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The Corrsin conjecture and anomalous transport

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Abstract. The present paper examines several models of turbulent transport in which the use of the Corrsin conjecture is effective. We discuss applying the diffusive approximation of correlation effects for a scaling description of turbulent transport in a system of random drift flows. The renormalization method of quasi-linear equations to treat the diffusion of magnetic force lines is considered. The factorization applied by Corrsin makes it possible to obtain transport estimates on the basis of power approximations of the spatial correlation function. The diffusive approximation of correlation effects is effectively used in percolation and fractal models of anomalous transport. We point out the close relation among the Corrsin conjecture, renormalized quasi-linear equations and scaling estimates in which the characteristic spatial scale also appears to be the diffusive displacement. In the present paper we focus on scaling arguments that play an important role in obtaining the estimates of transport effects.

1. Introduction

The problem of providing a description of turbulent transport still remains urgent [1–4]. One of the important directions is the research of correlation effects. Long-range correlations are responsible for anomalous transport in magnetized plasma and complex systems [5–10]. Factors such as anisotropy, different kinds of seed diffusion mechanisms and reconstruction of the flow topology significantly impact on the effective diffusivity. That is why it is difficult to choose a main correlation mechanism that is responsible for anomalous transport [1–10]. The number of scientific publications in this field has been constantly growing. Therefore, an important aspect of analysis is the consideration of the scalings that underlie complex calculations and computer simulations. Thus, the Corrsin conjecture [11] is often used to analyze turbulent transport in its different aspects [12–16]. From the formal standpoint, Corrsin suggested a simple approximation to establish the non-trivial relation between the Lagrangian and Eulerian correlation functions. It consists of two hypotheses. First, particle trajectories are statistically independent of the stochastic velocity field. Second, the displacements of particles have a Gaussian distribution. Corrsin applied the randomization method, where the Gaussian function was used as the probability density (the solution of the conventional

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diffusion equation). This implies a diffusive interpretation of the characteristic spatial scale that enters into the Eulerian correlation function. The Corrsin diffusive approximation looks very attractive, because it restores the possibility to visualize correlation effects. This is especially important in analyzing anisotropic models in which several different transport mechanisms are present simultaneously. This is not surprising, because the turbulent diffusion differs significantly from one-dimensional transport and we must introduce both different characteristic and temporal scales.

In this paper we consider several models of turbulent transport in which the use of the Corrsin conjecture is effective in both isotropic and anisotropic media. Thus, the separation of longitudinal and transverse correlation scales based on the diffusive approximation is analyzed for the systems of random shear flows and the 'braided magnetic field' model. A study is made of the relationship between the models of Corrsin and Dreizin–Dykhne; and the analogy between the Taylor method and the Kadomtsev–Pogutse diffusive renormalization approach. This gives the possibility of considering the fractional differential equations approximation and percolation estimates on the basis of the conventional theory of long-range correlations.

Many aspects of turbulent transport had to be, regretfully, left out of this paper. Moreover, even the Corrsin approach has many different aspects [1, 2, 6, 17, 18]. Thus, the direct calculation of correlation functions on the basis of the diffusive approximation suggested by Taylor and McNamara was considered in detail in the review [3], whereas the Howells form of effective diffusivity was discussed in [15]. More complex aspects of the correlation approximations in the framework of perturbation theory and the plasma physics problems can be found in the papers [7–12, 18]. Using rigorous analysis, Weinstock [18] and Kraichnan [19] shown that the Corrsin conjecture is equivalent to a first-order truncation of renormalization expansion, which can be considered systematically. Therefore, in this paper we will focus on scaling arguments that play a very important role in obtaining estimates of transport effects. For the convenience of the reader, we repeat the relevant material, thus making our exposition self-contained. Many important questions, which have no direct relation to obtaining the transport scalings, are omitted. However, the reader can easily find the necessary information in the extensive review [1–10] and other papers devoted to the different aspects of turbulent transport [20–26].

2. The Corrsin diffusive conjecture and correlations

The simplest correlation approach to the description of turbulent transport of a passive tracer is based on a stochastic equation of motion [27]

$$x(t) = \int_0^t V(t')\,dt'. \tag{1}$$

Here $V(x_0, t)$ is the Lagrangian velocity. Following the Langevin ideas, it is easy to write an expression for $x^2(t)$

$$\frac{d}{dt}x^2(t) = 2x\frac{dx(t)}{dt} = 2V(t)x(t) = 2V(t)\int_0^t V(t')\,dt'. \tag{2}$$

Then, averaging over the ensemble of Lagrangian trajectories yields the Taylor expression for the turbulent diffusion coefficient

$$D_{\mathrm{T}}(t) = \frac{d}{dt}\left(\frac{\langle x^2(t)\rangle}{2}\right) = \int_0^t \langle V(t)V(t')\rangle\,dt' = \int_0^t C(\tau)\,d\tau \approx V_0^2\tau_{\mathrm{c}}. \tag{3}$$

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The brackets $\langle \cdots \rangle$ indicate an average over the ensemble of Lagrangian trajectories. Here $C(\tau) = \langle V(t+\tau)V(t)\rangle$ is the correlation function, V_0 is the characteristic velocity and τ_c is the correlation time. The dependence $D_T \propto V_0^2$ of the turbulent diffusivity on the perturbation amplitude describes short-range correlation regimes well. On the other hand, the specific form of the expression for $D_T(t)$ depends on the behavior of the correlation function $C(t)$. There is much to be seen from approaching correlation and transport properties via the scaling method.

The expression (3) can be successfully applied not only for the transport description of a passive scalar, but also for the analysis of effects related to the stochasticity of the magnetic field, where there exists the same kind of magnetic diffusivity dependence on perturbation amplitude for short-range correlations. There is a clear physical model of the diffusive description of force lines in the braided magnetic field [3, 4, 6, 9]. This analysis is based on the stochastic equation for force lines, which is similar to the passive scalar expression (1),

$$\frac{d\mathbf{r}_\perp}{dz} = \mathbf{b}(z, \mathbf{r}_\perp), \quad \mathbf{b} = \frac{\mathbf{B}'}{B_0} \approx b_0. \tag{4}$$

Here a weak random field $\mathbf{B}'(B_x, B_Y, 0)$ is superimposed on a strong constant field $\mathbf{B}(0, 0, B_0)$ aligned with the z-axis and b_0 is the characteristic relative scale of perturbations. It is natural to introduce a force line correlation function $C_B = \langle \mathbf{b}(z, \mathbf{r}_1)\mathbf{b}(0, 0)\rangle$, which is analogous to the Taylor representation. Jokipii and Parker [28] suggested a quasi-linear expression (3) for the transverse diffusion coefficient of force lines of the magnetic field,

$$D_m = \frac{1}{4}\int_{-\infty}^{\infty} dz \langle \mathbf{b}(z, 0)\mathbf{b}(0, 0)\rangle \propto b_0^2 L_Z. \tag{5}$$

Here, L_Z is the longitudinal correlation length, $\Delta_\perp$ is the perpendicular correlation length and $b_0 L_Z > \Delta_\perp$, which corresponds to the small magnetic numbers $R_m = b_0 L_Z/\Delta_\perp < 1$.

The definition of the correlation function suggested by Taylor [27] is based on using Lagrangian velocities $V(x_0, t)$, but their experimental determination is a serious problem. That is why use is made of the Eulerian representation for the correlation function, which takes into consideration the velocity correlation at points separated by a distance λ,

$$C_E(\lambda, t) = \langle u(x_0, T)u(x_0 + \lambda, T + t)\rangle. \tag{6}$$

Here, $u(x, t)$ is the Eulerian velocity field. However, there is no simple relation between the Lagrangian correlation function and the Eulerian one [12,13]. Actually, there is no Lagrangian relation between the points x_0 and $x_0 + \lambda$ in expression (6). Here, λ is merely some arbitrary displacement. Corrsin [11] suggested an approximation formula in terms of the randomization of the Lagrangian correlation function with the probability density $\rho(x, t)$,

$$C(t) = \int_{-\infty}^{\infty} \rho(\lambda, t) C_E(\lambda, t)\, d\lambda, \tag{7}$$

in which he expressed the Lagrangian correlation function through the Eulerian one. However, a more important point is the idea of the diffusion nature of the displacement λ, because the classical solution of the diffusion equation in

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three-dimensional space was used for $\rho(\lambda, t)$,

$$\rho(\lambda, t) = \frac{1}{(4\pi D_0 t)^{3/2}} \exp\left(-\frac{\lambda^2}{4D_0 t}\right). \tag{8}$$

This formula also includes the molecular diffusion coefficient D_0. Hence, we can consider both the turbulent transport and the molecular diffusion. From this point of view, it is worth pointing out that λ is the distance and the diffusive displacement at the same time. In fact, instead of formal averaging in the form

$$\langle V(x(0))V(x(t))\rangle = \int \langle V(0)V(y)\delta(y - x(t))\rangle \, dy, \tag{9}$$

the factorization approach was used, where

$$\langle V(0)V(y)\delta(y - x(t))\rangle = \langle V(0)V(x)\rangle\langle\delta(y - x(t))\rangle. \tag{10}$$

Moreover, the Gaussian distribution (8) was employed for the description of trajectory correlations $\langle\delta(y - x(t))\rangle \approx \rho(x, t)\, dx$. Note that the factorization (10) offers an additional possibility of developing the scaling approximation of turbulent transport by the power approximations of the Eulerian correlation function and different kinds of probability density [6–10].

