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RISK ANALYSIS OF A ROBOT–SAFETY DEVICE SYSTEM SUBJECTED TO A PRIORITY RULE

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We introduce a robot–safety device system characterized by cold stand-by and by an admissible risky state. The system is attended by a single repairman and the robot has overall (break-in) priority in repair with regard to the safety device. We obtain an explicit formula for the point availability of the robot via an integral equation of the renewal-type. The explicit solution requires the notion of effective repair-versus-virtual repair. In order to decide whether the risky state is admissible, we also introduce a risk criterion. The criterion is always satisfied in the case of fast repair. As an example, we consider the case of Weibull–Gnedenko repair and we display a computer-plotted graph of the point availability obtained by a direct numerical solution of a convolution-type integral equation.

1. INTRODUCTION

Innovations in the field of microelectronics and micromechanics have enhanced the involvement of "smart" robots in all kinds of advanced technical applications, for example Brandin [2]. Unfortunately, no robot is completely reliable. Therefore, up-to-date robots are often connected with a safety device, for example Dhillon and

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Yang [5], Vanderperre and Makhanov [12]. Such a device prevents possible damage, caused by a robot failure or by hazardous man–machine interactions, in the robot's neighboring environment.

The usual "bugbears" are software failures, for example Gaskill and Went [7], common-cause failures, for example Dhillon and Yang [6], human errors, for example Dhillon and Liu [4] and physical failures, for example Birolini [1]. Moreover, the random behavior of the entire system (robot, safety device, repair facility) could jeopardize some prescribed safety requirements: for instance, if we allow the robot to operate during the repair time of the failed safety unit. Such a "risky" state is called *admissible* if the associated event, "The robot is operating and the safety device is under repair," constitutes a rare event. Therefore, an appropriate statistical analysis of robot–safety device systems is indispensable to support the system designer in problems of risk acceptance and safety assessments.

Apart from various robot–safety device systems introduced in the previous literature, for example Dhillon and Yang [5], Vanderperre and Makhanov [13], Vanderperre and Makhanov [14], we introduce a modified system (henceforth called the **M**-system) characterized by the following features:

- The **M**-system is attended by a single repairman.
- The robot has overall (break-in) priority in repair with regard to the safety device.
- The robot is allowed to operate when the safety unit is under repair, provided that a suitable risk-criterion is satisfied.
- The safety device needs not to operate when the robot is under repair. Therefore, upon failure of the robot, the *operative* safety device is put in cold stand-by until the repair of the robot has been completed.

The notion of "cold" stand-by signifies that the safety unit is failure-free in stand-by. We also introduce the notion of *virtual* repair time versus *effective* repair time.

In order to describe the random behavior of the **M**-system, we employ a stochastic process endowed with state probability functions satisfying integral equations of the convolution type. Next, we define a risk-function associated with the risky state of the robot. In order to decide whether the risky state is admissible, we introduce an associated risk-criterion compatible with the notion of rare events. We show that the criterion is always satisfied in the case of "fast" repair. As an example, we display the graph of the point availability of the robot obtained by a numerical solution of a convolution-type integral equation involving the case of Weibull–Gnedenko repair.

2. FORMULATION

Consider the **M**-system satisfying the following assumptions:

- The *operative* safety device has a constant failure rate λ_s, a zero failure rate in stand-by (the so-called cold stand-by) and a general repair time distribution

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$R_s(\cdot)$, $R_s(0) = 0$. Let f_s be the random variable corresponding to λ_s. The repair time is denoted by r_s.

- The robot has a constant failure rate λ and a general repair time distribution $R(\cdot)$, $R(0) = 0$. The failure-free time and the repair time are respectively denoted by f and r.
- The random variables f, f_s, r, r_s are supposed to be statistically independent and any repair is perfect. In order to introduce the virtual repair time of the safety device, we consider the random sum

$$r_v := \begin{cases} r_s + \sum_{i=1}^{n_{r_s}} r_i & \text{if } n_{r_s} > 0, \\ r_s & \text{if } n_{r_s} = 0, \end{cases}$$

where r_i; $i = 1, 2, \ldots$ denotes the ith repair time of the robot and n_{r_s} the number of robot failures during r_s. Clearly, r_v reduces to r_s if no failure occurs. Therefore, we call r_s the *effective* repair time and r_v the *virtual* repair time of the safety device. Let $V(t) := Pr\{r_v \leq t\}$.

