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DNS Study on Vortex and Vorticity in Late Boundary Layer Transition

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Abstract. Vortex and vorticity are two correlated but fundamentally different concepts which have been the central issues in fluid mechanics research. Vorticity has rigorous mathematical definition (curl of velocity), but no clear physical meaning. Vortex has clear physical meaning (rotation) but no rigorous mathematical definition. For a long time, many people treat them as a same thing. However, based on our high-order direct numerical simulation (DNS), we found that first, "vortex" is not "vorticity tube" or "vortex tube" which is widely defined as a bundle of vorticity lines without any vorticity line leak. Actually, vortex is an open area for vorticity line penetration. Second, vortex is not necessarily congregation of vorticity lines, but dispersion in many 3-dimensional cases. Some textbooks say that vortex cannot end inside the flow field but must end on the solid wall (and/or boundaries). Our DNS observation and many other numerical results show almost all vortices are ended inside the flow field. Finally, a more theoretical study shows that neither vortex nor vorticity line can attach to the solid wall and they must be detached from the wall.

AMS subject classifications: 76F06, 76F40, 76M23, 76D17

Key words: Vortex, vorticity, vorticity line, vorticity tube, Λ-vortex, hairpin vortex, DNS, flow transition, turbulence.

Nomenclature

$M_\infty =$ Mach number
$\mathrm{Re} =$ Reynolds number
$\delta_{in} =$ inflow displacement thickness

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$T_w =$ wall temperature
$T_\infty =$ free stream temperature
$Lz_{in} =$ height at inflow boundary
$Lz_{out} =$ height at outflow boundary
$Lx =$ length of computational domain along x direction
$Ly =$ length of computational domain along y direction
$x_{in} =$ distance between leading edge of flat plate and upstream boundary of computational domain
$A_{2d} =$ amplitude of 2D inlet disturbance
$A_{3d} =$ amplitude of 3D inlet disturbance
$\omega =$ frequency of inlet disturbance
$\alpha_{2d}, \alpha_{3d} =$ two and three dimensional streamwise wave number of inlet disturbance
$\beta =$ spanwise wave number of inlet disturbance
$R =$ ideal gas constant
$\gamma =$ ratio of specific heats
$\mu_\infty =$ viscosity
$x, y, z =$ streamwise, spanwise, normal directions

1 Introduction

Vorticity has rigorous mathematical definition (curl of velocity), but no clear physical meaning. On the other hand, vortex has clear physical meaning (rotation) but no rigorous mathematical definition. For a long time, many people treat them as a same thing.

Vorticity are ubiquitously seen in nature, ranging from smoke rings to clouds, from bubble rings to hurricanes, from swirls in a washing pool to whirlpools in the sea. It is intuitively acknowledged that a vortex is a fluid region with rotational motions. However, the problem of giving a precise and rational vortex definition is still an open issue [1, 2], even though people's understanding towards vortices has been greatly deepened ever since. Existing vortex identification criteria including λ_2, Q and $\tilde{\Delta}$ methods [3–5] have proved to be efficient in visualizing vortical structures. Nevertheless, these methods still fail to give a quantitative definition of vortices. Thus, a better understanding of vortices and a rational definition become more and more pressing since the discovery of coherent vortical structures and their key role in transitional and turbulent flows. Recently, a new vortex identification method called Ω-method is proposed by Liu et al. [6] based on the idea that vortices locate where vorticity overtake deformation and has advantages like no need for a case-related threshold, easiness to perform and the ability to capture strong and weak vortices at the same time.

Vorticity, on the other hand, is mathematically defined as the curl of velocity field.

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Helmholtz [7] further derived the well-known three vorticity theorems, along with which kelvin's circulation theorem and others paved the foundation in this field. The classical monograph of Lamb [8] summarized this era's theoretical achievements of incompressible, ideal fluid, dealing with mostly irrotational flow with only a small part of rotational flow. Prandtl's ground-breaking boundary layer theory [9] and the invention of aircraft in the beginning of 20th century opened a new stage of vortical flow research. Prandtl's work revealed the fundamental importance of viscosity in fluid motion and explained that the circulation around the wing is generated by the viscous shearing process within the thin boundary layer. The rapid development of aeronautical techniques at that time is largely due to people's understanding, utilization and control of vortices, as summarized by Durand [10] and Goldstein [11].

Almost at the same time, the important role of vortical structures in transitional and turbulent flows was recognized by researchers, as addressed by Kchemann's [12] famous quote "vortices are the sinews and muscles of turbulence". A few conceptual models are conjectured and proposed, e.g., Theodorsen's "horseshoe" model [13]. The development of experimental techniques provided researchers much richer information about the turbulent field. Kline et al. [14] observed the streamwise narrow streaks of high and low momentum fluid in boundary layer using hydrogen bubbles. Corino and Brodkey [15] concluded that the "ejections" and "sweeps" are driven by streamwise vortices. However, it was the DNS and PIV (Particle Image Velocimetry) techniques emerged in the late 1980s that really enabled researchers to directly observe the dynamical, detailed turbulent fields, especially DNS, which provides full information for 3-D (three dimension) turbulence. Since the first DNS of channel flow [16] and turbulent boundary layer [17], many significant discoveries have been made on the coherent structures in transitional and turbulent flows [18]. Wu and Moin [19] reported a new DNS for flow transition on a flat plate and confirmed the coherent structures found by previous researchers like Liu et al. [20] and Bake et al. [21]. Sayadi et al. [22] studied the vortices evolution in *H*- and *K*-type transition process using DNS. Particularly, Zagumennyi and Chashechkin [23] did a careful numerical study on the vortex pattern on a two dimensional plate. In parallel with these development, vorticity and vortex dynamics also received considerable attention from the community including Batchelor's classic [24] and more recent monographs by Safffman [25], lugt [26], and Wu et al. [2].

Based on the numerical data obtained from a verified DNS of boundary layer flow on a flat plate [27], some view points towards vortices and vorticity are examined. The relationship between vortices and vortex tubes or vortex lines is first investigated. As we know, vortex tubes and vortex lines are defined from vorticity and therefore these two terms have no direct connection with vortex, even historically Lamb [8] used vortex tubes to define vortices which has been proved to be inaccurate by many researchers [2,3,27,28]. However, the term vortex tubes or vortex lines themselves indicate certain connection to vortices and might be a source of confusions.

