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Drift–Alfvén vortices in dusty plasmas with non-zero ion-temperature effects

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Abstract. Reduced two-fluid equations governing the nonlinear dynamics of drift–Alfvén waves in dusty plasmas with non-zero ion temperature are derived. In the linear limit, we find a dispersion relation that shows the coupling between the ion-drift–Shukla–Varma mode, and electron-drift (magnetostatic) and (inertial or kinetic) Alfvén waves due to the finite collisionless electron skin depth or Larmor-radius corrections. In contrast to the case of an electron–ion plasma, when the nonlinear drift–Alfvén vortices are weakly localized, i.e. decrease at infinity as r^{-1}, the presence of the charged dust grains makes exponential localization possible. The physical meaning of such a localization is connected with the fact that charged massive dust granules provide an additional screening that results in stronger localization of the vortex. In several intermediate-β plasmas with $1 \gg \beta \gg m_e/m_i$ ($m_{e,i}$ is the electron or ion mass), the localization length approaches a minimum value when the vortex velocity is of the order of the ion diamagnetic drift. It then reaches the value $\rho_i \delta_d^{-1/4}$, where ρ_i is the ion Larmor radius and δ_d is the ratio of the dust to ion densities multiplied by the dust charge number Z_d. In the case of very low plasma pressure, $\beta \ll m_e/m_i$, the vortex is localized with typical scale $(\lambda_e \rho_i)^{1/2} \delta_d^{-1/4}$, where λ_e is the electron skin depth. Our investigation can thus predict the velocities of coherent nonlinear structures in space plasmas.

Dedication

This paper is dedicated to John Dougherty on the occasion of his 65th birthday.

1. Introduction

Dust grains are inherent in a number of space plasma systems and they are important in the formation of planets, planetary rings (Mendis and Axford 1974; Mendis 1997), and accretion discs (Bliokh et al. 1995), as well as in the dynamics of cometary tails (Horanyi and Mendis 1991). Because of the higher temperature and mobility of electrons, the dust charge is usually negative. The dust grains play a crucial role in the formation of noctilucent clouds (Thomas 1991) in the Earth's mesosphere. In situ observations of the dust in the upper atmosphere have recently

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been provided by Havnes et al. (1996) from radar measurements in the high-latitude atmosphere. In laboratories, charged dust grains have been observed in the vicinity of the walls of controlled thermonuclear device (CTD) (Winter 1998) and during the first experiments in dusty-plasma experiment device (DPX) (Thomas and Watson 1999). Much attention has been paid to the study of dust plasmas in the presence of microgravity, for example observations of the dust grains in active experiments on board the MIR orbital station (Fortov et al. 1998), where charging processes were induced by photoemission produced by solar UV radiation. In this case, the dust grains can also be positively charged.

In our recent paper (Pokhotelov et al. 1999), we showed that large-amplitude wave perturbations in dusty plasmas can exist in the form of strongly localized (with exponential fall-off at infinity) drift–Alfvén vortices. It was shown that the presence of dust in a system leads to additional shielding similar to that of Rossby waves in a shallow rotating atmosphere (Petviashvili and Pokhotelov 1992; Pokhotelov et al. 1996), when localization is defined by the deformation radius. This supplementary dust shielding appears because extremely massive charged dust grains provide an effective plasma screening similar to Debye screening.

It should be noted that the theoretical treatment of drift–Alfvén vortices presented by Pokhotelov et al. (1999) was carried out in the approximation where electron temperature T_e is much higher than the ion temperature T_i. In such plasmas the effects due to the electron diamagnetic drift are more important than those due to the ion diamagnetic drift. The latter can thus be neglected, and the corresponding theoretical description can be given in the framework of the ordinary drift approximation without considering the effects due to magnetic viscosity. According to Pokhotelov et al. (1999) the typical scale of vortex localization is proportional to the square root of the vortex velocity. Smaller velocity means stronger vortex localization. In the presence of non-zero ion temperature the typical dimension of a vortex becomes dependent on the vortex velocity as the square root of its difference with the ion diamagnetic drift velocity. Therefore, one should expect that the minimum value of the localization length can be obtained in the case where the vortex velocity approaches the ion diamagnetic drift velocity. Moreover, in a zero-ion-temperature dusty plasma, these vortices can exist only in the case where the dust is negatively charged, while the non-zero ion temperature provides the possibility for the existence of drift–Alfvén vortices also when the dust is positively charged. This results in the appearance of a wider class of possible vortex solutions that decrease exponentially at infinity. Thus, incorporation of non-zero ion temperature in the theory of drift–Alfvén vortices may lead to an extensive modification of the theory developed by Pokhotelov *et al.* (1999). The vortices with their minimum localization scale are less sensitive to magnetic shear and can accumulate a significant amount of energy to produce strong electric fields necessary for the plasma energization and subsequent particle precipitation in the upper atmosphere to produce a rich variety of auroral phenomena (Borovsky 1993).

The aim of our paper is to consider the nonlinear propagation of low-frequency (in comparison with the ion gyrofrequency), long-wavelength electromagnetic waves in non-uniform dusty plasmas with non-zero ion temperature.

The paper is organized in the following fashion. In Sec. 2, we present a hydrodynamic description of an inhomogeneous magnetized plasma, taking into account the effect of finite ion Larmor radius. In Sec. 3, we derive a set of nonlinear equa-

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tions that governs the nonlinear dynamics of drift–Alfvén waves in inhomogeneous dusty plasmas with non-zero ion temperature. Section 4 is devoted to an analysis of the linear dispersion relations. Quasistationary solutions of the nonlinear equations are presented in Sec. 5. Our discussion and conclusions are given in Sec. 6.

2. Hydrodynamic description of small-ion-Larmor-radius effects

The description of finite-ion-Larmor-radius effects can be found using two different approaches. One is based on the solution of the full kinetic Boltzmann–Vlasov equation by expanding the distribution function in a series of harmonics of the angles in the space of transverse velocities of the particles. In the course of this expansion it is necessary to take into account not only the first and second harmonics but the third harmonic (see e.g. Lakhin et al. 1987a). In addition to this the corresponding expansion has to include terms of the order of $\rho_i^4\nabla_\perp^4$, where ρ_i is the ion Larmor radius. A generalization of such hydrodynamics to the case of relativistic plasmas has been published by Onishchenko et al. (1999). However, such an approach suffers from a lack of clarity in the description of relevant physical processes. Another approach is based on the 13-moment Grad (1949) approximation. That method was used by Lakhin et al. (1987b) and Mikhailovskii and Onishchenko (1995) for the investigation of kinetic Alfvén waves in magnetoplasma. Below we shall follow the second approach.

