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REDUCING COMPOSITIONAL TO DISQUOTATIONAL TRUTH

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Abstract. Disquotational theories of truth, that is, theories of truth based on the T-sentences or similar equivalences as axioms are often thought to be deductively weak. This view is correct if the truth predicate is allowed to apply only to sentences not containing the truth predicate. By taking a slightly more liberal approach toward the paradoxes, I obtain a disquotational theory of truth that is proof theoretically as strong as compositional theories such as the Kripke–Feferman theory, although it doesn't probe the compositional axioms.

§1. Disquotational theories of truth. There have been various proposals to axiomatize truth using T-sentences or similar equivalence statements. The T-sentences can be formulated in the language of arithmetic with a new unary predicate symbol T as

$$T\ulcorner\varphi\urcorner \leftrightarrow \varphi \tag{1}$$

where $\ulcorner\varphi\urcorner$ is the numeral of the Gödel code of the sentence φ. In an arithmetical context one can also formulate a slightly generalized variant of the T-sentences, the so-called uniform T-sentences, by considering the following equivalences for formulae $\varphi(x)$ containing at most the free variable x:

$$\forall x \left(T\ulcorner\varphi(\dot{x})\urcorner \leftrightarrow \varphi(x)\right). \qquad \text{(uniform } T\text{-sentences)}$$

Here $\ulcorner\varphi(\dot{x})\urcorner$ stands here for a function expression naturally representing the function that yields applied to a number n (the code of) the sentence $\varphi(\overline{n})$ with $\overline{n}$ the numeral of n.

In a uniform T-sentence the formula $\varphi(x)$ contains at most one free variable. It would be more natural to consider also T-sentences with more than one free variable, as a referee pointed out:

$$\forall x_1 \ldots \forall x_n \left(T\ulcorner\varphi(\dot{x_1} \ldots \dot{x_n})\urcorner \leftrightarrow \varphi(x_1 \ldots x_n)\right).$$

For the purposes of this paper, however, this would yield only a slightly stronger theory and the strengthening wouldn't affect the main results below.

Theories of truth based on the local or uniform T-sentences are said to capture a disquotational conception of truth. This is an abuse of terminology because the expression $\ulcorner\varphi\urcorner$ isn't the quotation of φ but rather the numeral of the code of the sentence φ under some fixed coding. In the setting here, however, $\ulcorner\varphi\urcorner$ can be seen as serving the purpose of the quotation of φ. Using an arithmetical framework avoids the need for a formal theory of quotation and expressions, although working in a framework with real quotation might be philosophically preferable.

Following Quine (1970), various deflationists like Field (1994) put a disquotational conception of truth at the centre of truth theoretic deflationism. Hence various deflationary

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accounts of truth rely on an axiomatization of truth by the *T*-sentences or kindred equivalences.

If the local or uniform *T*-sentences are added to a theory proving the Gödel diagonal lemma, some restriction on the instantiating sentences φ is needed to block the liar paradox. To state his axioms for truth, the deflationist needs a clear policy on which instances of the *T*-schema are admissible. Many authors discussing deflationism followed Tarski's (1935) lead and admitted only instantiating sentences φ or formulae $\varphi(x)$ without the truth predicate. Others rejected this proposal based on Tarski' object and metalanguage distinction as too restrictive. However, it has proved difficult to find a sensible restriction on instances of the *T*-schema that is more liberal than Tarski's and that avoids awkward consequences and that is not too artificial. Maximality approaches, as proposed by Horwich (1990), fail badly: McGee (1992) proved that there are many different maximal consistent sets of *T*-sentences and that they may have unwanted consequences. He showed that for every sentence consistent with the base theory to which the *T*-sentences are added, there is a maximal consistent set of *T*-sentences that has these sentences as a consequence. In fact, any sentence independent from the underlying base theory can be decided in either way by a suitable *T*-sentence. Hence postulating the elements of a maximal consistent set of *T*-sentences neither yields a specific theory nor need it yield a sound theory with respect to its consequences not containing the truth predicate. Moreover, Cieśliński (2007) showed that maximal *conservative* sets of *T*-sentences don't fare much better.

A motive for searching for sets of *T*-sentences that are more inclusive than Tarski's set of typed *T*-sentences is the deductive weakness of the typed *T*-sentences. In contrast to certain axiomatizations based on Tarski's inductive clauses for truth and other compositional theories of truth, the typed *T*-sentences—whether they are of the local or the uniform variety—do not prove any new sentences not containing the truth predicate when they are added to a reasonable base theory. That is, the typed *T*-sentences are conservative over Peano arithmetic or other theories of arithmetic (see Halbach, 2001, for a discussion). Even worse for some deflationist accounts, Tarski's typed *T*-sentences do not prove any nontrivial generalizations although deflationists often advance the view that truth is a device for expressing generalizations.[1]

In this paper I propose a specific set of *T*-sentences as axioms for truth. The set of *T*-sentences that are used as axioms is fairly simple but it hasn't been seriously considered so far as an alternative to Tarski's very restrictive proposal of banning all instances φ that contain the truth predicate.

I propose to consider as axioms all *T*-sentences $T\ulcorner\varphi\urcorner \leftrightarrow \varphi$ for all sentences φ that do not contain negated occurrences of the truth predicate T. More specifically, I'll look at the uniform *T*-sentences with the restriction to formulae $\varphi(x)$ that contain the truth predicate only *positively*. On this less restrictive approach one may hope to obtain a consistent axiomatization of truth that is deductively stronger than the set of Tarski's typed *T*-sentences and that compares in its strength with compositional theories of truth.

In this paper I will study the theory PUTB which arises from the addition of the uniform *T*-sentences in which Tarski's restriction to *T*-free sentences is liberalized to formulae

[1] See for instance Halbach (1999b, proposition 1). Here I do not enter the discussion on what it means to express generalizations. One might object that a good theory of truth may express generalizations without proving them. This claim, however, seems dubitable and not to be shared by many authors. For a proof theoretic analysis of the typed *T*-sentences also in their uniform variant see Halbach (1999a).