3. The power approximation of the correlation function and transport

Results of numerous investigations point out essentially non-diffusive character of turbulent transport in the systems with a magnetically confirmed plasma [3, 4, 9]. The variety of analyzed regimes requires not only special description methods, but also applying universal approaches, such as percolation models [6], the self-organized critically approach [3] and correlation scalings [9]. Thus, in the framework of monoscale percolation models, the classical Taylor result, $D_{\mathrm{T}} \propto V_0^2$ or $D_{\mathrm{m}} \propto b_0^2$, is modified for the cases of long-range correlations by the scalings [6]

$$D_{\mathrm{T}} \approx \lambda^2\omega\left(\frac{V_0}{\lambda\omega}\right)^{\gamma_{\mathrm{T}}} \quad \text{or} \quad D_{\mathrm{m}} \approx \frac{\Delta_\perp^2}{L_Z}\left(\frac{b_0 L_Z}{\Delta_\perp}\right)^{\gamma_{\mathrm{T}}}, \tag{11}$$

where $\gamma_{\mathrm{T}} \approx 0.7$ and ω is the characteristic perturbation frequency. On the other hand, the scaling representation of the mean free path R

$$R \propto t^H \tag{12}$$

is sufficient to consider anomalous diffusion. Here H is the Hurst exponent [6–10]. In the case of classical diffusive behavior, we find $H = \frac{1}{2}$. The cases $0 < H < \frac{1}{2}$ lead to subdiffusive behavior. The cases $\frac{1}{2} < H < 1$ correspond to superdiffusive behavior. Thus, self-organized criticality models point on the superdiffusive character of transport in a turbulent magnetized plasma with $H = 0.62$–0.72 [29]. These results are naturally interpreted in terms of scaling for the Lagrangian correlation function $C(t) \propto 1/t^{2(1-H)}$ and hence $R^2 \propto C(t)t^2 \propto t^{2H}$.

Note that one of the simplest methods to obtain a scaling representation for the correlation function is the use of seed (molecular) diffusion as the decorrelation mechanism. Indeed, the Corrsin conjecture looks fairly formal; however, it restores the possibility to visualize correlation effects and to take into account the effects of molecular diffusion. The important point to note here is that the classical definition

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of Taylor's coefficient of turbulent diffusion does not contain any information on molecular diffusivity, which can be used to describe 'collective' correlation effects [30]. Since, interactions both create and destroy correlations, an assumption was made that the number of interactions N_{I} is proportional to the number of particles that are located in the correlation region W_{D}, $N_{\mathrm{I}} \approx n W_{\mathrm{D}} \approx n R_{\mathrm{D}}^d$. Here n is the density of particles in this region, R_{D} is the spatial scale of this region and d is the space dimensionality. Then, from the dimensional point of view, the correlation effects can be expressed in the form

$$C(t) = \langle V(0)V(t)\rangle \approx V_0 \frac{V_0}{N_{\mathrm{I}}} \approx \frac{V_0^2}{nW_{\mathrm{D}}}. \tag{13}$$

Here $V(t)$ is the velocity at the moment t and V_0 is the characteristic scale of the velocity. The estimate of W_{D} can be obtained from the Gaussian distribution for the probability density (8) of diffusing particles $\rho(x,t)$. To derive the estimate an assumption was made that the correlation scale R_{D} has a diffusive nature, $R_{\mathrm{D}} \propto (D_0 t)^{1/2}$ for $t \to \infty$. Here D_0 is the molecular coefficient of diffusion. Then, simple calculations yield the scalings

$$C(t) = \langle V(0)V(t)\rangle \approx \frac{V_0^2}{n(D_0 t)^{d/2}} \propto \frac{1}{t^{d/2}}. \tag{14}$$

Such 'long tails' of correlation functions are investigated in molecular dynamics and are related to 'the collective' (hydrodynamic) nature of the systems evolution [30].

The use of scaling for the Eulerian correlation function is also fairly effective; let us consider the scaling for the spatial correlation function of velocity

$$C_{\mathrm{E}}(\lambda) = \langle V(x)V(x+\lambda)\rangle \propto V_0^2 \left(\frac{\lambda_0}{\lambda}\right)^{\alpha_{\mathrm{C}}}. \tag{15}$$

Here, α_{C} is the correlation exponent, V_0 and λ_0 are the dimensional parameters of the model. Then the dimensional consideration yields

$$C \approx v^2 \approx \frac{\lambda^2}{t^2} \approx V_0^2 \left(\frac{\lambda_0}{\lambda}\right)^{\alpha_{\mathrm{C}}}. \tag{16}$$

If we assume that the spatial scale λ appears to be the correlation scale and the diffusive displacement at the same time (as in the Corrsin conjecture), then it is possible to treat (16) as the transport scaling. Now the diffusive estimate is given by

$$\lambda \propto V_0^{2/(2+\alpha_{\mathrm{C}})} \lambda_0^{\alpha_{\mathrm{C}}/(2+\alpha_{\mathrm{C}})} t^{2/(2+\alpha_{\mathrm{C}})} \quad \text{and} \quad H = \frac{2}{2+\alpha_{\mathrm{C}}}, \quad 0 < \alpha_{\mathrm{C}} < 2. \tag{17}$$

The result (17) was rigorously obtained for the incompressible random flow in [31]. This simple scaling allows us to analyze the turbulent transport in a random two-dimensional flow based on the information about the spatial correlations of the velocity. Thus, one can see that long-range correlations (the small values of α_{C}) lead to the higher values of the Hurst exponent (large convective contribution). Moreover, in the framework of the analogous ballistic approach $\lambda(t) \approx V(\lambda)t$, it is also possible to strongly consider anisotropic models of shear flows of turbulent transport by the modified expression $\lambda_\perp(t) \approx V_\perp(\lambda_\parallel)t$, where longitudinal and transverse correlation scales are separated.

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4. The diffusive tracer equation and the Taylor model of shear flow

In spite of the effectiveness of scaling estimates of correlation effects and transport, the equations describing the evolution of the tracer distribution density play an important role in the analysis of anomalous diffusion effects. In this section, we consider the effective diffusion of a passive scalar in a shear flow in the presence of seed diffusivity. Taylor suggested a new method [32, 33] to obtain the effective diffusion coefficient, which is based on averaging the equation

$$\frac{\partial n}{\partial t} + V_X(y,z)\frac{\partial n}{\partial x} = D_0 \nabla^2 n. \tag{18}$$

Here n is the scalar density, V_X is the longitudinal (along the x-axis) velocity and D_0 is the seed diffusivity. In the Taylor model the influence of transverse molecular diffusion on longitudinal convective transport was considered. The paper [32] deals with the Poiseuille (parabolic) flow in a cylindrical tube, but in order to simplify calculations we will analyze a flat model. Considering the profile of the longitudinal flow in the form $V_X(z) = V_0(L^2 - z^2)/L^2$, we can examine the scalar transport problem within the framework of the decomposition method

$$n = \langle n \rangle + n_1(x,z,t) = n_0 + n_1(x,z,t), \quad V_X = \langle V_X \rangle + V_1 \equiv V_0 + V_1. \tag{19}$$

Here V_0 is the characteristic velocity and L is the characteristic spatial scale. Use is made of the expression for mean values

$$\langle n \rangle \equiv \frac{1}{2L}\int_{-L}^{L} n(x,z,t)\,dz \equiv n_0(x,t), \quad \langle V_X \rangle = \frac{1}{2L}\int_{-L}^{L} V(z)\,dz = \frac{2}{3}V_0. \tag{20}$$

Hence, one obtains $V_1 = V_0(\frac{1}{3} - (z/L)^2)$. The substitution of the expressions for n and V_X into the initial equation (20) yields

$$\frac{\partial}{\partial t}n_0 + \frac{\partial}{\partial t}n_1 + (V_0 + V_1)\frac{\partial}{\partial x}(n_0 + n_1) = D_0\nabla^2[n_0 + n_1]. \tag{21}$$

Taking the average of this equation, it is a simple matter to obtain an expression for the mean density evolution as follows:

$$\frac{\partial}{\partial t}n_0 + V_0\frac{\partial}{\partial x}n_0 + \left\langle V_1\frac{\partial}{\partial x}n_1 \right\rangle = D_0\frac{\partial^2 n_0}{\partial x^2}. \tag{22}$$

In order to evaluate a closed equation for the mean density of a scalar, it would be necessary to find the expression for $n_1(x,z,t)$. Subtracting (22) from (21), one obtains the equation for the evolution of the density perturbation n_1,

$$\frac{\partial n_1}{\partial t} + V_1\frac{\partial n_0}{\partial x} + V_0\frac{\partial n_1}{\partial x} + V_1\frac{\partial n_1}{\partial x} - \left\langle V_1\frac{\partial n_1}{\partial x} \right\rangle = D\left(\frac{\partial^2 n_1}{\partial x^2} + \frac{\partial^2 n_1}{\partial z^2}\right). \tag{23}$$

This expression is too complex; therefore, as suggested by Taylor, use is made of the heuristic method to obtain the estimates of effective diffusion, which is based on several hypothesis: the quasi-steadiness of n_1 i.e. $\partial n_1/\partial t \approx 0$ and the smallness of $\partial n_1/\partial x$ and $\partial^2 n_1/\partial x^2$ in comparison with $\partial n_0/\partial x$ and $\partial^2 n_1/\partial z^2$. Taylor kept the term $\partial^2 n_1/\partial z^2$ in order to take into account the density gradient in the direction of walls, which has to be greater to satisfy the no-flux condition $\partial n_1/\partial z = 0$ at $z = L$ and $z = -L$. Then, solving the equation obtained from (29)

