J. Plasma Physics (2012), *vol.* 78, *part* 5, *pp.* 531–535. © Cambridge University Press 2012
doi:10.1017/S0022377812000244

Dissipative instability under weak beam–plasma coupling

EDUARD V. ROSTOMYAN

Institute of Radiophysics & Electronics, National Academy of Sciences of Armenia,
Alikhanyan Str. 1, Ashtarak, 0203, Armenia
(eduard_rostomyan@mail.ru)

(Received 22 December 2011; accepted 7 February 2012; first published online 7 March 2012)

Abstract. Beam–plasma interaction is investigated in a model of plasma microwave oscillator: waveguide with spatially separated plasma and beam layers of finite thicknesses. Investigation is carried out in general form without specifying shape of the waveguide in cross section. Approach is based on perturbation theory over wave coupling. Spatial separation implies weak beam–plasma interaction that exhibits many specific features. Developing instability is caused by the growth of the negative energy beam wave. It is shown that upon weak coupling presence, dissipation leads to a new type of dissipative beam instability. Its maximal growth rate is inversely proportional to collision frequency in plasma. The growth rate of this instability is obtained for an arbitrary level of dissipation. Basic parameters of instability show that its properties cannot be neglected upon design of high-power, high-frequency plasma-filled generators/amplifiers based on relativistic e-beams.

1. Introduction

The use of plasma pre-fill in microwave devices allows their operation at higher beam currents and leads to the ability to gradually change the frequency of output radiation. In addition, plasma mitigates voltage depression and improves the characteristics of output radiation. These and some other principal advantages make plasma-filled sources of microwave radiation based on relativistic electron beams very promising (see [1–9] and citations therein). The development of the theory of plasma-filled microwave devices considers a cylindrical waveguide with a thin annular electron beam and coaxial thin annular plasma spatially separated from the beam (no overlap) [2–3].

Interaction of an electron beam with plasma in a waveguide has been widely investigated. However, primary attention has been paid to the case that corresponds to full overlap of beam and plasma fields (strong beam–plasma coupling). In this case, physical nature of developing instability is due to the induced radiation of the system's normal mode oscillations by beam electrons. They are determined by plasma alone, as its density is assumed much higher than the beam density. The beam oscillations are actually suppressed and do not reveal themselves. The fields are actually detached from the beam in that they exist in the absence of beam.

The opposite case when the beam and plasma fields are slightly overlapped is the case of weak beam–plasma coupling. It may be realized, for instance, if the beam and the plasma in a waveguide are spatially separated. This transverse profile is of considerable interest to plasma microwave electronics but has not been actively studied. Along the advantages of plasma filling, this transverse geometry provides conditions for increasing the influence of the beam's normal mode oscillations.

Weak coupling changes the physical nature of the beam–plasma interaction. Electron beam is actually left to its own. Its oscillations come into play. Account of the beam's normal mode oscillation leads to substantially new effects. Moreover, there is negative energy beam wave (NEBW) among beam proper waves. Its growth causes instability due to the sign of its energy. The growth rate of this instability attains maximum in the resonance of plasma wave with NEBW. Resonance of this (wave–wave) type comes instead of wave–particle resonance (the usual Cherenkov Effect). This interaction was named the "Collective Cherenkov Effect" [3]. The transformation of the single particle Cherenkov Effect to that of collective type takes place in case of weak coupling as well as in very high beam currents when it exceeds limiting current in a vacuum waveguide. The oscillations of over-limiting electron beams reveal themselves significantly for both weak and strong coupling, and the beam–plasma instability is due to the growth of NEBW [2–3, 9–11]. The only condition for this is that the beam–plasma waveguide should be non-uniform in its cross section.

There is another factor that also leads to beam instability caused by the growth of NEBW. This is dissipation. It serves as a channel for energy removal for the growth of NEBW. This shows that dissipation is actually coupled with space charge phenomena and its effect also should be taken into account. In a system with electron beam, dissipation becomes a factor that not only limits growth of the fields but also influences the mode structure of instability and reduces the growth rate. Along with these properties of common application,

https://doi.org/10.1017/S0022377812000244 Published online by Cambridge University Press

dissipation plays a unique role. Regardless of its level, it never suppresses beam instability completely. With increase in the level of dissipation, conventional beam–plasma instability is transformed to dissipative beam–plasma instability (DBI). Actually, dissipation causes instability. Herewith all types of beam–plasma instabilities (Cherenkov, cyclotron, etc) are transformed to the same type of DBI. Its maximal growth rate depends on parameter characterizing dissipation as $\sim 1/\sqrt{\nu}$ (ν is the effective frequency of collisions in plasma). Up to recently, this was the single known type of DBI. It differs from conventional beam–plasma instabilities, in particular, by low growth rate as well as by relatively low energy of excited oscillations.

