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On unstable electromagnetic radiation through nonlinear wave–particle interactions in presence of drift wave turbulence

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Abstract. A study on the generation of unstable electromagnetic wave through nonlinear wave–particle interactions in an inhomogeneous plasma has been presented. Drift wave turbulence, which is one of the common features of inhomogeneous plasma, is found to be strongly in phase relation with thermal particles. The plasma particles, accelerated by drift wave turbulence field, may transfer their energy to electromagnetic O-mode through a modulated field. This process has been described by Vlasov Maxwell system of equations. From the nonlinear dispersion relation for O-mode the growth rate has been estimated. It has been observed that growth of unstable O-mode may predict magnetospheric electromagnetic radiation phenomena.

1. Introduction

The radiation phenomena associated with plasma inhomogeneity are important in space as well as in laboratory plasmas. There are inhomogeneous regions in magnetospheric plasma where drift mode instabilities are driven by the free energy associated with electron density gradient drifts (Gary 1993). The extremely low frequency (ELF) radiations observed at the magnetopause are closely associated with plasma inhomogeneity features at the magnetopause (Kintner and Gurnett 1978).

The reported intense high frequency radiations at magnetopause with equilibrium density gradients are connected with nonlinear interactions involving drift wave turbulent fields (Andre et al. 2001). The low frequency drift wave fields are found to be strongly in phase relation with thermal plasma particles. Such a drift wave field may transfer their wave energy to a non-resonant wave through nonlinear wave–particle interaction process (Singh and Deka 2005).

The study on the role of drift wave turbulence associated with plasma inhomogeneity for the generation of various unstable radiations is an interesting issue for plasma physicists. Efforts had been made earlier to explain the observed auroral radiation phenomena on the basis of cyclotron maser instability, which included the effect of inhomogeneity along the geomagnetic field lines (Wu and Lee 1979; Queau et al. 1985; Benson et al. 1988). On the basis of two fluid model, Shukla et al. (1995) considered coupling of non-thermal inertial convective effects as well as drift acoustic modes in non-uniform plasmas. In their analysis, it has been shown that free energy stored on the sheared plasma flow can be coupled to inertial convective cells and coupled electron drift-ion acoustic modes. As a result, purely growing and oscillatory modes are excited.

A number of authors considered the generation of unstable O-mode in auroral region (Hayes and Melrose 1986; Gurnett 1974). Investigations on enhanced electromagnetic radiations in top ionospheric regions of earth have been carried out thoroughly and are characterized. Such radiation phenomena are later termed as Auroral Kilometric Radiation (AKR). Later, it has been confirmed that AKR is composed of X-mode and O-mode radiations (Mellot et al. 1984). Afterwards, much interests have been paid to identify the underlying mechanism of the generation of such unstable O-mode in the auroral region.

Among the mode–mode coupling processes in a turbulent plasma, plasma maser effect (Tsytovich et al. 1975; Nambu 1983, 1986; Vladimirov and Yu 2004) is identified as most effective mechanism for wave energy transfer from low frequency turbulent wave field to generate intense high frequency non-resonant mode. Plasma maser effect occurs when both resonant and non-resonant waves are present in the system. The resonant modes are those modes which are in phase relation with thermal particles; for these modes Cherenkov resonance condition $\omega - \mathbf{k}\cdot\mathbf{v} = 0$ is satisfied. The non-resonant modes are those modes for which both the linear and nonlinear conditions are not satisfied, i.e. $\Omega - \mathbf{K}\cdot\mathbf{v} \neq 0$ and $\Omega - \omega - (\mathbf{K} - \mathbf{k})\cdot\mathbf{v} \neq 0$. Here, ω and Ω are frequencies of the resonant and the non-resonant waves, respectively, and $\mathbf{k}$ and $\mathbf{K}$ are the corresponding wave numbers. The thermal particles which are strongly in

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phase relation with resonant mode turbulent field are accelerated. These accelerated particles transfer their energy to a nonlinear wave through a modulated wave. As a result, wave energy of low frequency resonant wave is transferred to high frequency non-resonant mode and unstable intense radiation phenomenon is possible at the expense of low frequency turbulent field. Plasma maser effect is especially important in strongly magnetized plasma where $\Omega_e > \omega_{pe}$ (Nambu et al. 1993).

A mechanical model for plasma maser effect has been proposed by Vladimirov et al. (2001). This model explains nonlinear wave–particle interaction process and energy transfer process among waves through plasma maser effect on the basis of Fermi's pinball model (Fermi 1949) for acceleration of cosmic ray to high energies.

The most attractive feature of plasma maser effect is that the interaction between resonant and non-resonant waves is possible even for waves having large frequency gap (Krivitsky et al. 1992). Moreover, it does not require any frequency matching conditions. Plasma maser effect always coexists with reverse absorption effect due to slow change in distribution function. In absence of any embedded magnetic field this process cancels out the reverse absorption effect (Nambu et al. 1993).

In this paper, we wish to study amplification of O-mode in presence of drift wave turbulence (associated with plasma inhomogeneity in magnetosphere) through plasma maser effect. Most of the earlier works on plasma maser effect were considered for the homogeneous plasma system (Nambu et al. 1987; Vladimirov 1994; Deka et al. 1997; Deka 1998). Plasma maser effect as an effective upconversion process has been considered by many authors, which also covers explanation of radiation phenomena in the magnetosphere. Since most of the turbulent wave energy in magnetosphere is available in the form of low frequency turbulent field, plasma maser effect as energy upconversion process is capable of explaining many intense radiation phenomena in space. As plasma inhomogeneity is a common feature of a real plasma, from consideration of physical process in inhomogeneous plasma, it is more important to consider plasma maser effect for non-uniform systems. Singh and Deka (2005) and Deka and Borgohain (2011) considered plasma maser effect in inhomogeneous plasmas. In both of these works, it has been shown that plasma density gradient plays an effective role in space in enhancing electrostatic radiation through wave–particle interaction process. In Deka and Borgohain (2011), the authors also considered plasma maser effect in the context of tokamak plasma environment for amplification of ion acoustic wave including stochastic properties of the waves.

Here, we consider low frequency drift wave turbulence which is supported by strong drift of plasma particles and plasma shear flow along the magnetic field. The interaction process of such drift wave turbulence with O-mode has been described by Vlasov Maxwell system

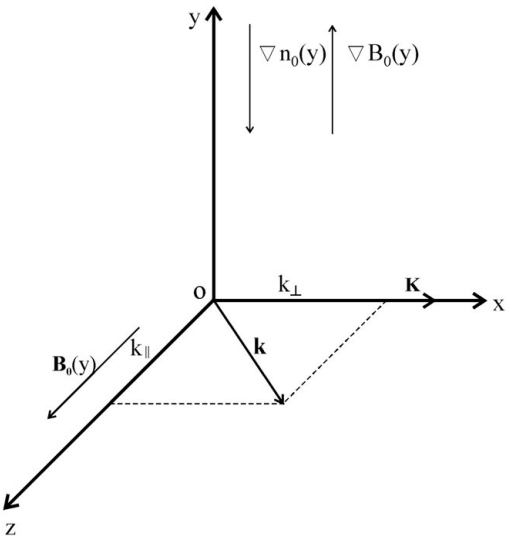


Figure 1. *Geometry of the model*; $\mathbf{k} = (k_\perp, 0, k_\parallel)$ is the propagating vector of drift wave, $\mathbf{K} = (K_\perp, 0, 0)$ is the propagating vector of O-mode, $\nabla n_0(y)$ is the density gradient along the negative y-direction, $\nabla B_0(y)$ is the magnetic gradient along the positive y-direction.

of equations. In the present paper, we have considered the same model for particle distribution function, based on local approximation for density gradient parameter, as it was considered in references Singh and Deka (2005) and Deka and Borgohain (2011). In these two investigations, the authors considered amplification of electrostatic waves in presence of drift wave turbulences whereas in the present case amplification of an electromagnetic radiation has been considered.

