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PROBLEMS AND SOLUTIONS

PROBLEMS

02.1.1. *LS and BLUE Are Algebraically Identical*, proposed by Richard W. Farebrother. Let X be a given $n \times q$ matrix of rank q, let y be a given $n \times 1$ matrix, and let b be a $q \times 1$ matrix of unknown elements. Then the least squares problem is defined by:

$\min e'e$ subject to $e = y - Xb$.

Further, let $c \neq 0$ be a given $q \times 1$ matrix and let a be an $n \times 1$ matrix of unknown elements. Then the minimum variance unbiased linear estimation problem is defined by:

$\min a'a$ subject to $X'a = c$.

It is well known that the *solutions* $a^* = X(X'X)^{-1}c$ and $b^* = (X'X)^{-1}X'y$ to these problems are connected by the relationship $a^{*\prime}y = c'b^*$. Readers are invited to show that the *problems* themselves are algebraically identical.

02.1.2. *A Particular Symmetric Idempotent Matrix*, proposed by Heinz Neudecker. Consider the $q \times q$ symmetric real matrices B and

$$C = \begin{pmatrix} rI_m & 0 \\ 0 & 0 \end{pmatrix},$$

$m < q$, integer $r > 1$. Let $tr(B^k) = tr(C^k)$, $k = 1,2,3,\ldots$. Show that $(1/r)B$ is idempotent.

CORRIGENDUM

Geert Dhaene has pointed out that his solution to Problem 00.1.1′, Determinant of a Skew-Symmetric Matrix, published in *Econometric Theory* 17, pp. 277–278, contains an error. A correction by the author is the following.

By the skew-symmetry of S, we have $|S_{j1,1i}| = (-1)^{n-2}|S_{i1,1j}|$ and

$$|S_{i1,1j}| + |S_{j1,1i}| = \begin{cases} 2|S_{i1,1j}| & \text{if } n \text{ is even;} \\ 0 & \text{if } n \text{ is odd.} \end{cases}$$

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Therefore, the statement $|S| = \sum_{i=2}^{k} s_{i1}^2 |S_{i1,1i}|$ at the beginning of page 278 (and the proof) is correct only when n is odd. A correct solution of the problem when n is even is as follows. Consider the $n \times n$ skew-symmetric matrix

$$S = \begin{pmatrix} S_{11} & -S_{21}' \\ S_{21} & S_{22} \end{pmatrix}, \quad \text{where } S_{22} = \begin{pmatrix} 0 & -s_{21} \\ s_{21} & 0 \end{pmatrix}$$

and s_{21} is a scalar. $|S_{22}| \geq 0$. Assume that skew-symmetric matrices of even order up to $(n-2) \times (n-2)$ have a nonnegative determinant. Then, if $s_{21} \neq 0$,

$$|S| = |S_{22}||S_{11} + S_{21}' S_{22}^{-1} S_{21}| \geq 0,$$

since $S_{11} + S_{21}' S_{22}^{-1} S_{21}$ is skew-symmetric. Moreover $|S| \geq 0$ also if $s_{21} = 0$, by the continuity of $|S|$ in s_{21}. The result follows by induction.

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