In classical vortex dynamics (see Encyclopedia of Mathematics and Physics, 2006 [29]), "a vortex is $\cdots$ defined by the vorticity in the fluid, which measures the rate of local fluid

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rotation". Some online lecture notes [30], which is open to the world, addresses:

"$\cdots$ 3-D vortex must be subjected to Helmholtz Vortex Theorems:

1. The strength Γ of the vortex is constant all along its length,
2. The vortex cannot end inside the fluid, it must either
   a) Extend to $\pm\infty$, or
   b) End at a solid boundary, or
   c) Form a closed loop, $\cdots$."

This clearly shows that they consider vortex to be a vorticity tube without vorticity line leak. Their points of views are:

1. Consider "vortex" as "vorticity tube";
2. Vortex ends on the solid wall;
3. Vorticity keeps constant along vortex;
4. Vortex cannot end inside the flow field.

The direct corollary of these online notes is that "vortex (vorticity tube) cannot break down forever", and then how turbulence can be generated by "vortex breakdown" as many textbooks suggest? In addition, the modernized misunderstanding appeared in many textbooks and research papers include:

1. Consider vortex is a congregation of vorticity lines [2,30];
2. Consider vortex has larger vorticity than the surrounding area.

Although the impropriety of these theorems had been pointed out as early as one hundred years ago by Hadamard [31], and had been elaborated with a detailed analysis of simple analytical flows very recently by Fuentes [32], these misunderstandings are still hindering researchers from a richer knowledge of turbulence. Vorticity, vorticity line and vorticity tube have very rigorous mathematical definitions as we define $\nabla\times\overrightarrow{V}$ as vorticity and vorticity line as a line where the vorticity vector has a direction in its tangent. Vorticity tube is a bundle of vorticity lines, which has no vorticity line leak. There are no rooms for us to argue. On the other hand, vortex is intuitively considered as a rotating core of fluids with vorticity line leaks, which is totally different from vorticity tube. As we show below, vortex CAN end inside the fluid field, but CANNOT have constant vorticity along its length. In addition, vortex could not be always a congregation of vorticity lines but dispersion of vorticity lines.

In pursuit of deeper understanding of vortices and vorticity, a high order direct numerical simulation (DNS) was conducted on a flat plate boundary layer flow with near

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60 million grid points and over 400,000 time steps at a free stream Mach number of 0.5. The paper is organized as follows. Numerical details of the calculation are presented in Section 2 with verification and validation. Section 3 provides some analysis on vortices and vorticity based on the numerical results, followed by Section 4 with conclusions.

2 Case setup and code validation

2.1 Case set-up

The computational domain is shown in Fig. 1(a). The mesh includes $1920\times128\times241$ points in streamwise (x), spanwise (y), and wall normal (z) directions respectively. A uniform grid is employed in both streamwise and spanwise directions, while a stretching grid is used in normal direction. The first grid interval is carefully chosen to make sure the grid is fine enough to capture all the small scales ($z^+ = 0.43$). The Message Passing Interface (MPI) plus the streamwise direction domain decomposition which is shown in Fig. 1(b) is utilized for parallel computation. The flow conditions, including Reynolds number, Mach number, etc. are listed in Table 1. Lx and Ly are the lengths of the computational domain in streamwise and spanwise directions, while Lz_{in} is the height of the computational domain at the inlet. x_{in} is the distance between inlet and the leading edge of the flat plate and Tw represents the wall temperature. δ_{in} represents the inflow displacement thickness and also acts as reference length for non-dimensioning. For our case, δ_{in} is estimated to be 2-4mm. In addition, the Reynolds number is defined as $\mathrm{Re} = \frac{\rho_\infty U_\infty \delta_{in}}{\mu_\infty}$ and the inflow Mach number is intentionally chosen to be 0.5 to avoid considering complicated compressible effects.

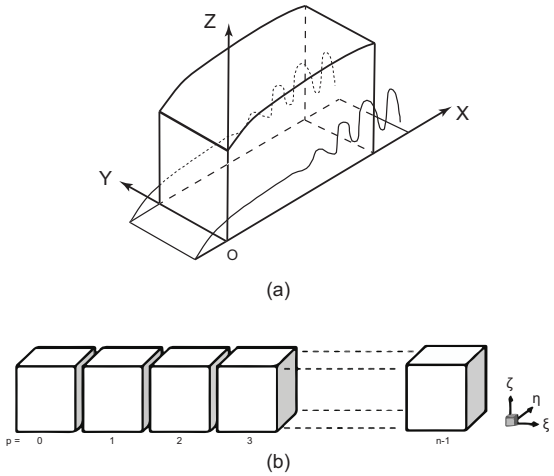


Figure 1: Computational domain and domain decomposition for MPI.

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Table 1: Flow parameters.

M_∞	Re	x_{in}	Lx	Ly	Lz_{in}	T_w	T_∞
0.5	1000	$300.79\delta_{in}$	$798.03\delta_{in}$	$22\delta_{in}$	$40\delta_{in}$	273.15K	273.15K

2.2 Numerical methods

The 3D compressible Navier-Stokes equations in curvilinear coordinates can be written in the following conservative form:

$$\frac{1}{J}\frac{\partial \mathbf{Q}}{\partial t}+\frac{\partial(\mathbf{F}-\mathbf{F}_V)}{\partial \xi}+\frac{\partial(\mathbf{G}-\mathbf{G}_V)}{\partial \eta}+\frac{\partial(\mathbf{H}-\mathbf{H}_V)}{\partial \zeta}=0, \tag{2.1a}$$