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with positive occurrences of the truth predicate only. I'll show that PUTB can define the truth predicate of the so-called Kripke–Feferman theory of truth, which is among the stronger theories of compositional truth. Nevertheless the truth predicate of PUTB is not compositional: it doesn't provably commute with conjunction, for instance, not even for T-free sentences. Finally I'll compare PUTB and KF in some aspects concerning their behavior toward the paradoxes. A philosophical evaluation of the formal results with respect to deflationism will be left to a future paper.

§2. Positive *T*-sentences. If λ is a liar sentence, that is, if $\lambda \leftrightarrow \neg T\ulcorner\lambda\urcorner$ is derivable in an arithmetical theory like Robinson's arithmetic Q, then the T-sentence $T\ulcorner\lambda\urcorner \leftrightarrow \lambda$ is inconsistent with that arithmetical theory.

Tarski offered an influential diagnosis of the paradox in terms of semantically closed languages. One main ingredient for generating the paradox is an instance of the T-schema with a sentence that contains the truth predicate. Clearly, the liar sentence λ does contain the truth predicate and I shall follow Tarski in rejecting the T-sentence $T\ulcorner\lambda\urcorner \leftrightarrow \lambda$ as a legitimate instance.

However, not all sentences containing the truth predicate need to be rejected as instances of the T-schema. Under a more detailed diagnosis of the liar paradox, one might be able to avoid Tarski's broad-spectrum remedy against the paradoxes and apply a more specific curative against the effects of the paradoxes. In the baneful instance λ the truth predicate appears in a *negated* position. One might suspect that only instantiating sentences φ that contain a negated occurrence of the truth predicate lead to an inconsistency. Of course one will have to exclude connectives that can be used to express negation.

I will show that this more refined diagnosis of the paradox is sound by proving that all uniform T-sentences $\forall x \left(T\ulcorner\varphi(\dot{x})\urcorner \leftrightarrow \varphi(x)\right)$ where $\varphi(x)$ is a formula without negated occurrences of the truth predicate are jointly consistent with an underlying theory of syntax.

For the formal proofs I first need to make my notation precise. The language $\mathscr{L}_{\mathrm{PA}}$ of arithmetic is assumed to contain the connectives $\neg$, $\wedge$, and $\vee$ only, and $\forall$ and $\exists$ as quantifiers. The language $\mathscr{L}_{\mathrm{T}}$ is $\mathscr{L}_{\mathrm{PA}}$ augmented by the unary predicate symbol T. I assume that some standard coding of all expressions of $\mathscr{L}_{\mathrm{T}}$ in the natural numbers is fixed. In what follows I will not distinguish between an expression of the language $\mathscr{L}_{\mathrm{T}}$ and its code.

DEFINITION 2.1. *A formula φ of $\mathscr{L}_{\mathrm{T}}$ is* T*-positive iff T doesn't occur in φ in the scope of an odd number of negation symbols.*

I propose now a restriction on the possible instances of the T-schema that is less stringent than Tarski's proposal:

DEFINITION 2.2 (positive uniform T-sentences). *The positive uniform T-sentences are all sentences of the form $\forall x \left(T\ulcorner\varphi(\dot{x})\urcorner \leftrightarrow \varphi(x)\right)$ where $\varphi(x)$ is a* T*-positive formula of $\mathscr{L}_{\mathrm{T}}$ with at most x free.*

The positive uniform T-sentences are added to an arithmetical base theory.

DEFINITION 2.3 (PUTB). *The axioms of* PUTB *are all positive uniform T-sentences and all axioms of* PA *including all instances of the induction schema in $\mathscr{L}_{\mathrm{T}}$.*

The acronym PUTB stands for *positive uniform Tarski-biconditionals*. Clearly all typed uniform T-sentences are also axioms of PUTB as their instantiating formulae don't contain any negated occurrences of the truth predicate and are therefore T-positive. Also a sentence $T\ulcorner T\ulcorner 0{=}0\urcorner\urcorner \leftrightarrow T\ulcorner 0{=}0\urcorner$ is an axiom of PUTB. Even a truth-teller sentence, although it's

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self-referential, is admitted as a legitimate instance as it doesn't contain a negated occurrence of the truth predicate.

As formulated in Definition 2.2, the instances $\varphi(x)$ of the *T*-sentences may contain only one free variable. In general, it's not possible to prove versions with more than one variable in PUTB. Quantification using more than one variable can't be replaced with quantification over a single sequence of objects and taking projections, as the truth predicate is not provably closed under substitution of equivalent terms. In the present framework, however, admitting instances with more than one free variable will not increase the proof theoretic strength of PUTB; and the main results below hold for the version with more than one free variable as well.

In nonarithmetical contexts such as set theory where not all objects have names, a satisfaction predicate can be employed to formulate axioms similar to the uniform *T*-sentences with more than one free variable. So the proposal can be applied to such languages and theories as well.

§3. Models for PUTB. If $\mathfrak{A}$ is a model of arithmetic and S is a subset of the domain of $\mathfrak{A}$, the pair $(\mathfrak{A}, S)$ is the expansion of $\mathfrak{A}$ to the language $\mathscr{L}_{\mathrm{T}}$ with S as the extension of the truth predicate. The symbol $\mathbb{N}$ denotes the standard model of arithmetic. Thus, if S a set of natural numbers, $(\mathbb{N}, S)$ is the model of $\mathscr{L}_{\mathrm{T}}$ that interprets the arithmetical vocabulary in the standard way and the truth predicate by the set S. All models are *classical* models. The set $\mathscr{L}_{\mathrm{T}}^{+}$ contains exactly all T-positive sentences of $\mathscr{L}_{\mathrm{T}}$.