2. Formulation

We consider a magnetically confined plasma in presence of drift wave turbulence. The O-mode, present in the system, is considered as super imposing perturbation field to the system. The confining magnetic field with negligibly small gradient is taken along the z-direction $\mathbf{B}_0 = \mathbf{B}_0(y)$ and the system has spatial gradient in y-direction. The geometry of the model is shown in Fig. 1.

For such an inhomogeneous plasma (Stix 1992), the particle distribution function (Krall and Trivelpiece 1973) is considered as

$$f_{0e}\left(v_\perp, v_\parallel, \frac{v_x}{\Omega_e} - y\right) \simeq f_{0e}(v, y) + \left(\frac{v_x}{\Omega_e} - y\right)\frac{df_{0e}}{dy}, \tag{2.1}$$

where $f_{0e}(v, y)$ is the distribution function for the guiding center, $v_x = v_\perp \cos\theta$, $v_\parallel$ and $v_\perp$ are the components of velocity along and perpendicular to the direction of the external magnetic field, θ is the phase angle of the particle in the orbit, and $\Omega_e = eB_0/mc$ is the electron cyclotron frequency.

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We introduce the density gradient as

$$\epsilon' = \left[-\frac{1}{f_{0e}}\frac{df_{0e}}{dy}\right]_{y=0}. \tag{2.2}$$

Thus, the particle distribution function reduces to the form

$$f_{0e}\left(v_\perp, v_\parallel, \frac{v_x}{\Omega_e} - y\right) \simeq f_{0e}(v,y)\left\{1 + \epsilon'\left(y - \frac{v_x}{\Omega_e}\right)\right\}. \tag{2.3}$$

The interaction of high-frequency O-mode with low-frequency drift wave turbulence is governed by Vlasov-Maxwell's equations

$$\left[\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}} - \frac{e}{m}\left(\mathbf{E} + \frac{\mathbf{v}\times\mathbf{B}}{c}\right)\cdot\frac{\partial}{\partial \mathbf{v}}\right] \times F_{0e}(r,v,t) = 0, \tag{2.4}$$

$$\nabla\times\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \tag{2.5}$$

$$\nabla\times\mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c}\mathbf{J}, \tag{2.6}$$

$$\mathbf{J} = -en_e\int \mathbf{v}\, f_{0e}(\mathbf{r},\mathbf{v},t)\, d\mathbf{v}, \tag{2.7}$$

$$\nabla\cdot\mathbf{E} = 4\pi e n_e \int f_{0e}(\mathbf{r},\mathbf{v},t)\, d\mathbf{v}. \tag{2.8}$$

According to the linear response theory (Nambu 1976) of a turbulent plasma, the unperturbed particle distribution function and fields are

$$F_{0e} = f_{0e} + \epsilon f_{1e} + \epsilon^2 f_{2e}, \tag{2.9}$$

$$\mathbf{E}_{0e} = \epsilon\mathbf{E}_l + \epsilon^2\mathbf{E}_2, \tag{2.10}$$

where ϵ is a small parameter associated with the drift wave turbulent field $\mathbf{E}_l = (E_{l\perp}, 0, E_{l\parallel})$ with propagation vector $\mathbf{k} = (k_\perp, 0, k_\parallel)$, f_{0e} is the space and time average part, f_{1e}, f_{2e} are fluctuating parts of the distribution function, E_2 is the second order electric field.

To the order of ϵ, we have from (2.4)

$$\left[\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}} - \frac{e}{m}\left(\frac{\mathbf{v}\times\mathbf{B}}{c}\right)\cdot\frac{\partial}{\partial \mathbf{v}}\right] f_{1e} = \frac{e}{m}\left(\mathbf{E}_l\cdot\frac{\partial}{\partial \mathbf{v}} f_{0e}\right). \tag{2.11}$$

Using Fourier transforms and the method of characteristics (Krall and Trivelpiece 1973), we have from (2.11)

$$f_{1e}(\mathbf{k},\omega) = \frac{ie}{m}\left[\frac{m}{T_e k_\perp}E_{l\perp}\left\{1 - \left(k_\parallel v_\parallel - \omega - \frac{\epsilon' T_e k_\perp}{m\Omega_e}\right)S_{a,b}\right\} \times f_{0e} - E_{l\parallel}\frac{\partial f_{0e}}{\partial v_\parallel}S_{a,b}\right], \tag{2.12}$$

where

$$S_{a,b} = \sum_{a,b}\frac{J_a(\alpha)J_b(\alpha)\exp[i\{b-a\}\theta]}{a\Omega_e + k_\parallel v_\parallel - \omega + i0^+},\ \alpha = \frac{k_\perp v_\perp}{\Omega_e},$$

and $i0^+$ is a small imaginary part of ω.

Now, we perturb the quasisteady state by a high-frequency test O-mode field $\mu\delta\mathbf{E}$ with a propagating vector $\mathbf{K} = (K_\perp, 0, 0)$, electric field $\delta\mathbf{E} = (0, 0, \delta E_h)$, magnetic field $\delta\mathbf{B}_h = (0, \delta B_h, 0)$, and a frequency Ω. Thus the total perturbed electric field, magnetic field, and the particle distribution functions are

$$\delta\mathbf{E} = \mu\,\delta\mathbf{E}_h + \mu\epsilon\,\delta\mathbf{E}_{lh} + \mu\epsilon^2\,\Delta\mathbf{E}, \tag{2.13}$$

$$\delta\mathbf{B} = \mu\,\delta\mathbf{B}_h + \mu\epsilon\,\delta\mathbf{B}_{lh} + \mu\epsilon^2\,\Delta\mathbf{B}, \tag{2.14}$$

$$\delta f = \mu\,\delta f_h + \mu\epsilon\,\delta f_{lh} + \mu\epsilon^2\,\Delta f, \tag{2.15}$$

where $\delta\mathbf{E}_{lh}$, $\Delta\mathbf{E}$, $\delta\mathbf{B}_{lh}$, $\Delta\mathbf{B}$ are the modulation fields, δf_h is the fluctuating part due to a high-frequency O-mode, and δf_{lh}, Δf are particle distribution functions corresponding to modulating fields.

To the order of μ, $\mu\epsilon$, $\mu\epsilon^2$, we have from (2.4)

$$P\delta f_h = \frac{e}{m}\left(\delta\mathbf{E}_h + \frac{\mathbf{v}\times\delta\mathbf{B}_h}{c}\right)\cdot\frac{\partial f_{oe}}{\partial \mathbf{v}}, \tag{2.16}$$

$$P\delta f_{lh} = \frac{e}{m}E_l\cdot\frac{\partial}{\partial \mathbf{v}}\delta f_h + \frac{e}{m}\left(\delta\mathbf{E}_h + \frac{\mathbf{v}\times\delta\mathbf{B}_h}{c}\right)\cdot\frac{\partial f_{1e}}{\partial \mathbf{v}} + \frac{e}{m}\left(\delta\mathbf{E}_{lh} + \frac{\mathbf{v}\times\delta\mathbf{B}_{lh}}{c}\right)\cdot\frac{\partial f_{oe}}{\partial \mathbf{v}}, \tag{2.17}$$

$$P\Delta f = \frac{e}{m}E_l\cdot\frac{\partial}{\partial \mathbf{v}}\,\delta f_{lh} + \frac{e}{m}\left(\delta\mathbf{E}_{lh} + \frac{\mathbf{v}\times\delta\mathbf{B}_{lh}}{c}\right)\cdot\frac{\partial f_{1e}}{\partial \mathbf{v}}, \tag{2.18}$$

where the operator P is given by

$$\mathrm{P} \equiv \frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}} - \frac{e}{m}\left(\frac{\mathbf{v}\times\mathbf{B}}{c}\right)\cdot\frac{\partial}{\partial \mathbf{v}}.$$

Here, we have omitted the second order field quantities which can be justified under random phase approximations.