$$\mathbf{Q}=\begin{pmatrix}\rho\\ \rho \mathrm{u}\\ \rho \mathrm{v}\\ \rho \mathrm{w}\\ \mathrm{e}\end{pmatrix},\qquad \mathbf{F}=\frac{1}{J}\begin{pmatrix}\rho \mathrm{U}\\ \rho \mathrm{u}\mathrm{U}+\mathrm{p}\xi_\mathrm{x}\\ \rho \mathrm{v}\mathrm{U}+\mathrm{p}\xi_\mathrm{y}\\ \rho \mathrm{w}\mathrm{U}+\mathrm{p}\xi_\mathrm{z}\\ \mathrm{U}(\mathrm{e}+\mathrm{p})\end{pmatrix},\qquad \mathbf{G}=\frac{1}{J}\begin{pmatrix}\rho \mathrm{V}\\ \rho \mathrm{u}\mathrm{V}+\mathrm{p}\eta_\mathrm{x}\\ \rho \mathrm{v}\mathrm{V}+\mathrm{p}\eta_\mathrm{y}\\ \rho \mathrm{w}\mathrm{V}+\mathrm{p}\eta_\mathrm{z}\\ \mathrm{V}(\mathrm{e}+\mathrm{p})\end{pmatrix}, \tag{2.1b}$$

$$\mathbf{H}=\frac{1}{J}\begin{pmatrix}\rho \mathrm{W}\\ \rho \mathrm{u}\mathrm{W}+\mathrm{p}\zeta_\mathrm{x}\\ \rho \mathrm{v}\mathrm{W}+\mathrm{p}\zeta_\mathrm{y}\\ \rho \mathrm{w}\mathrm{W}+\mathrm{p}\zeta_\mathrm{z}\\ \mathrm{W}(\mathrm{e}+\mathrm{p})\end{pmatrix},\qquad \mathbf{F}_V=\frac{1}{J}\begin{pmatrix}0\\ \xi_\mathrm{x}\tau_{xx}+\xi_\mathrm{y}\tau_{xy}+\xi_\mathrm{z}\tau_{xz}\\ \xi_\mathrm{x}\tau_{xy}+\xi_\mathrm{y}\tau_{yy}+\xi_\mathrm{z}\tau_{yz}\\ \xi_\mathrm{x}\tau_{xz}+\xi_\mathrm{y}\tau_{yz}+\xi_\mathrm{z}\tau_{zz}\\ \xi_\mathrm{x}b_x+\xi_\mathrm{y}b_y+\xi_\mathrm{z}b_z\end{pmatrix}, \tag{2.1c}$$

$$\mathbf{G}_V=\frac{1}{J}\begin{pmatrix}0\\ \eta_\mathrm{x}\tau_{xx}+\eta_\mathrm{y}\tau_{xy}+\eta_\mathrm{z}\tau_{xz}\\ \eta_\mathrm{x}\tau_{xy}+\eta_\mathrm{y}\tau_{yy}+\eta_\mathrm{z}\tau_{yz}\\ \eta_\mathrm{x}\tau_{xz}+\eta_\mathrm{y}\tau_{yz}+\eta_\mathrm{z}\tau_{zz}\\ \eta_\mathrm{x}b_x+\eta_\mathrm{y}b_y+\eta_\mathrm{z}b_z\end{pmatrix},\qquad \mathbf{F}_V=\frac{1}{J}\begin{pmatrix}0\\ \zeta_\mathrm{x}\tau_{xx}+\zeta_\mathrm{y}\tau_{xy}+\zeta_\mathrm{z}\tau_{xz}\\ \zeta_\mathrm{x}\tau_{xy}+\zeta_\mathrm{y}\tau_{yy}+\zeta_\mathrm{z}\tau_{yz}\\ \zeta_\mathrm{x}\tau_{xz}+\zeta_\mathrm{y}\tau_{yz}+\zeta_\mathrm{z}\tau_{zz}\\ \zeta_\mathrm{x}b_x+\zeta_\mathrm{y}b_y+\zeta_\mathrm{z}b_z\end{pmatrix}. \tag{2.1d}$$

In Eq. (2.1a), Q is the vector of conserved quantities, while F, G, H are inviscid flux vector, and F_V, G_V, H_V are viscous flux vector. The detailed form of the vectors is given in Eqs. (2.1b), (2.1c) and Eq. (2.1d), where J is Jacobian of the coordinate transformation, ξ_x, ξ_y, ξ_z, η_x, η_y, η_z, ζ_x, ζ_y and ζ_z are coordinate transformation metrics, U, V and W are contravariant velocity components, τ_{xx}, τ_{yy}, τ_{zz}, τ_{xy}, τ_{xz} and τ_{yz} are viscous stress components, while b_x, b_y and b_z are heat flux respectively. Six order compact scheme [33] is used for spatial discretization while a third-order TVD (total variation diminishing) Runge-Kutta method [34] is employed for time marching. A spatial filtering [33] is adopted to eliminate numerical oscillations caused by central difference scheme. 2D and 3D T-S (Tollmien-Schlichting) waves are obtained by stability analysis and enforced at the inflow boundary, and therefore the simulated transition belongs to the category of natural transition. It should also be noted that these unstable waves are enforced at the inflow computational boundary, which has a distance of x_{in} downstream the leading edge of the plate. In this way, the disturbances are introduced into the flow. Non-reflecting boundary conditions are applied at both outflow boundary and far field boundary. Periodic

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and non-slip conditions are naturally used at the spanwise boundaries and wall boundary respectively.

2.3 Code validation

The code "DNSUTA" was developed at the University of Texas at Arlington and carefully validated by NASA Langley and UTA researchers [35,36]. Only a short description of the validation would be addressed here and readers are encouraged to refer to these papers for details. A more detailed comparison is also reported in [27].

2.3.1 Velocity profiles and grid convergence

Time and spanwise-averaged streamwise velocity profiles for two different streamwise locations in two different grid levels $960\times64\times121$ and $1920\times128\times241$ are shown respectively in Fig. 2(a) and Fig. 2(b). The inflow velocity profiles at $x=300.79\delta_{in}$ is a typical laminar boundary layer velocity profile. And the mean velocity profile at $x=632.33\delta_{in}$ approaches a turbulent flow velocity profile (Log Law). This comparison shows that the grid convergence has been realized and the simulation from laminar flow to turbulence has also been achieved.

