J. Plasma Physics (2005), *vol.* 71, *part* 4, *pp.* 435–447. © 2005 Cambridge University Press
doi:10.1017/S002237780400323X Printed in the United Kingdom

Long-range correlations and percolation regimes in the system with drift flows

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(Received 7 May 2004, accepted 18 June 2004)

Abstract. Correlation effects play an important role in the description of turbulent transport. The present paper considers the influence of drift flow and time-dependence effects on the passive scalar behavior in the framework of the percolation approach. The renormalization method of a small parameter in continuum percolation models is reviewed. The modification is suggested of the renormalization condition of the small parameter of the percolation model in accordance with the additional external influences superimposed on the system. This approach makes it possible to consider simultaneously both parameters: the characteristic drift velocity U_{d} and the characteristic perturbation frequency ω. The effective diffusion coefficient $D_{\mathrm{eff}} \propto \omega^{1/7}$ satisfactorily describes the low-frequency region ω, where the long-range correlation effects play a significant role. The character of the dependence of D_{eff} on the drift flow amplitude U_{d} in different regimes is analyzed.

1. Introduction

The analysis of correlation effects plays an important role in the description of turbulent transport. In spite of considerable progress in this field of research [1–6], the problem is still awaiting a complete solution. One of the important directions of research is in obtaining scaling laws that characterize the turbulent diffusion of a passive scalar. In spite of the formal quasi-linear approach [1–4] permitting the expressing of the diffusivity through the velocity correlation function, the conventional diffusive equation cannot adequately describe anomalous transport mechanisms. However, in the framework of generalized probabilistic models (such as continuum time random walk (CTRW), Levy flight approach, etc.) it is possible to describe the essentially non-Markovian character of transport in terms of fractal differential equations and the Hurst exponent H [4–6]. Thus, the continuum time random walk approach allows a description of both superdiffusion ($H > 1/2$) and subdiffusion ($H < 1/2$) regimes on the basis of the special approximations of correlation functions. A variety of forms of turbulent transport require not only special description methods, but also an analysis of general mechanisms for different turbulence types. One such mechanism is percolation transport [5]. Its description is based on the idea of scaling representation of the correlation scale,

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borrowed from theory of phase transitions and critical phenomena [4–6]. It was suggested [7] that one could explain anomalous transport in two-dimensional cases in terms of the percolation threshold. The percolation model implies that there exists a percolation (fractal) streamline in the two-dimensional random flow under consideration, which embraces almost the whole plane. The convective transport of the passive scalar along this streamline defines the transport character in the system under analysis [5]. The percolation approach looks very attractive because it gives the simple and, at same time, universal model of behavior related to strong correlation effects. The percolation approach in fact gives the possibility of effectively realizing the scaling representation of correlation scale and obtaining dependences of transport coefficients on parameters characterizing common properties of a flow (velocity scale V_0, spatial scale λ, 'seed' diffusion D_0, etc.). Thus, in the framework of the percolation model, interesting results [5, 8, 9] were obtained that show the possibilities of the transition from the quasi-linear character (the Taylor formula) of dependence of the effective diffusion coefficient on the velocity scale

$$D_{\text{eff}} = \int_0^\infty \langle V(0)V(t)\rangle \, dt \approx V_0^2 \tau \approx \frac{V_0^2}{\omega} \propto V_0^2, \tag{1}$$

to the regime with the unconventional kind of dependence

$$D_{\text{eff}} \approx \lambda V_0 \left(\frac{\lambda\omega}{V_0}\right)^{3/10} \approx \lambda V_0 \left(\frac{1}{Ku}\right)^{1/(\nu+2)} \propto V_0^{7/10}. \tag{2}$$

Here, V_0 is the characteristic velocity, τ is the correlation time, λ is the characteristic size of structures, ω is the characteristic frequency of perturbations, $Ku \approx V_0/\lambda\omega$ is the Kubo number, and $\nu = 4/3$ is the percolation exponent [4, 5]. Note that the result (2) agrees well with numerical simulations that correspond to the dependence of the effective diffusion coefficient on the perturbation amplitude with the interval of exponents 0,6–0,8 [10–12]. Such a behavior also describes the alteration of regimes with the increase of perturbation amplitude: the quasi-linear regime with $D_{\text{eff}} \propto V_0^2 \propto Ku^2$, then the linear dependence $D_{\text{eff}} \propto V_0 \propto Ku$, which corresponds to the convective cells model [2, 3], and the regime with a slower dependence on V_0 than the linear one [2–5]. The analysis of transport in terms of the Kubo number is very important because it allows the possibility of using the correlation theory. For example, the dimensionless number $R_{\text{m}} = b_0 L_\parallel/\Delta_\perp$ introduced in [7] to describe a stochastic magnetic field, is the direct analogy with the Kubo number. Here b_0 is the relative amplitude of perturbations of magnetic field, $L_\parallel$ is the longitudinal correlation length, and $\Delta_\perp$ is the transverse correlation size. Therefore the magnetic diffusion coefficient D_{m} has the analogous character of the dependence on perturbation amplitude b_0; but instead of the Kubo number, the magnetic Kubo number R_{m} is used. If $R_{\text{m}} \ll 1$ then we deal with the quasi-linear regime [1–4] $D_{\text{m}} \propto b_0^2$. When $Ku \leq 1$, then we have the Kadomtsev–Pogutse regime [7] with $D_{\text{m}} \propto b_0$. The percolation limit was considered in [5] on the basis of the time-dependent model (2) $D_{\text{m}} \propto b_0^{7/10}$.