We consider a dusty plasma embedded in an external magnetic field $\mathbf{B}_0 = B_0\hat{\mathbf{z}}$, where $\hat{\mathbf{z}}$ is a unit vector. The plasma density is assumed to be inhomogeneous along the x axis, while the temperature is supposed to be homogeneous. The plasma inhomogeneity is assumed to be small, i.e the characteristic perpendicular wavelength $\lambda_\perp$ is much smaller than the typical scale of density inhomogeneity L. The wave frequency is small in comparison with the proton cyclotron frequency ω_{ci} ($\omega_{ci} = eB_0/m_ic$, where e is the magnitude of the electron charge, m_i the ion mass and c the speed of light).

The hydrodynamic description of small-ion-Larmor-radius effects requires the effects due to the transverse stress tensor (collisionless magnetic viscosity). In the drift approximation the perpendicular ion velocity is

$$\mathbf{v}_i \approx \mathbf{v}_E + \mathbf{v}_{Di} + \mathbf{v}_\pi + \omega_{ci}^{-1}\hat{\mathbf{z}} \times d_t(\mathbf{v}_E + \mathbf{v}_{Di} + \mathbf{v}_\pi), \tag{1}$$

where

$$\mathbf{v}_E = cB_0^{-1}\mathbf{E} \times \hat{\mathbf{z}}, \qquad \mathbf{v}_{Di} = (m_in_i\omega_{ci})^{-1}\hat{\mathbf{z}} \times \boldsymbol{\nabla}_\perp p_i, \qquad \mathbf{v}_\pi = (m_in_i\omega_{ci})^{-1}\hat{\mathbf{z}} \times \boldsymbol{\nabla}\boldsymbol{\pi}. \tag{2}$$

Here $\mathbf{v}_E$ is the $\mathbf{E} \times \mathbf{B}$ drift velocity, $\mathbf{E} = -\boldsymbol{\nabla}_\perp\phi$ is the perpendicular component of the wave electric field, ϕ is the scalar potential, $\boldsymbol{\pi}$ the magnetic viscosity tensor, $\mathbf{v}_{Di}$ is the ion diamagnetic drift velocity, and p_i and n_i are the ion pressure and number density respectively; $\mathbf{v}_\pi$ is the correction to ion diamagnetic drift velocity due to magnetic viscosity, and $d_t = \partial_t + (\mathbf{v}_E + \mathbf{v}_{Di} + \mathbf{v}_\pi) \cdot \boldsymbol{\nabla}$. The latter will be introduced below.

The last term on the right-hand side of (1), related to the perpendicular ion inertia, can be written as

$$\omega_{ci}^{-1}\hat{\mathbf{z}} \times d_t(\mathbf{v}_E + \mathbf{v}_{Di} + \mathbf{v}_\pi) \equiv \mathbf{v}_i^I = \mathbf{v}_E^I + \mathbf{v}_{Di}^I + \mathbf{v}_\pi^I, \tag{3}$$

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where

$$\mathbf{v}_E^I = \frac{1}{\omega_{ci}}\hat{\mathbf{z}} \times d_t^0 \mathbf{v}_E = -\frac{c}{B_0\omega_{ci}} d_t^0 \boldsymbol{\nabla}_\perp \phi, \tag{4}$$

$$\begin{aligned}\mathbf{v}_{Di}^I &= \frac{1}{\omega_{ci}}\hat{\mathbf{z}} \times [d_t^0 \mathbf{v}_{Di} + (\mathbf{v}_{Di}\cdot\boldsymbol{\nabla})(\mathbf{v}_E + \mathbf{v}_{Di})] \\ &= -\frac{1}{m_i n_{0i}\omega_{ci}^2} d_t^0 \boldsymbol{\nabla}_\perp p_i - \frac{c}{m_i n_{0i} B_0 \omega_{ci}^2}\{p_i, \boldsymbol{\nabla}_\perp \phi\},\end{aligned} \tag{5}$$

and

$$\mathbf{v}_\pi^I = \frac{1}{\omega_{ci}}\hat{\mathbf{z}} \times [d_t^0 \mathbf{v}_\pi + \mathbf{v}_{Di}\cdot\boldsymbol{\nabla}\mathbf{v}_\pi + (\mathbf{v}_\pi\cdot\boldsymbol{\nabla})(\mathbf{v}_E + \mathbf{v}_{Di})], \tag{6}$$

where

$$d_t^0 = \partial_t + \mathbf{v}_E\cdot\boldsymbol{\nabla}, \qquad \{A, B\} = \partial_x A \partial_y B - \partial_y A \partial_x B.$$

In (5) the term containing the scalar nonlinearity has been neglected, since it is small (of the order of $\lambda_\perp/L$) compared with the term containing the vector nonlinearity. This is valid for perturbations with wavelengths smaller than the typical plasma density inhomogeneity length.

The magnetic viscosity tensor satisfies the system of equations

$$d_t \pi_{xx} + 2\pi_{xx}\boldsymbol{\nabla}\cdot\mathbf{v}_i + \pi_{xy}(\boldsymbol{\nabla}\times\mathbf{v}_i)_z + p_i W_{xx} - 2\omega_{ci}\pi_{xy} = 0, \tag{7}$$

$$d_t \pi_{xy} + 2\pi_{xy}\boldsymbol{\nabla}\cdot\mathbf{v}_i - \pi_{xx}(\boldsymbol{\nabla}\times\mathbf{v}_i)_z + p_i W_{xy} + 2\omega_{ci}\pi_{xx} = 0, \tag{8}$$

where

$$\begin{aligned} W_{xy} &= \frac{\partial v_{ix}}{\partial y} + \frac{\partial v_{iy}}{\partial x}, \qquad W_{xx} = \frac{\partial v_{ix}}{\partial x} - \frac{\partial v_{iy}}{\partial y}, \\ \pi_{xx} &= -\pi_{yy}, \qquad \pi_{xy} = \pi_{yx}. \end{aligned} \tag{9}$$

Expansion of (7) and (8) gives

$$\pi_{ik} = \pi_{ik}^B + \pi_{ik}^P + \pi_{ik}^G \qquad (i, k = x, y), \tag{10}$$

where

$$\pi_{xx}^B = -\pi_{yy}^B = -\frac{p_i}{2\omega_{ci}} W_{xy}, \qquad \pi_{xy}^B = \pi_{yx}^B = \frac{p_i}{2\omega_{ci}} W_{xx}, \tag{11}$$

with the components of the tensor W_{ik} calculated at $\mathbf{v}_i = \mathbf{v}_E + \mathbf{v}_{Di} + \mathbf{v}_\pi$. The expression for the components π_{ik}^P has a similar form, with W_{ik} calculated at $\mathbf{v}_i = \mathbf{v}_i^I$. Finally, we have

$$\pi_{xx}^G = -\pi_{yy}^G = -\frac{1}{2\omega_{ci}}[d_t^0 \pi_{xy}^B + 2\pi_{xy}^B \boldsymbol{\nabla}\cdot\mathbf{v}_i^{(0)} - \pi_{xx}^B(\boldsymbol{\nabla}\times\mathbf{v}_i^{(0)})_z] \tag{12}$$

and

$$\pi_{xy}^G = \pi_{yx}^G = \frac{1}{2\omega_{ci}}[d_t^0 \pi_{xx}^B + 2\pi_{xx}^B \boldsymbol{\nabla}\cdot\mathbf{v}_i^{(0)} + \pi_{xy}^B(\boldsymbol{\nabla}\times\mathbf{v}_i^{(0)})_z]. \tag{13}$$

The transverse stress tensor π_{ik}^B originates from so-called Braginskii hydrodynamics (Braginskii 1965), π_{ik}^P corresponds to the correction due to ion polarization drift, and π_{ik}^G is known as the Grad correction to ion-drift hydrodynamics (Grad 1949).

