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# Prologue

More than 100 elements are now known to exist, distinguished from each other by the electric charge $Ze$ on the atomic nucleus. This charge is balanced by the charge carried by the $Z$ electrons which together with the nucleus make up the neutral atom. The elements are also distinguished by their mass, more than 99% of which resides in the nucleus. Are there other distinguishing properties of nuclei? Have the nuclei been in existence since the beginning of time? Are there elements in the Universe which do not exist on Earth? What physical principles underlie the properties of nuclei? Why are their masses so closely correlated with their electric charges? Why are some nuclei radioactive? Radioactivity is used to man's benefit in medicine. Nuclear fission is exploited in power generation. But man's use of nuclear physics has also posed the terrible threat of nuclear weapons.

This book aims to set out the basic concepts which have been developed by nuclear physicists in their attempts to understand the nucleus. Besides satisfying our appetite for knowledge, these concepts must be understood if we are to make an informed judgment on the benefits and problems of nuclear technology.

After the discovery of the neutron by Chadwick in 1932, it was accepted that a nucleus of atomic number $Z$ was made up of $Z$ protons and some number $N$ of neutrons. The proton and neutron were then thought to be elementary particles, although it is now clear that they are not but rather are themselves structured entities. We shall also see that in addition to neutrons and protons several other particles play an important, if indirect, role in the physics of nuclei. In this and the following two chapters, to provide a background to our subsequent study of the



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nucleus, we shall describe the elementary particles of nature, and their interactions, as they are at present understood.

## 1.1 Fermions and bosons

Elementary particles are classified as either *fermions* or *bosons*. Fermions are particles which satisfy the Pauli exclusion principle: if an assembly of identical fermions is described in terms of single-particle wave-functions, then no two fermions can have the same wave-function. For example, electrons are fermions. This rule explains the shell structure of atoms and hence underlies the whole of chemistry. Fermions are so called because they obey the Fermi–Dirac statistics of statistical mechanics.

Bosons are particles which obey Bose–Einstein statistics, and are characterised by the property that *any* number of particles may be assigned the same single-particle wave-function. Thus, in the case of bosons, coherent waves of macroscopic amplitude can be constructed, and such waves may to a good approximation be described classically. For example, photons are bosons and the corresponding classical field is the familiar electromagnetic field **E** and **B**, which satisfies Maxwell's equations.

At a more fundamental level, these properties are a consequence of the possible symmetries of the wave-function of a system of identical particles when the coordinates of any two particles are interchanged. In the case of fermions, the wave-function changes sign; it is completely anti-symmetric. In the case of bosons the wave-function is unchanged; it is completely symmetric.

There is also an observed relation between the intrinisc angular momentum, or spin, of a particle and its statistics. The intrinsic spin **s** is quantised, with spin quantum number $s$ (see Appendix C). For a fermion, $s$ takes one of the values $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$; for a boson, $s$ takes one of the values $0, 1, 2, \ldots$. A theoretical explanation of this relationship can be given within the framework of relativistic quantum field theory.

## 1.2 The particle physicist's picture of nature

Elementary particle physics describes the world in terms of elementary fermions, interacting through fields of which they are sources. The particles associated with the interaction fields are bosons. To take the most familiar example, an electron is an elementary fermion; it carries electric charge $-e$ and this charge produces an electromagnetic field **E**, **B**, which

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Table 1.1. *Types of interaction field*

| Interaction field | Boson | Spin |
|---|---|---|
| Gravitational field | 'Gravitons' postulated | 2 |
| Weak field | $W^+$, $W^-$, Z particles | 1 |
| Electromagnetic field | Photons | 1 |
| Strong field | 'Gluons' postulated | 1 |

exerts forces on other charged particles. The electromagnetic field, quantised according to the rules of quantum mechanics, corresponds to an assembly of *photons*, which are bosons. Indeed, Bose–Einstein statistics were first applied to photons.

Four types of interaction field may be distinguished in nature (see Table 1.1). All of these interactions are relevant to nuclear physics, though the gravitational field becomes important only in densely aggregated matter, such as stars. Gravitational forces act on all particles and are important for the physics on the large scale of macroscopic bodies. On the small scale of most terrestrial atomic and nuclear physics, gravitational forces are insignificant and except in Chapter 10 and Chapter 11 we shall ignore them.

Nature provides an even greater diversity of elementary fermions than of bosons. It is convenient to divide the elementary fermions into two classes: *leptons*, which are not sources of the strong fields and hence do not participate in the strong interaction; and *quarks*, which take part in all interactions.

The electron is an example of a lepton. Leptons and their interactions are described in Chapter 2. Quarks are always confined in compound systems which extend over distances of about 1 fm. The term *hadron* is used generically for a quark system. The proton and neutron are hadrons, as are mesons. The proton and neutron are the subject matter of Chapter 3.

## 1.3 Conservation laws and symmetries: parity

The total energy of an isolated system is constant in time. So also are its linear momentum and angular momentum. These conservation laws are derivable from Newton's laws of motion and Maxwell's equations, or from the laws of quantum mechanics, but they can also, at a deeper

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level, be regarded as consequences of 'symmetries' of space and time. Thus the law of conservation of linear momentum follows from the homogeneity of space, the law of conservation of angular momentum from the isotropy of space; it does not matter where we place the origin of our coordinate axes, or in which direction they are oriented.