DEFINITION 3.1. *For any set S natural numbers,* Γ *is defined as follows:*

$$\Gamma(S) = \{\varphi \in \mathscr{L}_{\mathrm{T}}^{+} : (\mathbb{N}, S) \models \varphi\}.$$

The proof of the following lemma relies on the fact that $\Gamma(S)$ contains only T-positive sentences and that connectives like $\rightarrow$ are not admitted.

LEMMA 3.2. *The operator* Γ *is monotone, that is,* $S_1 \subseteq S_2$ *implies* $\Gamma(S_1) \subseteq \Gamma(S_2)$.

The proof is by induction on the length of elements of $\Gamma(S_1)$.

The operator Γ bears some resemblance with Kripke's (1975) jump operator. The need for a nonclassical evaluation scheme is avoided as only T-positive sentences are considered.

The monotonicity of Γ implies the existence of fixed points.

LEMMA 3.3. *There are fixed points S of* Γ*, that is, there is a set S such that* $\Gamma(S) = S$ *obtains. The smallest fixed point* $\Gamma^{\Omega}(\emptyset)$ *is obtained by iterated applications of* Γ *to the empty set.*

THEOREM 3.4. *The model* $(\mathbb{N}, S)$ *is a model of* PUTB *if S is a fixed point of* Γ.

Proof. Assume $(\mathbb{N}, S) \models T\ulcorner\varphi\urcorner$ for a sentence φ of $\mathscr{L}_{\mathrm{T}}^{+}$. Then it follows that $\varphi \in S$ and, as S is a fixed point, also $\varphi \in \Gamma(S)$ and hence $(\mathbb{N}, S) \models \varphi$. □

COROLLARY 3.5. PUTB *is consistent.*

Theorem 3.4 may be strengthened to a characterization of all standard PUTB models.

THEOREM 3.6. *A model* $(\mathbb{N}, S')$ *is a model of* PUTB *if and only if the following condition is satisfied (where* $C(\mathscr{L}_{\mathrm{T}}^{+})$ *is the set of all natural numbers that are not* $\mathscr{L}_{\mathrm{T}}^{+}$ *sentences):*

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There is a set $A \subseteq C(\mathscr{L}_{\mathrm{T}}^{+})$ *and a fixed point* $S \subseteq \mathscr{L}_{\mathrm{T}}^{+}$ *of* Γ *such that* $S' = S \cup A$.

Thus extensions of the truth predicate in standard models of PUTB may be obtained by taking a fixed point of Γ, which will contain only sentences of $\mathscr{L}_{\mathrm{T}}^{+}$, and then adding arbitrary numbers that are not $\mathscr{L}_{\mathrm{T}}^{+}$-sentences.

Proof. If S is a fixed point of Γ and $A \subseteq C(\mathscr{L}_{\mathrm{T}}^{+})$, then $(\mathbb{N}, S \cup A)$ is a model of PUTB by Theorem 3.4 as numbers that are not codes of T-positive sentences do not matter for the verification of the axioms of PUTB.

For the other direction I need to show that $S'\backslash C(\mathscr{L}_{\mathrm{T}}^{+})$ is a fixed point of Γ if $(\mathbb{N}, S') \models$ PUTB. Assume to the contrary that $S = S'\backslash C(\mathscr{L}_{\mathrm{T}}^{+})$ isn't a fixed point. Then there must be a sentence $\varphi \in \mathscr{L}_{\mathrm{T}}^{+}$ such that either $\varphi \in S$ and $\varphi \notin \Gamma(S)$, or $\varphi \notin S$ and $\varphi \in \Gamma(S)$. In both cases it follows that $(\mathbb{N}, S') \not\models$ PUTB contrary to the assumption. □

§4. The Kripke–Feferman theory. In compositional axiomatizations of truth, the truth of a sentence is 'defined' in terms of the semantic values of the constituents of that sentence. An axiom stating that a conjunction of two sentences is true if and only if both conjuncts are true is a typical compositional axiom. Ever since Davidson work on truth, Tarski's inductive clauses for truth, turned into axioms, have been considered as a paradigm of a compositional theory of truth. For certain other theories of truth it's less clear whether they qualify as compositional. Here I don't try to specify precise criteria that allow one to classify axiomatic systems of truth as compositional or not. Under a suitable conception of compositionality, however, the so-called Kripke–Feferman theory of truth should qualify as compositional.

The Kripke–Feferman theory is a natural type-free extension of the classical typed theory of truth based on Tarski's clauses (see Halbach, forthcoming). If the goal is to reduce compositional to disquotational truth, I raise the yardstick for a successful reduction by classifying the Kripke–Feferman theory as compositional, as the Kripke–Feferman theory is much stronger than the usual typed compositional theory that results from converting Tarski's clauses into axioms.

In the literature various versions of KF can be found (Reinhardt, 1986; Cantini, 1989; McGee, 1991; Halbach & Horsten, 2006). Feferman, who was the first to formulate the theory, didn't call his theory KF but rather Ref(PA) in his 1991 paper (for more information on the history of KF see also Halbach, 2006).

Before formulating the axioms of KF, I need to introduce some abbreviations. The predicate expressions $\mathrm{Var}(x)$, $\mathrm{ClTerm}(x)$, $\mathrm{Sent}(x)$, and $\mathrm{For}(x, v)$ are natural representations of the sets of codes of all variables, closed terms, all sentences of $\mathscr{L}_{\mathrm{T}}$, and formulae with only the variable v free, respectively. The function $\mathrm{val}(x)$ takes (codes of) terms to what they denote; $\mathrm{val}(x)$ is never the code of an expression of $\mathscr{L}_{\mathrm{T}}$ if x isn't the code of a closed term. The function symbol $\dot{\neg}$ represents the function that sends the code of a formula to the code of its negation. Thus PA will prove $\dot{\neg}\ulcorner\varphi\urcorner = \ulcorner\neg\varphi\urcorner$ for all formulae of $\mathscr{L}_{\mathrm{T}}$. Moreover, I assume that $\mathrm{PA} \vdash \forall x(\mathrm{Sent}(\dot{\neg}x) \rightarrow \mathrm{Sent}(x))$. Analogous remarks apply to other underdotted symbols like $\dot{\wedge}$, $\dot{\vee}$, or $\dot{T}$. In particular, $\dot{\forall}$ represents the function that takes the code of a variable and of a formula and yields (the code of) the universal quantification of the formula by the variable. $x(\dot{y}/v)$ is a function expression for a function that takes the codes of a formula, a number, and a variable and gives the code of the formula where the numeral of y is substituted for the variable in the formula. Thus, for instance,

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I can express using this function expression as in KF8 below that a universally quantified sentence $\forall x \varphi(x)$ of $\mathscr{L}_{\mathrm{T}}$ is true if and only if all of its instances $\varphi(\overline{n})$ are true where $\overline{n}$ is the numeral of n.