Some more reasoning should be mentioned that evidently shows that dissipation must be taken into account. One of the basic trends of microwave electronics is increasing frequency of output radiation. The trend leads to a decrease in skin depth in the walls of resonators. Their Q-quality decreases. Actually, dissipation increases. One more basic trend of plasma microwave electronics is increasing of output power. The second trend leads to increasing of beam current along with the role of beam space charge oscillations. Dissipation is actually coupled with space charge oscillations and its role increases also. With increase in beam current, return current, which flows mainly via metallic surfaces, increases as well. Finite conductivity of metallic surfaces and decrease of the skin depth lead to significant increase of dissipation level. Here it is pertinent to note that dissipation of return current in metallic walls leads to the so-called resistive wall instability. This is an important special case of dissipative beam instabilities. The importance is due to its important role in particle accelerators [13].

From all this, it follows that the theory of future high-power, high-frequency microwave devices should be constructed with account of dissipation. Dissipative phenomena in the physics of beam–plasma interaction have been treated mainly for strong beam–plasma coupling, and only for particular cases of unbound beam–plasma systems with underlimiting electron beams (e.g. see [14]). Influence of dissipation on weak beam–plasma interaction is studied much less. This is due to complicated mathematical tool for correct and adequate consideration of non-uniform cross-section waveguide. First the problem was considered in [12], where the beam and plasma pipes in cylindrical waveguide were assumed to be infinitesimal thin. It was shown that dissipation leads to a new type of dissipative beam instability, whose growth rate had more critical, inverse proportional dependence on dissipation. More critical (as compared to conventional) dependence is the result of superposition of dissipation on instability that already is caused by the NEBW excitation.

There is no doubt that the assumption of infinitesimal thinness of beam and plasma layers imposes restrictions on the generality of considerations. Actually, there is a need to link to real systems. In other words, weak beam–plasma interaction should be considered in much more general form, without the assumption. Present investigation makes up the deficiency. It considers weak interaction of beam and plasma layers of finite thicknesses in presence of dissipation.

One more deficiency is made up in the present paper. Usually beam–plasma interaction theory treats dissipation in two limits: $\nu \to 0$, when no dissipative instability develops, and $\nu \gg \delta$ (δ is the growth rate of no dissipative instability). The last case corresponds to dissipative instability. Consideration of these extreme cases does not allow taking into account dissipation of an arbitrary level, as well as makes impossible to trace in detail the transition of no dissipative instability to dissipative with increase in dissipation level. Present paper considers influence of dissipation of arbitrary level on weak beam–plasma interaction. The growth rate is obtained for an arbitrary value of ν/δ ratio. This actually presents dynamics of gradual transition of no dissipative instability to that of dissipative type with increase in dissipation level. Consideration is carried out without preliminarily specifying the shape of the waveguide's cross section.

2. Statement of the problem

Consider interaction of monoenergetic electron beam and plasma in a waveguide in general form. The only assumption that we comply is as follows: The beam and plasma are separated spatially, which implies weak coupling. For a start, we do not particularize shape of the waveguide in its cross section. The beam current is assumed to be less than the limiting vacuum current. Dissipation in the system is taken into account by accounting collisions in plasma. We restrict ourselves by a case of a strong external longitudinal magnetic field that prevents transverse motion of beam and plasma particles.