In order to describe the random behavior of the **M**-system, we employ a stochastic process $\{N_t, t \geq 0\}$ with discrete state space $\{A, B, C, D\} \subset [0, \infty)$, characterized by the following set of mutual exclusive events:

$\{N_t = A\}$: "The robot and the safety device are jointly operative at time t."

State A is called the safe state.

$\{N_t = B\}$: "The robot is operative but the safety device is under progressive repair at time t."

State B is the so-called risky state. We tacitly assume that state B is admissible. See Section 5 for further details.

$\{N_t = C\}$: "The robot is under progressive repair and the safety device is in (cold) stand-by at time t."

$\{N_t = D\}$: "The robot is under progressive repair and the safety device is waiting for repair at time t."

Note that a transition from state C into state D is only possible via states A and B. We assume that the **M**-system starts operating in state A at some time origin $t = 0$, that is, let $N_0 = A$ with probability one.

The state probability functions are defined by

$$p_K(t) := Pr\{N_t = K\}, \qquad K = A, B, C, D,$$

where $p_A(t) + p_B(t) + p_C(t) + p_D(t) = 1$, $p_A(0) = 1$.

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We recall that the robot is only available in states A and B. Therefore, the point availability of the robot, denoted by $\mathcal{A}(t)$, is given by

$$\mathcal{A}(t) = p_{A\bigcup B}(t) = p_A(t) + p_B(t).$$

The long-run availability is denoted by $\mathcal{A}(\infty)$. Observe that

$$\mathcal{A}(\infty) = \lim_{T\to\infty} \frac{1}{T} \int_0^T \mathcal{A}(t)\, dt,$$

provided that the precious limit exists.

2.1. Notations

- The notation $N << 1$ signifies that the number N is considerably smaller than 1.
- The expectation operator, for example Brémaud [3, Appendix A, p. 268], is denoted by **E**. Note that, for instance,

$$\mathbf{E}e^{-zr} = \int_0^\infty e^{-zx}\, dR(x), \qquad \mathrm{Re}\, z \geq 0.$$

- The notation $X \sim Y$ means that the random variables X and Y are equally distributed.
- Let $F(t)$ be any probability distribution on $[0, \infty)$.
  The n-fold convolution of F, see for example, Gnedenko and Ushakov [8, pp. 25–26], is denoted by $F^{n*}(t)$. For $n = 0$, $F^{0*}(t)$ represents the Heaviside unit-step function with the unit-jump at $t = 0$, that is,

$$F^{0*}(t) := \begin{cases} 1 & \text{if } t \geq 0, \\ 0 & \text{if } t < 0. \end{cases}$$

- Consider a repairable robot attended by a single repairman but *without* the safety device, henceforth called the **R**-system. The point availability of the robot is denoted by $p_R(t)$. It is well known, see for example Shaked and Shanthikumar [10, p. 665], that

$$p_R(t) = \int_{0-}^t e^{-\lambda(t-x)} d \sum_{n=0}^\infty F^{n*}(x),$$

where

$$F(x) := \int_0^x (1 - e^{-\lambda(x-y)})\, dR(y).$$

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3. PRELIMINARY PROPERTIES

The following properties are stated for direct reference.

Property 3.1 (Vanderperre and Makhanov [13]):

$$p_R(\infty) = (1 + \lambda\, \mathbf{E}r)^{-1}, \qquad p_R(0) = 1,$$

$p_R(\cdot)$ satisfies the integral equation

$$p_R(t) + \lambda \int_0^t p_R(t-u)(1 - R(u))du = 1.$$

Next, we deal with the virtual repair time of the safety device.

THEOREM 3.1:

$$V(t) = \int_0^t \sum_{k=0}^{\infty} R^{k*}(t-u)e^{-\lambda u}\frac{(\lambda u)^k}{k!}dR_s(u),$$

$$\boldsymbol{E}r_v = \boldsymbol{E}r_s(1 + \lambda\, \boldsymbol{E}r).$$

PROOF: Applying the law of total probability yields

$$\begin{aligned} Pr\{r_v \le t\} &= \int_0^{\infty} Pr\{r_v \le t | r_s = u\}\, dR_s(u) \\ &= \int_0^t Pr\left\{\sum_{i=1}^{n_u} r_i \le t - u\right\} d\, R_s(u) \\ &= \int_0^t \sum_{k=0}^{\infty} R^{k*}(t-u)\, Pr\{n_u = k\}\, dR_s(u), \end{aligned}$$

where n_u denotes the number of robot failures during the interval $[0, u]$. But $\{n_u\}$ is a homogeneous Poisson process with parameter λ, that is,

$$Pr\{n_u = k\} = e^{-\lambda u}\frac{(\lambda u)^k}{k!};\ k = 0, 1, \ldots..$$

Hence,

$$Pr\{r_v \le t\} = \int_0^t \sum_{k=0}^{\infty} R^{k*}(t-u)e^{-\lambda u}\frac{(\lambda u)^k}{k!}\, dR_s(u).$$

