Student Problems

Students up to the age of 19 are invited to send solutions to either or both of the following problems to Stan Dolan, 126A Harpenden Road, St Albans, Herts., AL3 6BZ.

Two prizes will be awarded – a first prize of £25, and a second prize of £20 – to the senders of the most impressive solutions for either problem. It is not necessary to submit solutions to both. Solutions should arrive by January 20th 2019. Please give your School year, the name and address of your School or College, and the name of a teacher through whom the award will be made. Please print your own name clearly! The names of all successful solvers will be published in the March 2019 edition.

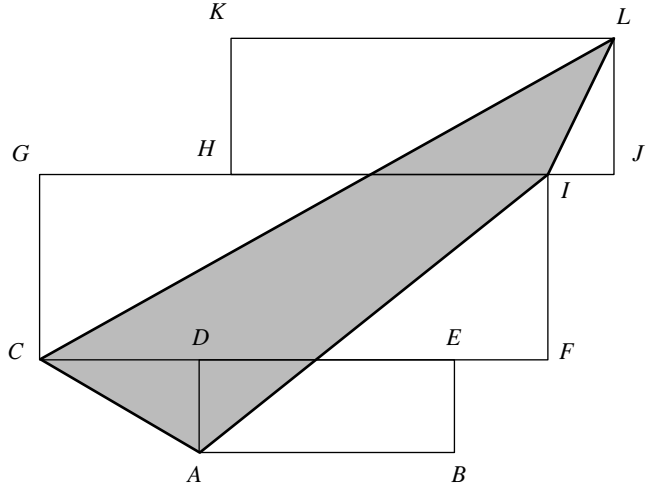
The MA and the *Gazette* comply fully with the provisions of the 2018 GDPR legislation. Submissions **must** be accompanied by the SPC permission form which is available on the Mathematical Association website

https://www.m-a.org.uk/the-mathematical-gazette

Note that if permission is not given, a pupil ***may still participate and will be eligible for a prize in the same way as others****.*

Problem 2018.5 **(Stan Dolan)**

The diagram shows three similar rectangles, *ABED*, *CFIG* and *HJLK*. Find, with proof, a trapezium with the same area as the shaded quadrilateral *AILC*.



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Problem 2018.6 **(Paul Stephenson)**

Consider the two infinite series

$$1+\frac{1}{3}+\frac{1}{6}+\ \dots\ +\frac{2}{n(n+1)}+\ \dots \qquad \text{(Reciprocals of the triangle numbers)}$$

$$1+\frac{1}{2}+\frac{1}{4}+\ \dots\ +\frac{1}{2^{n-1}}+\ \dots \qquad \text{(Reciprocals of powers of 2)}$$

Precisely when is a sum of successive terms of the first series equal to the sum of successive terms of the second series?

An example of two such sums is

$$\frac{1}{3}+\frac{1}{6}+\frac{1}{10}+\frac{1}{15}+\frac{1}{21}+\frac{1}{28}=\frac{1}{2}+\frac{1}{4}.$$

Solutions to 2018.3 and 2018.4

Both problems were solved by Sae Koyama (King's College London Mathematics School) and Andrew Ng (Tonbridge School).

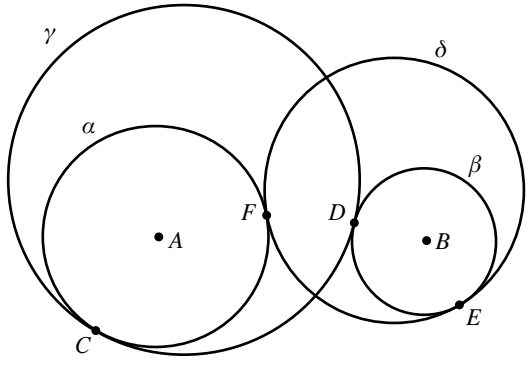
Problem 2018.3 **(Geoff Strickland)**

Two circles, α and β, have centres at A and B respectively.

Circle γ contains circle α and touches it at C. Circle γ also touches circle β (externally) at D.

Circle δ contains circle β and touches it at E. Circle δ also touches circle α (externally) at F.

Prove that AB, CD and EF are concurrent.

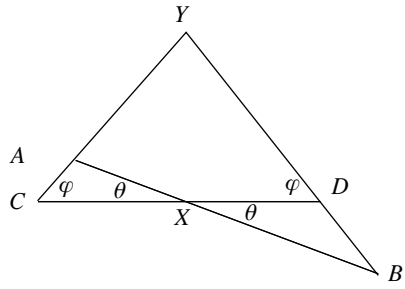


Solution

The following solution is based upon Sae's idea of using the Sine Rule.

Let AB intersect CD at X and EF at X'. Let CA and BD intersect at Y, the centre of circle γ.

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By the Sine Rule

$$\frac{AX}{AC} = \frac{\sin\varphi}{\sin\theta} = \frac{XB}{BD}$$
$$\Rightarrow \frac{AX}{XB} = \frac{AC}{BD}.$$

Similarly

$$\frac{AX'}{X'B} = \frac{AF}{BE} = \frac{AC}{BD}.$$

Therefore X and X' are the same point.

Problem 2018.4 **(Nick Lord)**

For positive real numbers x and y find the minimum value of

$$\frac{(1+x)(x+y)(y+8)}{xy}.$$

Solution

Sae and Andrew both gave a solution using calculus but this can be avoided by using the arithmetic-geometric mean inequality.

$$\frac{(1+x)(x+y)(y+8)}{xy} = (1+x)\left(1+\frac{y}{x}\right)\left(1+\frac{8}{y}\right).$$

Let $x = u, \frac{y}{x} = v, \frac{8}{y} = w$. Then we must maximise

$$(1+u)(1+v)(1+w) = 1 + \Sigma u + \Sigma uv + uvw$$

where u, v and w are positive numbers such that $uvw = 8$.

Using the AM-GM inequality twice, the maximum is $1 + 6 + 12 + 8 = 27$ and is attained for $u = v = w = 2$, i.e. when $1, x, y, 8$ form a geometric progression.

The first prize of £25 is awarded to Sae Koyama and the second prize of £20 is awarded to Andrew Ng.

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Note from the Editor

See *www.m-a.org.uk/the-mathematical-gazette* for the SPC questions.

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