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Acceleration of electrons and electromagnetic fields of highly intense laser pulses

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Abstract

The acceleration of electrons by very intense lasers in vacuum is studied for different forms of laser pulses: finite plane and three-dimensional laser pulses. The latter case is treated by introducing an approximation of the laser frequency as a function of the Cartesian components of the wave number. Various examples which lead to high accelerations of electrons are given.

Keywords: Acceleration of electrons in vacuum; Electromagnetic fields of laser pulses; Finite three-dimensional laser pulses

1. INTRODUCTION

The study of the acceleration of electrons by intense laser fields developed in different directions in the last years. For example, in the wakefield accelerator, highly intense laser pulse is focused into an underdense plasma and a resonant ponderomotive excitation of an electron plasma wave occurs. A breakthrough has been achieved by Pukhov and Meyer-ter-Vehn (2002) with the discovery of the so-called bubble acceleration regime. The laser pulse expels electrons ponderomotively outward from the region of highest intensity in such a way that a solitary laser-plasma cavity is formed that moves with the laser pulse.

Seifter *et al.* (2009) investigated implosions of high symmetry to achieve ignition for inertial confinement fusion on the National Ignition Facility. For pulsed power generators, the development of relativistic electron beam sources is important. Li *et al.* (2009) proposed a carbon-fiber aluminium cathode with high-density uniform emitters that was tested in a single pulsed power generator. This is a new class of plasma cathodes that offers promising prospects for relativistic beam sources. External injection of electron bunches longer than the plasma wavelength in a laser wakefield accelerator leads to the generation of femtosecond ultra relativistic bundles with a small energy spread (Luttikhof *et al.*, 2009). Niu *et al.* (2008) treated theoretically and numerically the resonant acceleration of plasma electrons in combined circularly polarized Gaussian laser fields and selfgenerated quasistatic fields.

The work described so far considers the acceleration of electrons inside a plasma environment. In this paper, we only want to describe the interaction of electrons with laser fields in the vacuum. The behavior of a high-density electron beam in the field of a super-intense laser pulse was studied by Kulagin *et al.* (2008). They considered a one-dimensional theory for the description of the interaction and found exact solutions for the equations of motion of the electrons. Mainly Salamin and Keitel (2002), Wang *et al.* (2002), and Bahari and Taranukhin (2003) treated theoretically the direct acceleration of electrons by intense focused laser beams with a single frequency (stationary beams) where mostly the electrons were injected sideways into the region of high intensity and gained energy of the order of GeV. Earlier measurements related to these calculations were done by Malka *et al.* (1997).

The problem arising with the sideward injection of electrons into the focus of the beam is that the electrons get mainly reflected from the intense laser beam and cannot enter the region of high electromagnetic fields. In order to circumvent this disadvantage, we let run a laser pulse of about the length of a half wavelength over a free electron, which is an optimal condition for the acceleration. This a

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challenging problem in two respects: (1) A finite laser pulse carries a frequency spectrum. Therefore, in this case, we cannot apply fields with a single frequency as usually used in literature (Davis, 1979). (2) Three-dimensional electromagnetic wave packets have to be constructed, which solve the Maxwell equations. The careful study of both aspects is the content of this paper. Until now we treated only plane electromagnetic waves (Scheid & Hora, 1989; Häuser *et al.*, 1994).

In Section 2, we first consider the acceleration of an electron in general. In Section 3, we study the acceleration with plane laser pulses, and in Section 4, the action of a finite plane laser pulse. Section 5 treats the electromagnetic fields of three-dimensional finite laser pulses. In order to find results for such laser pulses, we approximate the frequency in Section 6, and give results for laser acceleration in this approximation. We restrict the energies of the electrons to TeV energies to prevent the appearance of the problems of the quantum radiation regime (Ruhl, 2009).