The Laplace–Stieltjes convolution theorem, for example Gnedenko and Ushakov [8, pp. 25–26], entails that

$$\mathbf{E}e^{-zr_v} = \mathbf{E}e^{-(z+\lambda(1-\mathbf{E}e^{-zr}))r_s}.$$

Thus, $\mathbf{E}r_v$ follows from the relation,

$$\mathbf{E}r_v = -\frac{\partial}{\partial\, z}\, \mathbf{E}e^{-z\, r_v}|_{z=0}.$$

■

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4. POINT AVAILABILITY

The following Theorem 4.1 shows that the point availability of the robot in the **M**-system is identical to the point availability of the robot in the **R**-system!

THEOREM 4.1:

$$\mathcal{A}(t) = p_R(t),\ t \geq 0,$$
$$\mathcal{A}(\infty) = (1 + \lambda\, \boldsymbol{E}r)^{-1}.$$

PROOF: We recall that the safety device cannot fail if the robot is under repair (cold stand-by). Consequently, the **M**-system is either in state C or D if the **R**-system is down. Conversely, the **R**-system is down if the **M**-system is in state C or D. Therefore,

$$Pr\left\{N_t = C \bigcup D \mid N_0 = A\right\} = 1 - p_R(t).$$

Invoking the law $\sum_K p_K(t) = 1$ and the definition of $\mathcal{A}(t)$ entails that $\mathcal{A}(t) = p_R(t)$. Finally, since $p_R(\infty)$ exists, we have by Property 3.1,

$$\mathcal{A}(\infty) = p_R(\infty) = (1 + \lambda\, \mathbf{E}r)^{-1}.$$ ■

In order to determine the risk-function, we need the following.

Property 4.1: The function $p_A(t)$ satisfies the integral equation

$$p_A(t) = e^{-(\lambda+\lambda_s)t} + \int_0^t p_A(t-u)d\,\varphi(u), \tag{4.1}$$

where

$$\varphi(u) := \int_0^u \left(1 - e^{-(\lambda+\lambda_s)(u-x)}\right) d\left(\frac{\lambda}{\lambda+\lambda_s}R(x) + \frac{\lambda}{\lambda_s+\lambda_s}V(x)\right).$$

The unique solution is given by

$$p_A(t) = \int_{0-}^t e^{-(\lambda+\lambda_s)(t-u)}\, d \sum_{n=0}^{\infty} \varphi^{n*}(u). \tag{4.2}$$

Furthermore,

$$p_A(\infty) = (1 + \lambda \mathbf{E}r + \lambda_s \mathbf{E}r_v)^{-1}.$$

PROOF: First note that state A is *regenerative* for the process $\{N_t\}$, that is, the Markov property of the exponential distribution ensures that state A is a renewal state of the **M**-system. Furthermore, observe that a busy period of the repairman can either start via a robot failure with $Pr\{f < f_s\}$ or via a failure of the safety device with

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$Pr\{f_s < f\}$. Note that $Pr\{f = f_s\} = 0$ and that $Pr\{f < f_s\} = \lambda/(\lambda + \lambda_s)$, whereas $Pr\{f_s < f\} = \lambda_s/(\lambda + \lambda_s)$.

Let θ_a (respect. θ_b) be the (conditional) busy period of the repairman starting with a robot failure (respect. with a failure of the safety device). The random variable

$$\theta := \begin{cases} \theta_a & \text{if } f < f_s, \\ \theta_b & \text{if } f_s < f \end{cases}$$

is called the *global* busy period of the repairman. Clearly,

$$\theta \sim \begin{cases} r & \text{if } f < f_s, \\ r_v & \text{if } f_s < f \end{cases}$$

Hence,

$$Pr\{\theta \le t\} = \frac{\lambda}{\lambda + \lambda_s} R(t) + \frac{\lambda_s}{\lambda + \lambda_s} V(t).$$