The perpendicular ion pressure yields the equation for transverse ion heat balance (cf. Braginskii 1965):

$$d_t p_i + 2p_i \boldsymbol{\nabla}\cdot\mathbf{v}_i + (\boldsymbol{\pi}\cdot\boldsymbol{\nabla})\mathbf{v}_i = 0, \tag{14}$$

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where, according to our assumption that $\boldsymbol{\nabla} T_{i0} = 0$, the corrections due to the transverse thermal flux are neglected. The second term on the left-hand side of (14) describes the compressibility of the ion motion. The numerical factor 2 reflects the fact that transverse ion motion in our case is two-dimensional. Thus, the effective ratio of specific heats γ equals 2. The last term describes the transverse heat flux due to the magnetic viscosity.

One can show that the last term in (14) vanishes. This can be verified by substituting the explicit expressions for the components of the Braginskii magnetic viscosity tensor (11) into (14). Physically, this means that collisionless magnetic viscosity, at least to leading order, does not produce dissipation in our system.

Taking into account that $\boldsymbol{\nabla} \cdot (\mathbf{v}_E + \mathbf{v}_{Di}) = 0$ and $\mathbf{v}_{Di} \cdot \boldsymbol{\nabla} p_i = 0$, we obtain from (14)

$$[d_t^0 + (\mathbf{v}_\pi + \mathbf{v}_E^I + \mathbf{v}_{Di}^I + \mathbf{v}_\pi^I) \cdot \boldsymbol{\nabla}] p_i + 2 p_i \boldsymbol{\nabla} \cdot (\mathbf{v}_\pi + \mathbf{v}_E^I + \mathbf{v}_{Di}^I + \mathbf{v}_\pi^I) = 0. \tag{15}$$

Equations (1), (10)–(13), and (15) correspond to expansion in the parameter $\omega_{ci}^{-1} d_t$. In the following we consider that $\omega_{ci}^{-1} d_t \simeq \rho_i^2 \nabla_\perp^2 \simeq \epsilon \ll 1$. Thus, we have to expand also in the parameter $\rho_i^2 \nabla_\perp^2$, keeping in mind that both these parameters are of the same order of smallness.

Expanding the ion pressure in a power series in the small parameter ϵ, we find in the first approximation

$$d_t^0 p_i = 0. \tag{16}$$

Here, the ion pressure consists of unperturbed and perturbed parts, i.e. $p_i = p_{i0}(x) + p_h$. The latter obeys the equation

$$d_t^0 p_h = \frac{c}{B_0} (\boldsymbol{\nabla} p_{i0} \times \boldsymbol{\nabla} \phi)_z. \tag{17}$$

Equation (17) can be rewritten in the form that is generally used for the study of hydromagnetic plasma stability (Shukla 1985; Mikhailovskii 1998), namely $p_h = \boldsymbol{\xi}_\perp \cdot \boldsymbol{\nabla} p_{i0}$, with $\boldsymbol{\xi}_\perp$ being the transverse plasma displacement. The pressure p_h describes the so-called hydromagnetic part of perturbed ion pressure.

In the same approximation, the ion fluid velocity equals

$$\mathbf{v}_i^{(1)} = \mathbf{u}_{*i} + \mathbf{v}_E + \mathbf{v}_{Di}^h, \tag{18}$$

where

$$\mathbf{u}_{*i} = \frac{1}{m_i n_{0i} \omega_{ci}} \hat{\mathbf{z}} \times \boldsymbol{\nabla} p_{i0}, \tag{19}$$

and

$$\mathbf{v}_{Di}^h = \frac{1}{m_i n_{0i} \omega_{ci}} \hat{\mathbf{z}} \times \boldsymbol{\nabla} p_h. \tag{20}$$

Using the results of the first approximation of the expansion of the ion velocity and the ion pressure, we obtain at second order in the parameter ϵ the correction to the ion velocity:

$$\mathbf{v}_i^{(2)} = \mathbf{v}_E^I + \mathbf{v}_{Di}^{(2)} + \mathbf{v}_{Di}^{I(1)} + \mathbf{v}_\pi^{B(1)}. \tag{21}$$

Substituting $p_i = p_{i0} + p_h$ into (5) and using (17), we obtain

$$\mathbf{v}_{Di}^{I(1)} = -\frac{2c}{m_i n_{i0} B_0 \omega_{ci}^2} \{p_{i0} + p_h, \boldsymbol{\nabla}_\perp \phi\}. \tag{22}$$

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With the help of (2) and (11), we have

$$\mathbf{v}_\pi^B = \frac{p_i}{2m_i n_{i0}\omega_{ci}^2}\nabla_\perp^2 \mathbf{v}_i - \frac{c}{2m_i n_{i0} B_0 \omega_{ci}^2}\nabla_\perp^2\phi(\hat{\mathbf{z}}\times\boldsymbol{\nabla} p_i) - \frac{c}{m_i n_{i0} B_0 \omega_{ci}^2}\{\boldsymbol{\nabla}_\perp\phi, p_i\}. \quad (23)$$

Using the results of the first approximation, from (23), we have

$$\mathbf{v}_\pi^{B(1)} = -\frac{p_i}{2m_i n_{i0}\omega_{ci}^2}\nabla_\perp^2(\mathbf{v}_E + \mathbf{v}_{Di}^h) - \frac{c}{2m_i n_{i0} B_0 \omega_{ci}^2}\nabla_\perp^2\phi\hat{\mathbf{z}}\times\boldsymbol{\nabla}_\perp(p_{i0}+p_h) - \frac{c}{m_i n_{i0} B_0 \omega_{ci}^2}\{\boldsymbol{\nabla}_\perp\phi, p_{i0}+p_h\}, \quad (24)$$

Comparing (22) and (24), we find that already in the second approximation it is necessary to take into account the effects of the magnetic viscosity. This fact was already noted by Rudakov (1962) and Roberts and Taylor (1962), who pointed out the importance of these effects for obtaining the correct dispersion relation $\omega = k_y u_{i*}$ (where k_y is the y component of the wave vector) for the ion-drift mode.