2. ACCELERATION OF ELECTRONS IN VACUUM

The relativistic equation of motion of a free electron with velocity $\vec{v}$ in an electromagnetic field can be written as

$$\frac{d(\gamma\vec{\beta})}{d(\omega_0 t)} = \frac{eE_0}{m_0 c^2 k_0}\left(\frac{\vec{E}}{E_0} + \frac{\vec{\beta}\times c\vec{B}}{E_0}\right). \tag{1}$$

Here, $\vec{\beta} = \vec{v}/c$, $\gamma = \sqrt{1/(1-\beta^2)}$ and $\omega_0 = k_0 c$ is an arbitrary constant frequency, E_0 is a constant electric field strength, and $|eE_0|/(m_0 c^2 k_0) = a_0$ is the acceleration parameter. A field without an electric y-component moving mainly in z-direction has the form

$$\vec{E} = E_x\vec{e}_x + E_z\vec{e}_z, \tag{2}$$

$$\vec{B} = B_x\vec{e}_x + B_y\vec{e}_y + B_z\vec{e}_z. \tag{3}$$

In order to prove the correctness of ansatz (2) for the electric field, we take the Maxwellian equations in vacuum $\mathrm{div}\vec{B} = 0$, $\mathrm{div}\vec{E} = 0$, $\partial\vec{B}/\partial t = -\mathrm{rot}\vec{E}$, and $\partial\vec{E}/\partial t = c^2\mathrm{rot}\vec{B}$. These equations lead to the following differential equation for $\vec{E}$:

$$\frac{\partial^2\vec{E}}{\partial t^2} = -c^2\mathrm{rot}(\mathrm{rot}\vec{E}) = c^2\Delta\vec{E}.$$

Therefore, we can assume $E_y = 0$ in the following.

The parameter a_0 can be quite large. For example, for a laser with a wave number 1 $(\mu\mathrm{m})^{-1}$ (wavelength $\lambda = 2\pi\,\mu\mathrm{m}$) and an intensity of $I = (\epsilon_0/\mu_0)^{1/2}E_0^2 = 10^{22}\,\mathrm{Wcm}^{-2}$ we obtain $a_0 = 0.38\times 10^3$. The resulting value of the Lorentz factor γ is proportional to a_0^2 as we will show for example in Section 3.

In the next Section, we consider plane laser pulses (only $E_x \neq 0$ and $B_y \neq 0$) that are linearly polarized in x-direction and move in positive z-direction.

3. ACCELERATION BY PLANE LASER PULSES

A plane laser pulse linearly polarized in the x-direction and moving in positive z-direction has the electric and magnetic fields

$$\vec{E} = E_x(z-ct)\vec{e}_x, \quad \vec{B} = B_y(z-ct)\vec{e}_y = \frac{1}{c}E_x(z-ct)\vec{e}_y. \tag{4}$$

Here we consider laser pulses with a mixture of frequencies. In various research (Bahari & Taranukhin, 2003; Wang *et al.*, 2002; Davis, 1979) laser pulses with a single frequency were studied.

Then Eq. (1) has the form in components

$$\frac{d\gamma v_x/c}{d\omega_0 t} = -a_0\left(1-\frac{v_z}{c}\right)\frac{E_x}{E_0}, \tag{5}$$

$$\frac{d\gamma v_y/c}{d\omega_0 t} = 0, \tag{6}$$

$$\frac{d\gamma v_z/c}{d\omega_0 t} = -a_0\frac{v_x}{c}\frac{E_x}{E_0}. \tag{7}$$

Assuming the initial conditions for $t = 0$: $\vec{v}(0) = 0$, $\vec{r}(0) = 0$, we can calculate the velocity, energy, and the location of the electron. With the abbreviation

$$A(u) = a_0\int_0^u \frac{E_x(u')}{E_0}du', \tag{8}$$

where u is given by $u = k_0 z - \omega_0 t$, we obtain (Häuser *et al.*, 1994)

$$\frac{v_x}{c} = \frac{A}{1+A^2/2}, \quad \frac{v_y}{c} = 0, \quad \frac{v_z}{c} = \frac{A^2}{2+A^2}, \tag{9}$$

$$\gamma = 1 + \frac{A^2}{2}, \tag{10}$$

$$k_0 x = -\int_0^u A du', \quad k_0 y = 0, \quad k_0 z = -\int_0^u \frac{A^2}{2}du'. \tag{11}$$

In the next section, we will calculate these integrals for a special form of the plane wave. So far no approximations are used.