In general, the treatment of the problem may be developed in different ways. The traditional way is to consider a multi-layer structure of a given geometry. This leads to a very cumbersome dispersion relation. But in the case of weak coupling (namely, when the integral describing the overlap of the beam and the plasma fields is small), the interaction of beam with plasma may be considered by using another approach – perturbation theory over wave coupling. Parameter of weak beam–plasma coupling serves as small parameter that underlies this approach. It leads to a dispersion relation of much simpler form, which, in addition, clearly shows the interaction of the beam and plasma waves. Also, the procedure is not associated with a specific shape of the waveguide, i.e. obtained results may be easily adapted to waveguide of any cross section. The approach proceeds from an assumption of independence of beam and plasma in the zero-order approximation.

https://doi.org/10.1017/S0022377812000244 Published online by Cambridge University Press

In strong external magnetic field, perturbations in plasma and beam have longitudinal component only. In such a system it is expedient to describe perturbations by using polarization potential ψ. This actually is a single non-zero component of well-known Hertz vector. Longitudinal component of the electric field E_z is expressed as

$$E_z = \left(\frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\psi, \tag{1}$$

where z is a longitudinal coordinate, t is the time and c is the speed of light. For the problem stated above, the initial equations for polarization potential ψ, perturbations of the beam j_{bz} and the plasma j_{pz} currents are as follows:

$$\begin{gathered}\frac{\partial}{\partial t}\left(\Delta_\perp + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial}{\partial t^2}\right)\psi = -4\pi j_{pz} - 4\pi j_{bz},\\ \left(\frac{\partial}{\partial t} + \nu\right) j_p = \frac{\omega_p^2}{4\pi} E_z,\\ \left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial z}\right) j_b = \frac{\omega_b^2 \gamma^{-3}}{4\pi}\frac{\partial}{\partial t} E_z,\end{gathered} \tag{2}$$

where $\Delta_\perp$ is the Laplace operator over transverse coordinates, $\omega_{p,b}$ are the Langmuir frequencies for plasma and beam, respectively, ν is the effective collision frequency in plasma, $\gamma = (1 - u^2/c^2)^{-1/2}$ and u is the velocity of beam electrons. Transversal profiles of perturbations are given by functions $P_{p,b}(\mathbf{r}_\perp)$, i.e. $j_{pz}(z, \mathbf{r}_\perp) = j_p(z)\, P_p(\mathbf{r}_\perp)$ and $j_{bz}(z, \mathbf{r}_\perp) = j_b(z) P_b(\mathbf{r}_\perp)$ (e.g. for homogeneous beam–plasma, $P_{b,p} \equiv 1$).

The set of (2) reduces to the following eigenvalue problem:

$$\Delta_\perp \psi - \kappa^2 [1 - P_p(\mathbf{r}_\perp)\delta\varepsilon_p - P_b(\mathbf{r}_\perp)\delta\varepsilon_b]\psi = 0, \quad \psi|_\Sigma = 0, \tag{3}$$

where ψ is the proper function of the problem, Σ means the surface of the waveguide (it is not specified yet) $\kappa^2 = k^2 - \frac{\omega^2}{c^2}$, $\delta\varepsilon_p = \frac{\omega_p^2}{\omega(\omega + i\nu)}$ and $\delta\varepsilon_b = \frac{\omega_b^2}{\gamma^3(\omega - ku)^2}$. As we have mentioned above, direct solution of problem (3) presents considerable difficulties. In case of spatially separated beam and plasma, i.e. when $P_b(\mathbf{r}_\perp)P_p(\mathbf{r}_\perp) = 0$, and the integral describing of the overlap of the fields (see below) is small, it is possible to apply the perturbation theory. In zero-order approximation, the beam and plasma are considered as independent and they may be described by two independent eigenvalue problems for plasma and beam, respectively:

$$\Delta_\perp \psi_\alpha - \kappa^2 [1 - P_\alpha(\mathbf{r}_\perp)\delta\varepsilon_\alpha]\psi_\alpha = 0, \quad \psi_\alpha|_\Sigma = 0, \quad \alpha = p, b. \tag{4}$$

Proper functions ψ_p and ψ_b of these zero-order problems, as well as the zero-order dispersion relations,

$$D_{p,b}(\omega, k) = k_{\perp p,b}^2 - \kappa^2 \delta\varepsilon_{p,b} = 0, \tag{5}$$

are assumed to be known, where

$$k_{\perp\alpha}^2 = \frac{\left(\iint_{S_w} \left((\nabla_\perp \psi_\alpha)^2 + \kappa^2 \psi_\alpha^2\right) d\mathbf{r}_\perp\right)}{\left(\iint_{S_w} P_\alpha(\mathbf{r}_\perp)\psi_\alpha^2 d\mathbf{r}_\perp\right)}, \tag{6}$$

where S_w is the transverse cross-section area of the waveguide. If one applies perturbation theory to the zero-order problems that are described by

$$D_p(\omega = \omega_0, k = k_0) = D_b(\omega = \omega_0, k = k_0) = 0, \tag{7}$$