The velocity $\mathbf{v}_{Di}^{(2)}$ is defined by

$$\mathbf{v}_{Di}^{(2)} = \frac{1}{m_i n_{i0}\omega_{ci}}\hat{\mathbf{z}}\times\boldsymbol{\nabla}\delta p^{(1)}, \quad (25)$$

where $\delta p^{(1)}$ is a small correction of second order in ϵ, so that the ion pressure is represented as $p_i = p_{i0}(x) + p_h + \delta p^{(1)}$. To obtain $\delta p^{(1)}$, we use the equation for thermal heat balance (15), which in our approximation takes the form

$$d_t^0\delta p^{(1)} + (\mathbf{v}_E^I + \mathbf{v}_{Di}^{I(1)} + \mathbf{v}_\pi^{B(1)})\cdot\boldsymbol{\nabla}_\perp(p_{i0}+p_h) + 2(p_{i0}+p_h)\boldsymbol{\nabla}_\perp\cdot(\mathbf{v}_E^I + \mathbf{v}_{Di}^{I(1)} + \mathbf{v}_\pi^{B(1)}) = 0. \quad (26)$$

Using (4), (22), and (24), we reduce (26) to

$$d_t^0\delta p^{(1)} = \frac{2p_{i0}c}{B_0\omega_{ci}}d_t^0\nabla_\perp^2\phi + \frac{2\rho_i^2 c}{B_0}\boldsymbol{\nabla}_\perp\cdot\{p_{i0}+p_h, \boldsymbol{\nabla}_\perp\phi,\} + \frac{\rho_i^2 c}{B_0}\{\boldsymbol{\nabla}_\perp p_h, \boldsymbol{\nabla}_\perp\phi\}. \quad (27)$$

For quasistationary waves propagating along the y axis with constant velocity u (see Sec. 3) $p_h \simeq -(c\partial_x p_{i0}/uB_0)\phi$, and thus the last term on the right-hand side of (27) vanishes.

With the help of the equality

$$\frac{c}{B_0}\boldsymbol{\nabla}_\perp\cdot\{p_{i0}+p_h, \boldsymbol{\nabla}_\perp\phi,\} = d_t^0\nabla_\perp^2 p_h + \frac{c}{B_0}\{\boldsymbol{\nabla}_\perp p_h, \boldsymbol{\nabla}_\perp\phi\},$$

(27) now reads

$$d_t^0\left[\delta p^{(1)} - \frac{2p_{i0}c}{B_0\omega_{ci}}\nabla_\perp^2\left(\phi + \frac{p_h}{en_{i0}}\right)\right] = 0,$$

or

$$\delta p^{(1)} = \frac{2p_{i0}c}{B_0\omega_{ci}}\nabla_\perp^2\left(\phi + \frac{p_h}{en_{i0}}\right). \quad (28)$$

The second term in the parentheses describes the correction to the hydromagnetic part of ion pressure due to the finite ion Larmor radius, whereas the other term corresponds to the inertial correction. Since in our approximation $\omega_{ci}^{-1}d_t \simeq \rho_i^2\nabla_\perp^2 \simeq \epsilon$, both of these terms are of the same order of smallness. Substituting (28) into (25), we obtain

$$\mathbf{v}_{Di}^{(2)} = 2\rho_i^2\nabla_\perp^2(\mathbf{v}_E + \mathbf{v}_{Di}^h). \quad (29)$$

In order to proceed further, it is convenient to introduce the effective potential

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ϕ^* and effective velocity $\mathbf{v}_E^*$ defined by

$$\phi^* = \phi + \frac{p_h}{en_{i0}}, \qquad \mathbf{v}_E^* = \frac{c}{B_0}\hat{\mathbf{z}} \times \boldsymbol{\nabla}\phi^*. \tag{30}$$

With the help of this notation, we obtain

$$\mathbf{v}_{Di}^{(2)} = 2\rho_i^2 \nabla_\perp^2 \mathbf{v}_E^*.$$

At third order in the parameter ϵ, we have

$$\mathbf{v}_i^{(3)} = \mathbf{v}_{Di}^{I(2)} + \mathbf{v}_\pi^{I(1)} + \mathbf{v}_\pi^P + \mathbf{v}_\pi^G + \mathbf{v}_\pi^{B(2)} + \mathbf{v}_{Di}^{(3)}. \tag{31}$$

Using (4) and (28), we obtain

$$\mathbf{v}_{Di}^{I(2)} = -\frac{2c\rho_i^2}{B_0\omega_{ci}} d_t^0 \boldsymbol{\nabla}_\perp \nabla_\perp^2 \phi^* - \frac{2c^2\rho_i^2}{B_0^2\omega_{ci}} \{\nabla_\perp^2 \phi^*, \boldsymbol{\nabla}_\perp \phi\} \tag{32}$$

If one neglects the scalar nonlinearity, it follows from (6) that

$$\mathbf{v}_\pi^I = \frac{1}{\omega_{ci}} \hat{\mathbf{z}} \times [d_t^0 \mathbf{v}_\pi^{B(1)} + (\mathbf{v}_\pi^{B(1)} \cdot \boldsymbol{\nabla}) \mathbf{v}_E]. \tag{33}$$

Substituting (24) into (33) gives

$$\mathbf{v}_\pi^{I(1)} = \frac{c\rho_i^2}{2B_0\omega_{ci}} d_t^0 \boldsymbol{\nabla}_\perp \nabla_\perp^2 \phi^* + \frac{c^2\rho_i^2}{2B_0^2\omega_{ci}} \{\nabla_\perp^2 \phi^*, \boldsymbol{\nabla}_\perp \phi\}. \tag{34}$$

Calculating $\mathbf{v}_\pi^B$ at the velocity $\mathbf{v}_E^I + \mathbf{v}_{Di}^{I(1)}$, we obtain

$$\mathbf{v}_\pi^P = \frac{c\rho_i^2}{2B_0\omega_{ci}} \nabla_\perp^2 d_t^0 \boldsymbol{\nabla}_\perp \phi^* + \frac{c^2\rho_i^2}{2B_0^2\omega_{ci}} \nabla_\perp^2 \{p_{i0} + p_h, \boldsymbol{\nabla}_\perp \phi\}. \tag{35}$$

Using (2), (12), and (13), we find that

$$\mathbf{v}_\pi^G = \frac{c\rho_i^2}{4B_0\omega_{ci}} \nabla_\perp d_t^0 \nabla_\perp^2 \phi^* + \frac{c^2\rho_i^2}{2B_0^2\omega_{ci}} [\{\boldsymbol{\nabla}_\perp \phi, \nabla_\perp^2 \phi^*\} + \nabla_\perp^2 \phi^* (\hat{\mathbf{z}} \times \boldsymbol{\nabla}_\perp \nabla_\perp^2 \phi)]. \tag{36}$$

