, \tag{13}$$

and

$$\delta\boldsymbol{M} = M\left(\frac{\delta\boldsymbol{h}_\perp}{H} + \frac{\delta\rho}{\rho}\hat{z}\right), \tag{14}$$

where $\hat{z}$ is the unit vector along the z-direction, $\delta\boldsymbol{h}_\perp(\delta h_x, \delta h_y, 0)$, $M = (H/4\pi)\chi/(1+\chi)$, and the magnetic susceptibility χ is given by

$$\chi = \frac{H\mu_B}{m_i V_A^2}\tanh\left(\frac{\mu_B H}{k_B T}\right)/1 - \frac{H\mu_B}{m_i V_A^2}\tanh\left(\frac{\mu_B H}{k_B T}\right). \tag{15}$$

We assume the perturbation of all the quantities as $\exp i[k_x x + k_z z + \omega t]$, where ω is the harmonic frequency of the perturbation and $k_{x,z}$ are the wave numbers in perpendicular and parallel directions of the magnetic field, and related $k_x^2 + k_z^2 = k^2$. Eliminating the perturbations we solve (9)–(15) by putting variation of perturbation

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and write in terms of $\boldsymbol{v} = (\delta v_x, \delta v_y, \delta v_z)$, and get

$$\left[\omega^2 - i\omega\nu k^2 - k_x^2 T^2 + \frac{\omega^2}{\Lambda_1}\frac{k^2 V_A^2}{1+\chi} + \frac{k_x^2 V_A^2 \chi}{1+\chi}\right]\delta v_x + \frac{i\omega k_z^2 k^2 V_A^2 \zeta}{\Lambda_1(1+\chi)}\delta v_y - k_x k_z\left[T^2 - \frac{V_A^2\chi}{1+\chi}\right]\delta v_z = 0, \tag{16}$$

$$\frac{\zeta k_z^2 k^2 V_A^2}{\Lambda_1(1+\chi)}\delta v_x + \left[i\omega + \nu k^2 + \frac{i\omega}{\Lambda_1}\frac{k_z^2 V_A^2}{(1+\chi)}\right]\delta v_y = 0, \tag{17}$$

$$k_x k_z\left[T^2 + \frac{V_A^2\omega^2\chi}{\Lambda_1(1+\chi)}\right]\delta v_x + \frac{i\omega\zeta k_x k_z^3 V_A^2\chi}{\Lambda_1(1+\chi)}\delta v_y - \left[\omega^2 - i\omega\nu k^2 - k_z^2 T^2\right]\delta v_z = 0, \tag{18}$$

where $T^2 = c_s^2 - 4\pi G\rho/k^2 + \hbar^2 k^2/4m_e m_i$, $V_A = H/\sqrt{\{4\pi\rho\}}$ is the Alfven velocity, the Hall coefficient $\zeta = cH/4\pi Ne$, and $\Lambda_1 = \zeta^2 k_z^2 k^2 - \omega^2$. Equations (16)–(18) can be written in the matrix of $\delta v_x, \delta v_y, \delta v_z$, and the general dispersion relation using $i\omega = \sigma$ is obtained from the determinant of the matrix

$$\begin{aligned}
&\left(\sigma^2 + \sigma\nu k^2 + k_x^2 c_s^2 - \frac{4\pi G\rho k_x^2}{k^2} + \frac{\hbar^2 k^2 k_x^2}{4m_e m_i} + \frac{V_A^2\sigma^2 k^2}{\Lambda(1+\chi)} - \frac{k_x^2 V_A^2\chi}{1+\chi}\right)\\
&\times\left(\sigma + \nu k^2 + \frac{\sigma k_z^2 V_A^2}{\Lambda(1+\chi)}\right)\left(\sigma^2 + \sigma\nu k^2 + k_z^2 c_s^2 - \frac{4\pi G\rho k_z^2}{k^2} + \frac{\hbar^2 k^2 k_z^2}{4m_e m_i}\right)\\
&+\sigma\left(\frac{V_A^2 k_z^2 k^2\zeta}{\Lambda(1+\chi)}\right)^2\left(\sigma^2 + \sigma\nu k^2 + k_z^2 c_s^2 - \frac{4\pi G\rho k_z^2}{k^2} + \frac{\hbar^2 k^2 k_z^2}{4m_e m_i}\right)\\
&+k_x k_z\left(c_s^2 - \frac{4\pi G\rho}{k^2} + \frac{\hbar^2 k^2}{4m_e m_i} - \frac{V_A^2\chi}{1+\chi}\right)\\
&\times\left[\frac{\zeta^2 k_x k^2 k_z^5 V_A^4\chi}{\Lambda^2(1+\chi)^2} - k_x k_z\left(c_s^2 - \frac{4\pi G\rho}{k^2} + \frac{\hbar^2 k^2}{4m_e m_i} - \frac{\sigma^2 V_A^2\chi}{\Lambda(1+\chi)}\right)\right.\\
&\left.\times\left(\sigma^2 + \nu k^2 + \frac{\sigma k_z^2 V_A^2}{\Lambda(1+\chi)}\right)\right] = 0,
\end{aligned} \tag{19}$$

where $\Lambda = \zeta^2 k_z^2 k^2 + \sigma^2$. Equation (19) represents the general dispersion relation of the Jeans instability of cold quantum plasma, including the effects of viscosity, Hall current, and spin-induced magnetization. The dispersion relation depicts a linear coupling between various plasma modes at quantum scale in the presence of magnetic fields. The dispersion relation exhibits significant modifications in the Jeans instability of cold quantum magnetoplasma due to the inclusion of Hall current, viscosity, and spin-induced magnetization. It is important to mention that c_s^2 also contains quantum term. In the absence of viscosity and Hall current ($\nu = \zeta = 0$), dispersion relation (19) reduces to that of Lundin et al. (2008) with quantum effects. In the absence of magnetization, (19) reduces to that of Prajapati and Chhajlani (2010), where quantum effects are considered with Hall current and self-gravitation. If we ignore the effect of viscosity, magnetization, and Hall current, the dispersion relation given by Ren et al. (2009) is recovered, excluding resistive effects in that case. When we ignore spin magnetization and viscosity effects from the dispersion relation (19), it recovers the result given by Wu et al. (2010), excluding resistive effects in the system. In the absence of quantum effect and other included parameters, i.e. Hall current, magnetization, and viscosity, the dispersion relation given by (19) recovers the result given by Chandrasekhar (1961). Therefore, it is important to mention that dispersion