(the point $\{\omega_0, k_0\}$ is the intersection point of the plasma and beam curves) and search solution for (3) in the form $\psi = A\psi_p + B\psi_b$, $A, B = \text{const}$, one can obtain in first-order approximation the following dispersion relation:

$$D_p(\omega, k) D_b(\omega, k) = G\left(\kappa^4 \delta\varepsilon_p \delta\varepsilon_b\right)_{\omega=\omega_0,\ k=k_0}, \tag{8}$$

where

$$G = \frac{\left(\iint_{S_w} P_p \psi_p \psi_b d\mathbf{r}_\perp\right)\left(\iint_{S_w} P_b \psi_p \psi_b d\mathbf{r}_\perp\right)}{\left(\iint_{S_w} P_p \psi_p^2 d\mathbf{r}_\perp\right)\left(\iint_{S_w} P_b \psi_b^2 d\mathbf{r}_\perp\right)} > 0. \tag{9}$$

Actually, G is the coupling coefficient. It shows the efficiency of beam–plasma interaction. Mathematically it is expressed through integrals that represent the overlap of beam–plasma fields. Physically it determines how far the field of plasma wave penetrates into beam and *vice versa*. According to the conditions of our consideration, G is small, i.e. $G \ll 1$. Another condition for the applicability of the presented approach is homogeneity of beam and plasma inside layers.

Expressions (5) are the dispersion relations for individual magnetized plasma and beam in waveguide. Their solutions give the spectra of normal mode oscillations in the zero-order approximation. The spectra of the beam waves are given by

$$k_{\perp b}^2 - \kappa^2 \frac{\omega_b^2/\gamma^3}{(\omega - ku)^2} = 0, \tag{10}$$

and have the following form:

$$\omega_\pm = ku(1 + x_\pm); \quad x_\pm = \frac{\sqrt{\alpha}}{\gamma}(\pm\sqrt{\beta^4\gamma^2\alpha + 1} - \beta^2\gamma\sqrt{\alpha}), \tag{11}$$

where $\alpha = \omega_b^2/k_{\perp b}^2 u^2 \gamma^3$ and $\beta = u/c$. The parameter α actually determines the beam current as it corresponds (correct to the factor γ^{-2}) to the ratio of the beam current I_b to the limiting current in vacuum waveguide I_0. The order of I_0 is determined by $\omega_{b\,\text{lim}}^2 \sim \frac{u^2}{S}\gamma \sim k_{\perp b}^2 u^2 \gamma$, and $\alpha = \frac{I_b}{\gamma^2 I_0}$. In the limit of underlimiting beams, $I_b \ll I_0$ (or $\alpha \ll 1/\gamma^2$), expressions for beam waves take the simple form $x_\pm \to \pm\sqrt{\alpha}/\gamma$. The interaction of a negative energy beam wave in this form (lower sign) with plasma in the presence of dissipation is considered below.

https://doi.org/10.1017/S0022377812000244 Published online by Cambridge University Press

3. Growth rates

In the absence of dissipation, the conventional beam–plasma interaction results in instability that is due to the excitation of the system normal mode waves by the beam. Its maximal growth rate depends on beam density, $\sim n_b^{1/3}$. With increase in the level of dissipation, the conventional beam instability is gradually converted into that of dissipative-type. Its maximal growth rate depends on dissipation as $\sim \nu^{-1/2}$. For these instabilities the normal mode oscillations of the beam are neglected. The concept of negative energy wave is invoked to explain the physical meaning of dissipative instability. These results are only valid for the case of strong beam–plasma coupling. The dispersion relation (8) is obtained for the opposite case of weak beam–plasma coupling. Decrease in beam–plasma coupling leads to the exhibition of beam's normal mode oscillation. In this case the instability is caused by the excitation of NEBW. Specific features of weak beam–plasma interaction should appear in solutions of (8). If one looks these in the form $\omega = ku(1+x)$, then (8) becomes

$$\left(x + q + i\frac{\nu}{ku}\right)(x - x_+)(x - x_-) = G\frac{\alpha}{2\gamma^4}, \qquad (12)$$

where $q = \frac{1}{2\gamma^2}\left(\frac{k_\perp^2 u^2 \gamma^2}{\omega_p^2} - 1\right)$. The usual Cherenkov resonance of beam electrons with plasma wave corresponds to the condition $q = 0$; however, the resonance between the beam slow wave and plasma wave (the collective Cherenkov effect) corresponds to $q = -x_-$.