Using the Fourier transform and integrating along the unperturbed orbit (Gary 1993), we evaluate the various perturbed distribution functions δf_h, δf_{lh}, Δf from (2.16), (2.17), and (2.18) [see reference Singh and Deka (2005)].

Using Maxwell's equations for the modulated fields

$$\nabla\times\mathbf{B}_{lh} = \frac{1}{c}\frac{\partial}{\partial t}\delta\mathbf{E}_{lh} + \frac{4\pi}{c}\mathbf{J},$$

$$\mathbf{J} = -en_e\int \mathbf{v} f_{lh}\, d\mathbf{v},$$

in the form

$$\delta E_{lh} = \frac{4\pi i e n_e(\Omega-\omega)}{c^2|\mathbf{K}-\mathbf{k}|^2 - (\Omega-\omega)^2}\int v_\parallel f_{lh}\, d\mathbf{v},$$

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we get δE_{lh} as

$$\delta E_{lh} = \frac{4\pi i e n_e (\Omega-\omega)}{\mathrm{R}\{c^2|\mathbf{K}-\mathbf{k}|^2-(\Omega-\omega)^2\}}\left(\frac{e}{m}\right)^2 \delta E_h \times \int v_\parallel \left[E_{l\perp}\left[-\frac{K_\perp}{\Omega}\frac{\partial f_{0e}}{\partial v_\parallel}T_{p,q}Q_{s,t} + \left[\left(1-\frac{K_\perp v_\perp}{\Omega}\cos\theta\right)\frac{\partial f_{0e}}{\partial v_\parallel}Q_{s,t} - f_{0e}\left\{\frac{mv_\parallel}{\Omega T_e}+\left(\frac{mv_\parallel}{T_e}+\frac{\epsilon' K_\perp v_\parallel}{\Omega\Omega_e}\right)T_{p,q}\right\}\right] \times \left[\frac{m}{T_e(K_\perp-k_\perp)}\{1+(k_\parallel v_\parallel+\Omega-\omega)Q_{s,t}\} + \frac{\epsilon'}{\Omega_e}Q_{s,t}\right]\right] - E_{l\parallel}\frac{\partial}{\partial v_\parallel}\left[\left(1-\frac{K_\perp v_\perp}{\Omega}\cos\theta\right)\frac{\partial f_{0e}}{\partial v_\parallel}T_{p,q} - f_{0e}\left\{\frac{mv_\parallel}{\Omega T_e}+\left(\frac{mv_\parallel}{T_e}+\frac{\epsilon' K_\perp v_\parallel}{\Omega\Omega_e}\right)T_{p,q}\right\}\right]Q_{s,t} + \left(1-\frac{K_\perp v_\perp}{\Omega}\cos\theta\right)\frac{\partial}{\partial v_\parallel}\left[\frac{m}{T_e k_\perp}E_{l\perp} \times \left\{1-\left(k_\parallel v_\parallel-\omega-\frac{\epsilon' T_e k_\perp}{m\Omega_e}\right)S_{a,b}\right\}f_{0e} - E_{l\parallel}\frac{\partial f_{0e}}{\partial v_\parallel}S_{a,b}\right] - \frac{K_\perp v_\parallel}{\Omega}\left[\left[\frac{m}{T_e k_\perp}E_{l\perp} \times \left\{1-\left(k_\parallel v_\parallel-\omega-\frac{\epsilon' T_e k_\perp}{m\Omega_e}\right)S_{a,b}\right\}f_{0e}\right] \times \left[\frac{m}{T_e(K_\perp-k_\perp)}\{1+(k_\parallel v_\parallel+\Omega-\omega)Q_{s,t}\} + \frac{\epsilon'}{\Omega_e}Q_{s,t}\right] - E_{l\parallel}\frac{\partial f_{0e}}{\partial v_\parallel}S_{a,b}\left[\frac{m}{T_e(K_\perp-k_\perp)} \times \{1+(k_\parallel v_\parallel+\Omega-\omega)Q_{s,t}\}+\frac{\epsilon'}{\Omega_e}Q_{s,t}\right]\right], \tag{2.19}$$

where

$$\mathrm{R} = 1+\frac{4\pi e^2 n_e(\Omega-\omega)}{m\{c^2|\mathbf{K}-\mathbf{k}|^2-(\Omega-\omega)^2\}} \times \int v_\parallel \left[f_{0e}\left(\frac{k_\parallel}{|\mathbf{K}-\mathbf{k}|}+\frac{|\mathbf{K}-\mathbf{k}|}{\Omega-\omega}v_\parallel\right) \times \left[\frac{m}{T_e(K_\perp-k_\perp)}\{1+(k_\parallel v_\parallel+\Omega-\omega)Q_{s,t}\}+\frac{\epsilon'}{\Omega_e}Q_{s,t}\right] - \left(\frac{K_\perp-k_\perp}{|\mathbf{K}-\mathbf{k}|}-\frac{|\mathbf{K}-\mathbf{k}|}{\Omega-\omega}v_\perp\cos\theta\right)\frac{\partial f_{0e}}{\partial v_\parallel}Q_{s,t}\right]d\mathbf{v}, \tag{2.20}$$

$$T_{p,q} = \sum_{p,q}\frac{J_p(\alpha')J_q(\alpha')\exp[i\{q-p\}\theta]}{p\Omega_e-\Omega}, \quad \alpha' = \frac{K_\perp v_\perp}{\Omega_e},$$

$$Q_{s,t} = \sum_{s,t}\frac{J_s(\alpha'')J_t(\alpha'')\exp[i\{t-s\}\theta]}{s\Omega_e-(k_\parallel v_\parallel+\Omega-\omega)},$$

$$\alpha'' = \frac{(K_\perp-k_\perp)v_\perp}{\Omega_e}.$$

To derive the nonlinear dispersion relation for O-mode, we use Maxwell's equation, i.e.

$$\nabla\times\mathbf{B}_h = \frac{1}{c}\frac{\partial}{\partial t}\delta\mathbf{E}_h+\frac{4\pi}{c}\mathbf{J},$$

$$\mathbf{J} = -en_e\int v_\parallel(\delta f_h+\Delta f)\,d\mathbf{v},$$

in the form

$$\delta E_h = \frac{4\pi i e n_e(\Omega-\omega)}{c^2K_\perp^2-\Omega^2}\int v_\parallel(\delta f_h+\Delta f)\,d\mathbf{v}.$$