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relation (19) is a modified form of Chandrasekhar (1961) with effects due to Hall current, spin-induced magnetization, viscosity, and the quantum effect in the term c_s^2. In the absence of magnetization and quantum term, the above-general dispersion relation reduces to that given by Kalra and Talwar (1964), excluding electron inertia and resistive effects. If we neglect magnetization and quantum terms in dispersion relation (19), it recovers the classical result given by Hans (1966).

This dispersion relation (19) is complex and in order to study the effect of these parameters on the condition of instability, (19) is investigated for transverse and longitudinal modes of propagations. So we further reduce the dispersion relation (19) for two modes of propagation.

3.1. *Transverse propagation* ($k_x = k, k_z = 0$)

When the perturbation is assumed to be in the transverse direction of magnetic field, we get the following from (19):

$$\left(\sigma + \nu k^2\right)^2 \left(\sigma^2 + \sigma\nu k^2 + k^2 v_{\mathrm{ti}}^2 + k^2 \frac{m_i}{m_e}\left(v_{\mathrm{te}}^2 + \frac{3}{5}v_{\mathrm{Fe}}^2\right) - 4\pi G\rho + \frac{\hbar^2 k^4}{4m_e m_i} + \frac{V_A^2 k^2}{(1+\chi)} - \frac{\chi}{1+\chi}k^2 V_A^2\right) = 0. \tag{20}$$

The dispersion relation (20) has two different factors. It is clear that the propagation perpendicular to the direction of magnetic field separates wave propagation into two different factors: one for the viscosity term, and the other for the magnetization parameter, magnetic field, quantum term, and Jeans term. Here we discuss each factor separately.

The first factor in (20) gives

$$\sigma + \nu k^2 = 0. \tag{21}$$

This represents a stable damped mode due to the presence of the viscosity of the medium. Thus, viscous force is trying to stabilize the growth rate of the present system. The above mode is unaffected by the presence of quantum correction, self-gravitation, magnetization, magnetic field strength, and Hall current. This dispersion relation is identical to that given by Prajapati et al. (2010).

The second factor of (20) on putting various values gives

$$\sigma^2 + \nu k^2\sigma + k^2 v_{\mathrm{ti}}^2 + k^2\frac{m_i}{m_e}\left(v_{\mathrm{te}}^2 + \frac{3}{5}v_{\mathrm{Fe}}^2\right) - 4\pi G\rho + \frac{\hbar^2 k^4}{4m_e m_i} + k^2 V_A^2\frac{(1-\chi)}{(1+\chi)} = 0. \tag{22}$$

Relation (22) represents a gravitating mode modified due to the presence of magnetic field (Alfven velocity), magnetization effects, and viscosity with quantum corrections. The above dispersion relation is identical to the relation given by Sharma and Chhajlani (2014a), excluding finite electrical resistivity in the transverse mode of propagation. The dispersion relation (22) is in the quadratic form whereas in Sharma and Chhajlani (2014a) this relation is in cubic equation due to resistivity, and if we ignore resistivity, then we get the above relation. In the present case, the growth rate of Jeans instability in the presence of Fermi velocity, diffraction term, and magnetization depends upon the viscosity of the medium. The dispersion relation (22) may be easily verified with the previous known results if we ignore a particular parameter in the same. If we ignore viscosity and magnetization terms, then the dispersion relation is reduced to the one obtained by Ren et al. (2009), excluding the resistive effects in that case. In the absence of magnetization, (22) resembles dispersion relation obtained by Prajapati and Chhajlani (2010) in the transverse direction of propagation, and

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dispersion relation of Prajapati and Chhajlani (2010) is modified in the form of (22) due to spin-induced magnetization in the present case. In the absence of viscosity, (22) reduces to the result given by Lundin et al. (2008). So it is clear that in the present case the dispersion relation given by Lundin et al. (2008) is modified due to the presence of viscosity. Hence, these are the new findings in our case than those of Lundin et al. (2008) and Prajapati and Chhajlani (2010).

Further, the behavior of the system (nature of perturbation) will be governed by the roots of (22), i.e. whether it is growing, damped, or oscillatory. The values of the roots depend upon the coefficients of the equation, which is governed by magnetization, viscosity, and both the quantum effects, viz. Fermi pressure and quantum diffraction. Since (22) is a quadratic equation and it has two roots, it is clear that if the value of constant term is less than zero, then (22) will have at least one positive real root and the system described by the above equation will be unstable. Accordingly, from the constant term of (22) the condition of instability can be easily obtained as

$$v_{\mathrm{ti}}^2 + \frac{m_i}{m_e}\left(v_{\mathrm{te}}^2 + \frac{3}{5}v_{\mathrm{Fe}}^2\right) - \frac{4\pi G\rho}{k^2} + \frac{\hbar^2 k^2}{4m_e m_i} + V_A^2\frac{(1-\chi)}{(1+\chi)} < 0. \tag{23}$$