Expression (8) obviously shows the interaction of the beam and plasma waves. The interaction leads to instability. Mathematically, it is due to corrections to the expression for the slow wave (10). Using the condition of the collective Cherenkov resonance, one can easily obtain the following equation for the corrections x' to the expression of the slow wave $x = -\sqrt{\alpha}/\gamma + x'$,

$$\left(x' + \frac{i}{2\gamma^2}\frac{\nu}{ku}\right)x' = -G\frac{\sqrt{\alpha}}{4\gamma^3}. \qquad (13)$$

In the absence of dissipation, the growth rate of instability caused by growth NEBW $\mathrm{Im}\,\omega = ku\,\mathrm{Im}\,x' \equiv \delta_{\mathrm{nebw}}^{(\nu=0)}$ is given by

$$\delta_{\mathrm{nebw}}^{(\nu=0)} = \frac{ku}{2\gamma}\left(\frac{G\sqrt{\alpha}}{\gamma}\right)^{1/2}, \qquad (14)$$

and depends on beam density, $\sim n_b^{1/4}$. It should be mentioned that the system is stable under conventional Cherenkov resonance. Dissipation exhibits itself as additional factor that intensifies growth of NEBW. Equation (13) gives the following expression for growth rate in case of arbitrary level dissipation:

$$\mathrm{Im}\,\omega \equiv \delta(\lambda) = \delta_{\mathrm{nebw}}^{(\nu=0)}\left\{\sqrt{1 + \frac{\lambda^2}{4}} - \frac{\lambda}{2}\right\}, \qquad (15)$$

where $\lambda = \frac{\nu}{\delta_{\mathrm{nebw}}^{(\nu=0)}}\frac{1}{2\gamma^2}$ actually is the ratio of dissipation to the growth rate of no dissipative instability. Expression (15) shows gradual transition of no dissipative instability to that of dissipative instability with increase in the level of dissipation. Dependence of the growth rate on dissipation level is depicted in Fig. 1. In the limit of strong dissipation $\lambda \gg 1$, (15) becomes

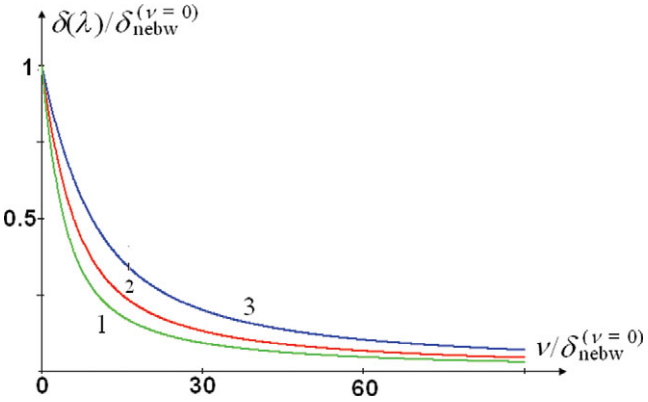


Figure 1. (Colour online) Dependence of the growth rate of instability caused by growing of negative energy beam slow wave on dissipation level. Curve 1 is plotted for $\gamma = 1.67$; curve 2 for $\gamma = 2.0$; and curve 3 is plotted for $\gamma = 2.5$.