After simplification, this equation can be written as $\delta E_h\,\epsilon_h(\mathbf{K},\Omega)=0$, where $\epsilon_h(\mathbf{K},\Omega)$ is the nonlinear dispersion relation of O-mode defined by

$$\epsilon_h(\mathbf{K},\Omega) = \epsilon_0(\mathbf{K},\Omega)+\epsilon_d(\mathbf{K},\Omega)+\epsilon_p(\mathbf{K},\Omega), \tag{2.21}$$

where $\epsilon_0(\mathbf{K},\Omega)$ is the linear part, $\epsilon_d(\mathbf{K},\Omega)$ is the direct coupling part, and $\epsilon_p(\mathbf{K},\Omega)$ is the polarization coupling part. The expressions of, $\epsilon_0(\mathbf{K},\Omega)$, $\epsilon_d(\mathbf{K},\Omega)$ and $\epsilon_p(\mathbf{K},\Omega)$ are derived as

$$\epsilon_0(\mathbf{K},\Omega) = 1-\frac{\omega_{pe}^2\Omega}{c^2K_\perp^2-\Omega^2}\int v_\parallel \times \left[\left(1-\frac{K_\perp v_\perp}{\Omega}\cos\theta\right)\frac{\partial f_{0e}}{\partial v_\parallel}T_{p,q} - f_{0e}\left\{\frac{mv_\parallel}{\Omega T_e}+\left(\frac{mv_\parallel}{T_e}+\frac{\epsilon' K_\perp v_\parallel}{\Omega\Omega_e}\right)T_{p,q}\right\}\right]d\mathbf{v}, \tag{2.22}$$

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$$\epsilon_d(\mathbf{K},\Omega) = -\frac{\omega_{pe}^2\Omega}{c^2K_\perp^2-\Omega^2}\left(\frac{e}{m}\right)^2\int v_\parallel\left[E_{l\perp}\left[E_{l\perp}\frac{K_\perp}{\Omega}\frac{\partial f_{0e}}{\partial v_\parallel}\;T_{p,q}Q_{s,t}\left\{\frac{m}{K_\perp T_e}(1+\Omega T_{p,q})+\frac{\epsilon'}{\Omega_e}T_{p,q}\right\}\right.\right.$$

$$+E_{l\perp}\left[\frac{m}{T_e(K_\perp-k_\perp)}\{1+(k_\parallel v_\parallel+\Omega-\omega)Q_{s,t}\}+\frac{\epsilon'}{\Omega_e}Q_{s,t}\right]\times\left[\frac{K_\perp}{\Omega}\;\frac{\partial f_{0e}}{\partial v_\parallel}T_{p,q}^2\right.$$

$$-\left[f_{0e}\left\{\frac{mv_\parallel}{\Omega T_e}+\left(\frac{mv_\parallel}{T_e}+\frac{\epsilon' K_\perp v_\parallel}{\Omega\Omega_e}\right)T_{p,q}\right\}-\left(1-\frac{K_\perp v_\perp}{\Omega}\cos\theta\right)\frac{\partial f_{0e}}{\partial v_\parallel}T_{p,q}\right]$$

$$\left.\times\left\{\frac{m}{K_\perp T_e}(1+\Omega T_{p,q})+\frac{\epsilon'}{\Omega_e}T_{p,q}\right\}\right]+E_{l\parallel}Q_{s,t}\left[-\frac{K_\perp}{\Omega}\;T_{p,q}\frac{\partial}{\partial v_\parallel}\left(T_{p,q}\frac{\partial f_{0e}}{\partial v_\parallel}\right)\right.$$

$$+\left[\left(1-\frac{K_\perp v_\perp}{\Omega}\cos\theta\right)\frac{\partial}{\partial v_\parallel}\left(T_{p,q}\frac{\partial f_{0e}}{\partial v_\parallel}\right)-\frac{\partial}{\partial v_\parallel}\left[f_{0e}\left\{\frac{mv_\parallel}{\Omega T_e}\right.\right.\right.$$

$$\left.\left.\left.+\;\left(\frac{mv_\parallel}{T_e}+\frac{\epsilon' K_\perp v_\parallel}{\Omega\Omega_e}\right)T_{p,q}\right\}\right]\right]$$

$$\left.\times\left\{\frac{m}{K_\perp T_e}(1+\Omega T_{p,q})+\frac{\epsilon'}{\Omega_e}T_{p,q}\right\}\right]\right]+E_{l\parallel}T_{p,q}\frac{\partial}{\partial v_\parallel}\left[E_{l\perp}\left[\frac{K_\perp}{\Omega}\frac{\partial f_{0e}}{\partial v_\parallel}T_{p,q}Q_{s,t}\right.\right.$$

$$+\left[f_{0e}\left\{\frac{mv_\parallel}{\Omega T_e}+\left(\frac{mv_\parallel}{T_e}+\frac{\epsilon' K_\perp v_\parallel}{\Omega\Omega_e}\right)T_{p,q}\right\}-\left(1-\frac{K_\perp v_\perp}{\Omega}\cos\theta\right)\frac{\partial f_{0e}}{\partial v_\parallel}T_{p,q}\right]$$

$$\left.\left.\times\left[\frac{m}{T_e(K_\perp-k_\perp)}\{1+(k_\parallel v_\parallel+\Omega-\omega)Q_{s,t}\}+\frac{\epsilon'}{\Omega_e}Q_{s,t}\right]\right]$$

$$+E_{l\parallel}\frac{\partial}{\partial v_\parallel}\left[\left(1-\frac{K_\perp v_\perp}{\Omega}\cos\theta\right)\frac{\partial f_{0e}}{\partial v_\parallel}T_{p,q}-f_{0e}\left\{\frac{mv_\parallel}{\Omega T_e}+\left(\frac{mv_\parallel}{T_e}+\frac{\epsilon' K_\perp v_\parallel}{\Omega\Omega_e}\right)T_{p,q}\right\}\right]Q_{s,t}\Bigg]$$

$$+E_{l\perp}\left[\frac{K_\perp}{\Omega}T_{p,q}Q_{s,t}\;\frac{\partial}{\partial v_\parallel}\left[\frac{m}{T_e k_\perp}\;E_{l\perp}\left\{1-\left(k_\parallel v_\parallel-\omega-\frac{\epsilon' T_e k_\perp}{m\Omega_e}\right)S_{a,b}\right\}f_{0e}\right.\right.$$

$$\left.-E_{l\parallel}\frac{\partial f_{0e}}{\partial v_\parallel}S_{a,b}\right]+\left[\left(1-\frac{K_\perp v_\perp}{\Omega}\cos\theta\right)Q_{s,t}\;\frac{\partial}{\partial v_\parallel}\left[\frac{m}{T_e k_\perp}\;E_{l\perp}\{1-\right.\right.$$

$$\left.\left.\left(k_\parallel v_\parallel-\omega-\frac{\epsilon' T_e k_\perp}{m\Omega_e}\right)S_{a,b}\right\}f_{0e}-E_{l\parallel}\frac{\partial f_{0e}}{\partial v_\parallel}S_{a,b}\right]$$

$$-\frac{K_\perp v_\parallel}{\Omega}\left[\frac{m}{T_e(K_\perp-k_\perp)}\{1+(k_\parallel v_\parallel+\Omega-\omega)Q_{s,t}\}+\frac{\epsilon'}{\Omega_e}Q_{s,t}\right]$$

$$\left.\times\left[\frac{m}{T_e k_\perp}\;E_{l\perp}\left\{1-\left(k_\parallel v_\parallel-\omega-\frac{\epsilon' T_e k_\perp}{m\Omega_e}\right)S_{a,b}\right\}f_{0e}-E_{l\parallel}\frac{\partial f_{0e}}{\partial v_\parallel}S_{a,b}\right]\right]$$

$$\left.\times\left\{\frac{m}{K_\perp T_e}(1+\Omega T_{p,q})+\frac{\epsilon'}{\Omega_e}T_{p,q}\right\}f_{0e}\right]-E_{l\parallel}T_{p,q}\frac{\partial}{\partial v_\parallel}\left[\left(1-\frac{K_\perp v_\perp}{\Omega}\cos\theta\right)Q_{s,t}\right.$$

$$\times\frac{\partial}{\partial v_\parallel}\left[\frac{m}{T_e k_\perp}\;E_{l\perp}\left\{1-\left(k_\parallel v_\parallel-\omega-\frac{\epsilon' T_e k_\perp}{m\Omega_e}\right)S_{a,b}\right\}f_{0e}-E_{l\parallel}\frac{\partial f_{0e}}{\partial v_\parallel}S_{a,b}\right]$$

$$-\frac{K_\perp v_\parallel}{\Omega}\left[\frac{m}{T_e(K_\perp-k_\perp)}\{1+(k_\parallel v_\parallel+\Omega-\omega)Q_{s,t}\}\;+\frac{\epsilon'}{\Omega_e}Q_{s,t}\right]$$

$$\left.\left.\times\left[\frac{m}{T_e k_\perp}\;E_{l\perp}\left\{1-\left(k_\parallel v_\parallel-\omega-\frac{\epsilon' T_e k_\perp}{m\Omega_e}\right)S_{a,b}\right\}f_{0e}-E_{l\parallel}\frac{\partial f_{0e}}{\partial v_\parallel}S_{a,b}\right]\right]\right]d\mathbf{v}, \tag{2.23}$$