To simplify the presentation, I assume that the language of Peano arithmetic contains function symbols for all these primitive recursive operations and the operations used for defining them. Of course, all required defining equations must be added as axioms to Peano arithmetic.

DEFINITION 4.1. *The Kripke–Feferman theory* KF *has all axioms of* PA *including all instances in* $\mathscr{L}_{\mathrm{T}}$ *of the induction scheme as its axioms. In addition, the following sentences are axioms:*

$$\forall x, y\big(\mathrm{ClTerm}(x) \wedge \mathrm{ClTerm}(y) \rightarrow (T\, x \doteq y \leftrightarrow \mathrm{val}(x) = \mathrm{val}(y))\big) \tag{KF1}$$

$$\forall x, y\big(\mathrm{ClTerm}(x) \wedge \mathrm{ClTerm}(y) \rightarrow (T\, \dot{\neg} x \doteq y \leftrightarrow \mathrm{val}(x) \neq \mathrm{val}(y))\big) \tag{KF2}$$

$$\forall x\big(\mathrm{Sent}(x) \rightarrow (T\, \dot{\neg}\dot{\neg} x \leftrightarrow T\, x)\big) \tag{KF3}$$

$$\forall x \forall y\big(\mathrm{Sent}(x) \wedge \mathrm{Sent}(y) \rightarrow (T(x \mathbin{\dot{\wedge}} y) \leftrightarrow T\, x \wedge T\, y)\big) \tag{KF4}$$

$$\forall x \forall y\big(\mathrm{Sent}(x) \wedge \mathrm{Sent}(y) \rightarrow (T\, \dot{\neg}(x \mathbin{\dot{\wedge}} y) \leftrightarrow T\, \dot{\neg} x \vee T\, \dot{\neg} y)\big) \tag{KF5}$$

$$\forall x \forall y\big(\mathrm{Sent}(x) \wedge \mathrm{Sent}(y) \rightarrow (T(x \mathbin{\dot{\vee}} y) \leftrightarrow T\, x \vee T\, y)\big) \tag{KF6}$$

$$\forall x \forall y\big(\mathrm{Sent}(x) \wedge \mathrm{Sent}(y) \rightarrow (T\, \dot{\neg}(x \vee y) \leftrightarrow T\, \dot{\neg} x \wedge T\, \dot{\neg} y)\big) \tag{KF7}$$

$$\forall v \forall x\big(\mathrm{Var}(v) \wedge \mathrm{For}(x, v) \rightarrow (T\, \dot{\forall} v x \leftrightarrow \forall y\, T\, x(\dot{y}/v))\big) \tag{KF8}$$

$$\forall v \forall x\big(\mathrm{Var}(v) \wedge \mathrm{For}(x, v) \rightarrow (T\, \dot{\neg}\dot{\forall} v x \leftrightarrow \exists y\, T\, \dot{\neg} x(\dot{y}/v))\big) \tag{KF9}$$

$$\forall v \forall x\big(\mathrm{Var}(v) \wedge \mathrm{For}(x, v) \rightarrow (T\, \dot{\exists} v x \leftrightarrow \exists y\, T\, x(\dot{y}/v))\big) \tag{KF10}$$

$$\forall v \forall x\big(\mathrm{Var}(v) \wedge \mathrm{For}(x, v) \rightarrow (T\, \dot{\neg}\dot{\exists} v x \leftrightarrow \forall y\, T\, \dot{\neg} x(\dot{y}/v))\big) \tag{KF11}$$

$$\forall x\big(\mathrm{ClTerm}(x) \rightarrow (T\, \dot{T} x \leftrightarrow T\, \mathrm{val}(x))\big) \tag{KF12}$$

$$\forall x\big(\mathrm{ClTerm}(x) \rightarrow (T\, \dot{\neg}\dot{T} x \leftrightarrow (T\, \dot{\neg}\, \mathrm{val}(x) \vee \neg\, \mathrm{Sent}(\mathrm{val}(x))))\big) \tag{KF13}$$

$$\forall x\big(T\, x \rightarrow \mathrm{Sent}(x)\big). \tag{KF14}$$

In many versions the consistency axiom $\forall x\big(\mathrm{Sent}(x) \rightarrow \neg(T\, x \wedge T\, \dot{\neg} x)\big)$ is added to the axioms of PA. It doesn't add anything to the proof theoretic strength of KF. More information on the consistency axiom and its effects have been provided by Cantini (1989) as well as Halbach & Horsten (2006).

In his 1991 paper Feferman provided the following proof theoretic analysis of KF:

THEOREM 4.2 (Feferman, 1991). *The theory* KF *is proof theoretically equivalent to ramified analysis up to* ϵ_0.

Consequently, KF is equivalent to ϵ_0-times iterated 'Tarskian' truth.

The positive uniform *T*-sentences are theorems of KF (see, for instance, Cantini, 1989, lemma 3.2 (ii)). Cantini's observation immediately implies the following lemma:

LEMMA 4.3. *The theory* PUTB *is a subtheory of* KF.

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§5. Defining the truth predicate of KF in PUTB. Cantini (1989, proposition 3.10) also showed that KF can handle positive inductive definitions via the positive uniform T-sentences. Cantini's proof can be applied to show that PUTB defines the truth predicate of KF.