The notion of the nature of the stochastic layer Δ corresponding to percolation (fractal) streamline is the foundation of percolation models. The simplest and, at the same time, most 'universal' model is the steady one. For the steady case with initial (background) diffusivity D_0, Isichenko et al. [8] obtained the percolation

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expression in the form

$$D_{\text{eff}} \approx \lambda V_0 \left(\frac{D_0}{\lambda V_0}\right)^{3/13} \approx \lambda V_0 \left(\frac{1}{Pe}\right)^{1/(\nu+3)} \propto V_0^{10/13}. \tag{3}$$

Here $Pe \approx \lambda V_0/D_0$ is the Peclet number. This result also differs significantly from the estimate of convective cells, $D_{\text{eff}} \propto V_0\Delta$ [2, 3]. The consideration of more complex situations (time-dependence, drift effects, etc.) is based on analyzing mechanisms responsible for the 'reorganization' of a stochastic layer.

The present paper considers the influence of drift flows and time-dependence effects on the turbulent transport in the framework of the percolation model. Yushmanov [13] suggested using the percolation approach to interpret experimental data characterizing neoclassical transport in a tokamak. The conventional Hamiltonian function has the form

$$H = H_0 + U_{\text{d}}(x\cos\theta + y\sin\theta). \tag{4}$$

Here, H_0 is the main fraction of the Hamiltonian function, U_{d} is the drift velocity, and θ is the poloidal angle. In the steady case, the percolation model agrees well with the experimental data. The complexity of the simultaneous incorporation of several factors often leads to the consideration of time dependence on the basis of only the quasi-linear expression. The percolation method suggested in [8, 9] is one of the important approaches that make it possible to analyze the long-range correlation effects that cannot be described in terms of the quasi-linear approach. In the present paper, we concentrate our attention on scaling arguments that play a very important role in obtaining estimates of transport effects. In the framework of the mean field theory, of course, many important problems [1–3, 6, 14] will be not considered. However, the aim of this paper is to establish the character of dependence of transport coefficient on parameters such as fluctuations amplitude, characteristic size, characteristic time, etc. The considered approach is especially effective for these purposes.

2. Percolation method and transport

A clear presentation of the fundamental ideas of the percolation theory and the fractal conception can be found in [4]. In the context of this paper, the streamlines of the two-dimensional random flow $\Psi = \Psi(x, y)$ are considered as the coastlines in a hilly landscape flooded by water. It is expected that there is a sharp transition from separated lakes on a boundless landscape to individual islands in an infinite ocean. The percolation theory requires the existence of at least one coastline of infinite length. This length is represented by the scaling, $L(\varepsilon) \propto 1/\varepsilon^{\beta}$. Here, β is the fractal exponent, and ε is a small dimensionless quantity characterizing the degree of deviation of the system from the critical state (the percolation threshold): $\varepsilon \approx h/\lambda V_0$, where h is the value of the streamline function $\Psi = \Psi(x, y)$ near the percolation threshold, λ is the characteristic scale, and V_0 is the characteristic velocity of the flow. The expression for $L(\varepsilon)$ corresponds exactly with the fractal representation of the curve length. From the formal standpoint [4–6] the length of the 'very intricate curve' (the fractal curve), $L(\delta)$, can be rewritten in the form, $L \approx \delta N(\delta)$, $N(\delta) \propto 1/\delta^{d_{\text{F}}}$. In this fractal approach the full length, L, is approximated by the small segments of the size, δ, $N(\delta)$ is the number of these segments, which are necessary for such an approximation, and d_{F} is the fractal dimensionality of

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the curve [4–6]. In the framework of the conventional representation, we have to use the value $d_{\mathrm{F}} = d = 1$. However, in this case the drawbacks of the conventional method of length measurement by the 'yardstick' (ruler) are conserved. Mandelbrot considered a problem of the measurement of a tortuous seacoast length in which the increase in accuracy of measurement (the decrease of the value δ) leads to the growth of the value $N(\delta)$ ($d_{\mathrm{F}} > 1$). From the formal standpoint, this approach yields, $L(\delta) \approx \delta N(\delta)|_{\delta \to 0} \to \infty$. This means that such a fractal line embraces almost the full plane.