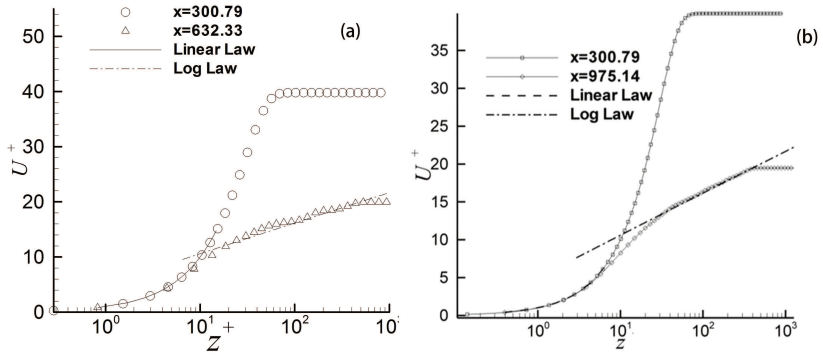


Figure 2: Time- and spanwise-averaged velocity profile in two grid levels.

2.3.2 Comparison with experiment

Utilizing the λ_2-eigenvalue method proposed by Jeong and Hussain [3], the vortical structures during transition is shown in Fig. 3. The formation of ring-like vortices chains is consistent with the experimental work by Lee and Li [37] as shown in Fig. 4.

2.3.3 Comparison with Rist's DNS data

Fig. 5 shows a comparison between our DNS results with Rist's DNS data [21] provided as his personal kindness. The comparison shows both DNS results are able to capture the typical vortical structures during transition.

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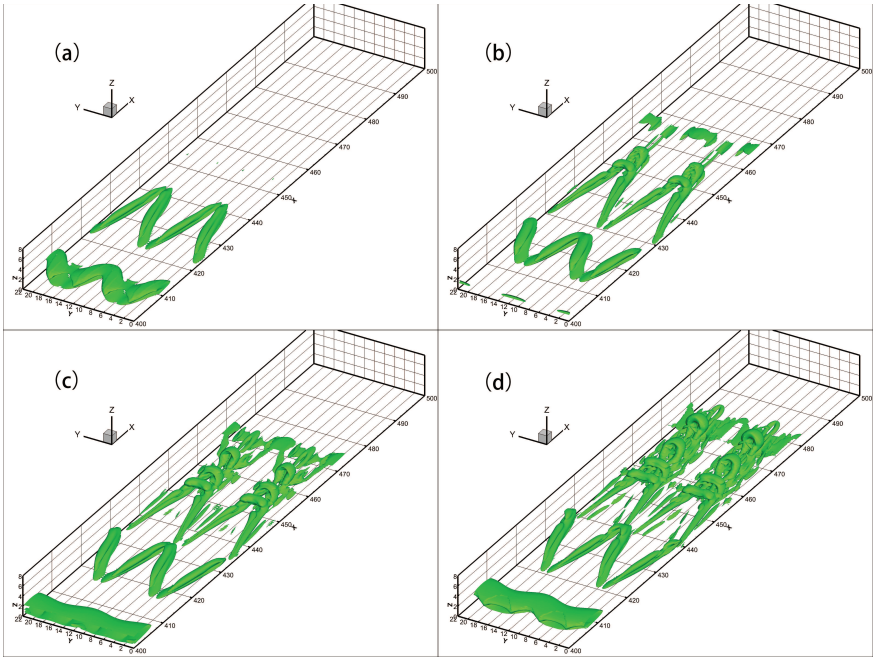


Figure 3: Evolution of vortical structures during transition.

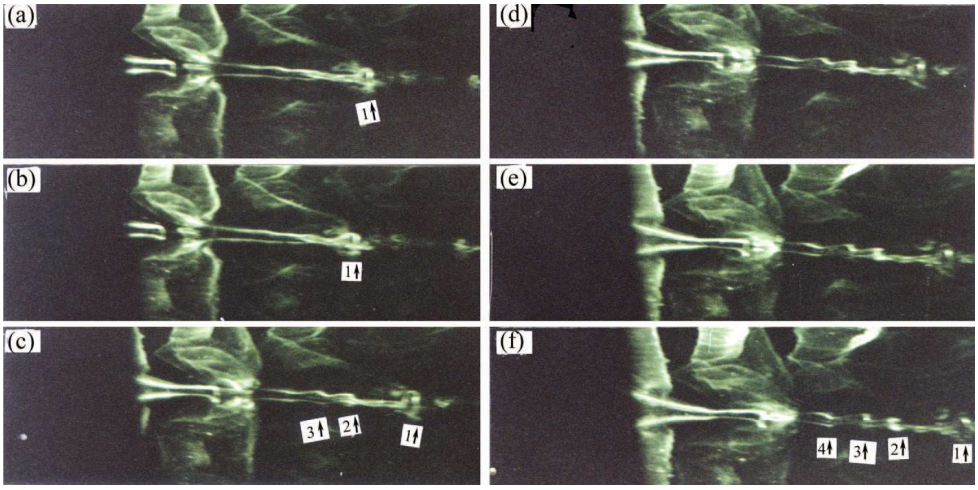


Figure 4: Evolution of ring-like vortices by experiment.

All above verifications and validations show that the DNS results are reliable and accurate.

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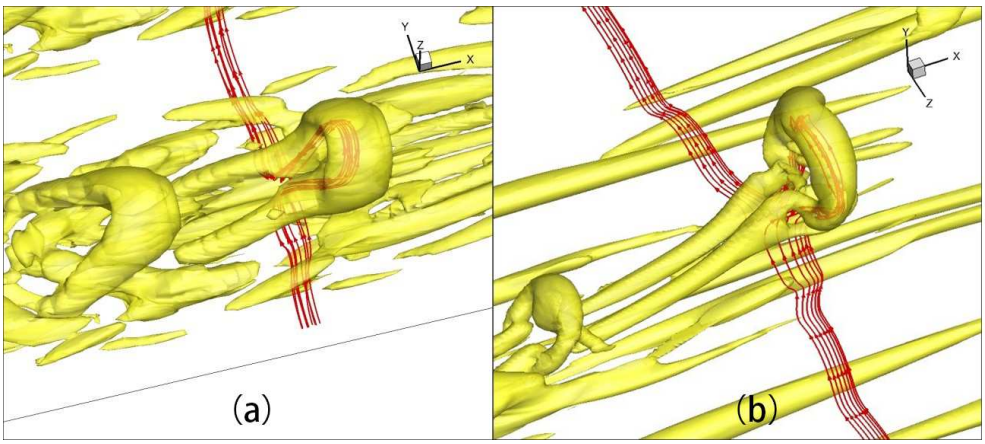


Figure 5: Comparison of our DNS results with Rist's DNS data.