THEOREM 5.1. *There is a formula $\zeta(x)$ in $\mathscr{L}_{\mathrm{T}}$ such that* PUTB *proves the following sentences:*

(i) $\forall x\,\forall y\,\big(\mathrm{ClTerm}(x)\wedge\mathrm{ClTerm}(y)\rightarrow(\zeta(x\doteq y)\leftrightarrow \mathrm{val}(x)=\mathrm{val}(y))\big)$

(ii) $\forall x\,\forall y\,\big(\mathrm{ClTerm}(x)\wedge\mathrm{ClTerm}(y)\rightarrow(\zeta(\dot\neg x\doteq y)\leftrightarrow \mathrm{val}(x)\neq\mathrm{val}(y))\big)$

(iii) $\forall x\,\big(\mathrm{Sent}(x)\rightarrow(\zeta(\dot\neg\dot\neg x)\leftrightarrow\zeta(x))\big)$

(iv) $\forall x\,\forall y\,\big(\mathrm{Sent}(x)\wedge\mathrm{Sent}(y)\rightarrow(\zeta(x\mathbin{\dot\wedge}y)\leftrightarrow\zeta(x)\wedge\zeta(y))\big)$

(v) $\forall x\,\forall y\,\big(\mathrm{Sent}(x)\wedge\mathrm{Sent}(y)\rightarrow(\zeta(\dot\neg(x\mathbin{\dot\wedge}y))\leftrightarrow\zeta(\dot\neg x)\vee\zeta(\dot\neg y))\big)$

(vi) $\forall x\,\forall y\,\big(\mathrm{Sent}(x)\wedge\mathrm{Sent}(y)\rightarrow(\zeta(x\mathbin{\dot\vee}y)\leftrightarrow\zeta(x)\vee\zeta(y))\big)$

(vii) $\forall x\,\forall y\,\big(\mathrm{Sent}(x)\wedge\mathrm{Sent}(y)\rightarrow(\zeta(\dot\neg(x\vee y))\leftrightarrow\zeta(\dot\neg x)\wedge\zeta(\dot\neg y))\big)$

(viii) $\forall v\,\forall x\,\big(\mathrm{Var}(v)\wedge\mathrm{For}(x,v)\rightarrow(\zeta(\dot\forall v x)\leftrightarrow\forall y\,\zeta(x(\dot y/v)))\big)$

(ix) $\forall v\,\forall x\,\big(\mathrm{Var}(v)\wedge\mathrm{For}(x,v)\rightarrow(\zeta(\dot\neg\dot\forall v x)\leftrightarrow\exists y\zeta(\dot\neg x(\dot y/v)))\big)$

(x) $\forall v\,\forall x\,\big(\mathrm{Var}(v)\wedge\mathrm{For}(x,v)\rightarrow(\zeta(\dot\exists v x)\leftrightarrow\exists y\,\zeta(x(\dot y/v)))\big)$

(xi) $\forall v\,\forall x\,\big(\mathrm{Var}(v)\wedge\mathrm{For}(x,v)\rightarrow(\zeta(\dot\neg\dot\exists v x)\leftrightarrow\forall y\,\zeta(\dot\neg x(\dot y/v)))\big)$

(xii) $\forall x\,\big(\mathrm{ClTerm}(x)\rightarrow(\zeta(\dot T x)\leftrightarrow\zeta(\mathrm{val}(x)))\big)$

(xiii) $\forall x\,\big(\mathrm{ClTerm}(x)\rightarrow(\zeta(\dot\neg\dot T x)\leftrightarrow(\zeta(\dot\neg\,\mathrm{val}(x))\vee\neg\,\mathrm{Sent}(\mathrm{val}(x)))))\big)$

Thus PUTB *proves all axioms of* KF *with the truth predicate T replaced by $\zeta(x)$.*

Thus the truth predicate of KF can be defined in PUTB.

Theorem 5.1 shows only that the disquotational theory PUTB has the conceptual resources for *defining* a compositional notion of truth; it doesn't show that the truth predicate of PUTB itself is compositional. However, Theorem 5.1 might be of some value to the deflationist as the theorem illustrates that disquotational truth is not bound to be conceptually weaker than compositional truth: any purpose with respect to T-free sentences that can be served by a compositional truth predicate of KF can be served by the disquotational truth predicate of PUTB.

Proof. As mentioned above, Cantini (1989) showed implicitly that PUTB can handle inductive definitions. I adapt his proof in order to show that PUTB allows one to define truth by a positive inductive definition in the style of Kripke (1975).

By the Diagonal theorem there is a formula $\zeta(x)$ such that $\zeta(x)$ is PA-provably equivalent to the following formula:

$$\exists y\,\exists z\,\big(\mathrm{ClTerm}(y)\wedge\mathrm{ClTerm}(z)\wedge x=(y\doteq z)\wedge\mathrm{val}(y)=\mathrm{val}(z)\big)\vee$$

$$\exists y\,\exists z\,\big(\mathrm{ClTerm}(y)\wedge\mathrm{ClTerm}(z)\wedge x=(\dot\neg y\doteq z)\wedge\mathrm{val}(y)\neq\mathrm{val}(z)\big)\vee$$

$$\exists y\,\big(\mathrm{Sent}(y)\wedge x=(\dot\neg\dot\neg y)\wedge T\ulcorner\zeta(\dot y)\urcorner\big)\vee$$

$$\exists y\,\exists z\,\big(\mathrm{Sent}(y)\wedge\mathrm{Sent}(z)\wedge x=(y\mathbin{\dot\wedge}z)\wedge T\ulcorner\zeta(\dot y)\urcorner\wedge T\ulcorner\zeta(\dot z)\urcorner\big)\vee$$

$$\exists y\,\exists z\,\big(\mathrm{Sent}(y)\wedge\mathrm{Sent}(z)\wedge x=(\dot\neg(y\mathbin{\dot\wedge}z))\wedge(T\ulcorner\zeta(\dot\neg\dot y)\urcorner\vee T\ulcorner\zeta(\dot\neg\dot z)\urcorner)\big)\vee$$

