A notational system for third-tone music

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1. THE NEED FOR A NEW NOTATIONAL SYSTEM

Evolution in musical notation has been intimately tied to the development of music theory and performance.

Invented by Guido d'Arezzo,[1] the four-line staff notation had an impact on both the quality and quantity of music composed during the Middle Ages and the Renaissance. Later in the Middle Ages, the five-line staff was used for keyboard music, but did not become standard until the seventeenth century.

At the close of the twentieth century, the computer, the latest advance in music media, stands to revolutionise performance and music theory. Not only do computers afford an unprecedented control over pitch and rhythm beyond the limitations imposed by human performers, but they also expand the horizons for music theory by making possible limitless microtonal and microdurational divisions, as well as unlimited resources in timbre. Obviously, the traditional five-line staff notation is inadequate for the task of representing the complex microdivisions of pitch that computers can achieve. Thus, it appears that computers will drive the next transformation in music notation.

During the twentieth century, many attempts have been made to represent music in ways other than the standard five-line staves, giving rise to such systems as Klavarscribo,[2] Equitone[3] and Notagraph[4], which continued to represent only 12 semitones of equal temperament. As early as 1950, graphic notation appeared and was widely used in compositions where precise pitch and definite duration were not required.

During attempts to use microtonal intervals before computers were readily available, a number of experimental systems employed signs other than the traditional accidental marks. None of these experiments led to a new standard of representing microtonal intervals, perhaps because they devised so many new symbols that they became too cumbersome to use.

When the computer is used as a musical instrument, music notation comes in the form of binary codes and programming language. However, binary codes and programming language are limited compared to traditional staff notation in assisting musicians, especially performers, who have a traditional musical education including harmony, counterpoint, keyboard harmony, sight singing, instrumentation and orchestration. Traditional staff notation is better suited to the illustration of such aspects as melody, rhythm, chords, dynamics and articulations.

Numeric notation was used to represent pitch long before the appearance of computer music, as in the figured bass[5] of the early seventeenth century. However, the computer uses binary codes which can be considered a form of numeric notation to represent all the different parameters of music, including dynamics, duration, articulations, timbre, as well as pitch. During a live concert, a human performer would be hopelessly overwhelmed trying to sight read from a purely binary code or a computer algorithm, or even the output data generated from a computer program.

At present, the growth of computer music requires a form of staff notation for the benefit of performers comparable to the standard notational system which all musicians are familiar with, but which can precisely represent the new microtonal possibilities of computer-generated sound.

2. A PROPOSAL FOR EQUAL TEMPERAMENT USING ONE-THIRD TONES

Before developing a new staff notational system that can accommodate computer music, it is necessary to

[1] Guido d'Arezzo (*c*.990–1050) was a music theorist and the author of *Micrologus*, the earliest comprehensive treatise on musical practice.

[2] The Klavarscribo system uses staves which run vertically with horizontal bar-lines, attempting to resemble the layout of a keyboard instrument.

[3] The Equitone system uses two lines to the octave with notes in five different positions relative to them to eliminate accidentals.

[4] The Notagraph system uses a seven-line staff with the space between them divided proportionally to represent intervals without leger lines for the most part.

[5] The figured bass is a reference term for the bass part of an ensemble work furnished with Arabic numbers and other signs indicating the harmonies implied above the bass.

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decide upon a microtonal system and to establish the intervals.

The history of tuning and temperament can be traced back almost three thousand years. The temperament of 12 equal divisions in the octave is one such system among hundreds of different possible tunings, and has only been practised for the last three hundred years of Western civilisation. The microtonal divisions of the octave by more than 12 notes is not a modern discovery. It dates back to more than two thousand years ago. For example, during the third century BC, King Fang in China calculated a scale based on 59 consecutive intervals of 3:2 that resulted in 60 *lu*.[6] King Fang also devised the 53-tone equal temperament.

In the West, Zarlino (1517–90) mentions a harpsichord made by Dominico Pesarese which permitted the playing of quarter tones. In 1555, Nicola Vicentino described the construction of his Arcicembalo with 35 keys in the octave, while in 1636 Mersenne described his keyboard with 32 notes in the octave. There were many others who advocated keyboards with any number of keys between 24 and 60 in the octave, including Salinas (1577), Fabio Colonna (1618), G. B. Doni (1635–40), Galeazzo Sabbatini (1650), and Athanasius Kircher (1650). In the nineteenth century, R. H. M. Bosanquet built the Enharmonic Harmonium that was based on 53 equally tempered tones per octave. Furthermore, there are countless numbers of ethnic tunings which have survived all over the world. For example, the Indian *sruti*[7] which consists of 22 pure intervals for the performance of *raga*[8] is still practised today.