Equation (23) is nothing but the Jeans criteria in the quantum system modified due to intrinsic magnetization corrections, and hence the system represented by (22) will be unstable if it holds the above condition (23). It is evident from (23) that the condition of Jeans instability depends upon magnetic field strength, both the quantum terms, viz. quantum diffraction and Fermi pressure, and the spin-generated magnetization of the system, and it can be clearly seen that the condition of instability is unaffected by the presence of Hall current and viscosity. Thus, we note that in addition to thermal velocity, the Fermi velocity has a role in deciding Jeans instability depending upon the range of temperature. Also, magnetization affects Jeans instability with values depending on temperature in respect of Fermi temperature. Condition (23) will also alter if we keep only quantum Fermi velocity and ignore the term due to Bohm potential. Similarly, if we neglect Fermi velocity and keep diffraction term, the condition will be different. In Neutron star both these effects can be considered simultaneously, or only one effect may be included depending upon the structure on Neutron star. But it is important to mention that in the absence of magnetization the condition of instability (23) is of the following form:

$$v_{\mathrm{ti}}^2 + \frac{m_i}{m_e}\left(v_{\mathrm{te}}^2 + \frac{3}{5}v_{\mathrm{Fe}}^2\right) - \frac{4\pi G\rho}{k^2} + \frac{\hbar^2 k^2}{4m_e m_i} + V_A^2 < 0. \tag{24}$$

This is the same as obtained by Ren et al. (2009). So it is to be mentioned that the condition of Jeans instability found in the present case is modified due to the magnetization term to the condition obtained by Ren et al. (2009) in the absence of resistivity in that case. Condition (23) in the present case is identical to the one obtained by Lundin et al. (2008). Thus, we find that in Lundin et al. (2008) the effect of viscosity and Hall current is not considered. Here we can conclude that with Hall current and viscosity the condition of instability remains the same but the growth rate of instability will be different due to the presence of these parameters in comparison to Lundin et al. (2008). It is to be noted that in the absence of magnetization term, the term related to magnetic field strength independently contributes to the condition of instability, while in the presence of spin effects the Alfven velocity term is coupled with the magnetization term χ. It is apparent from the condition of instability (23), the quantum term is added to magnetic field and sonic mode against the self-gravitation term in the condition of instability, and therefore in the transverse direction of

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propagation the quantum effects (in terms of the Bohm potential and the Fermi velocity) serve as stabilizer to the basic sonic mode of the system. So the quantum corrections have stabilizing influence on the growth rate of instability. It is also clear from condition (23) that the magnetization corrections have negative sign as that of gravitational term. Therefore, the magnetization parameter has destabilizing influence on the growth rate of instability of the system when the perturbation propagates to the transverse direction of magnetic field. In the transverse direction of propagation, the system defined may be unstable even if we neglect the gravitational effect from the system, only if the condition given below is satisfied:

$$\left(\frac{v_{\mathrm{ti}}^2}{V_A^2}+\frac{m_i}{m_e}\left(\frac{v_{\mathrm{te}}^2}{V_A^2}+\frac{3}{5}\frac{v_{\mathrm{Fe}}^2}{V_A^2}\right)+\frac{\hbar^2k^2}{4m_em_iV_A^2}+1\right)\frac{1}{2}<\frac{\chi}{(1+\chi)}. \tag{25}$$

This condition for non-gravitating system also depends upon the quantum corrections contribution in the form of the Fermi velocity and the Bohm potential. This is an important result at low temperature degenerate plasma system. This relation is also true for viscous, non-gravitating system, and the growth rate of the system will depend on the viscosity of non-gravitating medium. Thus, the magnetization is itself responsible for the instability of the system. If we ignore the magnetization term, then the above condition matches with Prajapati and Chhajlani (2010).

In the absence of magnetic field, magnetization, and quantum term, the above condition simply resembles the ordinary Jeans condition of instability given as

$$c_s^2=v_{\mathrm{ti}}^2+\frac{m_i}{m_e}\left(v_{\mathrm{te}}^2+\frac{3}{5}v_{\mathrm{Fe}}^2\right)<\frac{4\pi G\rho}{k^2}. \tag{26}$$

The term c_s^2 is also having quantum correction with the Fermi velocity whereas in the work done by Chandrasekhar (1961) the plasma is considered classical and the Fermi velocity is not present in c_s^2. For the applicability of the above condition the system must have the Jeans wave length, which is given as $\lambda_J=2\pi/k_J$. In particular, if we discard quantum terms, the critical wave number k_J is given as

$$k_J=\left[\frac{4\pi G\rho}{\left\{v_{\mathrm{ti}}^2+\frac{m_i}{m_e}\left(v_{\mathrm{te}}^2+\frac{3}{5}v_{\mathrm{Fe}}^2\right)+V_A^2\left\{(1-\chi)/(1+\chi)\right\}\right\}}\right]^{1/2}. \tag{27}$$

The system represented by (22) will be unstable for transverse propagation if the value of the wave number $k<k_J$. The above expression for critical wave number (when both the quantum effects and magnetization terms are neglected) is identical to the expression for critical wave number obtained by Sharma (1974) for the non-rotational case. It is also to be noted that k_J has simple sonic speed whereas in quantum plasmas this c_s^2 also includes the quantum correction due to Fermi velocity.

In dense astrophysical environment, the structure formation is mainly due to the unstable growing wave modes. We discuss numerically the gravitating mode obtained in the transverse direction to the magnetic field to study the effects of magnetization, viscosity, and quantum corrections on the growth rate of Jeans instability. The effect of each parameter on the growth rate of Jeans instability can be numerically observed in the gravitating mode. To solve (22) numerically, we introduce the following dimensionless quantities:

$$\sigma^*=\frac{\sigma}{\sqrt{4\pi G\rho}},\quad k^*=\frac{k\,c_s}{\sqrt{4\pi G\rho}},\quad Q=\frac{\hbar\sqrt{4\pi G\rho}}{m_em_ic_s^2},\quad V_A^*=\frac{V_A}{c_s},\quad \nu^*=\frac{\nu\sqrt{4\pi G\rho}}{c_s^2}. \tag{28}$$