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$$\epsilon_p(\mathbf{K},\Omega) = -\frac{\omega_{pe}^4 \Omega\,(\Omega-\omega)}{\mathrm{R}\left(c^2K_\perp^2-\Omega^2\right)\left\{c^2\left|\mathbf{K}-\mathbf{k}\right|^2-(\Omega-\omega)^2\right\}}\left(\frac{e}{m}\right)^2(\mathrm{A}+\mathrm{B})(\mathrm{C}+\mathrm{D}), \tag{2.24}$$

where

$$\begin{aligned}\mathrm{A} = \int v_\parallel \Bigg[& E_{l\perp}\left\{\frac{m}{K_\perp T_e}\left(1+\Omega T_{p,q}\right)+\frac{\epsilon'}{\Omega_e}T_{p,q}\right\}\times\left[f_{0e}\left(\frac{k_\parallel}{|\mathbf{K}-\mathbf{k}|}+\frac{|\mathbf{K}-\mathbf{k}|}{\Omega-\omega}v_\parallel\right)\right.\\ & \times\left[\frac{m}{T_e\left(K_\perp-k_\perp\right)}\{1+\left(k_\parallel v_\parallel+\Omega-\omega\right)Q_{s,t}\}+\frac{\epsilon'}{\Omega_e}Q_{s,t}\right]\\ & -\frac{\partial f_{0e}}{\partial v_\parallel}Q_{s,t}\left(\frac{K_\perp-k_\perp}{|\mathbf{K}-\mathbf{k}|}-\frac{|\mathbf{K}-\mathbf{k}|}{\Omega-\omega}v_\perp\cos\theta\right)\Bigg]-T_{p,q}\left[E_{l\perp}\frac{\partial f_{0e}}{\partial v_\parallel}\frac{|\mathbf{K}-\mathbf{k}|}{\Omega-\omega}Q_{s,t}\right.\\ & +E_{l\parallel}\frac{\partial}{\partial v_\parallel}\left[f_{0e}\left(\frac{k_\parallel}{|\mathbf{K}-\mathbf{k}|}+\frac{|\mathbf{K}-\mathbf{k}|}{\Omega-\omega}v_\parallel\right)\right.\\ & \times\left[\frac{m}{T_e\left(K_\perp-k_\perp\right)}\{1+(k_\parallel v_\parallel+\Omega-\omega)Q_{s,t}\}\right.\\ & \left.\left.\left.+\frac{\epsilon'}{\Omega_e}Q_{s,t}\right]\right]-E_{l\parallel}\left(\frac{K_{\perp-}k_\perp}{|\mathbf{K}-\mathbf{k}|}-\frac{|\mathbf{K}-\mathbf{k}|}{\Omega-\omega}v_\perp\cos\theta\right)\frac{\partial}{\partial v_\parallel}\left(\frac{\partial f_{0e}}{\partial v_\parallel}Q_{s,t}\right)\Bigg]\Bigg]\,d\mathbf{v},\end{aligned} \tag{2.25}$$

$$\begin{aligned}\mathrm{B} = \int v_\parallel \Bigg[& \left(\frac{k_\parallel}{|\mathbf{K}-\mathbf{k}|}+\frac{|\mathbf{K}-\mathbf{k}|}{\Omega-\omega}v_\parallel\right)\left\{\frac{m}{K_\perp T_e}\left(1+\Omega T_{p,q}\right)+\frac{\epsilon'}{\Omega_e}T_{p,q}\right\}\\ & \times\left[\frac{m}{T_e k_\perp}E_{l\perp}\left\{1-\left(k_\parallel v_\parallel-\omega-\frac{\epsilon' T_e k_\perp}{m\Omega_e}\right)S_{a,b}\right\}f_{0e}-E_{l\parallel}\frac{\partial f_{0e}}{\partial v_\parallel}S_{a,b}\right]\\ & +T_{p,q}\left(\frac{K_\perp-k_\perp}{|\mathbf{K}-\mathbf{k}|}+\frac{|\mathbf{K}-\mathbf{k}|}{\Omega-\omega}v_\perp\cos\theta\right)\frac{\partial}{\partial v_\parallel}\left[\frac{m}{T_e k_\perp}E_{l\perp}\right.\\ & \left.\times\left\{1-\left(k_\parallel v_\parallel-\omega-\frac{\epsilon' T_e k_\perp}{m\Omega_e}\right)S_{a,b}\right\}f_{0e}-E_{l\parallel}\frac{\partial f_{0e}}{\partial v_\parallel}S_{a,b}\right]\Bigg]\,d\mathbf{v},\end{aligned} \tag{2.26}$$

$$\begin{aligned}\mathrm{C} = \int v_\parallel \Bigg[& E_{l\perp}\left[-\frac{K_\perp}{\Omega}\frac{\partial f_{0e}}{\partial v_\parallel}T_{p,q}Q_{s,t}+\left[\left(1-\frac{K_\perp v_\perp}{\Omega}\cos\theta\right)\frac{\partial f_{0e}}{\partial v_\parallel}T_{p,q}\right.\right.\\ & \left.\left.-f_{0e}\left\{\frac{mv_\parallel}{\Omega T_e}+\left(\frac{mv_\parallel}{T_e}+\frac{\epsilon' K_\perp v_\parallel}{\Omega\Omega_e}\right)T_{p,q}\right\}\right]\times\left[\frac{m}{T_e\left(K_\perp-k_\perp\right)}\right.\\ & \left.\times\left\{1+(k_\parallel v_\parallel+\Omega-\omega)Q_{s,t}\right\}+\frac{\epsilon'}{\Omega_e}Q_{s,t}\right]\Bigg]-E_{l\parallel}\frac{\partial}{\partial v_\parallel}\left[\left(1-\frac{K_\perp v_\perp}{\Omega}\cos\theta\right)\frac{\partial f_{0e}}{\partial v_\parallel}T_{p,q}\right.\\ & \left.-f_{0e}\left\{\frac{mv_\parallel}{\Omega T_e}+\left(\frac{mv_\parallel}{T_e}+\frac{\epsilon' K_\perp v_\parallel}{\Omega\Omega_e}\right)T_{p,q}\right\}\right]Q_{s,t}\Bigg]\,d\mathbf{v},\end{aligned} \tag{2.27}$$