However, the serious investigation of microtonality in the West is a rather recent phenomenon. Microtonal music has given rise to many divergent developments in recent years. Among the famed microtonalists, Julian Carrillo divides the octave evenly into 96 tones, while Wendy Carlos uses various divisions of 12, 19, 31, 34, 53, 65 and 118 equal steps per octave. Also, Alois Haba, Ivan Alexandrovich Vyshnegradsky, Charles Ives, Joseph Yasser, Harry Partch and many others have made tremendous contributions to the development of microtonal music.

In this paper I propose the utilisation of a new third-tone system of equal temperament, or the divisions of whole tones by one-third. Since the Greek word *diatonos* translates as 'proceeding by whole tones' and chromatic refers to half tones, I propose using the term 'diathirdtonic' to refer to a whole interval in my third-tone system. A chromatic tone would then refer to a half interval of the diathirdtonic interval, or a one-sixth interval of a whole tone. The diathirdtonic interval then corresponds to an 18-tone scale and the chromatic interval corresponds to a 36-tone scale.

Table 1. Eugene Lee, *Theme and 14 Variations for Computer-generated Trumpet Sound*, the matrix of Variation XIV.

0	3	1	4	2	5	6	8	7	A	B	9	C	G	E	F	D	H
3	0	2	H	1	G	F	B	E	B	A	C	9	5	7	6	8	4
1	G	0	F	H	E	D	B	C	9	8	A	7	3	5	4	6	2
4	7	5	8	6	9	A	C	B	E	F	D	G	2	0	1	H	3
2	5	3	6	4	7	8	A	9	C	D	B	E	0	G	H	F	1
5	2	4	1	3	0	H	F	G	D	C	E	B	7	9	8	A	6
6	9	7	A	8	B	C	E	D	G	H	F	0	4	2	3	1	5
8	B	9	C	A	D	E	G	F	0	1	H	2	6	4	5	3	7
7	4	6	3	5	2	1	H	0	F	E	G	D	9	B	A	C	8
A	D	B	E	C	F	G	0	H	2	3	1	4	8	6	7	5	9
B	8	A	7	9	6	5	3	4	1	0	2	H	D	F	E	G	C
9	6	8	5	7	4	3	1	2	H	G	0	F	B	D	C	E	A
C	F	D	G	E	H	0	2	1	4	5	3	6	A	8	9	7	B
G	1	H	2	0	3	4	6	5	8	9	7	A	E	C	D	B	F
E	H	F	0	G	1	2	4	3	6	7	5	8	C	A	B	9	D
F	C	E	B	D	A	9	7	8	5	4	6	3	H	1	0	2	G
D	A	C	9	B	8	7	5	6	3	2	4	1	F	H	G	0	E
H	E	G	D	F	C	B	9	A	7	6	8	5	1	3	2	4	0

While it is not the intent or within the scope of this paper to elaborate upon the advantages of a third-tone system, it is not an arbitrary choice. A new third-tone system would have many advantages over other similar music already in existence. One very significant advantage is that the third-tone system would remain consistent with all the theoretical developments from the time of Schoenberg to the present. Most concepts of 12-tone music can be applied to third-tone music. Such concepts as combinatoriality, derived sets, and all-interval sets can be implemented for third-tone music. For example, PTO{0 3 1 4 2 5 6 8 7 A B 9 C G E F D H}[9] is used to generate combinatorialities of 2-, 3-, 6-, 9- and 18-voiced polyphonies in my *Theme and 14 Variations for Computer-generated Trumpet Sound*.[10]

The 18-voiced combinatorial polyphony which is used in the 14th Variation is represented in the matrix form in Table 1. The matrix grants only a glimpse into the tremendous possibilities inherent in a third-tone system. 'Diathirdtonic' music alone allows six thousand trillion more possible orderings[11] of its 18 distinct elements than 12-tone music allows with its 12 distinct elements.

An important aspect in this music is that the equally tempered diathirdtonic scale is well within the

[6] The *lu* is a Chinese term for a fundamental pitch. The 60 *lu* refers to a theoretical system of 60 pitches within the octave.

[7] The *sruti* is an Indian term for the distances between seven degrees in the scale which are measured by means of *22 sruties*, each roughly equal to a quarter tone.

[8] The *raga* is an Indian term often translated as mode, scale or melody type.