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$D_0(\partial^2 n_1/\partial z^2) = Q(x,z,t)$, where $Q = V_1(z)(\partial n_0(x,t)/\partial x)$, it is easy to find

$$n_1 = \frac{\partial n_0}{\partial x}\frac{V_0}{3D_0}\left[\frac{z^2}{2} - \frac{z^4}{4L^2}\right] + G. \tag{24}$$

Applying the condition $\langle n_1 \rangle = 0$, we arrive at

$$\frac{\partial n_1}{\partial x} = \frac{\partial^2 n_0}{\partial x^2}\frac{V_0}{3D_0}\left[\frac{z^2}{2} - \frac{z^4}{4L^2} - \frac{7}{60}L^2\right]. \tag{25}$$

The expression $\langle V_1 \partial n_1/\partial x \rangle$, which defines an additional contribution in longitudinal diffusive transport, can be rewritten in the form

$$\left\langle V_1 \frac{\partial n_1}{\partial x} \right\rangle = -D_1 \frac{\partial^2 n_0}{\partial x^2} = -\frac{8}{945}\frac{(V_0 L)^2}{D_0}\frac{\partial^2 n_0}{\partial x^2}. \tag{26}$$

The result obtained is non-trivial, because the additional diffusive contribution depends inversely on the seed diffusivity D_0. The physical interpretation of this result is in the limitation of the non-uniformity influence of the longitudinal velocity profile $V_X(z)$ by transverse diffusion. Hence, non-uniform longitudinal convection in combination with transverse diffusion leads to non-trivial longitudinal diffusion. Naturally, a new diffusive mechanism manifests itself at a large distance downstream only, since the equation obtained is correct when $t \gg \tau_D \approx L^2/D$. On the other hand, use was made of the condition of smallness of the transverse spatial scale in comparison with the longitudinal one l: $L \ll l$. The effective diffusion coefficient can be rewritten in terms of the Peclet number $Pe = V_0 L/D_0$,

$$D_{\text{eff}} = D_0 + \left(\frac{8}{945}\right)\frac{V_0^2 L^2}{D_0} = D_0 + D_0\left(\frac{8}{945}\right)Pe^2. \tag{27}$$

Despite the fact that Taylor considered a laminar flow, his paper allows the understanding of methods used for the description of both turbulent transport of a passive scalar [15–19] and magnetic diffusivity [3, 4], since application of diffusive terms to describe non-trivial correlation effects is widely used [6, 7]. Also the effective transport in the system with complex structures, such as convective cells, is described by the scaling in terms of the Peclet number $D_{\text{eff}} \propto Pe^{1/2}$, where the exponent 1/2 mirrors the topological differences of transport (convective) channels system from single laminar flow [15, 16].

5. Returns and correlation effects in random shear flow systems

In this section we pass from the laminar flow description to modeling transport in random flows with non-trivial correlation effects, which are also closely related to seed (molecular) diffusion. Corrsin [34] was probably the first to suggest using the probability of return particles to the initial flow for the description of turbulent diffusion. This correlation mechanism is very interesting, especially for the systems with random flows. Consider random fluctuations creating narrow convective flows of width a and velocity V_0 acting in the transverse direction on the diffusing test particle (see Fig. 1) together with the action of 'seed' diffusion in the longitudinal direction with diffusion coefficient D_0.

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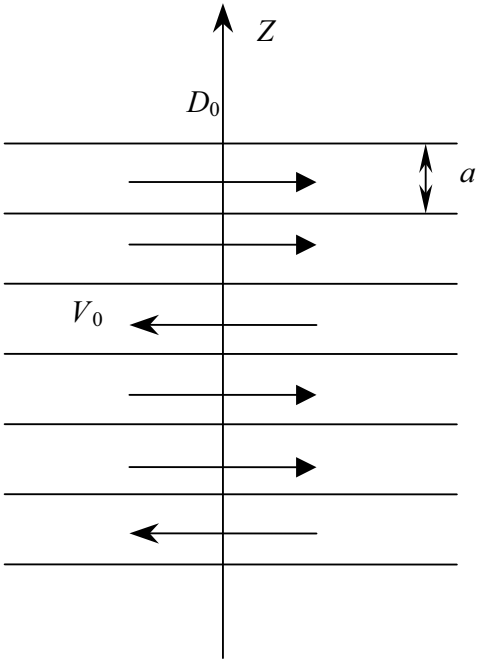


Figure 1. Random drift flow model.

To describe transport in the perpendicular direction, the correlation function in the factorization form has been considered in the Dreizin–Dykhne paper [35]:

$$C(t_1,t_2)=\int_{-\infty}^{\infty}\langle V_X(0)V_X(z)\rangle\rho(z,t_2-t_1)\,dz,$$
$$\rho=\frac{1}{(4\pi D_0(t_2-t_1))^{1/2}}\exp\left(-\frac{z^2}{4D_0(t_2-t_1)}\right). \tag{28}$$

Here $V_X(z)$ is the velocity of the flow at the point z. This method corresponds exactly to Corrsin's idea about the diffusive nature of decorrelation [11] with allowance for the anisotropy of the model. However, the main step in describing anomalous diffusion has become using the conjecture about the significant role of returns, since the condition $z\to 0$ for (28) corresponds to the return to the initial point. Thus, we have an expression

$$C(t_1,t_2)=C(\tau)\approx\frac{V_0^2 a}{\sqrt{4\pi D_0\tau}},\quad \tau=t_2-t_1. \tag{29}$$

Using the classical result from turbulent diffusion theory (3), the transport estimate was obtained as

$$\langle\lambda_\perp^2\rangle\approx\frac{V_0^2 a}{\sqrt{4\pi D_0}}\int_0^t\int_0^t\frac{dt_1\,dt_2}{\sqrt{t_1-t_2}}\approx\frac{V_0^2 a}{\sqrt{4\pi D_0}}t^{3/2}. \tag{30}$$

Here $\langle\cdots\rangle$ denotes the averaging symbol. This corresponds to superdiffusive transport with the Hurst exponent $H=\frac{3}{4}>\frac{1}{2}$, which is related to the random nature of shear flows. But in terms of the Peclet number we deal with a Taylor kind of dependence, $D_{\text{eff}}\propto Pe^2\propto V_0^2$. The representation of the correlation function in the

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form (30) is valid only for the case when the diffusive displacement in the transverse direction is much less than the transverse correlation scale length: $\lambda_\perp < \Delta_\perp \approx a$. This restriction yields the estimates

$$\lambda_\perp^2 \approx \frac{V_0^2 a}{\sqrt{D_0}} \tau_{\rm C}^{3/2} \leqslant a^2, \quad \tau_{\rm C} \approx \left(\frac{D_0 a^2}{V_0^4}\right)^{1/3}. \tag{31}$$

If timescales $t > \tau_{\rm C}$, then we deal with classical diffusion, $D_\perp \propto a^2/2\tau_{\rm C} \propto V_0^{4/3}/D_0^{1/3}$. Note that (29) includes the number of intersected flows $N_{\rm I}$ or the number of returns $N_{\rm B}$ for the particle walking along a straight-line trajectory for time t as $N_{\rm B}(t) \approx N_{\rm I}(t) \propto \sqrt{2D_0 t}/a$, which gives the possibility of representing the result (30) in the scaling form (32),

$$C(t) \propto \frac{V_0^2}{N_{\rm I}(t)} \quad \text{or} \quad C(t) \propto \frac{V_0^2}{N_{\rm B}(t)}. \tag{32}$$

Let us employ scaling arguments to the anisotropic form of the Corrsin functional by direct substitution of the power form of the correlation function $\langle V_X(0) V_X(z)\rangle \approx V_0^2/(1+(z/z_0)^{\alpha_{\rm C}})$ with the correlation exponent $\alpha_{\rm C}$,

$$C(t) \approx \frac{\lambda_\perp^2}{t^2} \approx \int_{-\infty}^{\infty} \frac{V_0^2}{1+(z/z_0)^{\alpha_{\rm C}}} \exp\left(-\frac{z^2}{4D_0 t}\right) \frac{dz}{\sqrt{4\pi D_0 t}}. \tag{33}$$

Here, V_0 and z_0 are the dimensional parameters of the correlation function. Using the dimensionless variable $z^2/4D_0 t$ in order to calculate the integral, we obtain the simple estimate in the scaling form: $C(t) \approx \lambda_\perp^2/t^2 \approx \text{constant}/t^{\alpha_{\rm C}/2}$ and $\lambda_\perp \approx t^{1-\alpha_{\rm C}/4}$. This yields the relationship between the Hurst exponent H, which describes transport in the perpendicular direction and the correlation exponent $\alpha_{\rm C}$, which describes correlation properties in the longitudinal direction [36–39],

$$H = 1 - \frac{\alpha_{\rm C}}{4}, \quad 0 < \alpha_{\rm C} < 2. \tag{34}$$

We can see that for the case $\alpha_{\rm C} = 0$, this expression leads to the Hurst exponent $H = 1$. This corresponds to particle transport with a ballistic character. The result (30) corresponds to the case $\alpha_{\rm C} = 1$. For the case $\alpha_{\rm C} = 2$ we obtain the classical diffusive scaling law $\lambda_\perp^2 \propto t$. Note that the considered system of correlated shear flows can be used as a transport model for a system of zonal flows [3,4] in a tokamak (Fig. 2). Thus even effects related to transport barriers could be taken into consideration by the substitution of a probability density based on subdiffusive scaling $z \propto t^{H_\parallel}$ instead of the Gaussian distribution ρ with the diffusive estimate $z^2 \propto D_0 t$. Then simple estimates yield the scaling relating longitudinal and transverse transport effects $H_\perp = 1 - \alpha_{\rm C} H_\parallel/2$, which is the generalization of the scaling (30).