4. FINITE PLANE LASER PULSE

Let us assume the electric field of the plane laser pulse as

$$\vec{E} = E_0\cos(\tilde{u})\exp(-\alpha\tilde{u}^2)\vec{e}_x \quad \text{with} \quad \tilde{u} = k_0 z - \omega_0 t + \varphi. \tag{12}$$

This plane pulse solves the Maxwellian equations exactly. The phase φ shifts the laser pulse in the negative z-direction. If we choose $\alpha \simeq 0.5$, only about one positive arc of the cosine function survives. For the function $A(u)$ we find

$$A(u) = a_0(g(u+\varphi,\alpha) - g(\varphi,\alpha)), \tag{13}$$

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with the odd function $g(\varphi, \alpha) = -g(-\varphi, \alpha)$

$$g(\varphi, \alpha) = \int_0^{\varphi} \cos(x) \exp(-\alpha x^2) dx. \tag{14}$$

The complete laser pulse has crossed the electron for $\tilde{u} \to -\infty$ and $\varphi \gg 1$. After crossing the integral $A = A_{fin}$ is given by

$$A_{fin} = a_0(g(-\infty, \alpha) - g(\infty, \alpha)) = -a_0\sqrt{\frac{\pi}{\alpha}}\exp\left(-\frac{1}{4\alpha}\right). \tag{15}$$

Therefore, we have the Lorentz factor

$$\gamma = 1 + \frac{1}{2}a_0^2\frac{\pi}{\alpha}\exp\left(-\frac{1}{2\alpha}\right). \tag{16}$$

In Figure 1, we present the scaled Lorentz factor $(\gamma - 1)/a_0^2$ as a function of the parameter α. The maximum of this curve is positioned at $\alpha = 0.5$ with the value $\pi \exp(-1) = 1.156$. We can compare the result with the case of an acceleration inside a half wavelength without the Gaussian factor in (12). This acceleration has the value $(\gamma - 1)/a_0^2 = 2$.

To construct a laser pulse of the form with $\alpha = 0.5$ running over the electron, we set $\varphi = 3\pi/2$ and the final value of $u = u_{fin} = -3\pi$. For $|u| > |u_{fin}|$, the electron moves with (nearly) constant velocity in z direction. At $u = -3\pi$ the electron reaches a distance from $\vec{r} = 0$ (see (11))

$$\begin{aligned} k_0 x &= -\int_0^{-3\pi} A(u')du', \quad k_0 y = 0, \\ k_0 z &= -\int_0^{-3\pi} \frac{1}{2}A(u')^2 du'. \end{aligned} \tag{17}$$

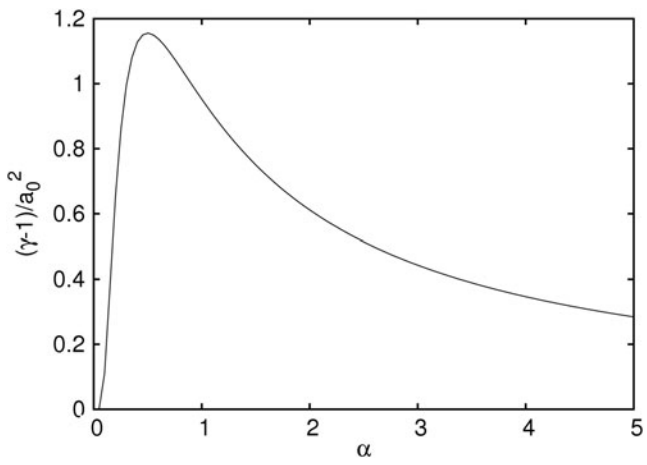


Fig. 1. Scaled Lorentz factor as a function of α for the polarized plane wave given in Eq. (12).

The first integral can be reduced to (with $\alpha = 0.5$)

$$\begin{aligned} k_0 x &= -3\pi a_0 g(3\pi/2, \alpha) \approx -3\pi a_0 g(\infty, \alpha) \\ &= -a_0\frac{3\pi}{2}\sqrt{\frac{\pi}{\alpha}}\exp\left(-\frac{1}{4\alpha}\right) = -a_0\pi \times 2.28. \end{aligned} \tag{18}$$

The last integral in (17) can only be solved numerically and yields for $\alpha = 0.5$

$$k_0 z = a_0^2\frac{3\pi}{4} \times 2.20. \tag{19}$$

The corresponding quantities for a half-wavelength acceleration are shorter, namely $k_0 x = -\pi a_0$ and $k_0 z = 3\pi a_0^2/4$ because of the smaller extension of the halt-wavelength pulse. For a maximum intensity of $I = 10^{22}\,\mathrm{Wcm}^{-2}$ and a wave number of $k_0 = 1\ (\mu\mathrm{m})^{-1}$, i.e., $a_0 = 0.38 \times 10^3$, and for $\alpha = 0.5$, we obtain a Lorentz factor of $\gamma = 1.67 \times 10^5$ and the acceleration lengths are $x = -2.72$ mm and $z = 0.75$ m.