[9] A, B, C, D, E, F, G and H represent 10, 11, 12, 13, 14, 15, 16 and 17, respectively.

[10] The *Theme and 14 Variations* was premiered at Chopin Hall in Seoul, 25 May, 1993.

[11] The statistical comparison of possible orderings of pitches between 12-tone and third-tone music is astonishing. While 12-tone music has 479,001,600 possible orderings, diathirdtonic music has 6,402,373,705,728,000 possibilities. Chromatic third-tone music has 371,993,326,789,901,217,467,999,448,150,835, 200,000,000 possible orderings of its 36 distinct elements.

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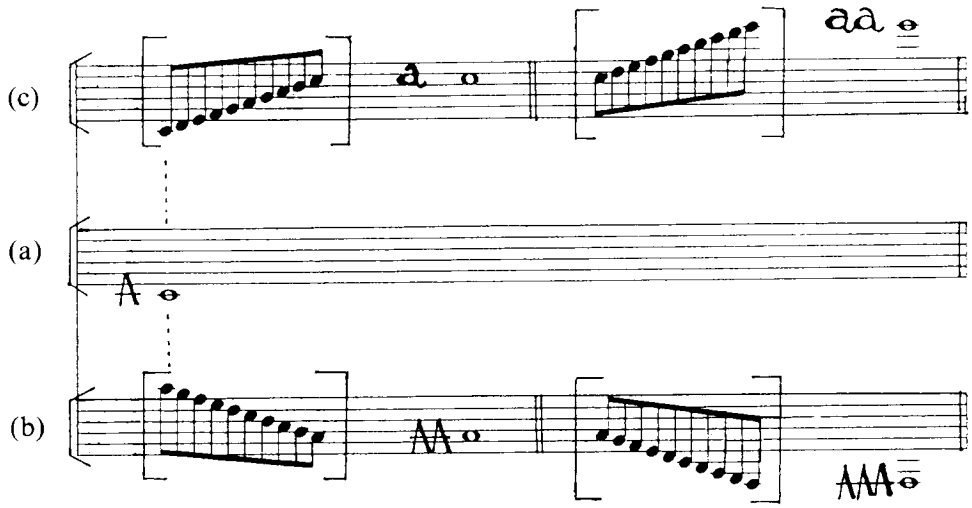


Figure 1. Positions of the A, AA, AAA, a and aa clefs.

range of easy audio-cognitive discrimination of the human ear. Diathirdtonic music and its chromatic counterpart have distinct linear and harmonic characteristics which add to the vocabulary available to the composer for psycho-emotional expression.

3. A NOTATIONAL SYSTEM FOR EQUALLY TEMPERED THIRD-TONE MUSIC

3.1. Staff

The staff in third-tone music is composed of six lines instead of the normal five lines in traditional music.

3.2. Clefs

The traditional G, F and C clefs are based on *hexachordum durum*, *hexachordum molle* and *hexachordum naturale* of the Medieval period, respectively. Since the reference pitch for the modern tuning system is A4, I propose the standard clef should be changed to represent A. Therefore, the most common clefs in third-tone music are A, AA, AAA, a and aa clefs instead of G, F and C clefs, as shown in figure 1.

The first leger line below the six-line staff, where the horizontal line of the A clef lies, represents 440 cycles per second (cps), as illustrated in figure 1(a). The second space of the six-line staff, where the horizontal line of the AA clef lies, represents 220 cps, which is nine steps below the locus of the A clef. The horizontal line of the AAA clef, which is placed another nine steps below the locus of the AA clef, i.e. the third leger line below the six-line staff, represents 110 cps, and so on, as illustrated in figure 1(b).

The fourth space of the six-line staff, i.e. nine steps above the locus of the A clef, where the belly of the a clef rests, represents 880 cps. The belly of the aa clef, which is placed another nine steps above the locus of the a clef, i.e. the third leger line above the

Table 2. Ratios, frequencies and cents of diathirdtonic notes.

Notes	Ratios	Frequencies	Cents
0	1:1	440.00	0.00
1	53:51	457.27	66.67
2	27:25	475.23	133.33
3	55:49	493.88	200.00
4	771:661	513.27	266.67
5	137:113	533.42	333.33
6	63:50	554.37	400.00
7	127:97	576.13	466.67
8	83:61	598.75	533.33
9	99:70	622.25	600.00
10	97:66	646.68	666.67
11	139:91	672.07	733.33
12	100:63	698.46	800.00
13	193:117	725.88	866.67
14	667:389	754.37	933.33
15	98:55	783.99	1000.00
16	50:27	814.77	1066.67
17	102:53	846.76	1133.33
18	2:1	880.00	1200.00

Table 3. Ratios, frequencies and cents of chromatic notes for the third-tone system.