$$\begin{aligned}\mathrm{D} = \int v_\parallel \Bigg[& \left(1-\frac{K_\perp v_\perp}{\Omega}\cos\theta\right)\frac{\partial}{\partial v_\parallel}\left[\frac{m}{T_e k_\perp}E_{l\perp}\left\{1-\left(k_\parallel v_\parallel-\omega-\frac{\epsilon' T_e k_\perp}{m\Omega_e}\right)S_{a,b}\right\}f_{0e}\right.\\ & \left.-E_{l\parallel}\frac{\partial f_{0e}}{\partial v_\parallel}S_{a,b}\right]Q_{s,t}-\frac{K_\perp v_\parallel}{\Omega}\left[\frac{m}{T_e k_\perp}E_{l\perp}\left\{1-\left(k_\parallel v_\parallel-\omega-\frac{\epsilon' T_e k_\perp}{m\Omega_e}\right)S_{a,b}\right\}f_{0e}\right.\\ & \left.-E_{l\parallel}\frac{\partial f_{0e}}{\partial v_\parallel}S_{a,b}\right]\times\left[\frac{m}{T_e\left(K_\perp-k_\perp\right)}\left\{1+\left(k_\parallel v_\parallel+\Omega-\omega\right)Q_{s,t}\right\}+\frac{\epsilon'}{\Omega_e}Q_{s,t}\right]\Bigg]\,d\mathbf{v}.\end{aligned} \tag{2.28}$$

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3. Plasma maser instability

The growth rate of O-mode is given by

$$\frac{\gamma_h}{\Omega} = -\left[\frac{\operatorname{Im}\epsilon_d(\mathbf{K},\Omega)+\operatorname{Im}\epsilon_p(\mathbf{K},\Omega)+(1/2)(\partial^2\epsilon_0)/(\partial\Omega\partial t)}{\Omega\partial\epsilon_0/\partial\Omega}\right]. \tag{3.1}$$

The third part in the numerator of the above formula is due to reverse absorption effect which in our case is

$$\begin{aligned}\frac{1}{2}\frac{\partial^2\epsilon_0}{\partial\Omega\partial t} = & -\frac{c^2K_\perp^2+\Omega^2}{\left(c^2K_\perp^2-\Omega^2\right)^2}\omega_{pe}^2\int v_\parallel \\ & \times\left[T_{p,q}\left(1-\frac{K_\perp v_\perp}{\Omega}\cos\theta\right)\frac{\partial}{\partial v_\parallel}\left(\frac{\partial f_{0e}}{\partial t}\right)\right. \\ & \left.-\frac{\partial f_{0e}}{\partial t}\left\{\frac{mv_\parallel}{\Omega T_e}+\left(\frac{mv_\parallel}{T_e}+\frac{\epsilon' K_\perp v_\parallel}{\Omega\Omega_e}\right)T_{p,q}\right\}\right]d\mathbf{v} \\ & -\frac{\omega_{pe}^2\Omega}{c^2K_\perp^2-\Omega^2} \\ & \times\int v_\parallel\left[\frac{\partial}{\partial v_\parallel}\left(\frac{\partial f_{0e}}{\partial t}\right)\frac{K_\perp v_\perp\cos\theta}{\Omega}T_{p,q}\right. \\ & \times\left(\frac{1}{\Omega}-\frac{1}{p\Omega_e-\Omega}\right)+\frac{\partial f_{0e}}{\partial t}\left\{\frac{mv_\parallel}{\Omega^2T_e}\right. \\ & \left.\left.+T_{p,q}\frac{\epsilon' K_\perp v_\parallel}{\Omega\Omega_e}-\left(\frac{mv_\parallel}{T_e}+\frac{\epsilon' K_\perp v_\parallel}{\Omega\Omega_e}\right)T_{p,q}\right\}\right]d\mathbf{v},\end{aligned} \tag{3.2}$$

where the slow time change of the electron distribution function is given by

$$\frac{\partial f_{0e}(v_\parallel)}{\partial t} = \pi\left(\frac{e}{m}\right)^2\frac{\partial}{\partial v_\parallel} \times\sum_{\mathbf{k},\omega}\left|E_{l\parallel}(\mathbf{k},\omega)\right|^2\delta(\omega-k_\parallel v_\parallel)\frac{\partial f_{0e}(v_\parallel)}{\partial v_\parallel}. \tag{3.3}$$

Putting this in (3.2) and then integrating partially, we find that the effect of reverse absorption, in our case, vanishes.

Here we have considered the plasma–maser interaction, the condition for which is $\omega = \mathbf{k}\cdot\mathbf{v}$, between O-mode and drift wave turbulence. Here, we consider two situations, first $\Omega < Kv_\parallel$ and second, the phase angle θ to be very very small, and with these assumptions we try to estimate the various parts of the dispersion relation of O-mode. In evaluation process, we consider the fact that for O-mode the most dominant contribution to Bessel's function comes from the terms $a = b = 0$ and $p = q = s = t = 1$.

From (2.22), we have

$$\epsilon_0(\mathbf{K},\Omega) = 1+\frac{\omega_{pe}^2}{c^2K_\perp^2-\Omega^2} \times\left\{1+\frac{1}{8}\frac{K_\perp V_e}{\Omega_e-\Omega}\left(\frac{K_\perp V_e}{\Omega_e}\right)^2\left(\frac{3\sqrt{\pi}}{2}+\frac{\epsilon'}{\Omega_e}v_e\right)\right\}. \tag{3.4}$$

Now, we calculate $\epsilon_p(\mathbf{K},\Omega)$ from (2.24). We observe that A and C are reals whereas B and D are complexes because of causality principle. Therefore,

$$\operatorname{Im}\epsilon_p = -\frac{\omega_{pe}^4\Omega(\Omega-\omega)}{\mathrm{R}\left(c^2K_\perp^2-\Omega^2\right)\left\{c^2|\mathbf{K}-\mathbf{k}|^2-(\Omega-\omega)^2\right\}} \times\left(\frac{e}{m}\right)^2(\mathrm{A}\times\operatorname{Im}D+\mathrm{C}\times\operatorname{Im}B). \tag{3.5}$$

From (2.26),

$$\begin{aligned}\operatorname{Im}B = & \frac{\sqrt{\pi}}{|k_\parallel|}\left[\frac{1}{4(\Omega_e-\omega)}\left(\frac{K_\perp v_e}{\Omega_e}\right)^2\right. \\ & \times\left(\frac{K_\perp-k_\perp}{|\mathbf{K}-\mathbf{k}|}+\frac{3\sqrt{\pi}}{4}\frac{|\mathbf{K}-\mathbf{k}|}{\Omega-\omega}v_e\right) \\ & -v_d\left\{\frac{2}{K_\perp v_e^2}+\frac{1}{4}\left(\frac{K_\perp v_e}{\Omega_e}\right)^2\right. \\ & \times\frac{1}{\Omega_e-\Omega}\left(\frac{2\Omega_e}{K_\perp v_e^2}+\frac{\epsilon'}{\Omega_e}\right) \\ & \left.\left.\times\left(\frac{k_\parallel}{|\mathbf{K}-\mathbf{k}|}+\frac{|\mathbf{K}-\mathbf{k}|}{\Omega-\omega}v_d\right)\right]\frac{1}{v_e}\right. \\ & \left.\times\left(E_{l\perp}\frac{\epsilon'}{\Omega_e}+2E_{l\parallel}\frac{v_d}{v_e^2}\right)\exp\left\{-\left(\frac{v_d}{v_e}\right)^2\right\},\end{aligned} \tag{3.6}$$

where $v_d = \omega/k_\parallel$ is the drift velocity.

