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ABCOLOGICAL ANECDOTES

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In June 1985 Joseph Oesterlé gave a lecture at the Max-Planck-Institut in Bonn (then the other side of the river). He discussed the conductor and discriminant of elliptic curves and a conjectural relationship between them due to Lucien Szpiro. He mentioned that for the particular curve defined by $y^2 = x(x-a)(x+b)$ with non-zero coprime rational integers $a \neq -b$ this amounted to an inequality $|abc| \leqslant C(\prod_{p|abc} p)^{\kappa}$, with c defined by $a+b+c=0$ and the product over primes p. Here C, κ are independent of a, b, c but I can no longer remember if this was just for some κ or for all $\kappa > 3$.

Anyway, one could clearly now forget about elliptic curves; and then if one is not interested in a precise value of κ one may as well estimate a, b, c separately using $\max\{|a|, |b|, |c|\}$. I recognized the subsequent inequality as a version of an analogue of a 1984 result of Richard Mason about polynomials (actually anticipated by Wilson Stothers). After the talk I rushed down the steps to the library and found his result, which (to highlight the analogy) can be stated in the exponential form

$$\max\{e^{\deg \mathcal{A}}, e^{\deg \mathcal{B}}, e^{\deg \mathcal{C}}\} \leqslant e^{-1} \prod_{\pi|\mathcal{ABC}} e$$

for all non-zero coprime $\mathcal{A}, \mathcal{B}, \mathcal{C}$ in $\mathbf{C}[t]$, not all in $\mathbf{C}$, with $\mathcal{A}+\mathcal{B}+\mathcal{C}=0$. Here the $\pi = t - \tau$ for τ in $\mathbf{C}$ are the normalized primes of $\mathbf{C}[t]$ and $e = \exp 1$ (by convention). Thus Mason has $\kappa = 1$, which was known to be best possible, and even a bit extra (also best possible). Converting back from $\mathbf{C}[t]$ to $\mathbf{Z}$, I followed standard practice by loosening up to any $\kappa > 1$ to accommodate archimedean valuations (and indeed it would be false with $\kappa = 1$, as is also believed for Klaus Roth's famous $|\alpha - r/s| \geqslant C^{-1}s^{-\kappa-1}$).

A couple of weeks later there occurred a Symposium on Analytic Number Theory in honour of Roth, and accordingly at a Problem Session I wrote on the blackboard the following:

Disprove (or prove) that for every $\epsilon > 0$ there exists $C(\epsilon)$ such that

$$\max\{|a|, |b|, |c|\} \leqslant C(\epsilon)\Big(\prod_{p|abc} p\Big)^{1+\epsilon}$$

for all coprime integers a, b, c with $a+b+c=0$.

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Of course I forgot then to say that a, b, c are all non-zero.

Since then, in connexion with the origin of the abc conjecture, several authors have referred to the Symposium Proceedings. In fact, the open problems raised during the symposium were circulated to the participants after the symposium but nothing was ever published. By the publication of the present note I hope to regularize this situation (especially in view of the developments of the last few years).

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