Notes	Ratios	Frequencies	Cents
0.5	105:103	448.55	33.33
1.5	89:84	466.16	100.00
2.5	98:89	484.46	166.67
3.5	111:97	503.48	233.33
4.5	44:37	523.25	300.00
5.5	110:89	543.79	366.67
6.5	131:102	565.14	433.33
7.5	275:206	587.33	500.00
8.5	197:142	610.39	566.67
9.5	173:120	634.35	633.33
10.5	409:273	659.26	700.00
11.5	109:70	685.14	766.67
12.5	89:55	712.03	833.33
13.5	37:22	739.99	900.00
14.5	187:107	769.03	966.67
15.5	89:49	799.23	1033.33
16.5	168:89	830.61	1100.00
17.5	155:79	863.21	1166.67

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Figure 2. Reference pitches of the five-line and six-line staff notations.

six-line staff, represents 1760 cps, and so on, as illustrated in figure 1(c).

It is also noteworthy that the loci of the reference pitches between the five-line and the six-line notational systems are analogous. C4 of the five-line staff with G clef and A4 of the six-line staff with A clef are both on the first leger lines below the staves, while C5 of the five-line staff and A5 of the six-line staff are both in the first spaces from the top of the staves, as shown in figure 2.

3.3. Diathirdtonic scale

3.3.1. Ratios, frequencies and cents

The ratios, frequencies and cents relative to reference A = 440 cps for 18 equally sized intervals contained within the octave are as shown in table 2.

3.3.2. Pitch notation

The diathirdtonic notes from 440 to 880 cps are represented on the six-line staves, as shown in figure 3.

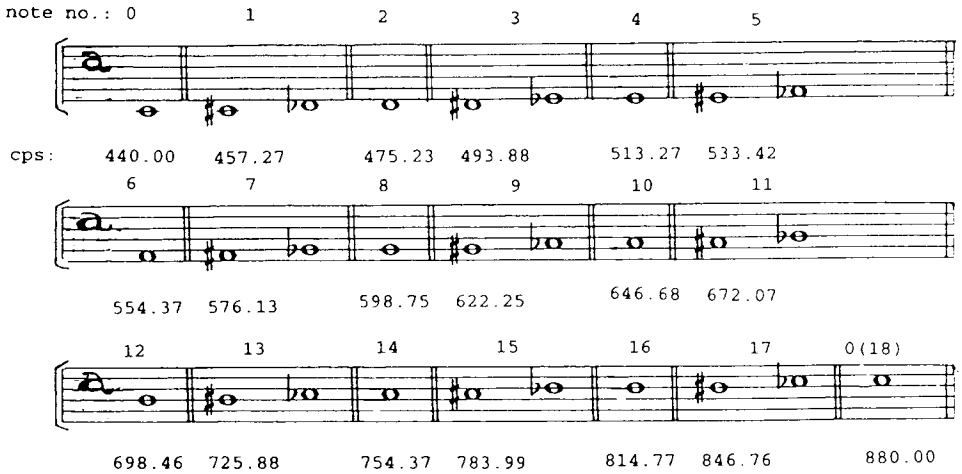


Figure 3. Diathirdtonic pitches.

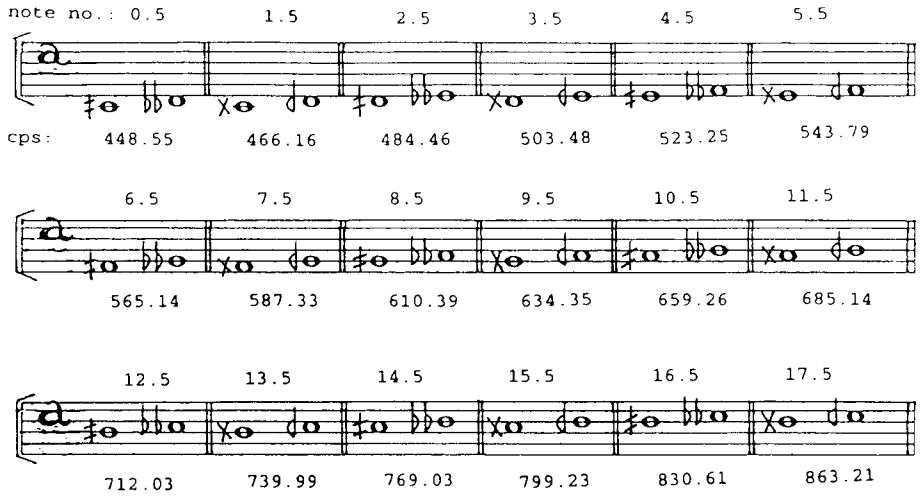


Figure 4. Chromatic pitches of the third-tone system.