3 DNS analysis

In this section, detailed analyses on vorticity and typical vortices i.e., Λ-vortex and hairpin vortex during late transition process of a flat plate boundary layer are given based on our DNS results. Both Λ-vortex and hairpin vortex are observed from experiments and named after their shapes. While Λ-vortex is mainly streamwise vortical structure and mostly seen in the vicinity of wall surfaces at the beginning stage of transition from laminar to turbulent flows, hairpin vortex is widely detected in both transitional and fully developed turbulent flows. These two typical vortices are believed to play a significant part in turbulence generation and sustenance. First, by carefully comparing the typical vortices and vortex tubes, it is reaffirmed that the vortex and the vorticity tube are two distinct concepts, and have no direct connection. Thus, the interpretation of "vorticity line" and "vorticity tube" which have certain link to vortex are improper in some respect since vortex is not a tube and cannot be a tube unless it is self-closed. Second, it is surprising to find that the Λ vortex is not located where vorticity is concentrated. Therefore, the intuitive idea that a vortex is a connected region with high concentration of vorticity is inaccurate. What's more, it is found by checking the DNS results that no vorticity tubes or vorticity lines can end on the solid boundary wall. Then further analytical reasoning on why "vortex tube" cannot exist will be given. Therefore, according to Helmholtz's theorem, vorticity lines must either extend to infinity or form closed loops, but cannot end at solid boundaries. Almost all vortices only end inside the flow field.

3.1 Vorticity tube is not vortex

Firstly, we revisit some concepts that are widely used and frequently appear in textbooks and research papers.

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Vorticity: Vorticity is rigorously defined as the curl of velocity field.

Vorticity line: a curve which is everywhere tangent to the local vorticity vector.

Vorticity tube: a tube-like surface formed by all vorticity lines passing through a closed curve, which could be considered as bundle of vorticity line without any vorticity line leakage.

Vortex: As mentioned in the introduction, a rigorous definition of vortex is not reached yet. However, there are several popular vortex identification methods based on the rotation essence of vortex.

The "Ω criterion" proposed by Liu et al. [6] is used in this paper to identify the vortices. The "Ω" is defined as the proportion of vorticity and deformation in fluid element motion:

$$\nabla V=\frac{1}{2}(\nabla V+\nabla V^T)+\frac{1}{2}(\nabla V-\nabla V^T)=S+W, \tag{3.1}$$

where S is the symmetric while W is the anti-symmetric part of the velocity gradient tensor. Let $a=trace(SS^T)$, $b=trace(WW^T)$, then

$$\Omega=\frac{b+\varepsilon}{(a+\varepsilon)+(b+\varepsilon)}. \tag{3.2}$$

$\Omega=0.52$ is set as the threshold to identify the region where rotation plays a dominant role rather than deformation as a vortex. Compared with traditional Q or λ_2 criteria, it doesn't need to tune the threshold and have clear physical meaning. In addition, the Ω method has the capability to capture both strong and weak vortices simultaneously while neither Q nor λ_2 criteria can. Actually, when $\Omega=1.0$, the flow has pure rotation and has no deformation which means the fluid is stiff like solid, but vortex itself is also very stiff like solid. Although Ω is a measurement of fluid stiffness, but it is a good measurement of flow rotation, i.e., vortex.

As we can see from the above definition that "vorticity tube" is composed of and bounded by a group of vorticity lines. On the other hand, "vorticity tube" is frequently referred as the definition of a vortex in textbooks and many research papers. This confusion has its roots from the classical non-vicious flow theory era, and some researchers have used the evolution of vorticity lines to study the vortex structure in boundary layer and proposed that the self-deformation of vorticity lines is the mechanism of the transformation from Λ-vortex to hairpin vortex [38]. However, our detailed visualisation of the DNS field shows that vortex is not a collection of vorticity lines and therefore, the concept of Λ vortex transform to hairpin vortex through a self-deformation is a misunderstanding which has been lasted for several decades. Actually, vortex rings and vortex legs of a Λ vortex are separated parts and they are generated by different mechanisms.

The Λ-shaped vortex was first observed in experiments by Hama [39] and Hama and Nutant [40] and has been believed to play a significant role during transition. It is shown in Fig. 6 that a Λ-vortex is formed at the beginning of essentially non-linear stage of

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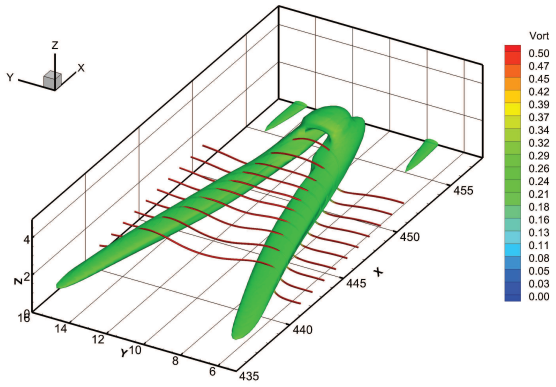


Figure 6: Vortex (green) and vorticity line (red) in Λ vortices.