From (23), we have

$$\begin{aligned} \mathbf{v}_\pi^{B(2)} = &-\frac{\rho_i^2}{2} \nabla_\perp^2 (\mathbf{v}_{Di}^{(2)} + \mathbf{v}_{Di}^{I(1)} + \mathbf{v}_\pi^{(1)}) \\ &- \frac{c}{2m_i n_{0i} B_0 \omega_{ci}^2} \nabla_\perp^2 \phi (\hat{\mathbf{z}} \times \boldsymbol{\nabla} \delta p^{(1)}) - \frac{c}{2m_i n_{0i} B_0 \omega_{ci}^2} \{\boldsymbol{\nabla}_\perp \phi, \delta p^{(1)}\}. \end{aligned} \tag{37}$$

Using the results of the second approximation from (37), we obtain

$$\begin{aligned} \mathbf{v}_\pi^{B(2)} = &-\rho_i^4 \nabla_\perp^4 \mathbf{v}_E^* - \frac{c^2\rho_i^2}{2B_0^2\omega_{ci}} [\{\boldsymbol{\nabla}_\perp \phi, \nabla_\perp^2 \phi^*\} + \nabla_\perp^2 \phi (\hat{\mathbf{z}} \times \boldsymbol{\nabla}_\perp \nabla_\perp^2 \phi^*)] \\ &+ \frac{c^2\rho_i^2}{2B_0^2\omega_{ci}} \nabla_\perp^2 [\nabla_\perp^2 \phi (\hat{\mathbf{z}} \times \boldsymbol{\nabla}_\perp \nabla_\perp^2 \phi^*) - \{p_{i0} + p_h, \boldsymbol{\nabla}_\perp \phi\}]. \end{aligned} \tag{38}$$

Finally,

$$\mathbf{v}_{Di}^{(3)} = \frac{1}{m_i n_{0i} \omega_{ci}} \hat{\mathbf{z}} \times \boldsymbol{\nabla} \delta p^{(2)}, \tag{39}$$

where $\delta p^{(2)}$ is the small correction to the ion pressure due to the higher-order ion-Larmor-radius effects ($\simeq \rho_i^4 \nabla_\perp^4$), which can be evaluated in the third approximation from the thermal balance equation (15). The contribution of this term to the

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divergence of the total electric current vanishes. Thus, we do not need the explicit expression for $\mathbf{v}_{Di}^{(3)}$.

We have obtained the expression for ion velocity $\mathbf{v}_i = \mathbf{v}_i^{(1)} + \mathbf{v}_i^{(2)} + \mathbf{v}_i^{(3)}$ (cf. (18), (21), and (31)), taking into account the terms proportional to ϵ^2. Using the results of the second approximation we obtain

$$\boldsymbol{\nabla} \cdot (\mathbf{v}_i^{(2)} n_i) = -\frac{n_{0i} c}{B_0 \omega_{ci}} d_t^0 \nabla_\perp^2 \phi^* + \frac{c n_{i0} \rho_i^2}{B_0 p_{i0}} \{\boldsymbol{\nabla}_\perp p_h, \boldsymbol{\nabla}_\perp \phi\}, \tag{40}$$

where we have used the fact that $(c/m_i B_0 \omega_{ci}) \boldsymbol{\nabla}_\perp \cdot \{p_{i0} + p_h, \boldsymbol{\nabla}_\perp \phi\} \simeq d_t^0 \nabla_\perp^2 p_h$, as noted above. With the help of (32), (34), (38), and (39), we obtain

$$\boldsymbol{\nabla} \cdot \mathbf{v}_{Di}^{I(2)} = -\frac{2c\rho_i^2}{B_0 \omega_{ci}} d_t^0 \nabla_\perp^4 \phi^*,$$

$$\boldsymbol{\nabla} \cdot \mathbf{v}_\pi^{I(1)} = \frac{c\rho_i^2}{2B_0 \omega_{ci}} d_t^0 \nabla_\perp^4 \phi^*,$$

and

$$\boldsymbol{\nabla} \cdot (\mathbf{v}_\pi^P + \mathbf{v}_\pi^G + \mathbf{v}_\pi^{B(2)} + \mathbf{v}_{Di}^{(3)}) = \frac{3c\rho_i^2}{4B_0 \omega_{ci}} d_t^0 \nabla_\perp^4 \phi^*, \tag{41}$$

so that

$$\boldsymbol{\nabla} \cdot (\mathbf{v}_i^{(3)} n_i) = -\frac{3c\rho_i^2}{4B_0 \omega_{ci}} d_t^0 \nabla_\perp^2 \phi^*. \tag{42}$$

Using (40) and (42), and noting that $n_i = n_{i0}(x) + \tilde{n}$, where $\tilde{n}$ is the ion density perturbation and $T_i = T_{i0} = \text{const}$, we can write the ion-density continuity equation in the form

$$\partial_t n_i + \mathbf{v}_E \cdot \boldsymbol{\nabla} n_{i0} - \frac{n_{i0} c}{B_0 \omega_{ci}} d_t^0 \nabla_\perp^2 \left(\phi + \frac{p_h}{e n_{i0}}\right) - \frac{3 n_{i0} c \rho_i^2}{4 B_0 \omega_{ci}} d_t^0 \nabla_\perp^4 \left(\phi + \frac{p_h}{e n_{i0}}\right)$$
$$+ \frac{c n_{i0} \rho_i^2}{B_0 p_{i0}} \{\boldsymbol{\nabla}_\perp p_h, \boldsymbol{\nabla}_\perp \phi\} \left(\phi + \frac{p_h}{e n_{i0}}\right) = 0. \tag{43}$$

In the linear approximation, from (17), (30), and (43) we have

$$\tilde{n} = -\frac{n_{i0} c \kappa_n}{u B_0} \phi + \frac{n_{i0} c}{B_0 \omega_{ci}} \left(1 - \frac{u_{*i}}{u}\right) (\nabla_\perp^2 \phi + \tfrac{3}{4} \rho_i^2 \nabla_\perp^4 \phi), \tag{44}$$

where $\kappa_n = d_x \ln n_{i0}$ and u is the wave velocity along the y axis. The expression (44) coincides with

$$\tilde{n} = -\frac{n_{i0} c \kappa_n}{u B_0} \phi + \left(1 - \frac{u_{*i}}{u}\right) (\Gamma_0 - 1) \frac{e n_{i0}}{T_{i0}} \phi, \tag{45}$$

which was obtained by Mikhailovskii (1974) by employing the full kinetic treatment. Here, $\Gamma_0 = I_0(z) \exp(-z)$, $z = k_\perp^2 \rho_i^2$, and I_0 is the modified Bessel function of zeroth order. In the long-wave approximation, namely $k_\perp^2 \rho_i^2 \ll 1$, we obtain from (45) the expression (44).