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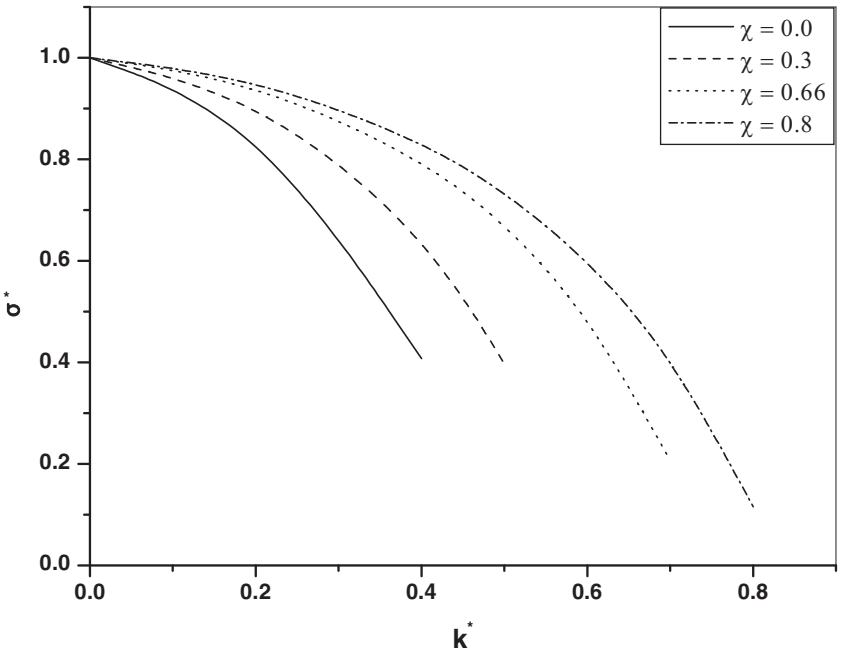


FIGURE 1. The normalized growth rate of Jeans instability versus normalized wave number k^* in the transverse direction of propagation, for various values of the dimensionless magnetization parameter (χ). The values of constant parameter are taken as $\nu^* = 0.5$, $Q = 0.5$, and $V_A^* = 2$.

Using (28), we write (22) in dimensionless form as

$$\sigma^{*2} + \sigma^* \nu^* k^{*2} + k^{*2} - 1 + \frac{Q^2}{4} k^{*4} + k^{*2} V_A^{*2} \frac{(1-\chi)}{(1+\chi)} = 0. \tag{29}$$

Numerical calculations of (29) are performed to investigate the growth rate of Jeans instability. The positive roots of σ^* from (29) are obtained to observe the behavior of instability as a function of wave number k^* for several values of different parameters involved. It is clear from (29) that growth rate in the transverse configuration of propagation depends upon the quantum parameter Q, spin-induced magnetization, and magnetic field strength.

First, to observe the effect of spin-induced magnetization on the growth rate of Jeans instability we have plotted the real positive roots against the normalized wave number. In Fig. 1, the growth rate of instability (positive root of σ^*) is shown to be varied with the normalized wave number (k^*) for various values of dimensionless magnetization parameter χ and for fixed value of dimensionless quantum parameter $Q = 0.5$, viscosity parameters $\nu^* = 0.5$ and $V_A^* = 2$. From Fig. 1, we conclude that the growth rate increases with increasing the value of magnetization. Thus, the effect of magnetization is destabilizing on the growth rate of instability. As it is obvious from (29), which shows that the magnetization parameter has negative sign in support of gravitational term. The magnetization parameter is found to play important share in the process of gravitational collapse, and increase in magnetization parameter tends the system to move toward destabilization. The figure shows four curves, the solid line is for $\chi = 0$ and further dashed and dotted lines are for $\chi = 0.3$ and $\chi = 0.66$ and one more observation, i.e. dashed dotted line is for $\chi = 0.8$. Figure 1 clearly depicts that initially the peak value of growth rate is the same for all the

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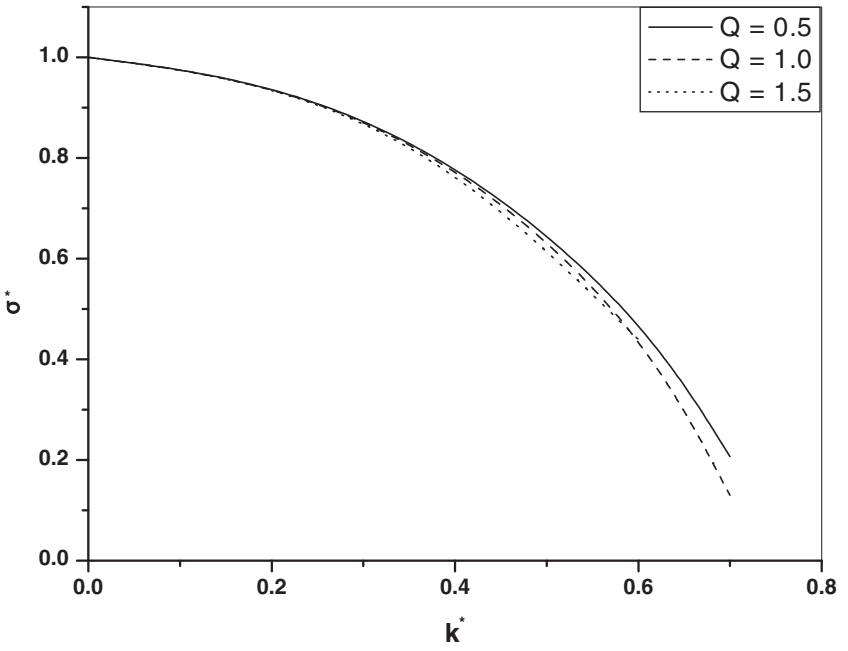


FIGURE 2. The normalized growth rate of Jeans instability σ^* versus normalized wave number k^* in the transverse direction of propagation for various values of the dimensionless parameter (Q). The values of constant parameter are taken as dimensionless viscosity parameter $\nu^* = 0.5$, $\chi = 0.66$, and $V_A^* = 2$.

values of magnetization but later the growth rate starts increasing for the larger values of magnetization parameter. Hence, the spin-induced magnetization increases the instability of self-gravitating quantum viscous plasma.