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$$\exists y\,\exists z\,\big(\mathrm{Sent}(y) \wedge \mathrm{Sent}(z) \wedge x = (y \dot{\vee} z) \wedge (T^{\ulcorner}\zeta(\dot{y})^{\urcorner} \vee T^{\ulcorner}\zeta(\dot{z})^{\urcorner})\big) \vee$$

$$\exists y\,\exists z\,\big(\mathrm{Sent}(y) \wedge \mathrm{Sent}(z) \wedge x = (\dot{\neg}(y \dot{\vee} z)) \wedge T^{\ulcorner}\zeta(\dot{\neg}\dot{y})^{\urcorner} \wedge T^{\ulcorner}\zeta(\dot{\neg}\dot{z})^{\urcorner}\big) \vee$$

$$\exists v\,\exists y\,\big(\mathrm{Var}(v) \wedge \mathrm{For}(y, v) \wedge x = (\dot{\forall} v\ y) \wedge \forall z\ T^{\ulcorner}\zeta(y(\dot{z}/v))^{\urcorner}\big) \vee$$

$$\exists v\,\exists y\,\big(\mathrm{Var}(v) \wedge \mathrm{For}(y, v) \wedge x = (\dot{\neg}\dot{\forall} v\ y) \wedge \exists z\ T^{\ulcorner}\zeta(\dot{\neg} y(\dot{z}/v))^{\urcorner}\big) \vee$$

$$\exists v\,\exists y\,\big(\mathrm{Var}(v) \wedge \mathrm{For}(y, v) \wedge x = (\dot{\exists} v\ y) \wedge \exists z\ T^{\ulcorner}\zeta(y(\dot{z}/v))^{\urcorner}\big) \vee$$

$$\exists v\,\exists y\,\big(\mathrm{Var}(v) \wedge \mathrm{For}(y, v) \wedge x = (\dot{\neg}\dot{\exists} v\ y) \wedge \forall z\ T^{\ulcorner}\zeta(\dot{\neg} y(\dot{z}/v))^{\urcorner}\big) \vee$$

$$\exists y\,\big(\mathrm{ClTerm}(y) \wedge x = (\dot{T} y) \wedge T^{\ulcorner}\zeta(\mathrm{val}(\dot{y}))^{\urcorner}\big) \vee$$

$$\exists y\,\big(\mathrm{ClTerm}(y) \wedge x = (\dot{\neg}\dot{T} y) \wedge (T^{\ulcorner}\zeta(\dot{\neg}\,\mathrm{val}(\dot{y}))^{\urcorner} \vee \neg\,\mathrm{Sent}(\mathrm{val}(y)))\big).$$

$\zeta(x)$ contains the truth predicate T only positively. Therefore

$$\forall x\ (T^{\ulcorner}\zeta(\dot{x})^{\urcorner} \leftrightarrow \zeta(x)) \tag{2}$$

is an axiom of PUTB.

As an example I show Claim (iv). Under the assumption $\mathrm{Sent}(x) \wedge \mathrm{Sent}(y)$ one reasons in PUTB as follows:

$$\zeta(x \dot{\wedge} y) \leftrightarrow T^{\ulcorner}\zeta(\dot{x})^{\urcorner} \wedge T^{\ulcorner}\zeta(\dot{y})^{\urcorner}$$

$$\leftrightarrow \zeta(x) \wedge \zeta(y)).$$

The first line follows from the definition of $\zeta(x)$, the second from (2).

The other Claims (i)–(xiii) can be proved in a similar way; even the quantifier cases require only the single-variable version of the positive uniform T-sentences. □

Lemma 4.3 and Theorem 5.1 establish the proof theoretic equivalence of PUTB and KF. More precisely, both theories have the same arithmetical content, PUTB is a subtheory of KF, and there is a relative interpretation of KF in PUTB that leaves arithmetical vocabulary unchanged and doesn't require a relativization of the quantifiers. The disquotational theory PUTB is thus as strong as one of the most powerful compositional theories of truth.

§6. The theory PUTB is not compositional. Even though in PUTB the truth predicate of KF can be defined, KF and PUTB are not the same theory. The disquotational theory PUTB is a *proper* subtheory of KF. There are various ways to demonstrate this claim.

Kentaro Fujimoto observed that PUTB doesn't prove KF13, that is, the axioms stating that a sentence $\neg T t$ is true, if and only if the value of the term t is a false sentence (a sentence hose negation is true) or not a sentence at all (private communication). This observation leaves open the possibility that PUTB proves axioms like KF4, that is, the axiom stating that truth commutes with conjunction.

I'll show that PUTB doesn't prove any of those compositional axioms. Thus the truth predicate of PUTB isn't compositional; it only *defines* a compositional truth predicate.

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Using the recursion theorem, I define a primitive recursive function s, which is represented in PA by the function symbol $\dot{s}$:

$$s(\varphi, n) = \begin{cases} \varphi & \text{if } n \text{ is an atomic sentence } \varphi \text{ of } \mathscr{L}_{\mathrm{PA}}, \\ T\,\dot{s}(t, \overline{n}) \wedge lh(t) \leq \overline{n} & \text{if } \varphi \text{ is } T\,t \text{ for some term } t, \\ \neg s(\psi) & \text{if } \varphi \text{ is } \neg\psi, \\ s(\psi) \wedge s(\psi) & \text{if } \varphi \text{ is } (\psi \wedge \chi), \\ s(\psi) \vee s(\psi) & \text{if } \varphi \text{ is } (\psi \vee \chi), \\ \forall x\ s(\psi) & \text{if } \varphi \text{ is } \forall x\ \psi, \\ \exists x\ s(\psi) & \text{if } \varphi \text{ is } \exists x\ \psi, \\ 0 & \text{else.} \end{cases}$$

The function symbol $lh(x)$ represents the function that yields, applied to a formula, its length (number of logical symbols, that is, connectives and quantifiers). The use of the recursion theorem for the definition of functions like $S(\varphi, n)$ is explained in more detail in Halbach (forthcoming).