$$\delta\left(\nu \gg \delta_{\mathrm{nebw}}^{(\nu=0)}\right) = \delta_{\mathrm{nebw}}^{(\nu\to\infty)}\left[1 - G\frac{(ku)^2}{\nu^2}\gamma\sqrt{\alpha}\right], \qquad (16)$$

where the factor before the brackets

$$\delta_{\mathrm{nebw}}^{(\nu\to\infty)} = 2\gamma^2\frac{\left(\delta_{\mathrm{nebw}}^{(\nu=0)}\right)^2}{\nu} = \frac{(ku)^2}{\nu}\frac{G}{2\gamma}\sqrt{\alpha} \qquad (17)$$

presents the maximal growth rate of the new type of dissipative instability, as shown up in [12]. It also follows from (13) by neglecting first term in parentheses. The new type of DBI is now substantiated for beam and plasma layers of finite thicknesses in waveguide of arbitrary cross section. It results from the superposition of dissipation on the instability that is caused by the growth of NEBW. The instability comes instead of the conventional DBI (with growth rate $\sim 1/\sqrt{\nu}$) when beam–plasma coupling becomes small. The dependence of its maximal growth rate on dissipation becomes more critical.

4. Conclusions

The interaction of e-beam and plasma under weak coupling substantially differs from that of conventional case of strong coupling. With decrease in the level of coupling, normal mode oscillations of the beam reveal themselves more significantly. The conventional beam–plasma instability changes to that caused by the growth of NEBW. Dissipation in the system exhibits itself as a second factor that also contributes growth. This superposition gives a new type of dissipative beam instability. Its maximal growth rate has a more critical (as compared to conventional) inversely proportional dependence on dissipation, i.e. $\nu^{-1/2} \to \nu^{-1}$. This dissipative instability develops not only in the case of weak beam–plasma coupling but also in other cases when

https://doi.org/10.1017/S0022377812000244 Published online by Cambridge University Press

dissipation superimposes on the instability caused by the growth of NEBW, e.g. for the case of an overlimiting electron beam interaction with plasma. Actually, this is a generalization of the instability discovered in [12] for the case of infinitesimal thin beam and plasma pipes. Obtained result may be easily adapted to the real geometry of plasma-filled microwave sources: cylindrical waveguide with thin annular electron beam and coaxial thin annular plasma. In this case the proper functions of initial eigenvalue problems are modified Bessel functions and the integrals in (6) and (9) may be easily calculated.

References

[1] Barker, R. J. and Schamiloglu, E., eds. 2001 *High-Power Microwave Sources and Technologies*. New York, NY: Willey-IEEE, 528 pp.

[2] Birau, M., Krasil'nikov, M. A., Kuzelev, M. V. and Rukhadze, A. A. 1997 *Phys. Usp.* **40**, 975.

[3] Kuzelev, M. V. and Rukhadze, A. A. 2000 *Plasma Phys. Rep.* **26**, 231; Kuzelev, M. V., Loza, O. T., Rukhadze, A. A., Strelkov, P. S. and Shkvarunets, A. G. 2001 *Plasma Phys. Rep.* **27**, 669.

[4] Goeble, D. M., Carmel, Y. C. and Nusinovich, G. S. 1999 *Phys. Plasmas* **6**, 2225.

[5] Nusinovich, G. S., Mitin, L. A. and Vlasov, A. N. 1997 *Phys. Plasmas* **4**, 4394.

[6] Gold, S. H. and Nusinovich, G. S. 1997 *Rev. Sci. Instrum.* **68**(11), 3945.

[7] Ponomarev, A. V. and Strelkov, P. S. 2004 *Plasma Phys. Rep.* **30**(1), 62.

[8] Bogdankevich, I. L., Ivanov, I. E., Loza, O. T., Strelkov, P. S., Ul'yanov D. K. and Garate, E. 2007 *Techn. Phys. Lett.* **33**(6), 480.

[9] Bogdankevich, I. L., Rukhadze, A. A., Strelkov, P. S. and Tarakanov, V. P. 2003 In: *Problems of Atomic Science and Technology*" series. *Plasma Phys.* **1**, 102.

[10] Rostomyan, E. V. 2001 *Europ. J. Appl. Phys.* **14**, 177.

[11] Karbushev, N. I. and Rostomyan, E. V. 2008 *Phys. Lett.* A **372**, 4484.

[12] Rostomyan, E. V. 2007 *Europhys. Lett.* **77**, 45001.

[13] Reiser, M. 1994 *Theory and Design of Charged Particle Beams*. New York: John Wiley.

[14] Bohmer, H., Chang, J. and Raether, M. 1971 *Phys. Fluids* **14**, 150.

https://doi.org/10.1017/S0022377812000244 Published online by Cambridge University Press