In Fig. 2, we have discussed the diffraction effect of quantum parameter (Q) by tracing a curve between the growth rate of Jeans gravitational instability and the wave number k^* for a fixed value of dimensionless magnetization parameter $\chi = 0.66$, viscosity parameters $\nu^* = 0.5$ and $V_A^* = 2$. Figure 2 clearly shows the effect of quantum parameter Q on the growth rate of Jeans instability in the presence of viscosity and magnetization. In Fig. 2, we have taken the values of constant parameters as $Q = 0.5$, $Q = 1$, and $Q = 1.5$. In the figure, the solid line is for $Q = 0.5$ and further dashed and dotted lines are for $Q = 1$ and 1.5 respectively. It is obvious from the figure that owing to an increase in quantum parameter, the growth rate of Jeans instability decreases. Hence, the finite quantum parameter has stabilizing influence on self-gravitating quantum plasma. It is clear that the quantum parameter plays an important role in the process of gravitational collapse, and thus increase in quantum parameter causes the system to move toward stabilization.

Figure 3 illustrates the growth rate of gravitational perturbation considering the different values of dimensionless viscosity parameter, i.e. the effect of parameter ν^* for the fixed value of dimensionless quantum parameter $Q = 0.5$, Alfven velocity $V_A^* = 2$, and magnetization parameter $\chi = 0.66$. The figure includes three curves, the solid line is for $\nu^* = 0.5$ and further dashed and dotted lines represent $\nu^* = 0.8$ and $\nu^* = 1.0$ respectively. It is clear from curves that the growth rate decreases as the value of viscosity parameter increases. Thus, increase in viscosity causes the system to move toward stabilization. Figure 3 clearly represents the stabilizing influence of

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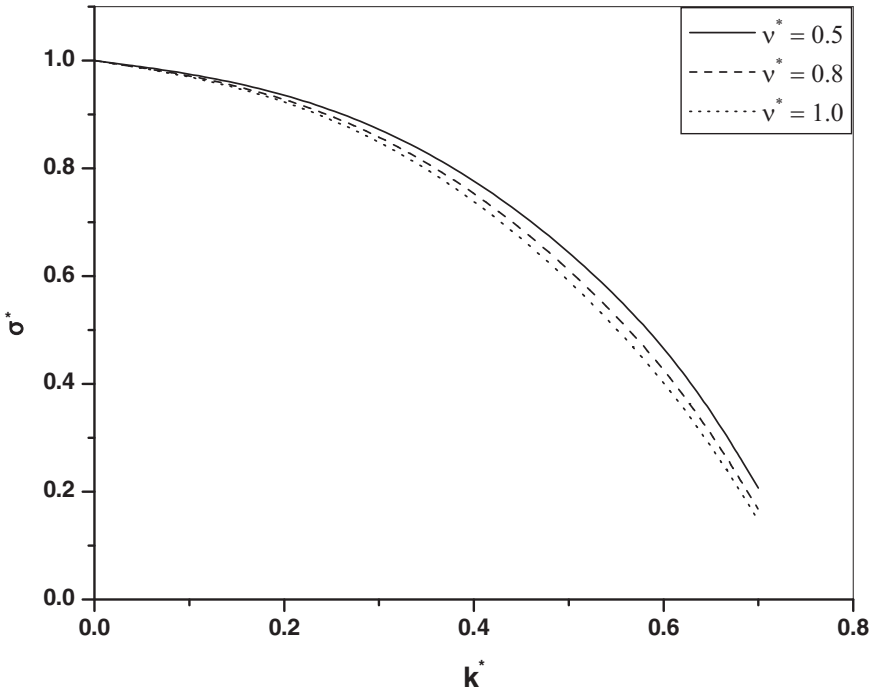


FIGURE 3. The normalized growth rate of Jeans instability σ^* versus the normalized wave number k^* in the transverse direction of propagation for various values of the dimensionless viscosity (ν^*). The values of constant parameter are taken as dimensionless magnetization parameter $\chi = 0.66$, $Q = 0.5$, and $V_A^* = 2$.

parameter ν^* on the growth rate of Jeans instability. The peak value of the growth rate is constant for all the values of viscosity parameter, but further, the growth rate of Jeans instability decreases with increase in parameter ν^*. We find that the unstable region decreases with increasing the value of ν^*. Although the peak value of growth rate for any ν^* is constant, the unstable region is larger for a minimum value of ν^*.

Next, in Fig. 4 the growth rate of gravitational perturbation is computed from dispersion relation given by (29) and is plotted against k^* by taking different values of magnetic field strength for a fixed value of dimensionless magnetization parameter $\chi = 0.66$, quantum parameter $Q = 0.5$, and viscosity parameter $\nu^* = 0.5$. The curves are drawn in Fig. 4, which clearly shows that the growth rate decreases as the value of magnetic field parameter increases. The figure includes three curves, the solid line is for $V_A^* = 1$ and further dashed and dotted lines represent $V_A^* = 2$ and $V_A^* = 3$ respectively. We find that the region of instability decreases by increasing the value of V_A^*. Thus, the magnetic field strength V_A^* has influence on the Jeans instability, and we conclude that magnetic field strength V_A^* has a stabilizing role in determining the growth rate of gravitational instability of quantum plasma.