LEMMA 6.1. *If φ a formula of $\mathscr{L}_{\mathrm{T}}$ and* PUTB $\vdash \varphi$ *then there is an n such that* PUTB $\vdash s(\varphi, n)$.

Proof. Assume that there is a proof of φ in PUTB. The proof contains only finitely many positive uniform T-sentences $\forall x \left(T\ulcorner\psi(\dot{x})\urcorner \leftrightarrow \psi(x)\right)$ as axioms. Let n be the maximal length of any such $\psi(x)$. The claim PUTB $\vdash s(\varphi, n)$ is then established by induction on the length of proofs.

If φ is an axiom $\forall x \left(T\ulcorner\psi(\dot{x})\urcorner \leftrightarrow \psi(x)\right)$ in the proof, $s(\varphi, n)$ is the following sentence:

$$\forall x \left((T\,\dot{s}\ulcorner\psi(\dot{x})\urcorner \wedge lh(\psi(x)) \leq \overline{n}) \leftrightarrow s(\psi(x), n)\right). \tag{3}$$

Since by the choice of n the length of $\psi(x)$ is at most n, $lh(\psi(x)) \leq \overline{n}$ is true and thus provable in PA. Since PA proves also $\dot{s}(\ulcorner\psi(x)\urcorner, \overline{n}) = \ulcorner s(\psi(x), n)\urcorner$, sentence (3) is provable in PUTB.

Since there are no other axioms and rules of PUTB specific to T, the translations of other axioms, and the induction step are easily proved. □

The lemma can be used to show that PUTB doesn't prove that truth commutes with connectives and quantifiers. In fact, it hardly proves any generalization concerning truth. I illustrate this with an example.

Define $h(n)$ as the conjunction $\overline{0} = \overline{0} \wedge \ldots \wedge \overline{0} = \overline{0}$ with n conjuncts. In PA the function h is represented by the function symbol $\dot{h}$.

EXAMPLE 6.2. PUTB $\nvdash \forall x\ T\,\dot{h}(x)$.

Proof. Assume to the contrary PUTB $\vdash \forall x\ T\,\dot{h}(x)$. From this assumption and Lemma 6.1 one can conclude that there is a number n so that PUTB $\vdash s(\forall x\ T\,\dot{h}(x), n)$ holds.

$$\text{PUTB} \vdash \forall x \left(T\,\dot{s}(\dot{h}(x), \overline{n})\ \wedge\ lh(\dot{h}(x)) \leq \overline{n}\right). \tag{4}$$

The claim $\forall x\, lh(\dot{h}(x)) = x$ is provable in PA, as the length of $h(k)$, that is, of the conjunction

$$\underbrace{\overline{0} = \overline{0} \wedge \ldots \wedge \overline{0} = \overline{0}}_{k}$$

is just k.

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Therefore (4) implies PUTB $\vdash \forall x\, x \leq \overline{n}$ and therefore also that PUTB is inconsistent, which contradicts Corollary 3.5. □

Using Lemma 6.1 one can also prove that compositional axioms like KF3–KF11 cannot be proved in PUTB, not even if the quantifiers in these sentences are restricted to sentences and formulae without the truth predicate.

§7. Symmetry. The Kripke–Feferman theory was criticized by various authors for the way the liar paradox is handled in the system and, more generally, for the discrepancy between the theorems of KF and what KF proves to be true. What KF proves cannot be brought in accordance with what it proves to be true; in this sense KF is highly asymmetric. In the presence of the consistency axiom, KF proves the liar sentence λ although it proves also that the liar sentence is not true. Even if the consistency axiom is dropped what is proved and what is proved to be true cannot be brought into agreement: KF cannot be consistently closed under the rules

$$\text{NEC}\ \frac{\varphi}{T\ulcorner\varphi\urcorner} \quad \text{and} \quad \frac{T\ulcorner\varphi\urcorner}{\varphi}\ \text{CONEC}$$

for all sentences φ of $\mathscr{L}_{\mathrm{T}}$, as was proved by Halbach & Horsten (2006, lemma 11). Thus there is not only a difference between what KF proves and what it proves to be true, rather it is not even possible to bring KF's 'internal' and its 'external' logic into agreement; KF is essentially asymmetric. In the following observation I sharpen a result from Halbach & Horsten (2006).

OBSERVATION 7.1. *If the following two axiom schemata, which are provable in* KF*, are added to a theory proving the diagonal lemma, then the resulting theory cannot be closed under* NEC *and* CONEC.

A1 $T\ulcorner\varphi \vee \psi\urcorner \rightarrow (T\ulcorner\varphi\urcorner \vee T\ulcorner\psi\urcorner)$ *for all sentences* φ *and* ψ *of* $\mathscr{L}_{\mathrm{T}}$.
A2 $T\ulcorner T\ulcorner\varphi\urcorner\urcorner \rightarrow T\ulcorner\varphi\urcorner$ *for all sentences* φ *of* $\mathscr{L}_{\mathrm{T}}$.