To describe the effects related to the considerable increase of transport coefficients, it is not sufficient to consider only the fractal presentation of a streamline. Moreover, the fractal character of the trajectory sometimes leads to slower diffusion (subdiffusion). Therefore, it is necessary to introduce another important value. In the percolation theory the correlation length $a(\varepsilon)$ is the main magnitude characterizing spatial scales of the system, which is located near the percolation threshold $\varepsilon \to 0$:

$$a = \frac{\lambda}{|\varepsilon|^{\nu}}, \quad L(a) \approx \lambda \left(\frac{a}{\lambda}\right)^{D_b}. \tag{5}$$

Here, $\nu = 4/3$ and $D_h = 1 + (1/\nu)$ are the percolation exponents that are exactly calculated for the two-dimensional case [4, 5, 15], λ is the geometric characteristic scale, and $L(a)$ is the length of the percolation streamline, which is also expressed through the small parameter $\varepsilon \to 0$. Thus, the idea of long-range correlations was realized in the percolation approach [4, 5]. However, there is a problem, since the diffusion coefficient is directly related to the conventional expression for the correlation length: $D_{\mathrm{C}} \approx l_{\mathrm{COR}}^2/\tau$. Here, τ is the correlation time. In the case under consideration, estimates yield $l_{\mathrm{COR}} \approx a(\varepsilon)_{\varepsilon \to 0} \to \infty$. For this reason, perhaps, Kadomtsev and Pogutse based their consideration on 'diffusion renormalization' of the quasi-linear equations [7]. However, in this approach the percolation character of correlation effects was lost. It is not surprising that in the framework of classical diffusion equations we cannot use the percolation exponents ν and d_{F}.

To develop the percolation approach, it is necessary to take into account that the percolation cluster occupies only a small fraction of the space. Therefore, the value $D_{\mathrm{eff}}(\varepsilon) \approx D_{\mathrm{C}} P_{\infty}(\varepsilon)$ can be the estimate of the effective diffusion coefficient. Here, D_{C} is the diffusion coefficient that corresponds to transport on the percolation cluster; and the value $P_{\infty}(\varepsilon)$ defines the fraction of the space that is occupied by the percolation cluster. In the continuum percolation theory [4, 5, 15] use is made of the scaling representations for $P_{\infty}(\varepsilon)$ in the form

$$P_{\infty}(\varepsilon) \approx \frac{L(\varepsilon)\Delta(\varepsilon)}{a^2(\varepsilon)} \approx \frac{\lambda}{a(\varepsilon)}. \tag{6}$$

Here, Δ is the width of the stochastic layer. Now we can calculate the diffusion coefficient that is based on the estimate of the finite correlation length a

$$D_{\mathrm{eff}} \approx \frac{a^2(\varepsilon)}{\tau(\varepsilon)} P_{\infty}(\varepsilon) \approx \frac{a(\varepsilon)}{\tau(\varepsilon)} \lambda. \tag{7}$$

One can see that in percolation models of turbulent diffusion the key problem is to determine the small parameter ε and to express the correlation time τ through characteristic flow parameters. Thus, for the steady model [8] the small parameter

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is defined by

$$\varepsilon_* = \left(\frac{1}{Pe}\right)^{1/(\nu+3)}. \tag{8}$$

For the time-dependent perturbations [9] the small parameter is given by the expression

$$\varepsilon_* \approx \left(\frac{1}{Ku}\right)^{1/(\nu+2)}. \tag{9}$$

These calculations (based on the renormalization method) will be considered below.

3. Percolation and renormalization of the small parameter

The important aspect of the percolation approach is the method of obtaining the small parameter, ε_*, that characterizes the closeness of a system to a percolation threshold, since in order to solve real physical problems the condition $\varepsilon \to 0$ looks too abstract. In the percolation models of turbulent diffusion, the key problem is to determine a small parameter, ε_0, and to find an adequate renormalization condition for ε_*. Thus, the correlation length is one of the most important values describing transport. However, in the system of the finite size, L_0, we cannot consider the infinite value $a(\varepsilon \to 0) \to \infty$. Here, it is relevant to introduce a new small 'renormalization' parameter ε_* [4] as the value that provides the condition $a(\varepsilon_*) \approx L_0$. Simplest calculations yield:

$$\varepsilon_* \approx \left(\frac{\lambda}{L_0}\right)^{1/\nu}. \tag{10}$$

This result can be interpreted in the framework of percolation experiments with finite size samples. In these conditions, the percolation threshold arises when the value ε_* is slightly different from zero and disposes in some $\Delta\varepsilon$ interval. The estimate obtained for ε_* can be considered as the characteristic width of this diapason $\Delta\varepsilon \approx \varepsilon_*$. Actually we are dealing with a small parameter $\varepsilon_0 \approx \lambda/L_0 \ll 1$, which describes the real physical system with the characteristic scales L_0 and λ. Upon 'renormalization' we obtain the new percolation parameter $\Delta\varepsilon \approx \varepsilon_* \approx \varepsilon_0^{1/\nu}$. It is natural that the value $\Delta\varepsilon$ decreases if the system size L_0 increases.