The Jeans instability is generally recognized as forming the basis of our understanding of gravitational condensation. The role of self-gravitational instability in the formation and in the different processes of astrophysical plasmas, such as atmospheres of Neutron stars and the interior and outer shell of massive white dwarfs, is well established. Neutron stars have very strong magnetic fields, even found with fields exceeding the order of 10^{14} G for young radio and x-ray pulsars, and 10^8–10^{14} G for much older millisecond pulsars. Further, it is very well established

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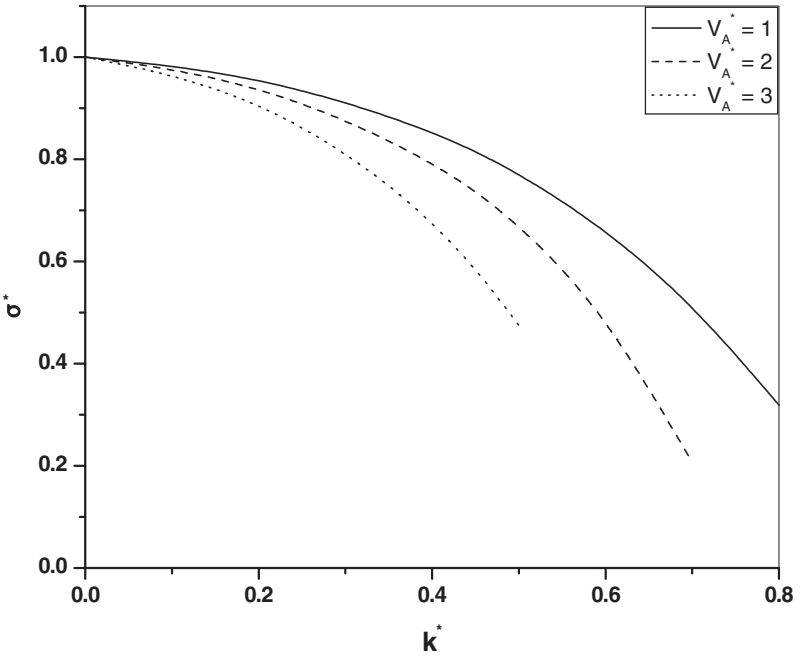


FIGURE 4. The normalized growth rate of Jeans instability σ^* versus normalized wave number k^* in the transverse direction of propagation for various values of the dimensionless Alfven velocity (V_A^*). The values of constant parameter are taken as dimensionless magnetization parameter $\chi = 0.66$, $Q = 0.5$, and $\upsilon^* = 0.5$.

that the Hall effect plays an important role in the dynamics of Neutron stars (Shaikh et al. 2008; Mengesha and Tessema 2013). Under these circumstances, the quantum theory comes into picture and the quantum hydrodynamics governs the dynamics of the Fermi system in which the Hall effect, viscosity, magnetic field, and other transport coefficients rule the process. It is important to mention that the Hall term appears in the general dispersion relation (19) in the denominator term and when we set apart the propagation in transverse and longitudinal directions, it does not appear in the dispersion relation of transverse and longitudinal directions. Thus, the Hall effect only affects the growth rate of Jeans instability but has no effect on the condition of Jeans instability and Jeans length. It is clear from the dispersion relation (22) that the gravitational mode was only affected by the magnetic susceptibility term introduced due to spin-generated magnetization. Further, if we take into account the spin-generated effects, we see that the Jeans wave number becomes $k_J = [4\pi G\rho/\{(1-\chi)/(1+\chi)\}\}]^{1/2}$. This expression shows that in the transverse mode of propagation the Jeans wave number decreases because of spin effects. It is clear that in the absence of susceptibility terms, the wave number takes the form of $k_J = [4\pi G\rho/(c_s^2 + V_A^2)]^{1/2}$. We can see here that the denominator terms are reduced due to the presence of susceptibility terms, and the modification due to susceptibility terms appears only in the Alfven term and not in the acoustic term. Further, the Jeans length $L_J = 2\pi/k_J$ is reduced due to the presence of magnetization effects, and hence this destabilizes the system. It is clearly visible that the spin effect terms decrease the pressure against the gravitational force and makes the system more likely to collapse. In order to find the time scales for gravitational collapse, we use $\omega_J^2 = (c_s^2 + (1-\chi)\, V_A^2/(1+\chi))/L_j$ and find the collapse time scale

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for the astrophysical structure. To estimate the numerical values of parameters, for a typical astrophysical region, the plasma density, $n_0 = 10^{30} - 10^{36}\,\mathrm{m}^{-3}$, temperature, $T \sim 10^5 - 10^8$ K, and magnetic field, $B_0 = 10^9 - 10^{14}$ G. Magnetic field appropriates to exist Hall effect and spin-generated effects in the system, and the mass density of the region is about 10^9 kg m^{-3} (the number density can be of the order of $10^{36}\,\mathrm{m}^{-3}$) and temperature $T \sim 10^7$ K (Shaikh et al. 2008; Mengesha and Tessema 2013). Using appropriate parameters for this situation, the typical Jeans length, $L_j = (\pi kT/G\rho m)^{1/2}$ comes out as of the order of $\sim 10^3$ to 10^5 m, which means that if the length is more than this range, the system will be unstable. It is important to mention that the presence of spin-induced magnetization reduces the Jeans length. Viscosity is also important in such a dense medium. The presence of viscosity with magnetization and magnetic field affects the growth rate of self-gravitational Jeans instability in the present quantum system.

3.2. *Longitudinal propagation* ($k_x = 0, k_z = k$)

In this case the perturbations are taken along the direction of the magnetic field $k_x = 0,\ \ k_z = k$. The dispersion relation (19) reduces to

$$\left(\sigma^2 + \sigma\nu k^2 + k^2 v_{\mathrm{ti}}^2 + k^2 \frac{m_i}{m_e}\left(v_{\mathrm{te}}^2 + \frac{3}{5}v_{\mathrm{Fe}}^2\right) - 4\pi G\rho + \frac{\hbar^2 k^4}{4m_e m_i}\right) \times \left\{\left(\sigma + \nu k^2 + \frac{V_A^2 k^2 \sigma}{\Lambda(1+\chi)}\right)^2 + \left(\frac{V_A^2 k^4 \zeta}{\Lambda(1+\chi)}\right)^2\right\} = 0. \tag{30}$$

The dispersion relation (30) gives the longitudinal wave propagation to the direction of the magnetic field for quantum plasma with Hall current, viscosity, and spin-induced magnetization. We find that the dispersion relation splits in two independent factors, one of which is gravitating mode influenced by the quantum term and the viscosity, and the other is independent of the quantum terms but depends upon the Hall current and spin effect of magnetization.