These conservation laws are as significant in nuclear physics as elsewhere, but there is another symmetry and conservation law which is of particular importance in quantum systems such as the nucleus: reflection symmetry and parity. By reflection symmetry we mean reflection about the origin, $\mathbf{r} \to \mathbf{r}' = -\mathbf{r}$. A single-particle wave-function $\psi(\mathbf{r})$ is said to have parity $+1$ if it is even under reflection, i.e.

$$\psi(-\mathbf{r}) = \psi(\mathbf{r}),$$

and parity $-1$ if it is odd under reflection, i.e.

$$\psi(-\mathbf{r}) = -\psi(\mathbf{r}).$$

More generally, a many-particle wave-function has parity $+1$ if it is even under reflection of all the particle coordinates, and parity $-1$ if it is odd under reflection.

Parity is an important concept because the laws of the electromagnetic and of the strong interaction are of exactly the same form if written with respect to a reflected left-handed coordinate system $(0x', 0y', 0z')$ as they are in the standard right-handed system $(0x, 0y, 0z)$ (Fig. 1.1). We shall see in Chapter 2 that this is not true of the weak interaction. Nevertheless, for many properties of atomic and nuclear systems the weak interaction is unimportant and wave-functions for such systems can be chosen to have a definite parity which does not change as the wave-function evolves in time according to Schrödinger's equation.

## 1.4 Units

Every branch of physics tends to find certain units particularly congenial. In nuclear physics, the size of the nucleus makes $10^{-15}\,\mathrm{m} = 1\,\mathrm{fm}$ (femtometre) convenient as a unit of length, usually called a *fermi*. However, nuclear cross-sections, which have the dimensions of area, are measured in *barns*; $1\,\mathrm{b} = 10^{-28}\,\mathrm{m}^2 = 100\,\mathrm{fm}^2$. Energies of interest are usually of the order of MeV. Since $mc^2$ has the dimensions of energy, it is convenient to quote masses in units of $\mathrm{MeV}/c^2$.

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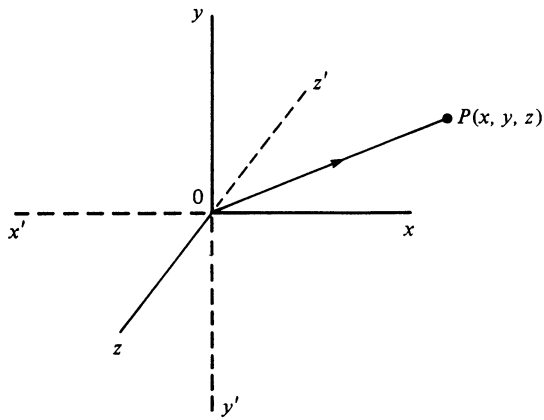



**Fig. 1.1** The point $P$ at $\mathbf{r}$ with coordinates $(x, y, z)$ has coordinates $(-x, -y, -z)$ in the primed, reflected coordinate axes. $(0x', 0y', 0z')$ make up a *left-handed* set of axes. In the figure, the $0z$ axis is out of the plane of the page.

For order-of-magnitude calculations, the masses $m_e$ and $m_p$ of the electron and proton may be taken as

$$m_e \approx 0.5 \text{ MeV}/c^2$$
$$m_p \approx 938 \text{ MeV}/c^2$$

and it is useful to remember that

$$\hbar c \approx 197 \text{ MeV fm}, \qquad e^2/4\pi\varepsilon_0 \approx 1.44 \text{ MeV fm},$$
$$e^2/4\pi\varepsilon_0 \hbar c \approx 1/137, \qquad c \approx 3 \times 10^{23} \text{ fm s}^{-1}.$$

The student will perhaps be surprised to find how easily many expressions in nuclear physics can be evaluated using these quantities.

## Problems

1.1 Show that the ratio of the gravitational potential energy to the Coulomb potential energy between two electrons is $\approx 2.4 \times 10^{-43}$.

1.2(*a*) Show that in polar coordinates $(r, \theta, \phi)$ the reflection $\mathbf{r} \to \mathbf{r}' = -\mathbf{r}$ is equivalent to $r \to r' = r$ $\theta \to \theta' = \pi - \theta, \phi \to \phi' = \phi + \pi$.

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(*b*) What are the parities of the following electron states of the hydrogen atom:

(*i*) $\psi_{100} = \frac{1}{\sqrt{\pi}}\left(\frac{1}{a_0}\right)^{\frac{3}{2}} \mathrm{e}^{-r/a_0}$,

(*ii*) $\psi_{210} = \frac{1}{4\sqrt{(2\pi)}}\left(\frac{1}{a_0}\right)^{\frac{3}{2}} \frac{r}{a_0} \mathrm{e}^{-r/2a_0} \cos\theta$,

(*iii*) $\psi_{21-1} = \frac{1}{8\sqrt{\pi}}\left(\frac{1}{a_0}\right)^{\frac{3}{2}} \frac{r}{a_0} \mathrm{e}^{-r/2a_0} \sin\theta\, \mathrm{e}^{-\mathrm{i}\phi}$?

($a_0 = (4\pi\varepsilon_0)\hbar^2/m_e e^2$ is the Bohr radius.)

1.3(*a*) Show that the wavelength of a photon of energy 1 MeV is $\approx 1240$ fm.

(*b*) The electrostatic self-energy of a uniformly charged sphere of total charge $e$, radius $R$, is $U = (3/5)e^2/(4\pi\varepsilon_0 R)$. Show that if $R = 1$ fm, $U \approx 0.86$ MeV.

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