5. ELECTROMAGNETIC FIELDS OF THREE-DIMENSIONAL LASER PULSES

We search for the electromagnetic fields of a three-dimensional laser pulse in vacuum, which is restricted to a given region in three dimensions at a fixed time. Let us start with a vector potential $\vec{A}$ in transverse gauge where

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t}, \quad \vec{B} = \mathrm{rot}\vec{A}, \quad \mathrm{div}\vec{A} = 0, \tag{20}$$

and where $\vec{A}$ has to satisfy the wave equation

$$\Delta \vec{A} = \frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2}. \tag{21}$$

The field $\vec{A}$ can be obtained from a further vector potential $\vec{U}$ by setting $\vec{A} = \mathrm{rot}\ \vec{U}$ where $\vec{U}$ has to obey the wave equation

$$\Delta \vec{U} = \frac{1}{c^2}\frac{\partial^2 \vec{U}}{\partial t^2}. \tag{22}$$

Choosing a certain three-dimensional wave packet for $\vec{U}$, the corresponding fields of $\vec{E}$ and $\vec{B}$ can be calculated by differentiation. If we set $\vec{U} = U_y\vec{e}_y$ for simplification and assume $\Delta U_y = (1/c^2)\partial^2 U_y/\partial t^2$, we calculate

$$\begin{aligned} \vec{E} &= \frac{\partial}{\partial z}\frac{\partial}{\partial t}U_y\vec{e}_x - \frac{\partial}{\partial x}\frac{\partial}{\partial t}U_y\vec{e}_z \\ \vec{B} &= \frac{\partial}{\partial x}\frac{\partial}{\partial y}U_y\vec{e}_x - \left(\frac{\partial^2}{\partial z^2}U_y + \frac{\partial^2}{\partial x^2}U_y\right)\vec{e}_y + \frac{\partial}{\partial z}\frac{\partial}{\partial y}U_y\vec{e}_z. \end{aligned} \tag{23}$$

Next we introduce an ansatz for U_y at time $t = 0$ and carry out

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a Fourier transformation of it:

$$U_y(x, y, z, t=0) = \frac{1}{\sqrt{8\pi^3}} \int f(k_x, k_y, k_z) \exp(i\vec{k}\vec{r}) d^3k, \quad (24)$$

with

$$f(k_x, k_y, k_z) = \frac{1}{\sqrt{8\pi^3}} \int U_y(x, y, z, t=0) \exp(-i\vec{k}\vec{r}) d^3r. \quad (25)$$

Then we find the time dependence if the wave proceeds in positive z-direction

$$\begin{aligned} U_y(\vec{r}, t) = & \frac{1}{\sqrt{8\pi^3}} \int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dk_y \int_{0}^{\infty} dk_z f(k_x, k_y, k_z) \\ & \times \exp(i(\vec{k}\vec{r} - \omega t)) \\ & + \frac{1}{\sqrt{8\pi^3}} \int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dk_y \int_{-\infty}^{0} dk_z f(k_x, k_y, k_z) \\ & \times \exp(i(\vec{k}\vec{r} + \omega t)), \end{aligned} \quad (26)$$

with

$$\frac{\omega}{c} = (k_x^2 + k_y^2 + k_z^2)^{1/2} = k. \quad (27)$$

It is quite easy to show that U_y fulfils Eq. (22). The Fourier transformation (24) can usually be calculated and is known. However, the time-dependent function $U_y(x,y,z,t)$ needs a three-dimensional numerical integration because of the special form (27) of the frequency, This numerical integration requires a high precision since finally the electromagnetic fields are obtained by second derivatives of U_y. If $f(k_x,k_y,k_z)$ is an even function in k_z with $f(k_x,k_y,k_z) = f(k_x,k_y, -k_z)$ then U_y can be simplified:

$$\begin{aligned} U_y(\vec{r}, t) = & \frac{1}{\sqrt{2\pi^3}} \int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dk_y \int_{0}^{\infty} dk_z f(k_x, k_y, k_z) \\ & \times \exp(i(k_x x + k_y y)) \cos(k_z z - \omega t). \end{aligned} \quad (28)$$