The first factor of (30) when equated to zero gives

$$\sigma^2 + \nu k^2 \sigma + k^2 v_{\mathrm{ti}}^2 + k^2 \frac{m_i}{m_e}\left(v_{\mathrm{te}}^2 + \frac{3}{5}v_{\mathrm{Fe}}^2\right) - 4\pi G\rho + \frac{\hbar^2 k^4}{4m_e m_i} = 0. \tag{31}$$

This dispersion relation represents a gravitating mode modified due to the presence of viscosity and both quantum corrections, viz. Fermi velocity and diffraction effect. This dispersion relation is identical to the dispersion relation given by Prajapati and Chhajlani (2010). In the absence of viscosity, quantum term, and the quantum term in c_s this relation is identical to that obtained by Chandrasekhar (1961). The dispersion relation (31) for gravitating mode in the presence of viscosity is similar to Sharma and Chhajlani (2014a) and effects of Hall current, viscosity, and magnetization are coupled with another Alfven mode. Thus, the present gravitating mode in quantum plasma is the modified form of classical plasma. It is clear that this mode has no influence on parameters such as Hall current and intrinsic magnetization, whereas it is influenced by viscosity and quantum parameter. Further, (31) is the quadratic equation and if its constant term is negative then at least one of the two roots will be real positive, which will make the system unstable. From the constant term of (31), the condition of Jeans instability can be easily obtained as

$$\frac{4\pi G\rho}{k^2} > v_{\mathrm{ti}}^2 + \frac{m_i}{m_e}\left(v_{\mathrm{te}}^2 + \frac{3}{5}v_{\mathrm{Fe}}^2\right) + \frac{\hbar^2 k^2}{4m_e m_i}. \tag{32}$$

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From the above condition and (31), we note that the magnetization and Hall current have no role on the growth rate of instability determined by this condition whereas viscosity has a role on the growth rate of Jeans instability. We also note that condition (32) is different from condition (23) of transverse propagation. The system represented by (31) will be unstable if it holds the above condition. This condition is identical to that given by Ren et al. (2009).

If we discard the quantum Bohm potential term and quantum term in c_s then the above condition simply becomes the ordinary Jeans instability condition, i.e.

$$\frac{4\pi G\rho}{k^2} > v_{\mathrm{ti}}^2 + \frac{m_i}{m_e} v_{\mathrm{te}}^2 \tag{33}$$

as given by Chandrasekhar (1961). Thus, we find that the viscosity parameter does not affect the condition of Jeans instability of magnetized, viscous quantum plasma when the perturbation is considered along the direction of magnetic field, but the growth rate of Jeans instability will be modified due to the presence of viscosity parameter in the dispersion relation (31).

Now to observe the effect of different parameters on perturbation along the longitudinal direction of propagation, we normalize (31). In order to study the effects of the quantum parameter Q and the viscosity parameter υ^* on the growth rate of Jeans instability, we write dispersion relation (31) in dimensionless form using dimensionless parameters from (28) as

$$\sigma^{*2} + \sigma^* \upsilon^* k^{*2} + k^{*2} + \frac{Q^2 k^{*4}}{4} - 1 = 0. \tag{34}$$

To study the effects of a particular parameter on the growth rate of perturbation in longitudinal direction, numerical calculations of the above-normalized equation have been performed. In Figs. 5 and 6 we depicted the growth rate of gravitational perturbation versus dimensionless wave number for various arbitrary values of viscosity and quantum parameter. Figure 5 is plotted for the growth rate of an unstable mode (positive root of (34)) against the non-dimensional wave number with variation in viscosity parameter υ^* (0.1, 0.3, and 0.5) by taking the value of Q as unity. We find that the growth rate of instability decreases with increase in υ^*. Hence, the viscosity has a stabilizing influence on the growth rate of instability in the longitudinal direction of propagation. Similarly, in Fig. 6, we plotted the growth rate of an unstable mode (positive root of σ^*) against the dimensionless wave number with variation in the value of dimensionless quantum parameter Q (1, 2, 3) by taking the values of $\upsilon^* = 0.5$. From the curves, it is observed that Q has the same effect on the growth of instability as that of the viscosity parameter υ^*, i.e. due to an increase in the value of Q, the growth rate of instability decreases. Figure 6 clearly depicts that initially the peak value of the growth rate is the same for all the values of quantum parameter but the growth rate starts decreasing for the larger value of quantum parameter. It is evident from the figure that the frequency of gravitational perturbation decreases as the quantum parameter Q increases. Hence, the value of Q has a stabilizing influence on the growth rate of instability in the longitudinal direction of propagation.

The second factor of (30) gives

$$\sigma^4 + 2\nu k^2 \sigma^3 + \left(\zeta^2 k^4 + \nu^2 k^4 + \frac{2V_A^2 k^2}{(1+\chi)}\right)\sigma^2 + \left(2\nu\zeta^2 k^6 + \frac{2\nu V_A^2 k^4}{(1+\chi)}\right)\sigma + \nu^2\zeta^2 k^8 + \frac{V_A^2 k^4}{(1+\chi)^2} = 0. \tag{35}$$