Proof. Reason in the theory in the following way:

$\lambda \leftrightarrow \neg T\ulcorner\lambda\urcorner$	diagonal lemma	(5)
$\lambda \vee \neg\lambda$	logic	(6)
$\lambda \vee T\ulcorner\lambda\urcorner$	from (5) and (6)	(7)
$T\ulcorner\lambda \vee T\ulcorner\lambda\urcorner\urcorner$	NEC	(8)
$T\ulcorner\lambda\urcorner \vee T\ulcorner T\ulcorner\lambda\urcorner\urcorner$	axiom schema 7.1	(9)
$\neg\lambda \vee T\ulcorner\lambda\urcorner$	axiom schema 7.1	(10)
$\lambda \rightarrow T\ulcorner\lambda\urcorner$	logic	(11)
$\neg\lambda$	from (5) and (11)	(12)
$T\ulcorner\lambda\urcorner$	from (5)	(13)
λ	CONEC	

□

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The theory PUTB is not closed under NEC. Using the standard model $\Gamma^{\Omega}(\emptyset)$, one can show PUTB $\nvdash T^{\ulcorner}\neg T^{\ulcorner}\overline{0} = \overline{0}^{\urcorner\urcorner}$; since PUTB $\vdash \neg T^{\ulcorner}\overline{0} = \overline{0}^{\urcorner}$ obtains, PUTB isn't closed under NEC.

One might still ask, however, whether PUTB, in contrast to KF, can be consistently closed under NEC and CONEC. In the following theorem I'll answer this question positively. So though PUTB is asymmetric in the sense that it is not closed under NEC, it is not essentially asymmetric, and thus it is in this respect superior to KF.

The idea for the proof is very simple: applications of NEC and CONEC to T-positive sentences can be replaced with uses of the positive uniform *T*-sentences. With respect to sentences that are not T-positive, the rules NEC and CONEC allow one only to write *T* in front of theorems using NEC and then to drop them again using CONEC, as the *T*-sentences cannot interfere with these applications of the rules. So adding NEC and CONEC will not yield new theorems that don't contain the truth predicate.

THEOREM 7.2. *The result of closing* PUTB *under* NEC *and* CONEC *is consistent. The theory is identical to* PUTB *closed under* NEC *only.*

Proof. Let a proof of a T-positive sentence in PUTB+NEC+CONEC be given. I assume that the proof has the form of a tree and that only sentences but not open formulae occur in the proof. Some axiomatic proof system would be suitable; if a Natural Deduction system is used NEC and CONEC can be applied only after all assumptions have been discharged.

Without any loss of generality I can assume that all applications to NEC and CONEC are to sentences that aren't T-positive as any application of NEC and CONEC to a T-positive sentence can be replaced by an appropriate use of a positive uniform *T*-sentence.

By induction on the number of applications of CONEC I'll prove first that all applications of CONEC can be eliminated from the proof.

Let P be the first subproof ending with an application $\frac{T^{\ulcorner}\varphi^{\urcorner}}{\varphi}$ of CONEC. I specify a model in which all sentences in P are satisfied. Let A be the set of all sentences ψ such that $\frac{\psi}{T^{\ulcorner}\psi^{\urcorner}}$ occurs in the subproof P. No sentence in A is T-positive by assumption.

By induction on the length of the proof P I show that

$$(\mathbb{N}, \Gamma^{\Omega}(\emptyset) \cup A) \models \chi$$

for all sentences χ in P.

The model $(\mathbb{N}, \Gamma^{\Omega}(\emptyset) \cup A)$, which I'll call $\mathfrak{A}$ for short, interprets the arithmetical vocabulary in the standard way and assigns $\Gamma^{\Omega}(\emptyset) \cup A$ as extension to T. The set $\Gamma^{\Omega}(\emptyset)$ is the minimal fixed point from Lemma 3.3.

Of course $\mathfrak{A}$ verifies all logical and arithmetical axioms and is closed under logic. The only axiom specific to truth are the positive uniform *T*-sentences. They are all satisfied in $\mathfrak{A}$ by Theorem 3.6.

By definition of $\mathfrak{A}$, any sentence $T^{\ulcorner}\psi^{\urcorner}$ that occurs as a conclusion in an application of NEC is true in $\mathfrak{A}$. Thus, I have established $\mathfrak{A} \models \chi$ for all sentences in P except for φ which is obtained as the last sentence in P from $T^{\ulcorner}\varphi^{\urcorner}$ by an application of CONEC. Since $\mathfrak{A} \models T^{\ulcorner}\varphi^{\urcorner}$ and φ isn't T-positive, φ must be in A; hence there must be a subproof P' of P ending with φ as conclusion. The proof P' doesn't contain an application of CONEC and the application of CONEC at the end of P is dispensable. It follows by induction that all applications of CONEC are dispensable.

Given a proof without an application of CONEC, a model $(\mathbb{N}, \Gamma^{\Omega}(\emptyset) \cup A)$ can be defined as above where A is the set of all sentences ψ such that an inference from ψ to $T^{\ulcorner}\psi^{\urcorner}$ occurs in the proof. Hence closing PUTB under NEC yields a consistent extension of PUTB. □

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§8. Conclusions. Many authors reject disquotational theories of truth because they fail to prove the compositional axioms of truth. Most famously Tarski (1935, p. 257) rejected an axiomatization of truth with the *T*-sentences as its only truth theoretic axioms for this reason. The theory PUTB suffers from the same defect, but at least it is not expressively weaker than the very strong compositional theory KF: the compositional truth predicate of KF is reducible to the disquotational truth predicate of KF.

In contrast to KF, the disquotational system PUTB is not essentially asymmetric: the internal and the external logic of PUTB can be brought into agreement by adding NEC and CONEC.

The results show that disquotational theories of truth need not be deductively weaker in their truth-free consequences than disquotational theories. Only because philosophers focused on typed theories of truth, disquotational theories appeared so awkwardly weak. Deflationists might not even be put off by the fact that PUTB doesn't prove the compositional axioms if they see truth as a mere instrument in proving and making generalization in the truth-free language. A more detailed philosophical evaluation of PUTB, however, will be left to another paper.

I'll conclude with an open problem. For the truth theoretic axioms of PUTB I have used *uniform T*-sentences. I think that uniform *T*-sentences are acceptable from a disquotational point of view, but if this claim is contested one might want to consider restricting the *T*-sentences of PUTB to local *T*-sentences $T\ulcorner\varphi\urcorner \leftrightarrow \varphi$ where φ is a T-positive sentence. I don't know, but I suspect that the resulting theory is conservative over PA.

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