3. Nonlinear equations for dusty plasmas with non-zero ion temperature

The dust grains are considered as immobile point charges. The equilibrium plasma state is neutral, that is,

$$n_{i0} = n_{e0} + Z_d n_{d0}, \tag{46}$$

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where n_{j0} is the unperturbed number density of the particle species j ($j = e$ for electrons, i for ions, and d for charged dust grains) and Z_d is the constant number of charges on the dust grain surface ($Z_d > 0$ for negatively charged and $Z_d < 0$ for positively charged dust grains).

In an electromagnetic field, the guiding-centre drift of electrons is

$$\mathbf{v}_e \approx \mathbf{v}_E + \mathbf{v}_{De} + v_{ez}\left(\hat{\mathbf{z}} + \frac{\mathbf{B}_\perp}{B_0}\right), \tag{47}$$

where $\mathbf{v}_{De} = -(c/eB_0n_e)\hat{\mathbf{z}} \times \boldsymbol{\nabla} p_e$ is the electron diamagnetic drift velocity. We regard the electrons as isothermal with temperature T_e, $p_e = T_e n_e$, p_e, T_e, and n_e are the electron pressure, the temperature, and the number density respectively; $\mathbf{B}_\perp = \boldsymbol{\nabla} A_z \times \hat{\mathbf{z}}$ is the perturbed magnetic field, A_z is the parallel (to $\hat{\mathbf{z}}$) component of the vector potential, and v_{ez} is given by

$$v_{ez} \approx \frac{c}{4\pi n_e e}\nabla_\perp^2 A_z. \tag{48}$$

Letting $p_e = p_{e0}(x) + p_{1e}$, where p_{1e} ($\ll p_{e0}$) is the electron pressure perturbation, with the help of (47) and (48) we obtain the equation for the electron heat balance, which, because $\boldsymbol{\nabla} \cdot (\mathbf{v}_E + \mathbf{v}_{De}) = 0$ and $\mathbf{v}_{De} \cdot \boldsymbol{\nabla} p_e = 0$ takes the form (Shukla 1985)

$$d_t^0 p_{e1} - \frac{c}{B_0}(\boldsymbol{\nabla} p_{e0} \times \boldsymbol{\nabla}\phi)_z + \frac{c p_{e0}}{4\pi e n_{e0}} d_z \nabla_\perp^2 A_z = 0, \tag{49}$$

where $d_z = \partial_z + B_0^{-1}\boldsymbol{\nabla} A_z \times \hat{\mathbf{z}} \cdot \boldsymbol{\nabla}$. We have assumed that $(\omega_{pe}^2/\omega_{ce})|\hat{\mathbf{z}} \times \boldsymbol{\nabla}\phi \cdot \boldsymbol{\nabla}| \gg c\partial_z \nabla_\perp^2 A_z$, where $\omega_{pe} = (4\pi n_{e0}e^2/m_e)^{1/2}$ is the electron plasma frequency, and $\omega_{ce} = eB_0/m_e c$ is the electron gyrofrequency. Owing to the high mobility of the electrons along the magnetic field, the contribution of the parallel thermal flux in (49) vanishes. Thus, it coincides, with an accuracy of the constant coefficient T_e, with the electron continuity equation.

Using (47) and (48), the parallel component of the electron momentum equation can be written as

$$(\partial_t + u_{e*}\partial_y)A_z - \lambda_e^2 d_t^0 \nabla_\perp^2 A_z + c d_z\left(\phi - \frac{p_{e1}}{e n_{e0}}\right) = 0, \tag{50}$$

where $u_{e*} = -(cT_e/eB_0)d\ln n_{e0}(x)/dx$ is the unperturbed electron diamagnetic-drift velocity, and $\lambda_e = c/\omega_{pe}$ is the collisionless electron skin depth.

In order to close the system of equations (49) and (50), it is necessary to take the ion motion into account.

Using (46)–(49) and (43), we represent the current closure equation $e\boldsymbol{\nabla} \cdot (n_i \mathbf{v}_i - n_e \mathbf{v}_e) = 0$ as

$$d_t^0 \nabla_\perp^2 \left[(1 + \tfrac{3}{4}\rho_i^2 \nabla_\perp^2)\left(\phi + \frac{p_h}{e n_{i0}}\right)\right] + \frac{c}{B_0 e n_{i0}}\boldsymbol{\nabla}_\perp \cdot \{p_{i0} + p_h,\ \boldsymbol{\nabla}_\perp \phi\} + \omega_{ci}\delta_d \kappa_d \partial_y \phi + \frac{v_A^2}{c} d_z \nabla_\perp^2 A_z = 0, \tag{51}$$

where $v_A = B_0/(4\pi n_{i0} m_i)^{1/2}$ is the Alfvén velocity, $\delta_d = Z_d n_{d0}/n_{i0}$, and $\kappa_d = d\ln[Z_d n_{d0}(x)]/dx$. The term $\omega_{ci}\delta_d\kappa_d\partial_y\phi$ is associated with the Shukla–Varma mode (Shukla and Varma 1993).

Equations (49), (50), (17), and (51) are the desired nonlinear equations for the coupled drift–Alfvén-Shukla–Varma modes in a non-uniform dusty plasmas with non-zero ion Larmor radius.

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4. Dispersion relations

In the linear approximation, Fourier transformation of (17), (49), (50), and (51) yields the local dispersion relation

$$[(\omega-\omega_{i*})(1-\tfrac{3}{4}k_\perp^2\rho_i^2)-\omega_{sv}][\omega(\omega-\omega_{e*})+k_\perp^2\lambda_e^2(\omega^2-k_z^2v_{Te}^2)]=k_z^2v_A^2(\omega-\omega_{e*}), \quad (52)$$

where ω, $\mathbf{k}_\perp$, and k_z are the wave frequency and the perpendicular and parallel wave numbers, $\omega_{i*}=k_yu_{i*}$ and $\omega_{e*}=k_yu_{e*}$ are the ion and electron diamagnetic-drift frequencies, $u_{i*}=(cT_i/eB_0)d\ln n_{i0}(x)/dx$ is the unperturbed ion diamagnetic drift, k_y is the wave number in the y direction, $\omega_{sv}=-\omega_{ci}\delta_d\kappa_dk_y/k_\perp^2$, $\rho_i=v_{Ti}/\omega_{ci}$ the ion Larmor radius, and $v_{Ti}=(T_i/m_i)^{1/2}$ the ion thermal velocity.