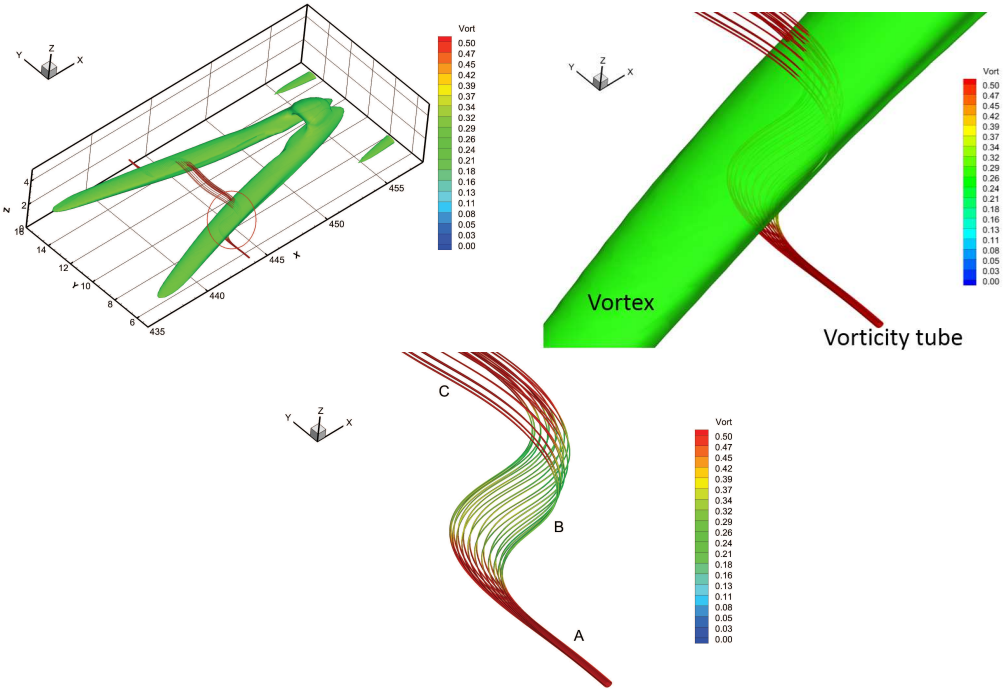


Figure 7: Vorticity tube and Λ vortex at $t=8.2$T.

boundary layer transition, and vorticity lines in the vicinity are also presented. Mathematically, non-linear stage means that the disturbance is growing so much that the second order terms of disturbance are large enough, which cannot be neglected anymore. We can see that the vorticity lines penetrate the vortex and the direction of vortex axis is different from that of vorticity lines. In Fig. 7, a "vorticity tube" is shown by drawing a group

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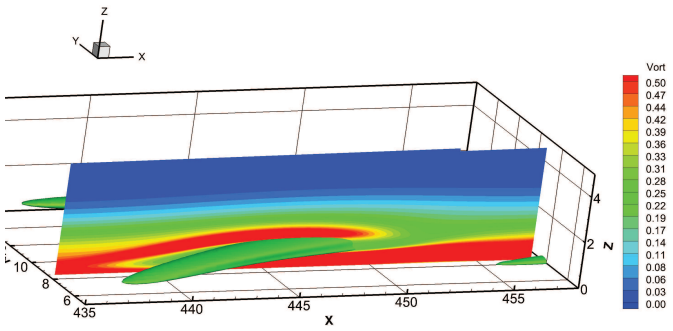


Figure 8: Λ vortex and shear layer (Vortex has smaller vorticity).

of vorticity lines through a small closed circle on x-z plane. It is obvious that "vorticity tube" does not equal to the Λ-vortex at all. Near the head of the Λ-vortex, the low speed zone generated by the ejections of the Λ-vortex would form a strong shear layer as shown in Fig. 8, which would further develop into a vortex ring through Kelvin-Helmholtz type instability and become the head of the hairpin vortex. This mechanism of vortex ring formation is different from the Λ-vortex leg formation, which is formed due to the shear layer rollup and stretching. As shown in Fig. 9, in the head of a hairpin vortex, part of the vorticity tube is contained in the vortices, however, most of the vorticity tube still belongs to the non-vortex region. Another obvious evidence is shown in Fig. 10, where a self-closed ring-like "vorticity tube" exists before the hairpin vortices, but not identified by the vortex identification criterion. It means that a self-closed ring of "vorticity tube" is not a vortex, which might be opposite to many people's intuition. All the above arguments and analysis clearly illustrate the difference between vortex and vorticity and testifies that vortex is in general not vorticity tube.

Actually, vorticity lines are only integral curves of vorticity vector field, and thus only related to the vorticity vector direction. However, vortex concerns not only the vorticity vector magnitude, but also its relative significance compared with local deformation. Although vorticity is thought to represent the rotation of micro fluid element which is not true, vorticity line and vorticity tube are not relevant to the concept of vortex. The source of misunderstanding of vortex as vorticity comes from solid mechanics. However, "fluid" is not "solid". Fluid flow is almost always a mixture of rotation and deformation even for vortex cores in fluids. That is the reason why vortex cores in fluids are not circle but elliptic shaped and others with deformation.

3.2 Vortex is not the congregation of vorticity

Although the mathematical definition of vortex is still an open issue, many researchers took vortex as the congregation of vorticity to be self-evident. This viewpoint is reflected in many textbooks and research papers, explicitly or implicitly. As mentioned in the in-

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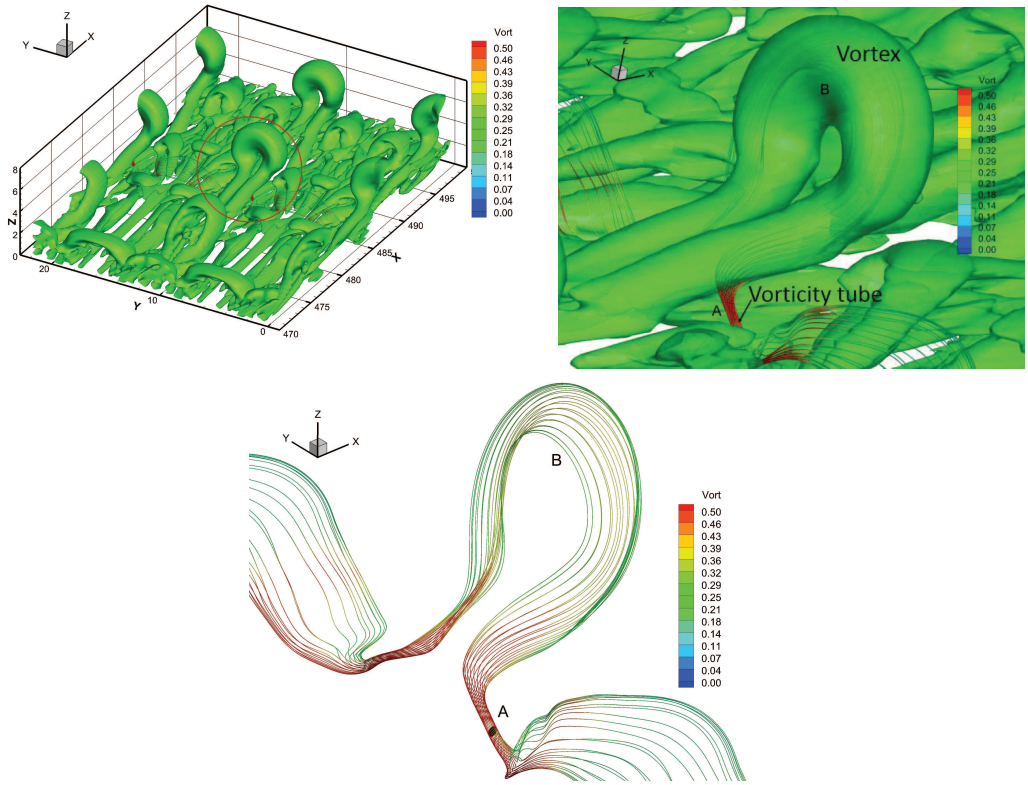