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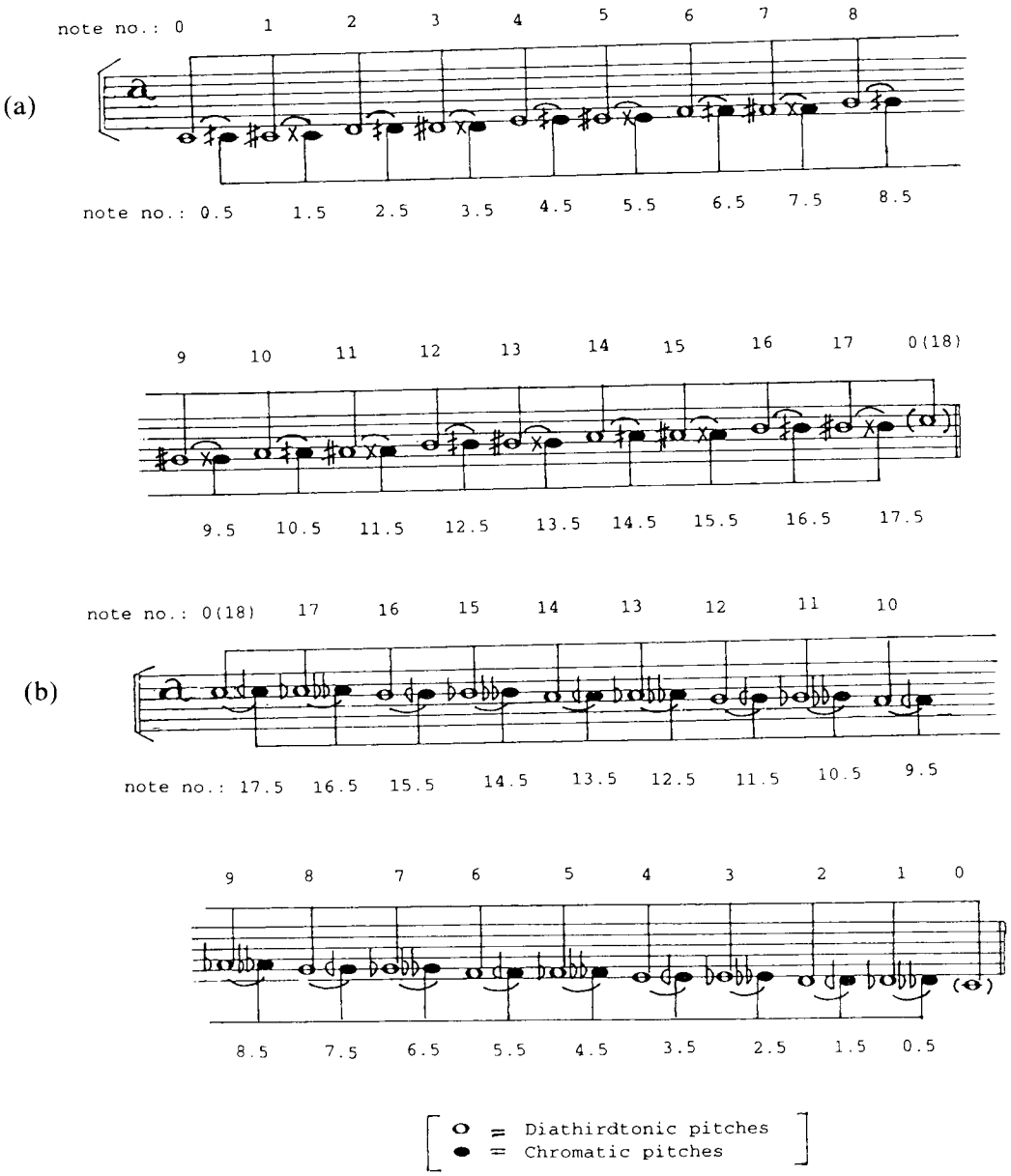


Figure 5. The notes of (a) the ascending, and (b) the descending chromatic scale of the third-tone system.