Another typical example of the renormalization small parameter consists of the consideration of percolation in models of graded type [16]. The graded character of the model corresponds to the assumption that the system undergoes a small external influence, which does not in general destroy the percolation character of the system behavior, but it can essentially change its properties. First, we will consider this method from a formal point of view. Let us introduce a parameter ε_0 characterizing the smallness of influence. In contrast to the renormalization, which uses the dependence of a percolation parameter on a system size $\Delta\varepsilon \approx \Delta\varepsilon(L_0) \approx \varepsilon_*$, here, we will deal with the spatial dependence that is related to the graded character of the problem $\varepsilon \approx \varepsilon(x)$. From the dimensional consideration, we can obtain the expression that characterizes uncertainty of choice of the small parameter in these conditions $\varepsilon_* \approx \Delta\varepsilon \approx \varepsilon'(x)a(\varepsilon_*)$. Then simple calculations yield

$$a(\varepsilon_*) \approx \lambda \frac{1}{\varepsilon_*^v} \approx \frac{\lambda}{[\varepsilon'(x)a(\varepsilon_*)]^v}. \tag{11}$$

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After the dimensional estimate in the form $\varepsilon'(x) \approx \varepsilon_0/\lambda$, we obtain the Trugman renormalization condition for the correlation scale

$$a(\varepsilon_*) \approx \lambda \frac{1}{\varepsilon_*^v} \approx \frac{\lambda}{\varepsilon_0^{v/(1+v)}}. \tag{12}$$

Here the value

$$\varepsilon_* = \varepsilon_0^{1/(1+v)} \gg \varepsilon_0 \tag{13}$$

is the new small percolation parameter. Note that the direct use of the value ε_0 as a parameter in the percolation dependences is incorrect, since the value ε_0 characterizes the destructive influence of the superimposed perturbation and not the degree of departure of the system from the percolation threshold.

This method looks quite formal, but renormalization (13) was repeatedly used to obtain the information about the critical exponents that describe the hull of a percolation cluster, to analyze transport in a system with shear flows, and to consider models of multiscale percolation. In the framework of the graded percolation Yushmanov [13] considered a problem of influence of small drift velocity U_d on the fractal topology of streamlines

$$V = V_0 + U_\mathrm{d}, \quad U_\mathrm{d} \ll V_0. \tag{14}$$

The simplest way to alter the small parameter is to use the value $\varepsilon_* \approx \varepsilon_0 = U_\mathrm{d}/V_0$. However, in this approach the fractal character of percolation streamlines is completely lost. Yushmanov suggested the use of the following dimensional estimate for the drift velocity

$$U_\mathrm{d} = \frac{a(\varepsilon)}{\tau(\varepsilon)} P_\infty(a), \quad a(\varepsilon) \approx \lambda|\varepsilon|^{-\nu}, \quad \tau(\varepsilon) \approx \frac{L(\varepsilon)}{V_0}. \tag{15}$$

The expression suggested for the steady case, $P_\infty \approx \lambda/a$, was used for P_∞. Simple calculations allow one to obtain the parametric dependence for the renormalized small parameter ε_* on the flow parameters V_0 and U_d,

$$\varepsilon_* = \left(\frac{U_\mathrm{d}}{V_0}\right)^{1/(1+\nu)}, \quad \text{where } \nu = 4/3. \tag{16}$$

It is easy to see that this expression coincides completely with the Trugman result (13) and can be interpreted in terms of the streamline function Ψ,

$$U_\mathrm{d} \approx \frac{\Psi_1}{a(\varepsilon)} \approx \frac{\varepsilon \Psi_0}{a(\varepsilon)}. \tag{17}$$

where Ψ_0 is the characteristic value of the streamline function and Ψ_1 is the perturbation. Here we deal with the conditions: $\Psi_1 \ll \Psi_0 \approx \lambda V_0$ and $a(\varepsilon) \gg \lambda$.