Note that the separation of longitudinal and transverse correlation scales plays an important role for the description of turbulent transport in strongly magnetized plasma. However, since in a strong magnetic field we can consider transport in the transverse direction within the framework of a two-dimensional approach, and longitudinal particle motion has a ballistic character; therefore the model of transverse pulsations requires an essential modification.

6. The diffusive approximation and the 'braided' magnetic field

The time dependence of a flow is an important factor that leads to the reconstruction of the streamline topology and influences the transport processes. To describe

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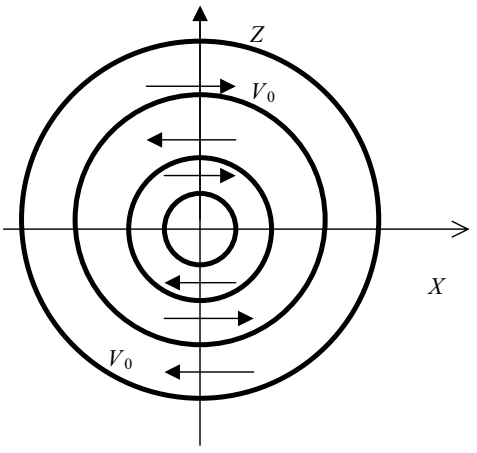


Figure 2. Zonal flow system.

this situation we need a dimensionless parameter, which includes the characteristic time of the flow topology reconstruction $T_0 \approx 1/\omega$. In the case of high frequencies ω, the path of a test particle can be estimated ballistically as $\Delta_\omega \approx V_0/\omega$. Then the convective fraction of transport can be characterized by the dimensionless Kubo number $Ku = \Delta_\omega/\lambda \approx V_0/\omega\lambda$, where λ is the spatial scale of the flow under consideration. In the conventional case we obtain the estimate of the diffusion coefficient with $\tau_c \approx 1/\omega$ and $\Delta_{cor} \approx \Delta_\omega$ as follows:

$$D_T \approx \frac{\Delta_{cor}^2}{\tau_c} \approx \frac{V_0^2}{\omega} \propto Ku^2. \tag{35}$$

There exists a question about the dependence of the effective diffusion coefficient on the Kubo number. Thus, increasing Ku mirrors the growth of correlation effects [1,3,6], and in such conditions the real correlation scale is smaller than the frequency path Δ_ω due to the alteration of the flow topology. This corresponds to low-frequency regimes with $Ku > 1$.

Analyzing magnetic diffusion models, in which transverse correlations are significant, we deal with the analogous situation (5), but the Kubo number is replaced by $R_m = b_0 L_Z/\Delta_\perp$, $b_0 L_Z \geqslant \Delta_\perp$ and $R_m > 1$. Thus, when describing transport in a strongly magnetized plasma, one deals with the braided magnetic field and particles move in the longitudinal direction almost ballistically, whereas magnetic perturbations act in the perpendicular direction. This gives the possibility of interpreting the longitudinal coordinate in terms of time and to use the diffusive approximation of long-range correlations in the transverse direction. Indeed, Kadomtsev and Pogutse [40] introduced a continuity equation for the density of magnetic field lines,

$$\frac{\partial n_b}{\partial z} + \mathbf{b}\nabla_\perp n_b(\mathbf{r}_\perp, z) = 0, \quad \mathbf{b} = \frac{\mathbf{B}'}{B_0} \approx b_0, \tag{36}$$

and represented n_b as a sum of the mean density $n_0 = \langle n_b \rangle$ and the fluctuation component n_1, $n_b = n_0 + n_1$. The problem as formulated would be reminiscent of the problem of quasi-linear diffusion of a passive scalar, where the value **b** corresponds to the velocity value. Consider the continuity equation for the density of passive

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scalar characterizing the incompressible flow:

$$\frac{\partial n}{\partial t} + V\frac{\partial n}{\partial x} = 0, \tag{37}$$

where $n(x,t)$ is the spatial density of the passive scalar and $V(t)$ is the random velocity field.

We also set $n = n_0 + n_1$, where $\langle n_1 \rangle = 0$ and $\langle V \rangle = 0$. As a result of simple manipulations (which are frequently used in the literature [3, 4, 41]), we arrive at the following two equations:

$$\frac{\partial n_0}{\partial t} + \left\langle V\frac{\partial n_1}{\partial x} \right\rangle = 0; \quad \frac{\partial n_1}{\partial t} + V\frac{\partial n_0}{\partial x} + V\frac{\partial n_1}{\partial x} - \left\langle V\frac{\partial n_1}{\partial x} \right\rangle = 0. \tag{38}$$

An assumption was made that the fluctuations n_1 and V are as small as ε in comparison with the mean density n_0. The quasi-linear character of the approximation mirrors itself in the equation for n_0, we keep the nonlinear term of the order of ε^2 but, in the equation for n_1, we keep only the terms that are of first order in ε. At this point, such assumptions and transformations are justified because, on the one hand, we wish to see how the equation for n_0 changes when fluctuations are taken into account and, on the other hand, we wish to obtain a solvable equation for n_1. As a result, the transformations put (38) for n_1 into the form

$$\frac{\partial n_1}{\partial t} = -V\frac{\partial n_0}{\partial x}. \tag{39}$$

The obvious drawback of the quasi-linear theory is that the nonlinear term in the equation for n_0 is retained while the nonlinear terms in the equation for n_1 are omitted. There are many papers whose authors tried to refine the quasi-linear approximation [42, 43]. A detailed analysis of such papers was carried out in [3, 4].

Within the framework of the Corrsin approach it is possible to describe the omitted correlation effects by including the additional diffusive term. The authors of [40] also wrote the equation for n_0 in the traditional form:

$$\frac{\partial n_0}{\partial z} + \nabla_\perp \langle \mathbf{b} n_1 \rangle = 0. \tag{40}$$

However, in the equation for n_1 (38), they replaced the second-order terms $\mathbf{b}\nabla_\perp n_1 - \langle \mathbf{b}\nabla_\perp n_1 \rangle$ (which were omitted in earlier studies) by a diffusion term $D\nabla_\perp^2 n_1$. Let us keep the equation for perturbation n_1 linear but pass from a hyperbolic equation of the form (39) to the parabolic equation for n_1. Moreover, the effective diffusion coefficient D_m in the additional diffusive term is used instead of the seed diffusion coefficient, which was applied in the Corrsin approach. Now the equation describing the long-range correlation effects in terms of the diffusive spreading of trajectories takes the form

$$\frac{\partial n_1}{\partial z} - D_\mathrm{m}\nabla_\perp^2 n_1 = -\mathbf{b}\nabla_\perp n_0. \tag{41}$$

Substitution of the solution for n_1, which is expressed through the Green function, into (41) yields the final equation for the mean density. The equation for n_0 is given by

$$\frac{\partial n_0}{\partial z} = D_\mathrm{m}\nabla_\perp^2 n_0. \tag{42}$$

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The magnetic diffusion coefficient and the Fourier spectrum of perturbation amplitudes are given by

$$D_{\rm m} = \frac{1}{2}\int \frac{b^2(\mathbf{k})}{ik_z + k_\perp^2 D_{\rm m}}\, d\mathbf{k}, \quad b^2(\mathbf{k}) = \frac{1}{(2\pi)^2}\int \langle \mathbf{b}(0)\mathbf{b}(r)\rangle \exp(-i\mathbf{k}\cdot\mathbf{r})\, d\mathbf{r}. \tag{43}$$

For $\Delta k_z > k_\perp^2 D_{\rm m}$, we have the quasi-linear expression

$$D_{\rm m} = \frac{\pi}{2}\int d\mathbf{k}\, b^2(\mathbf{k})\delta(k_z) \propto b_0^2 \Delta_Z, \tag{44}$$

where Δ_Z is the longitudinal correlation length. This could be interpreted as the Taylor result (3) in terms of the characteristic frequency ω, because of the analogy between the timescale and the longitudinal displacement. In the case of strong correlations $\Delta k_z < k_\perp^2 D_{\rm m}$, one arrives at

$$D_{\rm m}^2 = \frac{1}{2}\int \frac{b^2(\mathbf{k})}{k_\perp^2}\, d\mathbf{k} \propto b_0^2 \Delta_\perp^2. \tag{45}$$

Here $\Delta_\perp$ is the transverse correlation scale. In terms of the Kubo number (or the magnetic Kubo number $R_{\rm m}$) these results can be represented in the scaling form $D_{\rm m} \approx b_0\Delta_\perp \propto R_{\rm m}$, which differs markedly from the Taylor scaling (3) $D_{\rm m} \propto R_{\rm m}^2$. Note that the linear dependence $D_{\rm m} \approx b_0\Delta_\perp$ or $D_{\rm eff} \approx V_0\Delta_\perp$ are very effective to describe transport in systems with complex flow topology or low-frequency regimes [3, 4, 6]. Moreover, the expressions obtained include the spectrum of magnetic turbulence $b^2(\mathbf{k})$, which is essential for the interpretation of experimental results and comparison of the hydrodynamic description and spectral turbulent models.