The new system includes only those symbols used in the traditional five-line staff notations, sharps ( ) and flats ( ), to accurately represent all 18 different pitches of the diathirdtonic scale. However, unlike the traditional five-line staff notation, sharps and flats never represent chromatic pitches or accidentals, but rather the diathirdtonic pitches of the new system.

3.4. Chromatic pitches

3.4.1. Ratios, frequencies and cents

The ratios, frequencies and cents of the chromatic pitches relative to reference A = 440 cps of the diathirdtonic scale contained within the octave are as shown in table 3. The total number of pitches in the resultant chromatic scale of third-tone music becomes 36, including 18 diathirdtonic notes and 18 chromatic notes.

3.4.2. Pitch notation

The chromatic notes between 440 and 880 cps in the diathirdtonic scale are represented on the six-line staves, as shown in figure 4. The new chromatic system uses the symbols £ and ", as well as their enharmonic symbols, the double flat (𝄫) and the double sharp (x), used in traditional five-line staff notation. The symbols £ and x represent a half step up from their corresponding diathirdtonic pitches, as

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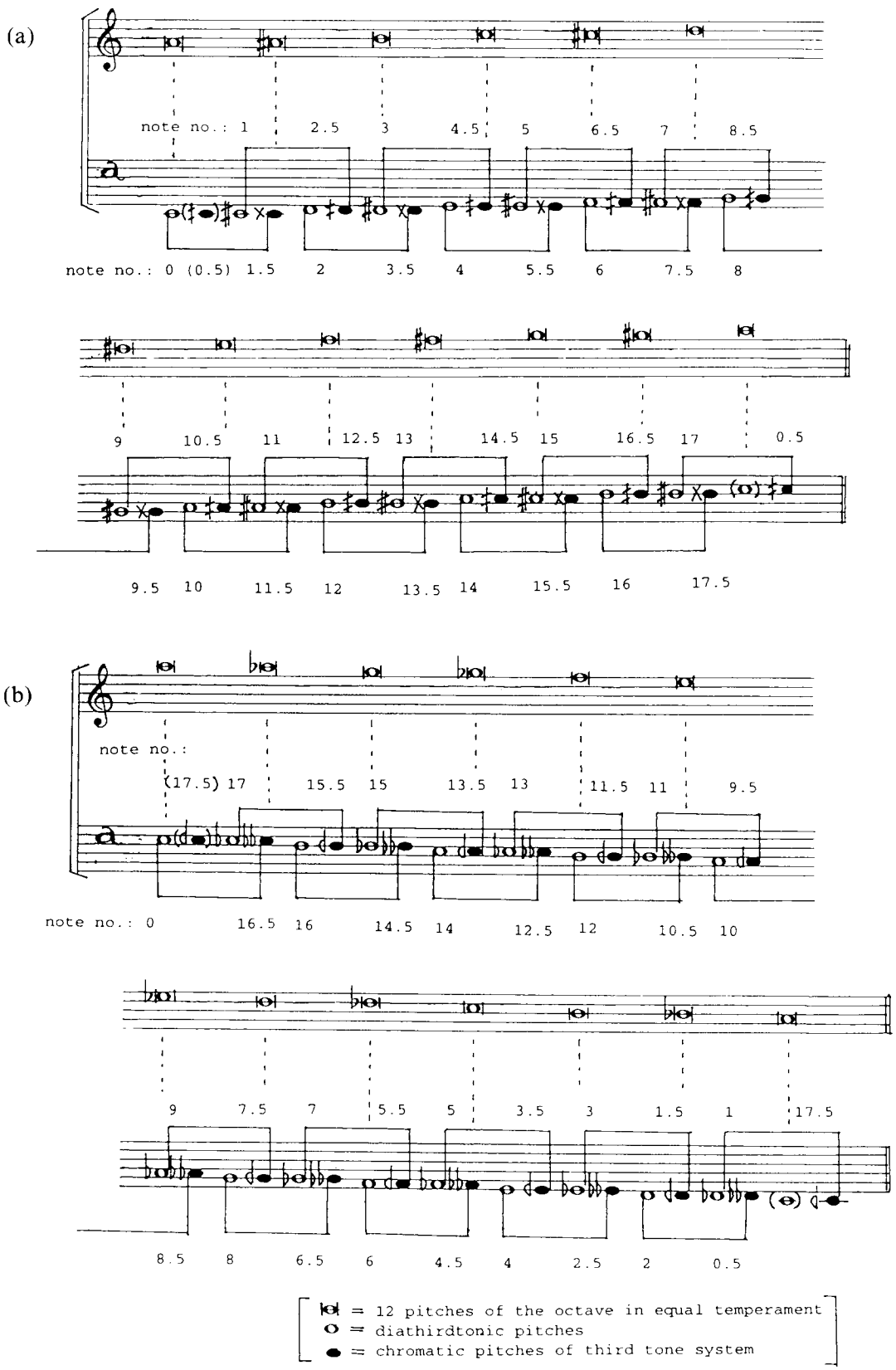