Several comments are in order. First, in a homogeneous plasma, (52) correctly reproduces the spectra of the dispersive Alfvén waves, namely,

$$\omega=k_zv_A(1+k_\perp^2\lambda_e^2)^{-1/2}\left[1+k_\perp^2\rho_s^2\left(1+\frac{3T_i}{4T_e}\right)\right]^{1/2}, \quad (53)$$

where $\rho_s^2=\rho_i^2\,T_e/T_i$ is the ion Larmor radius defined at the electron temperature. In a plasma of intermediate pressure, namely in a plasma with $m_e/m_i\ll\beta\ll1$, we have $\rho_s\gg\lambda_e$, and (53) reduces to

$$\omega=k_zv_A\left[1+k_\perp^2\rho_s^2\left(1+\frac{3T_i}{4T_e}\right)\right]^{1/2},$$

which is the frequency of the kinetic Alfvén waves (KAW); the KAW parallel phase velocity is much smaller than the electron thermal velocity $v_{te}=(T_e/m_e)^{1/2}$. On the other hand, in a very low-β plasma ($\beta\ll m_e/m_i$), we have $\rho_s\ll\lambda_e$, and (53) gives $\omega=k_zv_A(1+k_\perp^2\lambda_e^2)^{-1/2}$, which is the frequency of the inertial Alfvén waves, whose parallel phase velocity is much larger than v_{te}.

Secondly, in the case of waves propagating perpendicular to the external magnetic field ($k_z=0$), the dispersion relation (53) is decoupled into the ion-drift–Shukla–Varma mode with dispersion relation $\omega=\omega_{*i}+\omega_{sv}(1-\frac{3}{4}k_\perp^2\rho_i^2)^{-1}$ (which corresponds to the Shukla–Varma mode $\omega=\omega_{sv}$ (cf. Shukla and Varma 1993) in the cold-ion approximation) and the magnetic drift mode with dispersion relation $\omega=\omega_m\equiv\omega_{e*}(1+k_\perp^2\lambda_e^2)^{-1}$.

In a cold ($T_j\to0$) dusty plasma, from (19) we have

$$\omega^2-\omega\omega_{sv}-k_z^2v_A^2(1+k_\perp^2\lambda_e^2)^{-1}=0.$$

The latter shows that coupling between the Shukla–Varma mode and the inertial Alfvén wave arises owing to the parallel electron motion in the electromagnetic fields.

5. Quasistationary nonlinear waves and dipolar vortices

In a plasma with $1\gg\beta\gg m_e/m_i$ we can neglect the skin-length corrections compared with those due to the ion Larmor radius. Let us now consider stationary solutions of the nonlinear equations (17) and (49)–(51), assuming that all the field variables depend on x and $\eta=y+\alpha z-ut$, where u is the translation speed of the wave along the y axis, and α is the angle between the wave front normal and the (x,y) plane. For convenience, instead of n_{1e}, we introduce $\hat{n}\equiv n_{1e}+c\kappa_nn_0\phi/uB_0$.

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In the stationary η frame, (17) and (49)–(51) reduce to

$$\hat{D}_\phi(\hat{n}) = \frac{\alpha c}{4\pi e u}\hat{D}_A\nabla_\perp^2 A_z, \tag{54}$$

$$\hat{D}_A\left[\left(1-\frac{u_{e*}}{u}\right)\left(\phi-\frac{u}{\alpha c}A_z\right)-\frac{T_e}{en_{e0}}\hat{n}\right]=0, \tag{55}$$

$$\hat{D}_\phi\left(p_h+\frac{c\partial_x p_{i0}}{uB_0}\phi\right)=0, \tag{56}$$

and

$$\frac{\alpha v_A^2}{cu}\hat{D}_A\nabla_\perp^2 A_z=\hat{D}_\phi\left[\nabla_\perp^2\left(1+\tfrac{3}{4}\rho_i^2\nabla_\perp^2\right)\left(\phi+\frac{p_h}{en_{i0}}\right)-\frac{\omega_{ci}\delta_d\kappa_d}{u}\phi\right], \tag{57}$$

where

$$\hat{D}_\phi=\partial_\eta-\frac{c}{uB_0}[(\partial_x\phi)\partial_\eta-(\partial_\eta\phi)\partial_x],\qquad \hat{D}_A=\partial_\eta+\frac{1}{\alpha B_0}[(\partial_\eta A_z)\partial_x-(\partial_x A_z)\partial_\eta].$$

Since in drift–Alfvén waves the plasma density perturbation in the absence of drift effects equals $n_{1e}^{(0)}=-c\kappa_n n_0\phi/uB_0$, the value $\hat{n}$ describes the dispersion part of the electron-density perturbation. Equations (54)–(57) describe a wide range of quasistationary nonlinear waves in such a plasma. One of the possible solutions of (55) is

$$\hat{n}=\frac{n_{e0}e}{T_e}\left(1-\frac{u_{e*}}{u}\right)\left(\phi-\frac{u}{\alpha c}A_z\right). \tag{58}$$

Substituting (58) into (54) and using $\hat{D}_\phi(\phi-uA_z/\alpha c)=\hat{D}_A(\phi-uA_z/\alpha c)$, we obtain

$$\hat{D}_A\left[\left(1-\frac{u_{e*}}{u}\right)\left(\phi-\frac{u}{\alpha c}A_z\right)-\frac{\alpha v_A^2}{cu}\rho_s^2\nabla_\perp^2 A_z\right]=0, \tag{59}$$

from which follows the expression that connects A and ϕ:

$$\left(1-\frac{u_{e*}}{u}\right)\left(\phi-\frac{u}{\alpha c}A_z\right)-\frac{\alpha v_A^2}{cu}\rho_s^2\nabla_\perp^2 A_z=0. \tag{60}$$

Expanding the first term on the left-hand side of (60) in powers of the small parameter $\rho_s^2\nabla_\perp^2$, we have

$$\left(1-\frac{u_{e*}}{u}\right)\left(\phi-\frac{u}{\alpha c}A_z\right)\simeq\frac{\alpha^2 v_A^2}{u^2}\rho_s^2\left[\nabla_\perp^2\phi-\frac{\alpha^2 v_A^2}{u^2}\left(1-\frac{u_{e*}}{u}\right)^{-1}\rho_s^2\nabla_\perp^4\phi\right]. \tag{61}$$

Using (59)–(61), we obtain from (57)

$$\hat{D}_\phi\left\{\left[\frac{\alpha^4 v_A^4}{u^4}\rho_s^4\left(1-\frac{u_{e*}}{u}\right)^{-1}+\frac{3}{4}\rho_s^2\rho_i^2\left(1-\frac{u_{i*}}{u}\right)\right]\nabla_\perp^4\phi+\left(1-\frac{u_{i*}}{u}-\frac{\alpha^2 v_A^2}{u^2}\right)\rho_s^2\nabla_\perp^2\phi-\frac{\omega_{ci}\delta_d\kappa_d}{u}\rho_s^2\phi\right\}=0. \tag{62}$$