Observe that the *idle* time of the repairman, denoted by i_R, has distribution

$$Pr\{i_R \le t\} = Pr\{\min(f, f_s) \le t\}.$$

Consequently, the *return* time from state A to state A, via states B or C, denoted by r_{AA}, has distribution

$$Pr\{r_{AA} \le t\} = Pr\{\theta + i_R \le t\}.$$

Finally, note that $\mathbf{E}i_R = (\lambda + \lambda_s)^{-1}$, whereas,

$$\mathbf{E}\theta = \frac{\lambda}{\lambda + \lambda_s} \mathbf{E}r + \frac{\lambda_s}{\lambda + \lambda_s} \mathbf{E}r_v.$$

Therefore, by Renewal Theory, for example Serfoso [9, pp. 45–46],

$$p_A(t) = Pr\{\min(f, f_s) > t\} + \int_0^t p_A(t-u) dPr\{r_{AA} \le t\}. \quad (4.3)$$

But,

$$Pr\{\min(f, f_s) > t\} = e^{-(\lambda+\lambda_s)t}, \quad (4.4)$$

whereas

$$Pr\{r_{AA} \le t\} = \varphi(t). \quad (4.5)$$

Hence, Eq. (4.1) follows from Eqs. (4.3), (4.4), and (4.5). Note that the unique solution of Eq. (4.1), given by Eq. (4.2), can be obtained by iteration or via the Laplace-transform. (Such solution procedures are well-known and therefore omitted.)

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In order to obtain $p_A(\infty)$, we remark that $\varphi(t)$ is non-lattice, whereas $e^{-(\lambda+\lambda_s)t}$ is *directly* Riemann integrable. Whence, by the key renewal theorem, for example Serfoso [9, p. 47],

$$p_A(\infty) = \frac{\mathbf{E}i_R}{\mathbf{E}i_R + \mathbf{E}\theta} = \frac{1}{1 + \lambda\,\mathbf{E}r + \lambda_s\,\mathbf{E}r_v}. \qquad \blacksquare$$

Remark 4.1: Clearly, $p_A(\infty)$ is distribution-free, that is, $p_A(\infty)$ only depends on λ, λ_s, r, r_s and *not* on the structure of the repair time distributions. It should be noted that this kind of *insensitivity* property is rather an exception than a rule, cf. Vanderperre [12, Theorem, p. 70].

5. RISK-FUNCTION AND RISK-CRITERION

An obvious risk-function, associated with risky state B, is given by $p_B(t)$, henceforth denoted by $\mathfrak{R}(t)$, $t \geq 0$.

By Theorem 4.1, we have

$$\mathfrak{R}(t) = p_R(t) - p_A(t),$$

where $p_R(t)$ and $p_A(t)$ are determined by Properties 3.1 and 4.1. We also have $\mathfrak{R}(\infty) = p_R(\infty) - p_A(\infty)$. The quantity $\mathfrak{R}(\infty)$ is called the *long-run* risk. In order to obtain a suitable risk-criterion, compatible with the notion of rare events, we use the supremum norm

$$||\mathfrak{R}(t)|| := \sup\{\mathfrak{R}(t) : t \in [0, \infty]\}.$$

We propose the following risk-criterion:

$$\text{State } B \text{ is admissible if } ||\mathfrak{R}(t)|| << 1.$$

6. FAST REPAIR

Next, we consider the case of fast repair, that is, let $\lambda_s\,\mathbf{E}r_s << 1$. Observe that the notion of "fast" repair does not necessarily imply a small average repair time of the failed safety device. The mean $\mathbf{E}r_s$ is only supposed to be considerably smaller than the average lifetime $1/\lambda_s$ of the operative safety device. A condition that is usually satisfied in practice!

Note that

$$\mathfrak{R}(\infty) = \frac{\lambda_s\,\mathbf{E}r_s}{(1 + \lambda\,\mathbf{E}r)(1 + \lambda_s\,\mathbf{E}r_s)} < \lambda_s\,\mathbf{E}r_s.$$

Furthermore, we can show that the inequality $\mathfrak{R}(t) < \lambda_s\,\mathbf{E}\,r_s$ also holds for any $t \in [0, \infty)$. The proof is purily technical and therefore omitted. Consequently, our risk-criterion is always satisfied in the case of fast repair.

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CONCLUSION

The proposed risk-criterion provides a simple R_s-insensitive test to verify whether the risky state is admissible.

7. APPLICATION EXAMPLE

Finally, we compute the point availability of the robot. Let

$$R(t) = 1 - e^{-\mu t^{\beta}}, \quad \mu > 0, \quad \beta > 1,$$

be the Weibull–Gnedenko distribution with increasing repair rate $\mu\,\beta\,t^{\beta-1}$.