From (2.28),

$$\begin{aligned}\operatorname{Im}D = & \frac{\sqrt{\pi}}{|k_\parallel|}\left[\frac{K_\perp}{\Omega}v_d^2\left[\frac{2}{v_e^2(K_\perp-k_\perp)}+\frac{1}{4}\left(\frac{K_\perp v_e}{\Omega_e}\right)^2\right.\right. \\ & \times\frac{1}{k_\parallel v_d+\Omega-\omega-\Omega_e} \\ & \left.\left.\times\left\{\frac{2}{v_e^2(K_\perp-k_\perp)}\left(k_\parallel v_d+\Omega-\omega\right)+\frac{\epsilon'}{\Omega_e}\right\}\right] \\ & -\frac{\Omega-\omega-\Omega_e}{4(k_\parallel v_d+\Omega-\omega-\Omega_e)^2}\left(\frac{K_\perp-k_\perp}{\Omega_e}v_e\right)^2\end{aligned}$$

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$$\times \left(1 - \frac{3\sqrt{\pi}}{4}\frac{K_\perp v_e}{\Omega_e}\right)\Bigg] \frac{1}{v_e}$$

$$\times \left(E_{l\perp}\frac{\epsilon'}{\Omega_e} + 2E_{l\|}\frac{v_d}{v_e^2}\right) \exp\left\{-\left(\frac{v_d}{v_e}\right)^2\right\}. \quad (3.7)$$

We expand R from (2.20) about the small argument $\mathbf{k}$ and ω and using the relation $\epsilon_0(\mathbf{K}, \Omega) = 0$, we obtain

$$\frac{1}{\mathrm{R}\{c^2|\mathbf{K}-\mathbf{k}|^2 - (\Omega-\omega)^2\}} \simeq \frac{1}{c^2 k_\|^2}, \quad (3.8)$$

as lowest order approximation.

Using (2.22), (3.6), (3.7), and (3.8) and expressions for A and C (after completing integration from (2.25) and (2.27), respectively) in (3.1) [in view of (3.5)], the growth rate of O-mode due to the polarization coupling term, retaining the dominant terms only, is calculated as

$$\frac{\gamma_p}{\Omega} = \frac{\pi}{2}\left(\frac{\omega_{pe}}{\Omega_e}\right)^2 \frac{c^2 k_\|^2}{c^2 K_\perp^2 - \Omega^2}\frac{v_d}{v_e}\frac{1}{\Omega_e |k_\| | v_e^2}\left(\frac{e}{m}\right)^2 E_{l\|}$$

$$\times \left(E_{l\perp}\frac{\epsilon'}{\Omega_e} + 2E_{l\|}\frac{v_d}{v_e^2}\right) \exp\left\{-\left(\frac{v_d}{v_e}\right)^2\right\}. \quad (3.9)$$

From the considerations of $|\mathbf{K}| > |\mathbf{k}|$ and $\Omega > \omega$, we have identified the dominant terms in the above deductions.

Also integrating from (2.23) and taking the dominant terms only, we estimate the growth rate from direct coupling term [in view of (3.1)] as

$$\frac{\gamma_d}{\Omega} = \frac{\sqrt{\pi}}{16}\frac{K_\perp}{|k_\||}\frac{c^2K_\perp^2 - \Omega^2}{\Omega^2}\left(\frac{K_\perp v_e}{\Omega_e}\right)^3 \frac{K_\perp}{\Omega_e^3}\left(\frac{e}{m}\right)^2 E_{l\perp}$$

$$\times \left(E_{l\perp}\frac{\epsilon'}{\Omega_e} + 2E_{l\|}\frac{v_d}{v_e^2}\right) \exp\left\{-\left(\frac{v_d}{v_e}\right)^2\right\}. \quad (3.10)$$

It has been observed that in an inhomogeneous plasma drift wave turbulence is a common feature and its turbulence wave energy is mostly available in the perpendicular direction of the confining field. It is, therefore, justified to take $|E_{l\perp}| \gg |E_{l\|}|$ and we have from (3.9)

$$\frac{\gamma_p}{\Omega} = \frac{\pi}{2}\left(\frac{\omega_{pe}}{\Omega_e}\right)^2 \frac{c^2 k_\|^2}{c^2 K_\perp^2 - \Omega^2}\frac{v_d}{v_e}\frac{k_\perp \epsilon'}{k_\| |k_\||}\frac{E_{l\|}^2}{\Omega_e^2 v_e^2}$$

$$\times \left(\frac{e}{m}\right)^2 \exp\left\{-\left(\frac{v_d}{v_e}\right)^2\right\}, \quad (3.11)$$

and from (3.10)

$$\frac{\gamma_d}{\Omega} = \frac{\sqrt{\pi}}{16}\frac{K_\perp}{k_\|}\left(\frac{k_\perp}{k_\|}\right)^2 \frac{c^2K_\perp^2 - \Omega^2}{\Omega^2}\left(\frac{K_\perp v_e}{\Omega_e}\right)^3 \frac{K_\perp \epsilon'}{\Omega_e^4}$$

$$\times \left(\frac{e}{m}\right)^2 E_{l\|}^2 \exp\left\{-\left(\frac{v_d}{v_e}\right)^2\right\}. \quad (3.12)$$

In evaluating these growth rates, we have used the relation $E_{l\|}/E_{l\perp} = k_\|/k_\perp$.

For the case of very weak gradient, by putting $\epsilon' \sim 0$ in (3.9), we obtain the growth rate of O-mode for such case as

$$\frac{\gamma_p}{\Omega} = \pi\left(\frac{\omega_{pe}}{\Omega_e}\right)^2 \frac{c^2 k_\|^2}{c^2 K_\perp^2 - \Omega^2}\left(\frac{v_d}{v_e}\right)^2 \frac{E_{l\|}^2}{\Omega_e |k_\|| v_e^3}$$

$$\times \left(\frac{e}{m}\right)^2 \exp\left\{-\left(\frac{v_d}{v_e}\right)^2\right\}. \quad (3.13)$$

4. Discussion

This paper considers amplification of O-mode in inhomogeneous plasma in presence of drift wave turbulence. As the phase velocity of such an transverse wave is found to be larger than velocity of light, it is purely non-resonant and in this study we have excluded wave stochasticity aspects in the amplification process through plasma maser effect as it was considered in earlier studies (Vladimirov and Popel 1994; Deka and Borgohain 2011).

Plasma maser effect considers amplification of non-resonant wave through nonlinear wave–particle interaction process. In such process, particle heating is compensated by cooling and can lead to a stationary distribution of resonant particles. As a consequence, particle distribution function will no longer be in thermal equilibrium (Krivitsky et al. 1992; Vladimirov et al. 1999). Thus, the possibility of amplification of electromagnetic wave, as in the present case, is related to the non-stationarity of the distribution function. So in the nonlinear interaction of resonant and non-resonant waves, inhomogeneity and non-stationarity of the system, either open or closed, essentially have to be taken into account. It has been shown by Vladimirov et al. (1999) that the contribution of non-stationarity to the intensities of the resonant and non-resonant waves can be of same order as that of the nonlinear wave–wave interaction.

In the expression of dispersion relation [(2.21)], the contribution from polarization term is very sensitive for plasma anisotropy. In an unmagnetized plasma, the contribution to $\mathrm{Im}\,\epsilon_p$ vanishes (Krivitsky et al. 1992; Nambu et al. 1993). Its vanishing in the case of an unmagnetized plasma is connected with causality as well as homogeneity and isotropy. If one of the above conditions is violated, the contribution from the polarization coupling part may be significant even when the contributions from direct coupling term and quasilinear non-stationarity term (3.2) still vanish (Nambu et al. 1993; Vladimirov and Popel 1994). In a closed system, the contribution from direct coupling term $\mathrm{Im}\,\epsilon_d$ cancels out the quasilinear effect $(1/2)\left(\partial^2\epsilon_0\right)/(\partial\Omega\partial t)$ which arises from quasilinear effect due to non-stationarity.