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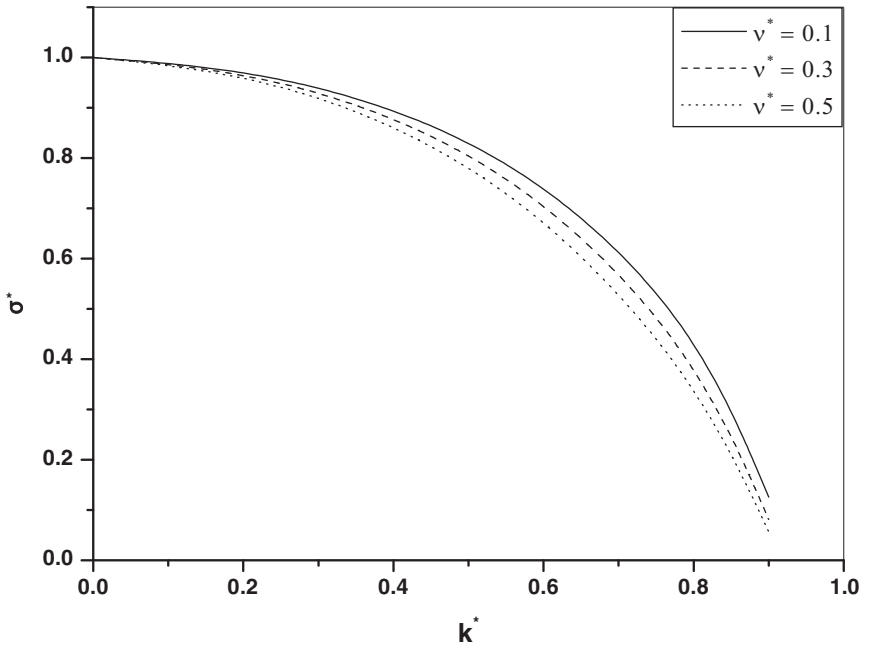


FIGURE 5. The growth rate (positive values of σ^*) in longitudinal direction against wave number k^* at different values of quantum parameter ν^* for a fixed value of dimensionless quantum parameter $Q = 1$.

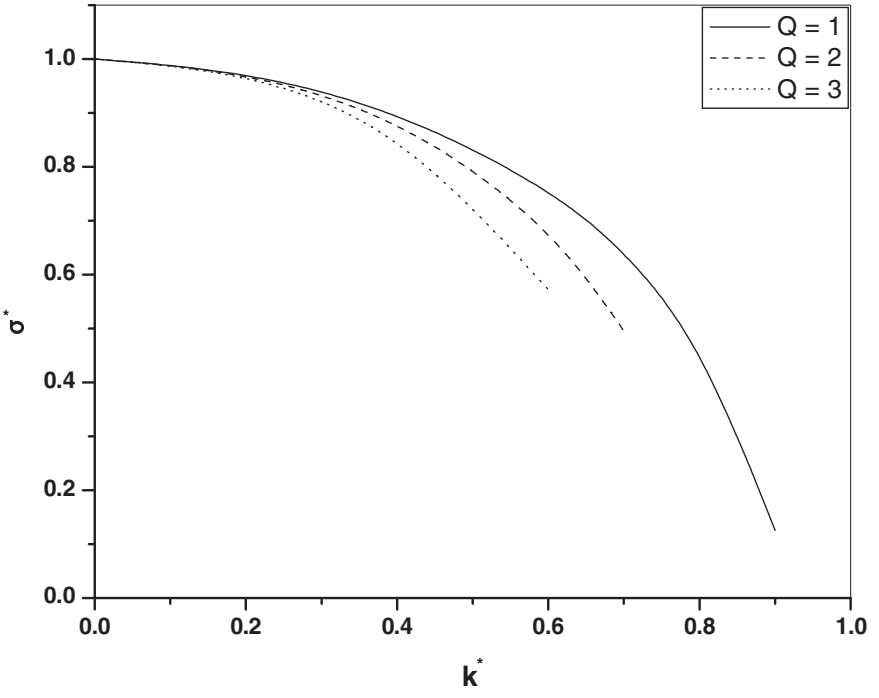


FIGURE 6. The growth rate (positive values of σ^*) in longitudinal direction against wave number k^* at different values of quantum parameter Q for a fixed value of dimensionless viscosity parameter $\nu^* = 0.5$.

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Equation (35) represents a stable non-gravitating Alfven mode modified due to viscosity, magnetization, and Hall current. Thus, magnetization effect is coupled with Alfven mode in this direction. The presence of quantum corrections has no influence in this mode. Thus, viscous force is capable to stabilize the growth rate of the considered system. The above mode is unaffected by the presence of quantum correction and self-gravitation. This dispersion relation is identical to that of (30) given by Prajapati and Chhajlani (2010) with the modification of spin-induced magnetization in the present case. If we ignore the spin-induced magnetization and the quantum term, then the dispersion relation obtained and given by (35) reduces to that of Chhajlani and Parihar (1993), excluding rotation, electron inertia, porosity, and radiative heat loss function from that dispersion relation.

4. Conclusions

In the present work, the effects of Hall current, viscosity, and spin-induced magnetization on the self-gravitational instability of quantum plasma are investigated. The general dispersion relation is obtained and further reduced for both longitudinal and transverse modes of propagation.

In the case of transverse mode of propagation, we get two separate factors. The first factor is usual viscous damped mode unaffected by the Hall current, quantum effect, and spin-induced magnetization. The second one is gravitating mode influenced by viscosity, magnetic field strength, quantum parameter, and magnetization. It is found that the condition of Jeans instability is modified by the presence of magnetization, magnetic field strength, and quantum parameter, while it is independent of viscosity and Hall current. Further, it is observed that the growth rate is affected by the presence of viscosity along with quantum effect and magnetization. From the curves, we find that quantum parameter and viscosity of plasma have a stabilizing influence, while magnetization has destabilizing effect on the growth rate of instability in the transverse mode of propagation.

In the case of longitudinal mode of propagation, we get two separate factors: The first one is the gravitating mode having the effects of viscosity and quantum parameter. We find that the condition of instability is unaffected by the presence of spin-induced magnetization and viscosity but modified by quantum parameter. It is evident from numerical results that the quantum parameter and viscosity of plasma have a stabilizing influence on the growth rate of the system. The second factor is a non-gravitating Alfven mode modified by the presence of magnetization and viscosity with no influence of quantum effect.

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