4. Temporal scales and renormalization

To calculate the effective diffusion coefficient D_eff it is necessary to obtain the expression for the correlation time τ (7). Moreover, consideration of characteristic times allows one to find the equation for the percolation parameter, ε_*. For the purposes of this paper we need to treat the characteristic times balance that underlies the calculation method [8, 9]. Thus, assuming that in the steady case the particle motion time along the percolation streamline $\tau_\mathrm{B} \approx L/V_0$ has to be of the same order as the characteristic diffusion time $\tau_\mathrm{D} \approx \Delta^2/D_0$ of the particle

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escape from the percolation stochastic layer of width Δ, Ishichenko et al. [8] obtained the expression

$$\frac{\Delta^2(\varepsilon_*)}{D_0} = \frac{L(\varepsilon_*)}{V_0}. \tag{18}$$

In fact this representation was repeatedly used to consider transport in systems with convective cells [17–19]. Expression (18) is based upon the particle balance in a cell of size λ

$$D_0 \frac{n}{\Delta^2} \approx V_0 \frac{n}{\lambda}. \tag{19}$$

Here n is the passive scalar density. Consideration of the balance between convective transport in the narrow layer of width Δ and the diffusive flow of particles leaving the cell, leads to expression (18). In the case of percolation, the characteristic size λ is replaced by the length of the percolation streamline L.

To obtain the equation that characterizes the value ε_* it is necessary to introduce the dependence $\Delta = \Delta(\varepsilon_*)$. Ishichenko et al. [8] and Bakunin [20] suggested using the following simple definition:

$$\Delta(\varepsilon_*) = \lambda\varepsilon_*. \tag{20}$$

Simple calculations lead to (3) and the estimate for D_{eff} in the form, $D_{\text{eff}} \approx V_0\Delta(\varepsilon_*) \approx \lambda V_0\varepsilon_*$. By substituting (20) in (18) one obtains the new small parameter, ε_*, in the form (8) and the effective diffusivity in the form (3). It should be noted that the small parameter, ε_*, is not an infinitesimal quantity as in the classical percolation theory. Therefore the width of the percolation (stochastic) layer depends on the parameters of the model V_0, λ, and D_0.

The value ε_* has significant physical interpretation in terms of the stream function Ψ that describes the velocity field in the framework of the two-dimensional approach. The values of the amplitude of the stream function Ψ corresponding to the percolation stochastic layer Δ lie in the interval

$$\Delta\Psi \approx \lambda V_0\varepsilon_*. \tag{21}$$

Then the estimate of the characteristic diffusion time τ_{D} can be rewritten in the form

$$\tau_{\text{D}} \approx \frac{(\Delta\Psi)^2}{V_0^2 D_0} \approx \frac{(\Delta\Psi)^2}{D_\Psi}. \tag{22}$$

Here, D_Ψ characterizes the 'diffusion of streamlines'. Moreover, for two-dimensional incompressible flows the formulation of the problem in terms of the stream function is equivalent to the Hamilton formulation. This allows consideration of the equation for the small percolation parameter in the form

$$\frac{\Delta H(\varepsilon_*)^2}{D_{\text{H}}} \approx \frac{L(\varepsilon_*)}{V_0}. \tag{23}$$

Here ΔH is the diapason of the Hamiltonian alteration in the stochastic layer and D_{H} characterizes the corresponding diffusion process.

The considered estimate makes it possible to use the Trugman expression to analyze the force line distortion mechanism by drift flows

$$D_\Psi \approx \frac{(\Delta\Psi)^2}{\tau_{\text{COR}}} \approx \frac{U_{\text{d}}^2 a(\varepsilon_*)^2}{\tau_{\text{COR}}}. \tag{24}$$

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Here τ_{COR} is the correlation time. Now we deal with the new parameter U_{d} to treat transport effects in the framework of the percolation approach. The simplest estimate is the expression for the effective diffusion coefficient [5, 13]

$$D_{\mathrm{eff}} \approx V_0 \Delta(\varepsilon_*) \approx \lambda V_0 \varepsilon_* \approx \lambda V_0 \left(\frac{U_{\mathrm{d}}}{V_0}\right)^{1(1+\nu)} \propto U_{\mathrm{d}}^{3/7}. \tag{25}$$

Yushmanov [13] suggested using this result to interpret experimental results characterizing neoclassical transport in tokamaks. The experimental data considered in [13] are satisfactory described by the scaling $D_{\mathrm{eff}} \propto U_{\mathrm{d}}^{3/7}$ that corresponds to the Trugman model [16].

The next step is the incorporation of the time-dependence effects that play a significant role in the analysis of transport processes. Often the complexity of simultaneous incorporation of several factors leads to the consideration of time-dependence on the basis only the quasi-linear expression (1). The percolation method suggested in [8, 9, 21–25] is an important approach that makes it possible to analyze the long-range correlation effects, which cannot be described in the framework of the quasi-linear approach. In the following sections we will consider possible modifications of equations for the small parameter (22), (23), and (24) for time-dependent drift flows.