7. Scaling for the longitudinal correlation function and the fractional differential equation

The Kadomtsev–Pogutse analysis was based on the ballistic representation of longitudinal (z-axis) motion and the diffusion approximation $D_{\rm eff}\partial^2 n_1/\partial r_\perp^2$ of transverse correlation effects. The opposite case, in fact corresponds to the Dreizin–Dykne model, where longitudinal motions have a diffusion character $\lambda_\parallel \approx \sqrt{2D_0 t}$ and in the transverse direction particles move under the influence of convective flows. From this standpoint it is easy to obtain an equation for the passive tracer density under the condition when longitudinal correlation effects can be approximated by the diffusive term. Thus, in the two-dimensional case the corresponding renormalized equations have the form

$$\frac{\partial n_0}{\partial t} = -\left\langle V_X(z)\frac{\partial n_1}{\partial x}\right\rangle; \quad \frac{\partial n_1}{\partial t} = D_0\frac{\partial^2 n_1}{\partial z^2} - V_X(z)\frac{\partial n_0}{\partial x}. \tag{46}$$

Here, in contrast to [40], the diffusion coefficient D_0 in the approximating term $D_0\partial^2 n_1/\partial z^2$ characterizes the seed diffusion. The dependences $n_0 = n_0(x,t)$ and $n_1 = n_1(x,z,t)$ were used to describe the two-dimensional case. Applying the Laplace transformation over t and the Fourier transformation over z, one obtains

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from (47),

$$s\tilde{n}_0(s,x) - n_0(x,0) = \tilde{D}(s)\frac{\partial^2 \tilde{n}_0}{\partial x^2}, \tag{47}$$

$$\tilde{D}(s) = \lim_{L_0\to\infty} \frac{1}{2L_0}\int_{-L_0}^{L_0} dz \int_{-\infty}^{\infty} dz' \left\{ \frac{\exp[-\sqrt{s|z-z'|^2/D_0}]}{\sqrt{D_0 s}} V_X(z)V_X(z') \right\}. \tag{48}$$

It is possible to derive a diffusion equation for the model of random shear flows [35]. This corresponds to the condition $z \to z'$ (the condition of particle 'return' to the initial point). A fractional differential equation was found [37],

$$\frac{\partial^{3/2} n_0(t,x)}{\partial t^{3/2}} = \frac{\partial^2}{\partial t^2}\int_0^t \frac{n_0(t',x)\,dt'}{\sqrt{\pi(t-t')}} = \frac{V_0^2 a}{\sqrt{2D}}\frac{\partial^2 n_0(t,x)}{\partial x^2} - \frac{n_0(0,x)}{2\sqrt{\pi}t^{3/2}}. \tag{49}$$

Indeed, the 'diffusive renormalization' of the quasi-linear equations allows us to obtain the transport equations, which differ significantly from the classical diffusive equation [9,10,24,39]. Now we consider using the correlation properties of a system of random flows to determine the equation, which describes both superdiffusive and conventional diffusive behavior. We can change the form of the equation for n_0 by means of a more detailed consideration of the function $F(|z-z'|) = V_X(z)V_X(z')$ in (48). This function describes the correlation properties. Let us consider the power approximation of the function $F(w)$,

$$F(w) = F(|z-z'|) \propto \frac{V_0^2}{1+|z-z'|^{\alpha_C}}. \tag{50}$$

Using the symmetry of integral (48), we can easily simplify it. In terms of the Laplace transformation, this equation takes the form

$$s\tilde{n}_0(s,x) - n_0(x,0) = \frac{V_0^2}{\sqrt{2D}}\left(\frac{s}{2}\right)^{\frac{1}{2}\alpha_C - 1}\frac{\partial^2 \tilde{n}_0}{\partial x^2}. \tag{51}$$

Changing to the dependence in time, we obtain the subdiffusion equation,

$$\frac{\partial^\gamma n_0}{\partial t^\gamma} = D_{\text{eff}}\frac{\partial^2 n_0}{\partial x^2} - Q(t,x), \quad D_{\text{eff}} = \frac{V_0^2 a}{2^{(\alpha_C-3)/2}\sqrt{D}}, \quad Q = \frac{n_0(0,x)}{2\sqrt{\pi}t^\gamma}. \tag{52}$$

Here, the order of the derivative with respect to time γ depends on the parameter α_C,

$$\gamma = 2H = 2 - \frac{\alpha_C}{2}, \quad 0 < \alpha_C < 2, \tag{53}$$

which describes correlation properties in the longitudinal direction (33). These calculations show that the application of scalings to examine correlation effects gives the possibility to accord the hydrodynamics transport description and the models related to the direct calculation of correlation functions.

On the other hand, because of the character of anisotropy, the scaling (53) can be obtained by the modification of the Taylor model: if instead of a laminar velocity profile one considers the system of random flows with the Eulerian correlation function $C(z-z') = \langle V_X(z)V_X(z')\rangle$. Indeed, applying the method of Green's functions to the equation for density perturbation n_1 yields

$$n_1 = -\int dz' \int dt' \left\{ V_X(z')\frac{\partial n_0(x,t')}{\partial x}\int \frac{dk}{2\pi} e^{ik(z-z')} e^{-D_0 k^2 (t-t')} \right\}. \tag{54}$$

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The substitution of (54) in the formula for flux yields the expression in terms of the memory function:

$$q_X \approx \left\langle -V_X(z)\frac{\partial n_1}{\partial x}\right\rangle = \int_0^t dt' \frac{\partial^2 n(x,t')}{\partial x^2} M(t-t'). \tag{55}$$

Here $M(t-t')$ is the memory function which is represented in the form

$$M(t-t') = \int \frac{dk}{2\pi}\left\{ e^{-D_0 k^2 (t-t')} \int d(z-z')\langle V_X(z)V_X(z')\rangle e^{-ik(z'-z)}\right\}. \tag{56}$$

Using the Fourier transform $\tilde{C}(k)$ of the function $C(z-z') = \langle V_X(z)V_X(z')\rangle$ we can rewrite (55) in the form

$$q_X \approx \int_0^t dt' \int \frac{dk}{2\pi}\left\{ \tilde{C}(k) e^{-D_0 k^2 (t-t')} \frac{\partial n_0(x,t')}{\partial x^2}\right\} \approx \int \frac{dk}{2\pi}\left\{ \frac{\tilde{C}(k)}{D_0 k^2}\right\} \frac{\partial^2 n_0(x,t)}{\partial x^2} \tag{57}$$

for the case of a smooth profile $n_0(x,t)$. The effective diffusion coefficient is given by the expression that coincides with the Howells form [44] but for anisotropic model,

$$D_{\text{eff}} \approx \int \frac{\tilde{C}(k)}{D_0 k^2}\frac{dk}{2\pi}. \tag{58}$$

The case under consideration can be interpreted in terms of a scaling representation of the spectrum $\tilde{C}(k) \approx k^{\alpha_C - 1}$ and the assumption about the diffusive character of the wave numbers, which make a main contribution to transport $k \propto 1/\lambda_\parallel \approx 1/\sqrt{D_0 t}$. Then simple calculations yield a scaling:

$$\lambda_\perp^2 \approx D_{\text{eff}} t \approx \frac{\tilde{C}(k)t}{k} \approx \lambda_\parallel^{1-\alpha_C}\lambda_\parallel t \propto t^{2-\alpha_C/2}, \tag{59}$$

which relates the Hurst exponent H describing anomalous transport in the transverse direction to the α_C exponent characterizing the spatial correlation properties of a system of random shear flows: $H = 1 - \alpha_C/4$.

8. Multiscale flows and transport scaling

Within the framework of the Corrsin conjecture, the value λ plays the role of both the diffusive displacement of the particle and the characteristic scale. This gives an additional possibility to treat transport scalings for a passive scalar. For instance, to obtain the scaling (59), we can also use the direct dimensional approach that is analogous to (16). Actually, from this standpoint the expression (59) can be represented in the ballistic form

$$\lambda_\perp(t) \propto V_\perp t. \tag{60}$$

Taking into account the anisotropy of the medium by consideration of both the longitudinal characteristic scale $\lambda_\parallel$ and the transverse characteristic scale $\lambda_\perp$, which are separated in our case,

$$V_\perp = V_\perp(\lambda_\parallel, \lambda_\perp) \approx V_\perp(\lambda_\parallel) \approx V_0\left(\frac{\lambda_\parallel}{\lambda_0}\right)^{\alpha_C/2}, \tag{61}$$

and the diffusion character of the approximation of longitudinal motion $\lambda_\parallel \approx \sqrt{2D_0 t}$, upon substitution we obtain the following expression $H = 1 - \alpha_C/4$ (see [45]). It is

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important to note that the power approximation of velocity by the scaling $V(\lambda) \approx V_0(\lambda/\lambda_0)^{\alpha_C/2}$ was recently applied [46,47] for the multiscale percolation model with the scaling form of a stream function,

$$\Psi_\lambda \approx \Psi_0\left(\frac{\lambda}{\lambda_0}\right)^M, \quad V_\lambda \approx \frac{\Psi_\lambda}{\lambda} \approx V_0\left(\frac{\lambda}{\lambda_0}\right)^{M-1}. \tag{62}$$