In estimating contributions from direct coupling term and polarization coupling term, we have considered

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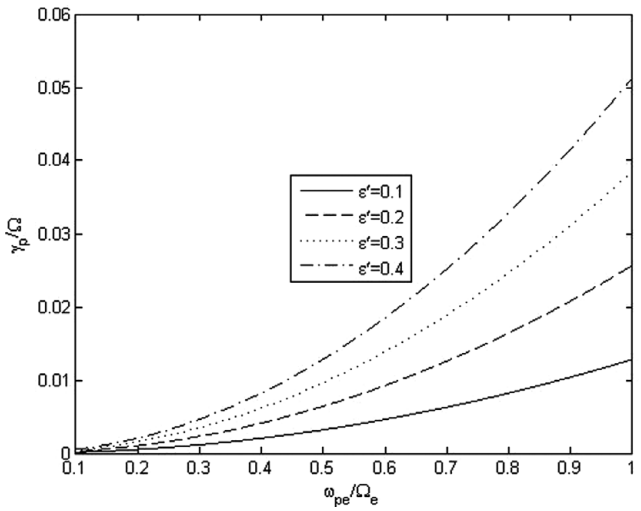


Figure 2. The growth rate of O-mode: Density gradient of plasma regions influences the growth rate.

$\Omega \sim \Omega_e \sim 10^6$ Hz, $\omega_{pe}/\Omega_e \sim 0.3$, $v_e \sim 4.19 \times 10^9$ m/s, $K_\perp \sim 10^{-3}\text{m}^{-1}$ (Benson et al. 1988).

We have also considered following typical drift wave data (Shukla et al. 1995) in space: $k_\perp \sim 10^{-3}\,\text{m}^{-1}, k_\parallel \sim 10^{-5}\,\text{m}^{-1}, v_d \sim 10^6\,\text{ms}^{-1}$.

We further consider observational electrostatic plasma wave electric field intensity (Gurnett and Frank 1977) $E_{l\parallel} \sim 10^{-4}\,\text{Vm}^{-1}$ and $e/m \sim 1.75 \times 10^{11}\,\text{Ckg}^{-1}$.

The estimated growth rates, respectively, from polarization coupling term [from (3.11)] and direct coupling term [from (3.12)] are found as

$$\frac{\gamma_p}{\Omega} \sim 10^{-3}\epsilon', \tag{4.1}$$

and

$$\frac{\gamma_d}{\Omega} \sim 10^{-10}\epsilon'. \tag{4.2}$$

Using typical value (Kintner and Gurnett 1978) of density gradient scale length in magnetosphere near the auroral region and considering its associated relation with ϵ', we put $\epsilon' = 0.1$ in (4.1) and (4.2), to get the growth rates for O-mode in inhomogeneous plasma, respectively, from polarization and direct coupling terms as

$$\frac{\gamma_p}{\Omega} \sim 10^{-4}, \tag{4.3}$$

and

$$\frac{\gamma_d}{\Omega} \sim 10^{-11}. \tag{4.4}$$

These estimations are sufficient to justify the amplification process of electromagnetic radiation in space. The plasma maser effect requires no frequency matching condition among O-mode and drift wave turbulent field. Therefore, amplification process and wave energy transfer are possible at any power level.

We have plotted in Fig. 2 the normalized growth rate [(3.11)] vs. ω_{pe}/Ω_e for O-mode with different values of the density gradient parameter ϵ'. It has been observed from the plotting that the growth rate is enhanced with increase in the value of density gradient parameter ϵ'.

In this study of amplification of electromagnetic wave in presence of drift wave turbulence in the ionospheric auroral region, polarization contribution is found to be dominant, whereas in Deka and Borgohain (2011) contribution from direct coupling term was found to be dominant.

This study suggests that plasma maser effect may be an effective mechanism for unstable generation of electromagnetic radiations associated with plasma inhomogeneity in auroral region.

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References

Andre, M., Behlke, R., Wahlund, J. E., Vaivads, A., Eriksson, A. I., Tjulin, A., Carozzi, T. D., Cully, C., Gustafsson, G. et al. 2001 *Ann. Geophys.* **19**, 1471–1481.
Benson, R. F., Mellott, M. M., Huff, R. L. and Gurnett, D. A. 1988 *J. Geophys. Res.* **93**, 7515–7520.
Deka, P. N. 1998 *Pramana* **50**, 345–354.
Deka, P. N. and Borgohain, A. 2011 *Phys. Plasmas* **18**, 042311.
Deka, P. N., Goswami, K. S. and Bujarbarua, S. 1997 *Planet. Space Sci.* **45**, 1443–1447.
Fermi, E. 1949 *Phys. Rev.* **75**, 1169.
Gary, S. P. 1993 *Theory of Space Plasma Microinstabilities.* New York: Cambridge University Press.
Gurnett, D. A. 1974 *J. Geophys. Res.* **79**, 4227.
Gurnett, D. A. and Frank, L. A. 1977 *J. Geophys. Res.* **82**, 1031–1050.
Hayes, L. M. and Melrose, D. B. 1986 *J. Geophys. Res.* **91**, 211–217.
Kintner, P. M. and Gurnett, D. A. 1978 *J. Geophys. Res.* **83**, 39–44.
Krall, N. A. and Trivelpiece, A. W. 1973 *Principles of Plasma Physics.* New York: McGraw-Hill Kogakusha Ltd.
Krivitsky, V. S., Tsytovich, V. N. and Vladimirov, S. V. 1992 *Phys. Rep.* **218**, 141.
Mellot, M. M., Calvert, W., Huff, R. L., Gurnett, D. A. and Shawhan, S. D. 1984 *Geophys. Res. Lett.* **11**, 1188–1191.
Nambu, M. 1976 *Phys. Fluids* **19**, 412.
Nambu, M. 1983 *Laser Part. Beams* **1**, 427.
Nambu, M. 1986 *Space Sci. Rev.* **44**, 357–391.
Nambu, M., Bujarbaruah, S. and Sarma, S. N. 1987 *Phys. Rev.* A **35**, 798.
Nambu, M., Vladimirov, S. V. and Schamel, H. 1993 *Phys. Lett.* A **178**, 400–406.
Queau, L., Pellat, D. R. and Roux, A. 1985 *Ann. Geophys.* **3**, 273–291.
Shukla, P. K., Birk, G. T. and Bingham, R. 1995 *Geophys. Res. Lett.* **22**, 671–674.
Singh, M. and Deka, P. N. 2005 *Phys. Plasmas* **12**, 102304.

https://doi.org/10.1017/S0022377812000207 Published online by Cambridge University Press

Stix, T. H. 1992 *Waves in Plasma*. New York: AIP.
Tsytovich, V. N., Stenflo, L. and Wilhelmsson, H. 1975 *Phys. Scr*. **11**, 251.
Vladimirov, S. V. 1994 *Phys. Scr*. **50**, 561–565.
Vladimirov, S. V., Maiorov, S. A., Yu, M. Y. and Stenflo, L. 2001 *Phys. Rev*. E **63**, 067401.
Vladimirov, S. V. and Popel, S. I. 1994 *Phys. Lett*. A **184**, 454–458.
Vladimirov, S. V., Stenflo, L. and Yu, M. Y. 1999 *J. Phys. Soc. Jpn*. **68**, 3881.
Vladimirov, S. V. and Yu, M. Y. 2004 *Phys. Scr*. T**113**, 32–35.
Wu, C. S. and Lee, C. L. 1979 *Astrophys. J*. **230**, 621–626.

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