Figure 9: Vorticity tube and ring-like vortices at $t=8.2$T.

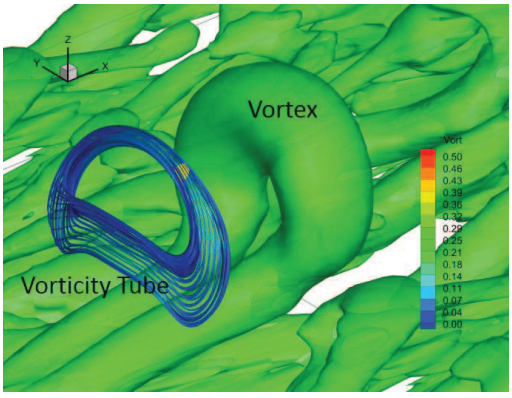


Figure 10: Ring-like vorticity tube and ring-like vortices.

troduction, Wu define vortices as "connected region with high concentration of vorticity compared with its surrounding" in the opening of his recent vorticity dynamics monograph [2]. In his recent book on the coherent structure of turbulence, Sedat Tardu also have such claim as "vortices must indubitably contain intense vorticity" [41].

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However, a scrutiny of the vorticity magnitude around the vortex region would prove that this widely-spread belief is inaccurate. Fig. 8 can also serves as an example that vorticity is smaller inside the Λ-vortex than the surrounding red shear layers with high vorticity. All the vorticity lines in this paper are coloured by local vorticity magnitude. It is easy to observe that vorticity magnitude is smaller in the vortex region than that of the surrounding non-vortex region (Figs. 7-9). In Fig. 7(c), we mark three positions as A, B, C at different parts of the vorticity tube, where B is within the vortices region while A and C are outside the vortices region. The numeric of vorticity magnitude and Ω at each position are given in Table 2. It can be seen that the vorticity at B is much smaller than A and C while it has a larger Ω than A and C. In Fig. 8, we can see that the vortex region identified by larger Ω is located between two strong shear layers. For the case of hairpin vortex, we also draw the vortex and vorticity tube in Fig. 9(a) correspondingly. Fig. 9(b) is a enlarged view near the hairpin vortex head and two positions A and B are marked at different positions of the vorticity tube, where A is outside the vortices region and B is within the head of hairpin vortex. The numeric of Ω and vorticity at A and B are given in Table 3, which again testifies that vortex doesn't necessarily correspond to large vorticity, but large Ω. This can also be observed from Fig. 11, which gives a comparison of Ω and vorticity contour on the middle spanwise slice. The fact that vorticity is small within the vortex, is also testified by the distribution of vorticity lines. It is clearly seen that the vorticity lines are quite concentrated outside the vortex, but become dispersed within the vortex region, as shown in both Fig. 7 and Fig. 9. Actually, the vortex axis direction is not aligned with the vorticity direction.

Table 2: Ω and vorticity within and outside the lambda vortices.

	A	B	C
Ω	≈0.5	≈0.6	≈0.5
Vorticity	0.7-0.8	0.2-0.3	0.7-0.8

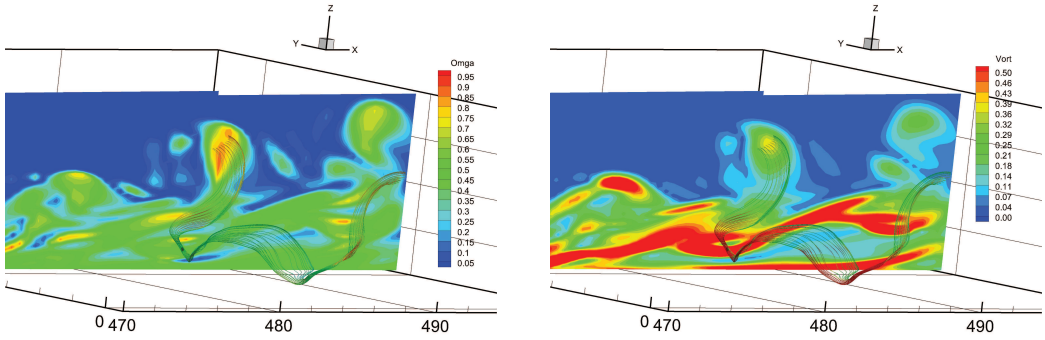
Table 3: Ω and Vorticity within and outside the ring vortices.

	A	B
Ω	≈0.5	≈0.6
Vorticity	0.7-0.8	0.2-0.3

Fig. 12 gives the Λ-vortex at t=5.85T and the slice x=446. The velocity vector is given on the slice and circles around the vortex core and the slice is contoured by vorticity magnitude. This is an important concept to distinguish vorticity and vortex: vorticity lines are abstract mathematical concept without clear physical meaning and vortex is more related with visual streamlines but without rigorous mathematical definition.

Note that for the current case the vortex structure is given by Ω method. However, the Q-criterion and λ_2 vortex identification method would give similar vortex structure if a proper threshold is selected.

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(a) Ω is large in vortex (b) Vorticity is relatively small in vortex

Figure 11: Ω and Vorticity Magnitude comparison for hairpin vortices.