Here Ψ_0 is the scale of the streamline function that corresponds to the characteristic spatial scale λ_0, M is the streamline exponent and V_0 is the characteristic velocity. Since the most intensive streamlines will contribute most to the transport, two different situations occur with respect to the value of M. Let us consider the first case, $M > 1$. The characteristic flow pattern is determined by the maximum value of $\lambda \approx \lambda_m$ for such a system. The second case of greatest interest arises when $M < 1$ and appears to be more complex. Consider the hierarchy of spatial scales $\lambda_0 \lessdot \lambda_1 \lessdot \lambda_2 \lessdot \cdots$, and the corresponding hierarchy of velocities, $V_0 \gtrdot V_1 \gtrdot V_2 \gtrdot \cdots$. The total velocity $V = V_0 + V_1 + V_2 + \cdots$ is the superposition of flows entering in the hierarchy. Isichenko and Kalda based their consideration of the hierarchy on Trugman's ideas of 'graded' percolation [48]. To describe turbulent transport, use was made [10, 46, 47] of the concept of distortion of a small-scale related flow, by way of superimposition of a weaker drift flow related to a large scale over $V = V_0 + V_1$. In the monoscale case this corresponds to the initial small parameter $\varepsilon = V_1/V_0$. The correlation scale $a \approx \Delta_{cor}$, which characterizes transport effects in the dimensional expression $D \approx a^2/\tau_c$, is also represented in the percolation theory by the Trugman scaling [48],

$$a(\varepsilon_*) = \lambda\varepsilon_*^{-\nu} = \lambda\left(\frac{V_0}{V_1}\right)^{\nu/(\nu+1)} = \lambda\varepsilon^{-\nu/(\nu+1)}. \tag{63}$$

Here, the value $\nu = \frac{4}{3}$ is the universal percolation exponent, which was calculated for the two-dimensional case in [6,9]. Following the ideas of the monoscale percolation approach [10,49], consider the simplest generalization $a_i(\varepsilon_*(i)) = \lambda_i\varepsilon_*(i)^{-\nu}$. In the case of the scale hierarchy, the Trugman renormalization (63) can be represented in the form

$$\varepsilon_*(i) = \varepsilon(i)^{1/(1+\nu)} \gtrdot \varepsilon(i) \approx V_{i+1}/V_i. \tag{64}$$

It is necessary to take into account the 'nested' hierarchy of scales. An assumption was made about the hierarchical flow pattern based on the system of nested scales in the form $a_i(\varepsilon_*) \leqslant \lambda_{i+1}$. This condition means that the assumption of 'nested' scales is valid only for $-1/\nu < M$. In fact, we have the double inequality for the value M that describes the case of interest $-1/\nu < M < 1$. Completing the consideration of the basic assumptions of the multiscale approach let us focus on the physical meaning of the parameter M. Obviously, this parameter governs the character of a flow. We can relate it to the correlation properties of a flow [49]. The simple estimate for the spatial correlation function in the scaling form is given by

$$C_E(\lambda) \propto V(\lambda)^2 \propto \lambda^{2(M-1)} \propto \frac{1}{\lambda^{\alpha_C}}. \tag{65}$$

Here α_C is the correlation exponent that describes the rate of decay of the correlation function $\alpha_C = 2(1 - M)$. The condition of applicability of the multiscale hierarchical model takes the following form [45, 49], $0 < \alpha_C < \frac{7}{2}$. We can see that there are interesting relations among the multiscale approach, the renormalized quasi-linear equations and the Corrsin diffusive approximation.

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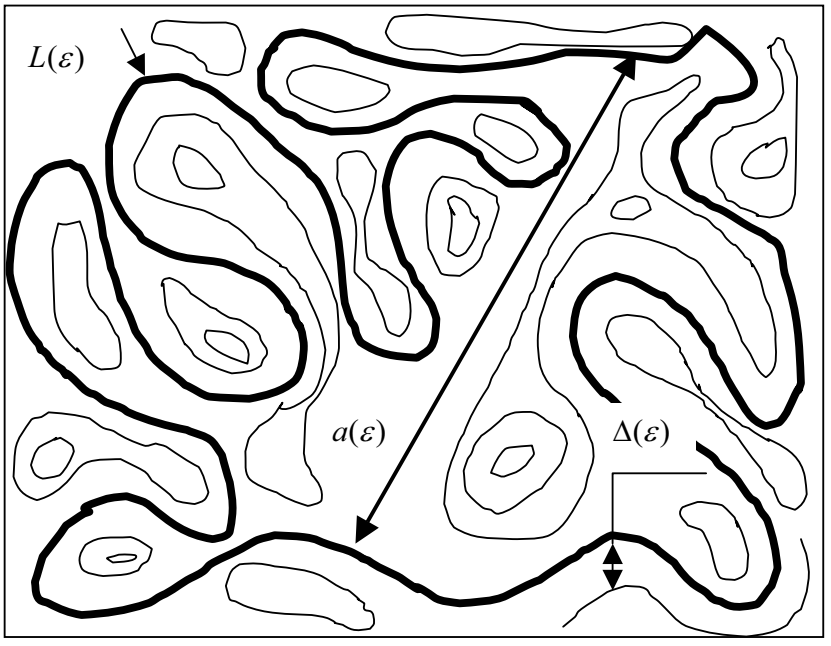


Figure 3. Two-dimensional random flow and percolation streamline.

9. The intermediate percolation regime and correlation renormalization

The power approximation of the correlation function is not the only possible scaling approach to describe correlation effects. Thus, in the framework of percolation theory [6] the scaling representation of the spatial correlation scale $a \approx \lambda/|\varepsilon|^{\nu}$ is used. Here λ is the characteristic scale, ε is the small percolation parameter, which describes the closeness of the system to the percolation threshold, and ν is the percolation exponent. Anomalous diffusion in the two-dimensional monoscale turbulent flow could be explained in terms of the percolation threshold. To describe anomalous transport in a two-dimensional random flow, it is necessary to consider a long percolation streamline [6] of the length $L(\varepsilon) \approx \lambda(a(\varepsilon)/\lambda)^{D_{\rm h}}$, where $D_{\rm h} = 1+1/\nu$ is the hull exponent, which establishes the relationship between $a(\varepsilon)$ and $L(\varepsilon)$ (see Fig. 3). For two-dimensional models the exponent ν was obtained rigorously: $\nu = \frac{4}{3}$ [6, 9]. This long percolation (fractal) streamline is responsible for the main convective contribution to transport. To estimate this contribution we have to analyze the effective diffusivity in the form $D_{\rm eff} \approx (a^2/\tau_{\rm c})P_{\infty}$. Here $\tau_{\rm c} \approx L(\varepsilon)/V_0$ is the correlation time, $P_{\infty} = \Delta(\varepsilon)L(\varepsilon)/a^2(\varepsilon) \approx \lambda/a(\varepsilon)$ is the part of the space that is responsible for percolation transport, $\Delta(\varepsilon) \approx \lambda\varepsilon$ is the percolation layer width and V_0 is the characteristic velocity. The percolation estimates of turbulent transport are usually obtained by the renormalization of the small parameter ε on the basis of the comparison of characteristic times [6, 50].

However, another approach based on the consideration of the dynamics of spatial correlation scales is possible. The simplest estimate, as obtained by Isichenko in [6], yields the intermediate regime with the motion of particles along the percolation streamline during the times $t < \tau_{\rm B} \approx L/V_0$, which is analogous to the fractal

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diffusion (this case differs significantly from the percolation model of turbulent diffusion with $t \geqslant \tau_{\mathrm{B}}$). The author of [6] used the estimate of the correlation scale in the form

$$a_{\mathrm{I}}(t) \approx \lambda\left(\frac{L(t)}{\lambda}\right)^{1/D_{\mathrm{h}}} \approx \lambda\left(\frac{V_0 t}{\lambda}\right)^{1/D_{\mathrm{h}}}. \tag{66}$$

An assumption was made that the test particle path at this stage is approximately ballistic $L(t) \approx V_0 t$. To describe this initial stage of percolation in the two-dimensional random flow, the expression $R^2 \approx D_{\mathrm{eff}} t \approx a^2 P_\infty \approx \lambda a \approx \lambda^2 (L/\lambda)^{1/D_{\mathrm{h}}} \approx \lambda^2 (V_0 t/\lambda)^{1/D_{\mathrm{h}}}$ was offered [6]. Here R is the mean free path of the particle. The new subdiffusive regime is described by the Hurst exponent $H = 1/D_{\mathrm{h}} = \frac{2}{7}$.

The model of evolution of correlation scale $a_{\mathrm{I}}(t)$ can be modified for the interpretation and analysis of other percolation regimes [51]. Thus, simultaneously with the increase of the correlation scale $a_{\mathrm{I}}(t) \approx (L(t)/\lambda)^{1/D_{\mathrm{h}}}$, it is necessary to take into account the increase of the stochastic layer width $\Delta = \Delta(t)$; which, in the framework of percolation models of turbulent diffusion, is closely related to the value of the small parameter $\varepsilon_* \approx \Delta/\lambda$, and hence to the definition of the correlation scale $a \approx \lambda/\varepsilon^\nu$. Easy calculations lead to the expression describing the increase of the correlation scale $a_{\mathrm{D}}(t)$ due to the growth of the stochastic layer width

$$a_{\mathrm{D}}(t) \approx \lambda\left(\frac{\Delta(t)}{\lambda}\right)^{-\nu}. \tag{67}$$

Within the framework of mean-field theory the consideration of the balance between $a_{\mathrm{D}}(t)$ and $a_{\mathrm{I}}(t)$ makes it possible to find an estimate of the characteristic time t_0, which has to be used to define the effective diffusion coefficient D_{eff}. Thus, in the case of a diffusive increase of the stochastic layer width $\Delta^2 \approx D_0 t$, one obtains

$$\lambda\left(\frac{V_0 t_0}{\lambda}\right)^{1/D_{\mathrm{h}}} \approx \lambda\left(\frac{\sqrt{D_0 t_0}}{\lambda}\right)^{-\nu}. \tag{68}$$

Calculations lead to the estimate

$$t_0 \approx \frac{\lambda^2}{D_0}\left(\frac{1}{Pe}\right)^{1/(\gamma+3)} = \frac{\lambda^2}{D_0}\left(\frac{D_0}{\lambda V_0}\right)^{1/(\gamma+3)}, \tag{69}$$

which, after substitution in the formula for the stochastic layer width, yields

$$\Delta \approx (D_0 t_0)^{1/2} \approx \lambda\left(\frac{1}{Pe}\right)^{1/(\gamma+3)}. \tag{70}$$

It is easy to note that this expression coincides exactly with the expression for the percolation steady case [6], and the corresponding estimate of the effective diffusion coefficient is given by the formula $D_{\mathrm{eff}} \approx V_0 \Delta \propto V_0^{10/13}$.