An exact evaluation of $\mathcal{A}(t)$ in terms of *finite* linear combinations of algebraic and/or transcendental functions is totally excluded. Therefore, we propose computing $\mathcal{A}(t)$ by a direct numerical solution of the integral equation
(cf. Property 3.1 and Theorem 4.1)

$$\mathcal{A}(t) + \lambda \int_0^t \mathcal{A}(t-x) e^{-\mu x^{\beta}} dx = 1. \tag{7.1}$$

A straightforward application of the composite trapezoidal rule yields the *second-order* numerical scheme

$$\begin{aligned} \mathcal{A}_{\mathrm{o}} &= 1, \\ \mathcal{A}_1 + \lambda(\mathcal{A}_1\alpha_0 + \mathcal{A}_0\alpha_1)\Delta t_1 &= 1, \\ \mathcal{A}_i + \lambda(\mathcal{A}_i\alpha_0 + \mathcal{A}_{i-1}\alpha_1)\Delta t_i + & \\ \lambda(\mathcal{A}_{i-1}\alpha_1 + \mathcal{A}_{i-2}\alpha_2)\Delta t_{i-1} + \ldots & \\ + \lambda(\mathcal{A}_1\alpha_{i-1} + \mathcal{A}_{\mathrm{o}}\alpha_i)\Delta t_1 &= 1, \end{aligned} \tag{7.2}$$

where $\mathcal{A}_i := \mathcal{A}(t_i)$, $\alpha_i := \alpha(t_i) := e^{-\mu t_i^{\beta}}$, $i = 0, 1, 2, \ldots, n$ and where Δt_i denotes the step of the finite difference grid $\{t_0, t_1, \ldots\}$, $t_0 \equiv 0$, $\Delta t_{i+1} := t_{i+1} - t_i$.

The linear system (7.2) is triangular and can be solved by back substitution. The second-order quadrature invokes the second-order scheme. Unfortunately, only a one-point or a two-point rule leads to an explicit non-iterative scheme. However, the accuracy of the scheme can be enhanced by constructing a finite difference grid characterized by high resolution at those regions where the numerical error is large (the so-called equidistribution principle). See further details in Thomson, Soni, and Weatherill [11].

Finally, note that the proposed numerical scheme is applicable to an *arbitrary* repair time distribution. As a comparison, we also consider the case of Erlang-repair

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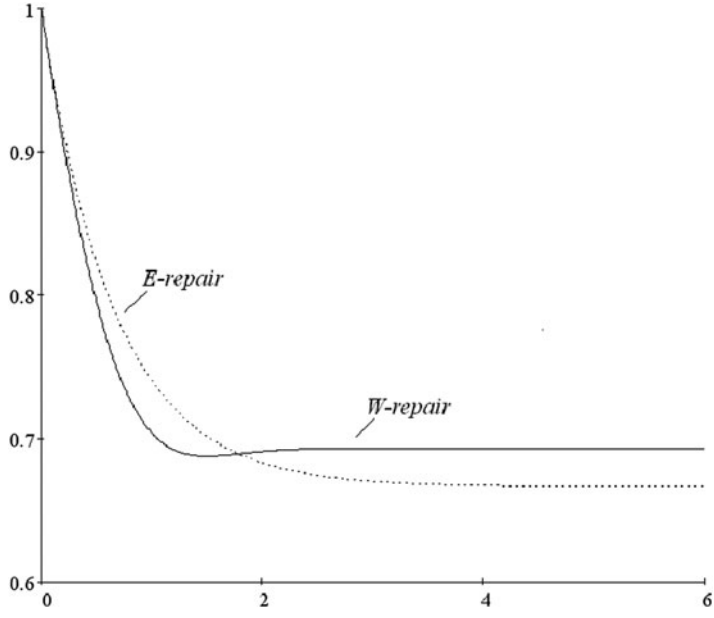


FIGURE 1. Graph of $\mathcal{A}(t)$ for **W**-repair versus **E**-repair, Case $\lambda = 0.5, \mu = 1, \beta = 1, K = 2$.

(**E**-repair), that is,

$$R(t) = 1 - e^{-t} \sum_{k=0}^{K-1} \frac{t^k}{k!}; \quad K = 1, 2, \ldots.$$

As a numerical example, let $\lambda = 0.5$, $\mu = 1$, $\beta = 2$, $K = 2$. First, note that $\mathcal{A}(\infty) = 0.693$ for **W**-repair and that $\mathcal{A}(\infty) = 0.5$ for **E**-repair ($K = 2$).