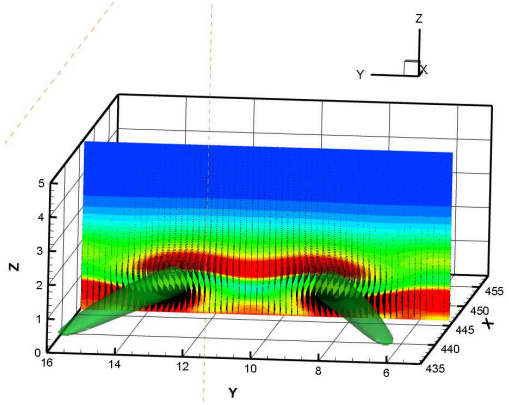


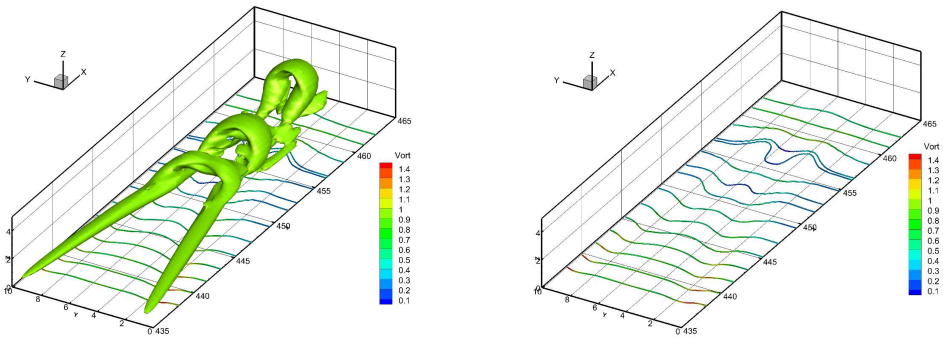
Figure 12: The Λ-vortex at $t=5.85$T (vortex has small vorticity with closed streamlines).

3.3 Vortex is never attached to the wall

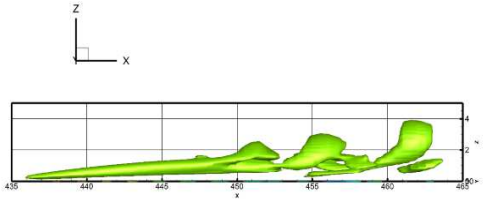
In general, the process from attachment to detachment is believed by many as a very important phenomenon during transition. Head and Bandyopadhyay [42] observed the hairpin-like vortex from the smoke visualisation of a turbulent boundary layer in their experiment. Townsend [43] proposed his "attached eddy" hypothesis in his classic book. Based on the idea of Head and Bandyopadhyay, Perry and Chong [44] proposed a hierarchy of hairpin vortices "forest" structure of boundary layer, that all the vortices of different scales are attached to the wall. Adrian [45] also gave similar ideas of vortices detached from the wall in his recent paper.

However, our DNS flow field visualization shows that vortex is generated within the fluid and never attached to the wall. Firstly, we prove that the vorticity lines on the wall cannot leave the wall surface. Let $V=(u,v,w)$, where u,v,w are streamwise (x) spanwise (y) and normal (z) velocity. Let, which represents the vorticity, vector. On the solid wall, $V=0$ and $\omega_z=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}=0$ since $\frac{\partial v}{\partial x}=\frac{\partial u}{\partial y}=0$. It means the vorticity on the wall has no

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(a) Vortex and vorticity lines on the wall (b) Vorticity line cannot depart from the wall



(c) Vortex is always detached from the wall

Figure 13: Vorticity tube and Λ vortex at $t=8.2$T.

normal component and vorticity lines are fitted on the wall. As the velocity on the wall is zero according to no-slip condition, the vorticity lines cannot lift up from the wall.

In Fig. 13, we give the vorticity lines on the wall and it is clear that they are all fitted on the surface (Fig. 13(b)). Since all vortices are not attached to the wall, they are generated within the fluid field and stay above the wall (Fig. 13(c)).

4 Conclusions

The differences between vortex and vorticity are reiterated and emphasized in this paper based on our DNS results for late boundary layer transition. It is first pointed out that the historically assumed idea that a vortex equals to a vortex tube is incorrect. What we found is that "vortex" is not "vorticity tube" or "vortex tube" which is widely defined as a bundle of vorticity lines without any vorticity line leak [46,47]. In addition, the so-called "vortex lines" and "vortex tubes" have no direct connection to vortices. Therefore, the names of "vortex line" and "vortex tube" is improper, and we propose using "vorticity line" and "vorticity tube" instead. Second, Λ-vortex which is typically seen in transitional boundary layer flow acts as a counter example of the common view that vortices must indubitably contain intense vorticity than its surroundings, which indicates that our understanding on vortices should be further deepened before we can reach a rational and quantitative definition of a vortex. In the end, the widely spread idea vortex lines can end on solid boundaries is proved to be incorrect based on the numerical results and

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analytical deduction. It should be pointed out that the compressibility of the numerical simulation in this paper is almost negligible since the inflow Mach number is around 0.5. In summary, we can conclude:

1. Vortex is a place where vorticity overtakes deformation ($\Omega > 0.5$);
2. Vortex is not vorticity tube but open area for vorticity line penetration;
3. Vortex is not always congregation of vorticity lines, but dispersion of vorticity lines in many 3-D cases;
4. Vorticity of a vortex is not necessarily larger than the surrounding area in 3-D;
5. Vortex has different direction from the vorticity vector in general 3-D cases;
6. Vortex cannot end on the solid boundary, but must end inside the fluid field;
7. Q or λ_2-method need a so-called proper threshold and we have no way to judge which threshold is proper and then have no way to judge which vortex structure is correct or incorrect;
8. Ω method can well capture the both strong and weak vortical structures without much adjustment to a threshold, while Q or λ_2-method are not capable to do so;
9. The concept of "Lambda vortex transformation to hairpin vortex through a self-deformation" is a misunderstanding to consider Lambda vortex as a bundle of vorticity lines, which is a misleading for several decades;
10. The concept that "the strength Γ of the vortex is constant all along its length" is a misunderstanding to consider "vortex" as "vorticity tube".

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