5. Non-quasi-linear effects and percolation

The analysis of the steady percolation model is based on the supposition of the presence of 'seed' diffusion, D_0, in the stochastic (percolation) layer, Δ [26]. New physical situations in which the stochastic layer nature is related to external influences (such as time-dependence, drift flows, etc.) could be analyzed by modifying the equation for the percolation small parameter that has already been studied. A typical example is the consideration of the influence of time-dependent effects on the effective transport. Thus, the quasi-linear estimate [1–4] can be used for flows with the characteristic frequency $\omega \approx 1/T_0$ of the form

$$D_{\mathrm{eff}} = \frac{1}{2}\frac{d\langle \Delta^2 \rangle}{dt} = \int_0^\infty C(t)\, dt \approx \frac{V_0^2}{\omega}. \tag{26}$$

Here, $C(t)$ is the autocorrelation function of velocity and $\langle \Delta^2 \rangle$ is the mean-square displacement. However, this approach does not mirror the physical essence of processes in the case of small frequencies (the low-frequency limit), when the particle path

$$l \approx V_0 T_0 \approx \frac{V_0}{\omega} \tag{27}$$

during the time T_0 can be essentially greater than the characteristic spatial scale λ. It is natural that in this case 'long' streamlines play an important role. To describe such models Gruzinov et al. [9] suggested using the percolation approach. For the simplest monoscale model [9] the equation for the percolation parameter ε_* was offered in the form

$$\tau_{\mathrm{B}} \approx \frac{L(\varepsilon_*)}{V_0} \approx \varepsilon_* T_0 = \frac{\varepsilon_*}{\omega}. \tag{28}$$

This implies that the ballistic motion time of the particle along the percolation streamline, which is approximately the lifetime of this streamline, is much less

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than the characteristic time T_0 (which is the time taken to change the entire flow pattern). At the same time, this corresponds to the correlation scale $a \approx \varepsilon_*^2 l \approx \varepsilon_*^2 (V_0/\omega) \ll l = (V_0/\omega)$. Upon the solution of this algebraic equation for ε_*, relationship (2) was found:

$$\varepsilon_* = \varepsilon_*(\lambda, V_0, \omega) = \left(\frac{1}{Ku}\right)^{1/(\nu+2)}, \tag{29}$$

which makes it possible to obtain the estimate of the effective diffusion coefficient (7) in accordance with the ideas developed in Ishichenko et al. [8] about the linear dependence of D_{eff} on the stochastic layer width Δ:

$$D_{\text{eff}} \approx V_0 \Delta(\varepsilon_*) \approx \lambda \varepsilon_* V_0 \propto \omega^{1/(\nu+2)}. \tag{30}$$

This expression differs significantly from the quasi-linear one. It is important to note that in the analyzed model of time-dependent perturbations [9, 20] the following condition was used:

$$\tau_{\text{B}} \approx \varepsilon_* T_0 < \tau_{\text{D}} \approx \frac{(\Delta\Psi)^2}{D_\Psi} < T_0. \tag{31}$$

Calculations yield the estimate of the characteristic diffusion time in which the frequency ω enters as a parameter

$$\tau_{\text{D}} \approx \frac{(\varepsilon_* \lambda V_0)^2}{V_0^2 D_0} \approx \frac{V_0^2}{D_0} \left(\frac{\lambda}{V_0}\right)^2 \left(\frac{\lambda\omega}{V_0}\right)^{2/(\nu+2)}. \tag{32}$$

The non-quasi-linear character of the dependence of the characteristic time on the perturbation frequency $\tau_{\text{D}} \propto \omega^{2/(\nu+2)}$ leads to considerable changes of the transport estimates. One of the important examples is the transport in a stochastic magnetic field that was considered in [23, 24]. Unfortunately, the value of seed diffusion D_0 that was used in (32) is too abstract. New physical situations, where time-dependent effects have an essential influence on the transport character, can be analyzed by way of more detailed consideration of mechanisms that are responsible for processes in the stochastic layer. In fact, it is necessary to examine the change of character of percolation transport under an external influence.

6. Transport in a system with drift flows and time-dependence effects

Yushmanov [13] considered the Hamiltonian function accounting for the simultaneous influence of drift flows and time-dependence effects:

$$H(r_\perp, t) = \frac{c\tilde{\varphi}(r_\perp, t) + v_\parallel(t) A(r_\perp, t)}{B_0} + U_{\text{d}}(x \cos\theta(t) + y \sin\theta(t)). \tag{33}$$