Note that the approach under consideration allows the use of the correlation scales balance as the basis for the construction of new models of percolation transport by the approximation $\Delta(t) \propto t^H$. Thus the linear estimate $\Delta \propto t$ with $H = 1$ is widely applied in dynamical system theory [10]. In the context of the description of time-dependent effects, this expression can be easy represented in the simplest dimensional form $\Delta(t) = \lambda\omega t$, where ω is the characteristic frequency. Then the consideration of the correlation scales balance yields

$$\Delta = \lambda\varepsilon_* = \lambda\left(\frac{1}{Ku}\right)^{1/(\nu+2)}, \tag{71}$$

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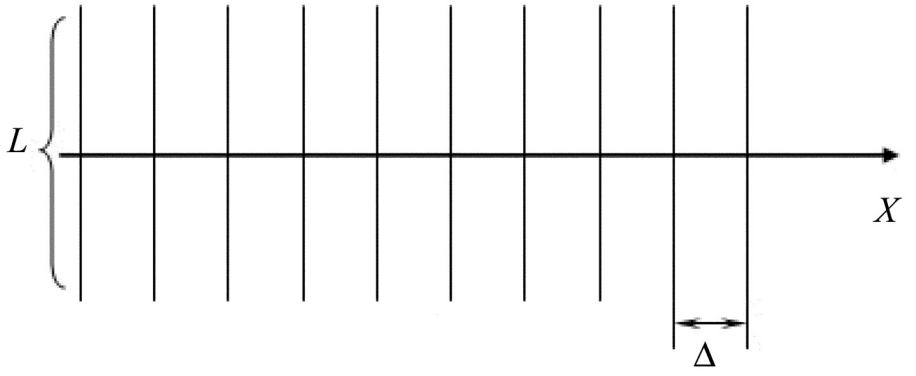


Figure 4. Comb-like structure.

and hence the estimate of the turbulent diffusion coefficient is given by the percolation expression $D_{\text{eff}} \approx V_0 \Delta \propto V_0^{7/10}$. Actually, a method is suggested for small-parameter renormalization on the basis of correlation scales balance, which allows us to obtain not only the known percolation results [6], but also new scalings describing dissipation and compressibility effects [50, 51].

10. Trapping and phase-space modification of the Corrsin conjecture

Naturally, the character of transport (superdiffusion or subdiffusion) depends essentially on the topological properties of the model. Therefore, it is interesting to consider not only the models with a superdiffusive regime, but also subdiffusive systems. The rigorous theory for such problems is based on the continuum-time random walks (CTRW) approach or on the diffusive models for medium with 'traps' [7–10]. To describe transport in a stochastic magnetic field, this approach was very successfully used in [9]. The main assumption is introducing the probability $\Phi(t)$ for a particle to remain in a trap over a time t. The scaling form of the function $\Phi(t) \propto 1/t^{\gamma}$ is usually used. Here, γ is the parameter of the probability function. There exists a simple estimate for $\Phi(t)$ based on the return probability $\Phi(t) \propto \rho(0,t)\Delta \propto \Delta/\sqrt{4\pi D_0 t}$. Here Δ is the correlation scale. We obtain the relation between the probability of finding the particle in the trap and the 'return' probability. The 'traps' can be imagined as teeth in a 'comb-like structure' (see Fig. 4). If the teeth length L tends to infinity, then diffusive expression is satisfactory. The models with teeth that have a limited length L and teeth that have fractal structures, appear to be the natural generalization of the 'comb-like' model.

Cardoso et al. [52] carried out experimental research on anomalous diffusion of scalars in the field with several vortices and studied trapping effects related to the capture of test particles by vortices (see Fig. 5). Naturally, in such a formulation of the problem, the continuous-time random walks is an adequate theoretical model. From the formal standpoint we must introduce a scaling to describe the waiting time distribution,

$$\psi(t) = \frac{d\Phi}{dt} \propto \frac{1}{t^{\gamma-1}}, \quad \Phi(t) = 1 - \int_0^t \psi(t)\, dt. \tag{72}$$

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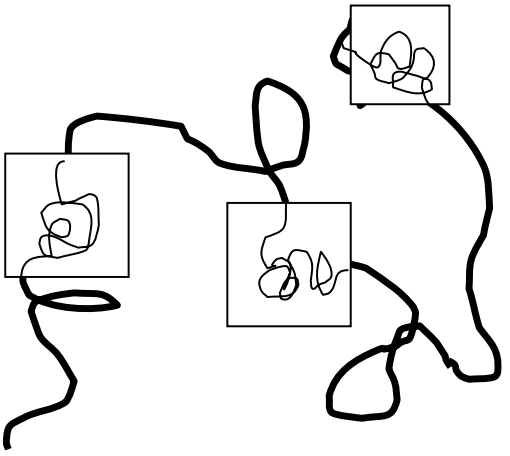


Figure 5. Vortex trapping regions.

Then, the mean waiting time between two 'jumps' is estimated by the expression

$$\tau_{\mathrm{T}} \approx \int_0^{t_{\max}} t\psi(t)\,dt \propto t_{\max}^{3-\gamma} \approx t^{3-\gamma}. \tag{73}$$

On the other hand, the complete walking time can be estimated through τ_{T} and the number of pauses N, $t \approx N\tau_{\mathrm{T}} \approx t^{3-\gamma}N$. Hence, one obtains the scaling for N in the form $N \propto t^{\gamma-2}$. Since the mean squared displacement is related to N by the scaling $R^2 \propto N$, we arrive at the relationship $R \propto \sqrt{N} \approx t^{\frac{1}{2}\gamma-1}$. Using the experimental results [52] yields the scaling for the mean squared displacement in the form $R \propto t^{1/3}$, where the Hurst exponent $H = \frac{1}{3}$ and the exponent $\gamma = \frac{8}{3}$. Moreover, the probability density function $\psi(t)$ corresponding to these scalings was successfully measured [52], which is evidence of the correctness of the continuous-time random walk prediction.

The model of subdiffusion transport considered above was based on using the formal probabilistic model related to the exit time distribution $\psi(t)$. However, in such an approach there is no direct connection with correlation functions, which appear to be a more adequate tool for turbulent analysis. Long-range correlations play a significant role in considering trapping. Vlad, Spineanu, Misguich and Balescu suggested that the trapping of test particles could be analyzed using a special model representation of a streamline system. Moreover, the essential modification of the Corrsin conjecture was made in [53, 54]. The good way is to keep the factorization of expression (10) suggested in [11], but to considerably modify the trajectory ensemble under consideration. Indeed, the sub-ensemble of trajectories in which the result displacement corresponds to some fixed value λ could be described as follows:

$$\langle V(x(0),0)V(x(t),t)\rangle|_{X(t)=\lambda}. \tag{74}$$

Consider a system of sub-ensembles, where the value of the initial velocity is fixed in each such system

$$L(V_0,t) = \langle V(x(0),0)V(x(t),t)\rangle|_{X'(t)=V_0} = V_0\langle V(x(t),t)\rangle|_{X'(t)=V_0}. \tag{75}$$

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On the one hand, this model leads to averaging sub-ensembles over the velocity with the kinetic distribution $F(V_0, t)$

$$C(t) = \int_{-\infty}^{\infty} F(V_0, t) L(V_0, t)\, dV_0. \tag{76}$$

On the other hand, this gives the possibility of a more detailed analysis of the streamline behavior. This is not surprising because for the new conditions the approximation of expression (75) by means of the specially chosen function $W(x, t)$ permits one to investigate the specific system of trajectories in the framework of the following equation:

$$\frac{dx}{dt} = W(x, t) = \langle V(x(t), t)\rangle|_{V_0}. \tag{77}$$

The option of an approximating function defines the character of trajectories and makes it possible to 'visualize' correlation effects, which in this formulation of the problem are also determined by the expression for W. Actually, in this approach the value $V_0 W(x, t)$ replaces the expression for the Eulerian correlation function $C_{\mathrm{E}}(x, t)$. The simplest example of the approximation $W(x, t)$ was examined in [54] in the form that allows one to easily solve (77) and at the same time to satisfy trapping character of transport $W(x, t) = V_0^2 \exp(-x/\lambda) \exp(-t/\tau)$. Here λ is the characteristic spatial scale and τ is the characteristic time. Simple transformations of (77) yield the anisotropc expression for the displacement $x(t \to \infty) \approx \lambda \ln(1 + |V_0|\tau/\lambda) \approx \lambda \ln(1 + Ku)$, which mirrors trapping. Vlad, Spineanu, Misguich and Balescu carried out calculations of the correlation function (76) and the diffusion coefficient using the Maxwellian distribution $F(V_0) = (1/\sqrt{\pi} V_{\mathrm{T}}) \exp(-V_0^2/V_{\mathrm{T}}^2)$. Here V_T is the characteristic velocity of the one-dimensional Maxwellian distribution. Performing calculations yields

$$D_{\mathrm{T}} = \frac{\tau}{\sqrt{\pi} V_{\mathrm{T}}} \int_{-\infty}^{\infty} dV_0 \frac{V_0^2}{(1 + |V_0|\tau/\lambda)^2} \exp\left(-\frac{V_0^2}{V_{\mathrm{T}}^2}\right). \tag{78}$$