Figure 1 clearly indicates that the overall availability (i.e., the point *and* the long-run availability) is not worse than $\inf_{t \geq 0} \mathcal{A}(t) = 0.61$ for **W**-repair and not worse than $\inf_{t \geq 0} \mathcal{A}(t) = 0.497$ for **E**-repair. Figure 2 compares the values of $\mathcal{A}(t)$ for **W**-repair versus **E**-repair and for various values of λ in the region $0.1 \leq \lambda \leq 0.9$.

Finally, Figure 3 compares the values of $\mathcal{A}(t)$ for **E**-repair. Case $K = 2$ versus $K = 10$.

FINAL REMARK

Note that the integral equation (7.1) can be written as

$$1 - \mathcal{A}(t) = \lambda \, \mathbf{E}r \int_0^t \mathcal{A}(t - x) \frac{1 - R(x)}{\mathbf{E}r} dx.$$

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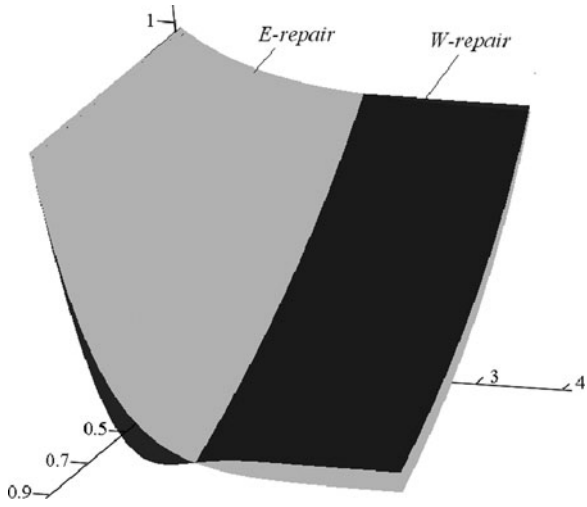


FIGURE 2. Graph of $\mathcal{A}(t)$ for **W**-repair versus **E**-repair, Case $0.1 \leq \lambda \leq 0.9, \mu = 1, \beta = 1, K = 2$.

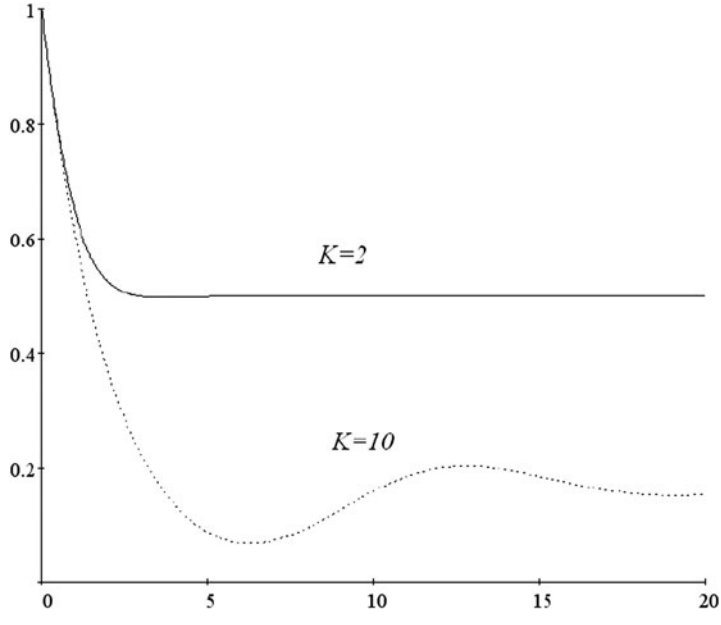


FIGURE 3. Graph of $\mathcal{A}(t)$ for **E**-repair, $K = 2$ versus $K = 10$.

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But,

$$\int_0^t \mathcal{A}(t-x)\frac{1-R(x)}{\mathbf{E}r}dx \leq \int_0^\infty \frac{1-R(x)}{\mathbf{E}r}dx = 1.$$

Thus, $\lambda \mathbf{E}r << 1$ implies that the overall availability is very close to 1 for *all* $t \geq 0$ and for *all* R. Hence, the impact of fast repair on the overall availability of the robot is apparently much larger than the impact of the repair rate.

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