Figure 6. Intervallic relationships of three-sixth or half tones in (a) the ascending, and (b) the descending chromatic scale of the third-tone system.

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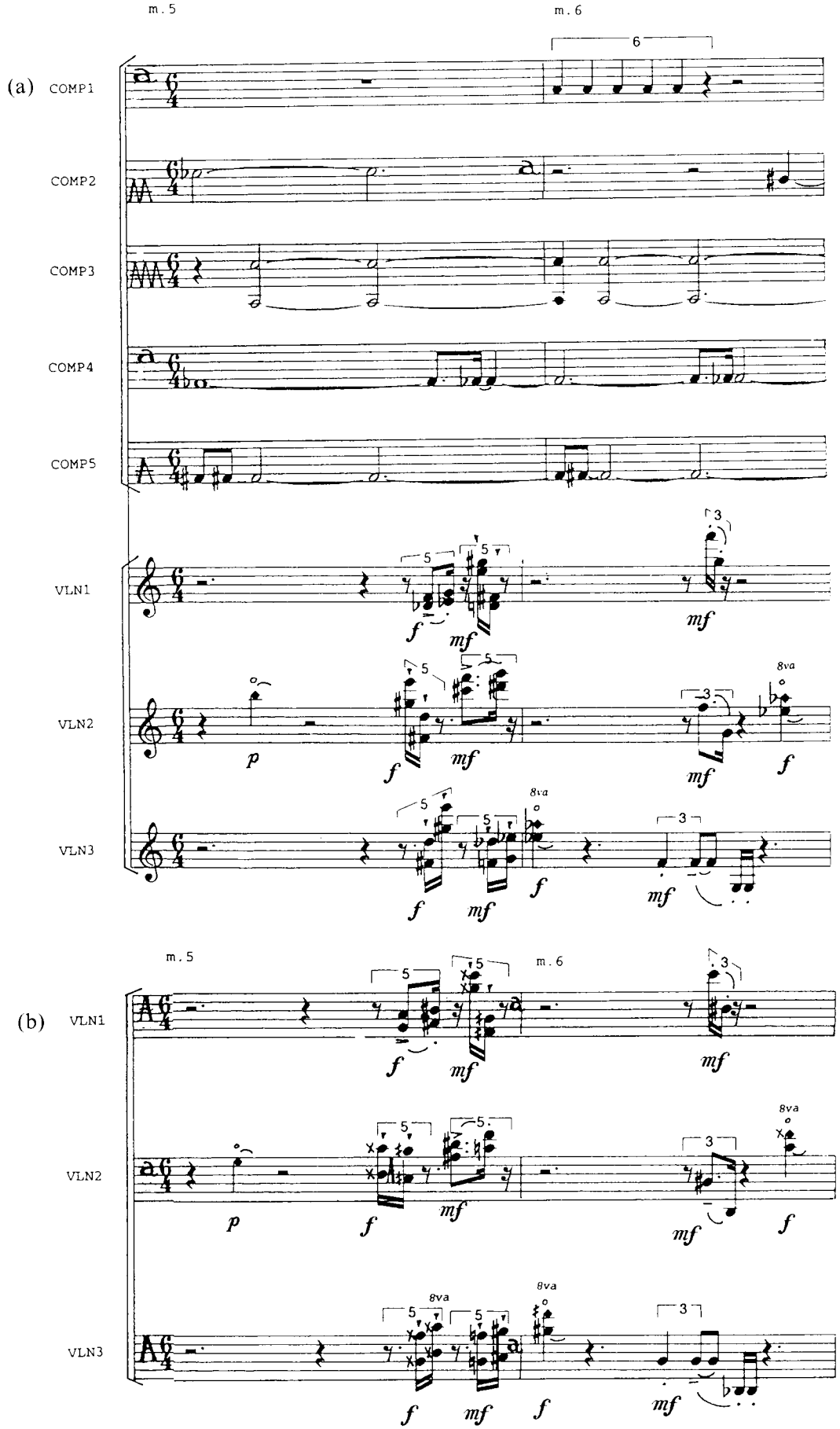


Figure 7. Eugene Lee, *Trihexadral Ballad for Three Violins and Computer-generated Tape*, (a) Movement III, mm. 5–6, (b) Ibid., violin parts only.