Here, $\tilde{\varphi}$ and A characterize the fluctuation amplitudes of electric and magnetic potentials, B_0 is the tore magnetic field, and $v_\parallel$ is the longitudinal velocity. As mentioned above, the simplest estimate for time-dependent effects is the quasi-linear expression (1). Yushmanov kept the Trugman result for the small percolation parameter

$$\varepsilon_* = \left(\frac{U_{\text{d}}}{V_0}\right)^{1/(1+\nu)} \tag{34}$$

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and transformed the expression for $D_{\rm eff}$ (7) to the quasi-linear form

$$D_{\rm eff} \approx P_\infty \frac{a^2}{\tau} \approx \frac{U_{\rm d}^2 \tau}{P_\infty} \approx U_{\rm d}^2 \tau \left(\frac{1}{\varepsilon_*}\right)^\nu. \tag{35}$$

The correlation estimate (15) of the drift velocity $U_{\rm d} \approx aP_\infty/\tau$ and the approximation $P_\infty \approx \lambda/a(\varepsilon_*)$ of the space fraction occupied by percolation streamlines (6) were used here. Applying the substitution of $\tau \approx 1/\omega$, we obtain the Yushmanov result [13, 27]:

$$D_{\rm eff} \approx \frac{U_{\rm d}^2}{\omega} \left(\frac{1}{\varepsilon_0}\right)^{\nu/(1+\nu)} \approx \frac{U_{\rm d}^2}{\omega} \left(\frac{V_0}{U_{\rm d}}\right)^{4/7} \propto U_{\rm d}^{10/7} V_0^{4/7} \frac{1}{\omega}. \tag{36}$$

Obviously, the substitution of $\tau \approx 1/\omega$ and the use of the small parameter (16) corresponding to the steady model is a fairly rough approximation. From the standpoint of the dependence of $D_{\rm eff}$ on the perturbation amplitude $U_{\rm d}$, this expression corresponds to the transfer from the quasi-linear regime with $D_{\rm eff} \propto U_{\rm d}^2$ to the linear one with $D_{\rm eff} \propto U_{\rm d}$. Moreover, the estimates under consideration for the conventional (without drift) time-dependent case [9] yield the expression for the characteristic time $\tau_{\rm D}$,

$$\tau_{\rm D} \approx \frac{(\Delta\Psi)^2}{V_0^2 D_0} \approx \varepsilon_*^2(\omega) \propto \omega^{6/10}, \tag{37}$$

which differs significantly from the quasi-linear result (1). Following the methods developed by Isichenko et al. [8], we can find the equation for the small parameter, ε_*, that incorporates the perturbation frequency ω. Consider the balance of characteristic times by using (24)

$$\frac{(\varepsilon_* \lambda V_0)^2}{D_\Psi} \approx \frac{(\Delta\Psi)^2}{D_\Psi} \approx \frac{L(\varepsilon)}{V_0}. \tag{38}$$

This is a fairly common representation that characterizes transport in the stochastic layer. Let us express the value D_Ψ in the dimensional form, which mirrors the certain character of external influences (drift and time-dependence)

$$D_\Psi \approx (\Delta\Psi)^2 \omega \approx U_{\rm d}^2 a(\varepsilon)^2 \omega. \tag{39}$$

Note that the expression for $\Delta\Psi$ is similar to the Trugman result (16). This is natural, since $a(\varepsilon)$ characterizes correlation properties of a system. However, here we do not use the Trugman small parameter, ε_* (16). The new value of the percolation parameter that characterizes the system when there is the simultaneous influence of both drift flows and time-dependence effects, will be obtained as a result of solution of the algebraic equation

$$\frac{(\varepsilon_* \lambda V_0)^2}{(U_{\rm d} a(\varepsilon_*))^2 \omega} \approx \frac{L(\varepsilon_*)}{V_0}. \tag{40}$$

In fact, we have renormalized the value $D_\Psi \approx V_0^2 D_0$ in (22) in accordance with the mechanism's distorting streamlines (the drift flow with characteristic velocity $U_{\rm d}$ and temporal fluctuations with the frequency ω). The new expression for the small parameter has the form

$$\varepsilon_* \approx \left(\frac{U_{\rm d}}{V_0}\right)^{2/(3(1+\nu))} \left(\frac{1}{Ku}\right)^{1/(3(\nu+1))} \propto U_{\rm d}^{2/7} V_0^{-3/7} \omega^{1/7}. \tag{41}$$

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Here we simultaneously use two dimensionless complexes: $\varepsilon_0 = U_d/V_0$ and the Kubo number $Ku = V_0/\lambda\omega$. The corresponding expression for the effective diffusion coefficient is given by

$$D_{\text{eff}} \approx V_0 \Delta(\varepsilon_*) \approx \left(\frac{U_d}{V_0}\right)^{2/(3(1+\nu))} \left(\frac{\lambda\omega}{V_0}\right)^{1/(3(\nu+1))} \propto U_d^{2/7} V_0^{4/7} \omega^{1/7}. \tag{42}$$