After vector integration of (62), we have

$$\kappa_1^{-2}\nabla_\perp^4\phi-\nabla_\perp^2\phi+\kappa_2^2+C\left(\phi-\frac{uB_0}{c}x\right)=0, \tag{63}$$

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where

$$\kappa_1^2 = -\rho_s^{-2}\left(1-\frac{u_{i*}}{u}-\frac{\alpha^2 v_A^2}{u^2}\right)\left[\frac{3T_i}{4T_e}\left(1-\frac{u_{i*}}{u}\right)+\frac{\alpha^4 v_A^4}{u^4}\left(1-\frac{u_{e*}}{u}\right)^{-1}\right]^{-1}, \quad (64)$$

and

$$\kappa_2^2 = \frac{\omega_{ci}\delta_d\kappa_d}{u}\left(1-\frac{u_{i*}}{u}-\frac{\alpha^2 v_A^2}{u^2}\right)^{-1}. \quad (65)$$

Equation (63) has a solution in the form of a dipolar vortex. Such a solution exists if $\kappa_1^2 > \kappa_2^2 > 0$. Its localization scale is of the order of $l \simeq (\kappa_1\kappa_2)^{-1/2} \simeq \rho_i\delta_d^{-1/4}$.

In plasmas with $\beta \ll m_e/m_i$, we can neglect the ion Larmor radius compared with the skin length. In the η frame, (54) and (56) take the forms

$$\hat{D}_\phi\left(n_{1e}+\frac{c\kappa_n n_{e0}}{uB_0}\phi\right)=0, \qquad \hat{D}_\phi\left(n_h+\frac{c\kappa_n n_{i0}}{uB_0}\phi\right)=0, \quad (66)$$

from which we obtain

$$\frac{n_{1e}}{n_{0e}} = \frac{n_h}{n_{i0}} = -\frac{c\kappa_n}{uB_0}\phi. \quad (67)$$

Using (67), we represent (50) in the form

$$\hat{D}_\phi\left[\left(1-\frac{u_{e*}}{u}\right)\left(A_z-\frac{\alpha c}{u}\phi\right)-\lambda_e^2\nabla_\perp^2 A_z\right]=0. \quad (68)$$

From (68) we obtain

$$\lambda_e^2\nabla_\perp^2 A_z = \left(1-\frac{u_{e*}}{u}\right)\left(A_z-\frac{\alpha c}{u}\phi\right). \quad (69)$$

With the help of (67) and (69), the current closure equation (51) reduces to

$$\hat{D}_\phi\left[\frac{\alpha^2 v_A^2}{u^2}\left(1-\frac{u_{e*}}{u}\right)^{-1}\lambda_e^4\nabla_\perp^4\phi-\left(1-\frac{u_{i*}}{u}-\frac{\alpha^4 v_A^4}{u^4}\right)\lambda_e^2\nabla_\perp^2\phi+\frac{\omega_{ci}\delta_d\kappa_d}{u}\lambda_e^2\right]=0 \quad (70)$$

After vector integration of (70) we obtain (63) with

$$\kappa_1^2 = -\lambda_e^{-2}\frac{u^2}{\alpha^2 v_A^2}\left(1-\frac{u_{i*}}{u}-\frac{\alpha^2 v_A^2}{u^2}\right)\left(1-\frac{u_{e*}}{u}\right), \quad (71)$$

and

$$\kappa_2^2 = \frac{\omega_{ci}\delta_d\kappa_d}{u}\left(1-\frac{u_{i*}}{u}-\frac{\alpha^2 v_A^2}{u^2}\right)^{-1}. \quad (72)$$

The characteristic localization scale of the vortex is of the order of $(\lambda_e\rho_i)^{1/2}\delta_d^{-1/4}$.

In the case of flute perturbations ($\alpha = 0$), the nonlinear equations (69) and (70) are decoupled into two independent equations describing the electrostatic waves ($\phi \neq 0$, $n_1 \neq 0$ and $A_z = 0$), the solution of which was discussed above, and an equation for magnetic holes (with $\phi = 0$, $n_1 = 0$, and $A_z \neq 0$),

$$\lambda_e^2\nabla_\perp^2 A_z = \left(1-\frac{u_{e*}}{u}\right)A_z. \quad (73)$$

The holes propagate in the direction of the electron diamagnetic drift with velocity $u > u_{e*}$. The characteristic scale of the magnetic holes is of the order of λ_e.

The analyses presented above shows that the nonlinearly coupled drift–Alfvén–Shukla–Varma modes in a non-uniform warm dusty plasma as well as the nonlinearly coupled inertial Alfvén–Shukla–Varma modes in a non-uniform cold dusty

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plasma can be represented in the form of dipolar vortices, which are well behaved in both the outer and inner regions of the vortex core. The dust also limits the propagation speeds of the dipolar vortices.

6. Discussion and conclusions

In this paper, we have derived the nonlinear equations (49), (50), (17) and (51), which govern the dynamics of the drift–Alfvén waves in a dusty plasma with non-zero ion temperature. In the linear limit, Fourier transformation of these equations yields a dispersion relation that exhibits the coupling between drift–Alfvén waves and the Shukla–Varma mode.

We have also discussed possible stationary solutions of nonlinearly coupled modes. Specifically, we have focused on dipolar vortex solutions. It is found that when velocity of the vortex approaches the ion diamagnetic drift velocity, the vortex is strongly localized. Its dimension l is different in plasmas with intermediate β ($1 \gg \beta \gg m_e/m_i$) and very low β ($\beta \ll m_e/m_i$). In the first case, $l \simeq \rho_i \delta_d^{-1/4}$, while in the second case, $l \simeq (\lambda_e \rho_i)^{1/2} \delta_d^{-1/4}$. Such vortices propagate along the y direction with velocity slightly exceeding the ion diamagnetic drift velocity.

In conclusion, the present study of the reduced nonlinear equations of the drift–Alfvén waves gives sufficient grounds to claim the existence of two-scale dipolar vortices in dusty plasmas. We have demonstrated that in dusty plasmas, the weakly localized drift–Alfvén vortices (decreasing as r^{-1}) are modified into exponentially decreasing vortices. The results of the present investigation should be useful in identifying the salient properties of coherent vortical structures as well as their translational speeds in the Earth's auroral ionosphere and the magnetosphere, as well as in low-temperature laboratory dusty plasmas. The drift–Alfvén waves considered in this paper are quite common in space plasmas. They are easily excited through a number of resonance processes, and may play an efficient role, for example, in ionosphere–magnetosphere coupling (see e.g. Goertz and Boswell 1979). They can also play an essential role in Saturn rings, where charged dust grains are invariably present.

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