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Table 4. Eugene Lee, *Trihexadral Ballad*, Movement III, mm. 5–6, in cps.

	Measure 5					Measure 6		
COMP1						646.68		
COMP2		288.06						725.88
COMP3		128.32 64.16						
COMP4		533.42						
COMP5		392.00						
VLN1		377.19 299.37	423.38 336.04	799.23 634.35	356.01 282.57	1508.75	846.76	
VLN2	2053.09	1370.26 863.21	610.38 384.52	1451.75 1152.26	1629.54 1293.37	725.88	407.38	2586.73
VLN3		587.32 369.99	1318.50 830.60	554.37 349.23	622.25 392.00	2489.02	349.23	196.00

illustrated in figure 5(a), while the symbols " and 𝄫 represent a half step down from their corresponding diathirdtonic pitches, as illustrated in figure 5(b). The advantage to the sight reader is that ascending and descending linear progressions of the chromatic scale can be represented in a logical order of either sharp or flat symbols, as illustrated in figure 5.

Double sharps and double flats, as well as £ and ", never represent diathirdtonic pitches of the new system; instead they represent chromatic pitches of the new system. The x and £ represent a three-sixth tone or half tone up from their corresponding diathirdtonic pitch, as illustrated in figure 6(a), while the 𝄫 and " represent a three-sixth tone or half tone down from their corresponding diathirdtonic pitch, as illustrated in figure 6(b).

4. MUSIC EXAMPLES UTILISING THE NEW NOTATIONAL SYSTEM

Measures 5 and 6 in the Third Movement of my *Trihexadral Ballad for Three Violins and Computer-generated Tape*[12] are represented in figure 7(a) to illustrate an actual music score written in the equal temperament of third-tone music. COMP1 to COMP5 represent the computer parts, while VLN1 to VLN3 represent the three violin parts. Table 4 illustrates the consequent output in cps of pitches in figure 7(a).

As with traditional convention, the clef can be changed within a single voice line, and/or a particular clef can be changed from its normal locus up to the next higher clef locus or down to the next lower clef locus in the staff for notational convenience. For example, within COMP2 of figure 7(a), the AA clef changes to the a clef, and within COMP3, the AAA clef is placed at the AA clef locus in order to eliminate excess leger lines.

[12] The *Trihexadral Ballad* was premiered by the CGNJ Performance Ensemble for the Capital Music Festival on 20 May, 1997 at Princeton University's Taplin Auditorium.

Another advantage of composing in the third-tone system is that slight alterations in the tuning of conventional instruments can achieve the appropriate third-tone pitches without requiring performers to learn to sight read the new six-line staves at the present time. For example, in figure 7(a) the three violin parts are notated in the traditional five-line staves. However, by tuning the three violins as indicated below, the resultant output of the pitches in the notation are those cps displayed in table 4:

Violin 1: A4 = 475.23 cps,
Violin 2: A4 = 457.27 cps, and
Violin 3: A4 = 440.00 cps.

Figure 7(b) is a transcription of the violin parts in figure 7(a) in the new six-line staff notation.

5. FINAL REMARKS

The computer has made it possible to compose music in microtonal divisions beyond the 12 of equal temperament. Composing in the third-tone system seems the logical choice, because it remains consistent with 12-tone music theory while remaining within the range of easy auditory discrimination.

However, while composing third-tone music, it becomes evident that there is a need for a new notational system capable of accurately representing computer-generated microtones while facilitating human performance of third-tone music.

This proposal for a notational system for third-tone music endeavours to permit sight-reading by maintaining a staff notation with the loci of reference pitches analogous to traditional five-line staves, and by representing linear progressions along the diathirdtonic and chromatic scales in a logical fashion while limiting the use of new confusing symbols. As with traditional staff notation, this system allows a view of the composition from the perspective of its totality. By easing the illustration of melody, rhythm, chords, dynamics and articulations, it may

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help to advance the meaningful discussion of microtonal music theory.

By being able to write microtonal pitches produced by computers together with specially tuned instruments in musical notation, microtonal music theory can be readily manipulated and exploited in the same fashion as 12-tone music. Then, computer music can move beyond the role of producing sound effects and back into the logical continuum of traditional music theory.

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