This result corresponds to the low-frequency limit, where the effective diffusion coefficient grows with ω, which differs significantly from the quasi-linear dependence $D_{\text{eff}} \propto 1/\omega$. This is the main fact in the framework of the percolation approach to the description of the low-frequency perturbations [9]. As mentioned in the introduction, such a form of dependence makes it possible to describe adequately the long-range correlation effects [4, 5]. In the case under consideration the Trugman regime with

$$D_{\text{eff}} \approx V_0 \lambda \left(\frac{U_d}{V_0}\right)^{1/(1+\nu)} \tag{43}$$

passes to the low-frequency regime, where

$$D_{\text{eff}} \propto U_d^{2/7} V_0^{4/7} \omega^{1/7} \tag{44}$$

in the region $\omega \approx U_d/\lambda$. Then in the region $\omega \approx U_d/\lambda (V_0/U_d)^{\nu/(\nu+1)}$ the character of the dependence is altered in accordance with [13]

$$D_{\text{eff}} \approx \frac{U_d^2}{\omega} \left(\frac{V_0}{U_d}\right)^{4/7}. \tag{45}$$

Calculations yield the estimate for the characteristic time:

$$\tau_D \approx \frac{(\Delta\Psi)^2}{D_\Psi} \approx \frac{\varepsilon_*(\omega)^2}{a(\varepsilon_*)^2 \omega} \approx \frac{1}{\omega^{11/21}}. \tag{46}$$

Yushmanov and Skoliakov [27] carried out detailed analysis of neoclassical transport on the basis of the percolation model and pointed out the absence of a correct result for the low-frequency case. Moreover they suggested an upper estimate for the dependence of the effective diffusion coefficient on frequency:

$$D_{\text{eff}} \propto \omega^{1/2}. \tag{47}$$

Indeed formula (42) suggested in the present paper satisfies this criterion.

The consideration of the dependence D_{eff} on the amplitude of drift flow velocity U_d also points to the correct character of change of regimes [5]. Thus, the quasi-linear regime [1–3] is described by the steeper dependence $D_{\text{eff}} \propto U_d^{10/7}$ than the low-frequency regime with $D_{\text{eff}} \propto U_d^{2/7}$. The dependence of D_{eff} on the characteristic scale of the velocity V_0 is the same as in all the regimes with drift flow.

In the framework of the percolation approach there is another good example. Isichenko [23, 24] considered the alteration of the transport regime depending on the value ω in the case of time-dependent perturbations of the magnetic field δB. In spite of that, the estimate considered in [23, 24] has a fairly rough (dimensional) character $\delta B(\omega, \delta t) \approx B_0 \omega \delta t$. This result agrees well with the model representation of transition for the value $Ku \approx 1$ from the low-frequency regime, where D_{eff} increases with ω, to the quasi-linear regime $D_{\text{eff}} \propto 1/\omega$.

https://doi.org/10.1017/S002237780400323X Published online by Cambridge University Press

7. Conclusions

In the present paper we have considered the influence of drift flow and time-dependence effects on the passive scalar behavior in the framework of the percolation approach. The renormalization method of a small parameter in continuum percolation models is reviewed. It is shown that the estimate $D_{\text{eff}} \approx U_{\text{d}}^{10/7}/\omega$ suggested in [13,27] is quasi-linear in character and it is based on using the Trugman results [16] for the steady case. Following the methods developed in [8,9] modifying the renormalization condition of the small parameter of the percolation model is suggested:

$$\tau_{\text{COR}} \approx \frac{(\Delta\Psi)^2}{D_\Psi} \approx \frac{L(\varepsilon)}{V_0}$$

in accordance with additional external influences superimposed on the system. This approach makes it possible to consider both parameters simultaneously: the characteristic drift velocity U_{d} and the characteristic perturbation frequency ω. However, in contrast to [13], the effective diffusion coefficient $D_{\text{eff}} \propto \omega^{1/7}$ adequately describes the low-frequency region ω, in which the long-range correlation effects play a significant role. The changes of regimes with the alterations of perturbations frequency, ω, were considered. The character of the dependence of D_{eff} on the drift flow amplitude U_{d} in different regimes is discussed.

Note that the influence of the small drift velocity is also important for the analysis of transport in systems with intrinsic trapping [28–30]. Thus, in recent papers [28–32] the possibilities of using model approximations of velocity field were discussed. It was shown that anisotropy significantly complicates an interpretation of the results due to the presence of numerous regimes. Moreover, the study of multiscale drift flows [3, 34] also shows a non-diffusion character of transport [5,21,22] and has to be related to the detailed consideration of correlation functions that considerably complicates the analysis.

Acknowledgements

The author thanks Professor V. Shafranov, Professor R. Balescu, G. Zaslavsky, Professor T. Schep and Dr H. de Blank for the useful discussions and support.

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