The results of calculations lead to the quasi-linear expression $D_{\mathrm{T}} \approx V_0^2 \tau_{\mathrm{c}} \propto Ku^2$ for the case $Ku \ll 1$, and 'flat' scaling $D_{\mathrm{T}} \propto Ku^{\gamma_{\mathrm{T}}}$ with $\gamma_{\mathrm{T}} \approx 0$ for the long-range correlation case $Ku \gg 1$. The approach considered was subsequently developed in a number of papers [53–56] where the two-dimensional model is analyzed in the framework of the special Hamiltonian function Φ_{H} (the stream function) characterizing the trapping in the flow $\dot{\mathbf{x}} = -\nabla\Phi_{\mathrm{H}}(\mathbf{x}, t) \times \mathbf{e}_Z = \langle \mathbf{V}(\mathbf{x}(t), t)\rangle|_{V_0, \Phi_0}$. Here Φ_0 is the initial value of the streamline function. Streamlines obtained in this approach appear to be closed curves, except for the single streamline, which is a straight line along the initial velocity V_0. In other words, if the considered trajectories are localized, the particles moving along closed streamlines make an inessential contribution to the transport. The authors of [53] found the universal character of the dependence of the effective diffusion coefficient D_T on the Kubo number for $Ku \gg 1$, $D_{\mathrm{T}} \propto Ku^{\gamma_{\mathrm{T}}}$. Here $\gamma_{\mathrm{T}} \approx 0.62$.

11. The simplified Corrsin conjecture

The diffusive approximation of correlation effects considered before takes the intermediate place between the rigorous description of correlation effects and empirical models based on the analysis of the correlation function shape. Such a problem

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was discussed by Hay and Pasquill in [57] (almost simultaneously with Corrsin), where Eulerian and Lagrangian correlation functions are assumed to be similar shapes but, at the same time, their characteristic scales are different. Thus, the characteristic temporal scale, which corresponds to the Lagrangian correlation function, is defined by the expression

$$\tau_{\mathrm{L}} = \frac{1}{\langle V^2 \rangle} \int_0^\infty C_{\mathrm{L}}(t)\, dt. \tag{79}$$

The characteristic Eulerian temporal and spatial scales are found analogously

$$\tau_{\mathrm{E}} = \frac{1}{\langle U^2 \rangle} \int_0^\infty C_{\mathrm{E}}(\Delta, t)\, dt, \tag{80}$$

$$l_{\mathrm{E}} = \frac{1}{\langle U^2 \rangle} \int_0^\infty C_{\mathrm{E}}(\Delta, t)\, d\Delta. \tag{81}$$

An assumption was made that there exists a certain universal constant β_{C}, which allows us to relate Lagrangian and Eulerian scales [57]

$$\tau_{\mathrm{L}} = \beta_{\mathrm{C}} \tau_{\mathrm{E}}, \quad l_{\mathrm{L}}(\beta_{\mathrm{C}} t) = \beta_{\mathrm{C}} l_{\mathrm{E}}(t). \tag{82}$$

A variety of turbulence types leads to the fact that the values of β_{C} defined experimentally lie over a wide interval $1 \leqslant \beta_{\mathrm{C}} < 8.5$. In spite of the obvious simplicity of the approach suggested by Hay and Pasquill, recent investigations show that within the framework of the consideration of one-particle vertical diffusion in strongly stratified turbulence, the Eulerian and Lagrangian velocity correlation functions are almost the same

$$\langle V_i(0) V_j(t) \rangle = \langle U_i(x, 0) U_j(x, t) \rangle. \tag{83}$$

Thus, Kaneda and Ishida [58] considered the Fourier transformation of the Corrsin conjecture (11) in the form

$$\langle V_i(t) V_j(t') \rangle = \int d^3\mathbf{k}\, \tilde{R}_{ij}(\mathbf{k}, t, t') \langle e^{-i\mathbf{k}(x(t') - x(t))} \rangle, \tag{84}$$

where

$$\tilde{R}_{ij}(\mathbf{k}, t, t') = \frac{1}{(2\pi)^3} \int \langle U_i(\mathbf{x} + \mathbf{r}, t) U_j(\mathbf{x}, t') e^{-i\mathbf{k}\cdot\mathbf{r}}\, d^3\mathbf{r} \rangle. \tag{85}$$

From the physical point of view, the Eulerian velocity correlations must dominate by large eddies. This corresponds to small values of $\mathbf{k}$. Therefore, for $\mathbf{k} \approx 0$ one can expect that

$$\langle e^{-i\mathbf{k}(x(t') - x(t))} \rangle \approx 1. \tag{86}$$

This simple estimate gives the simplified Corrsin conjecture

$$\langle V_i(t) V_j(t') \rangle = \langle U_i(\mathbf{x}, t) U_j(\mathbf{x}, t') \rangle. \tag{87}$$

This new representation

$$C_{\mathrm{L}}(\tau) \approx C_{\mathrm{E}}(\lambda, \tau)|_{\lambda = 0}, \tag{88}$$

was also discussed preliminarily in [59]. Within the framework of the Boussinesq approximation of strongly stratified flows, the validity of the simplified Corrsin conjecture (86) was confirmed by direct numerical simulations [58–60], which agree well with the hypothesis (86). This method proves to be very fruitful because even

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the simplest correlation approximation can be equally valid for special cases, and the search for such models is an urgent problem.

12. Conclusions

The models considered in this paper demonstrate the efficiency of the diffusive approximation of correlation effects for obtaining a scaling description of turbulent transport. The Corrsin conjecture finds fruitful application in receiving the scaling for anomalous transport in a system of random drift flows [35] where anisotropic effects are essential. The detailed analysis of the generalized Dreizin–Dykhne approach, where longitudinal correlations are described by scaling, allows us to obtain the relationship $H = 1 - \alpha_C/4$ between the Hurst exponent H and the correlation exponent α_C by several independent methods. The discussed model of random drift flows, which is described by the power form of the longitudinal correlation function, points out the close relation among the Corrsin conjecture, renormalized quasi-linear equations and scaling estimates in which the characteristic spatial scale is simultaneously the diffusive displacement. The comparison of different approaches shows that one of them using the fractional differential equation to describe anomalous transport in random shear flows system is rather adequate and does not even allow one to analyze non-Gaussian probability distribution function flows. In spite of the simplified character of the factorization approach, the separating method of longitudinal and transverse correlation effects was repeatedly used to describe turbulent transport in anisotropic media by the combination of both ballistic and diffusive estimates of spatial correlation scales $\lambda_\perp \approx V_\perp(\lambda_\parallel)t \approx V_\perp(\sqrt{D_0 t})t$. Another possible generalization of the shear flows model is using the return effects as the decorrelation mechanism within the framework of a more complex flow topology.

Here we have discussed the renormalization method of quasi-linear equations, which was suggested by Kadomtsev and Pogutse to describe diffusion of magnetic force lines. In contrast to [35], the authors of [40] used the diffusion approximation of transverse correlation effects and obtained the scaling in the Howells form [44], but for the anisotropic case. Moreover, the linear character of the dependence of the effective diffusion coefficient on the characteristic correlation scale $D_{\text{eff}} \approx V_0/k_\perp \approx V_0\Delta$ became one of the important assumptions for the percolation approach [6]. Here, we suggest new mean-field arguments to obtain percolation results within the framework of the monoscale model. This method is based on the comparison of spatial correlation scales $a_D(\Delta) \approx a_I(L)$, where one of them depends on the percolation line length, whereas another one is related to the percolation layer width. Calculations yield the steady percolation result [51] for the diffusive approximation of the percolation layer width $\Delta(t) \propto \sqrt{D_0 t}$. Other regimes could be obtained by consideration of a different scaling representation of the layer width by introducing new flow parameters through the characteristic times $\Delta(t) \approx \lambda(\omega t)^H$ or frequencies [50], where the Hurst exponent H is the parameter of the problem.

Naturally, the Corrsin assumption is not a universal approximation. Even in the framework of the statistical approach it is possible to essentially modify the representation of correlation effects. To illustrate the new trends we briefly consider here the interesting 'phase-space' model developed in a number of papers [53–56]. This approach is essentially guided by the specific idea of trapping behavior of streamlines that allows the determination of a new type of dependence of the

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effective diffusion coefficient on the Kubo number. Besides that, in the framework of the Boussinesq approximation to the description of strongly stratified flows, it is possible to use the simplified Corrsin conjecture $C_{\mathrm{E}}(\lambda,\tau)|_{\lambda\approx 0} \approx C_{\mathrm{L}}(t)$ based on the consideration of large scale structures [59, 60]. In spite of the simplicity of this approach, the simulations show its efficiency, and the search for such models is an important problem.

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