For studying those pulses, we choose a simple half wave packet restricted in the x- and y-directions by Gaussians (see Figs. 2 and 3):

$$\begin{aligned} U_y(\vec{r}, t=0) &= \frac{E_0}{k_0^2 c} \cos(k_0 z) \exp\left(-\frac{x^2 + y^2}{w_0^2}\right) \\ &\quad \text{for} \quad |k_0 z| \le \frac{\pi}{2}. \\ U_y(\vec{r}, t=0) &= 0 \quad \text{for} \quad |k_0 z| \ge \frac{\pi}{2}. \end{aligned} \quad (29)$$

Here, we excluded and do not discuss terms including a small parameter ε with $\varepsilon \to 0$, which eliminate the singularity in the second derivative with respect to z at $|k_0 z| = \pi/2$. Then the result with $k_0 w_0 \to \infty$ for the fields is $\vec{E} = E_0 \cos(u)\vec{e}_x$

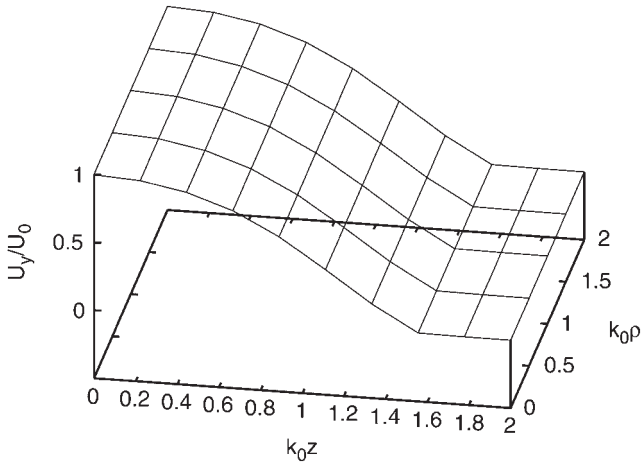


Fig. 2. The function U_y/U_0 with $U_0 = E_0/(k_0^2 c)$ as a function of $k_0 z$ and $k_0 \varrho$ for $k_0 ct = 0$ and $k_0 w_0 = 1000$.

and $\vec{B} = cE_0 \cos(u)\vec{e}_y$ with $u = k_0 z - \omega_0 t$. Now we calculate the Fourier transformation of U_y and obtain

$$\begin{aligned} f(\vec{k}) = & \frac{E_0}{k_0^2 c} \frac{w_0^2}{\sqrt{8\pi}} \left(\frac{\sin((k_0 - k_z)\frac{\pi}{2k_0})}{k_0 - k_z} + \frac{\sin((k_0 + k_z)\frac{\pi}{2k_0})}{k_0 + k_z} \right) \\ & \times \exp\left(-\frac{w_0^2}{4}(k_x^2 + k_y^2) \right). \end{aligned} \quad (30)$$

With the Fourier transform, we can calculate the wave packet $U_y(\vec{r}, t)$ by treating the following integral where we inserted cylindrical coordinates of $\vec{k}$ and $\vec{r}$ and made some substitutions ($\varrho = \sqrt{x^2 + y^2}$):

$$\begin{aligned} U_y(\vec{r}, t) = & \frac{E_0}{k_0^2 c} \frac{2}{\pi^2} \int_0^{\infty} dk_z \int_0^{\infty} dv \int_0^{\pi/2} d\varphi \exp(-v) \\ & \times \cos\left(\frac{2}{w_0} \varrho \sqrt{v} \cos(\varphi) \right) \\ & \times \left(\frac{\sin((k_0 - k_z)\frac{\pi}{2k_0})}{k_0 - k_z} + \frac{\sin((k_0 + k_z)\frac{\pi}{2k_0})}{k_0 + k_z} \right) \\ & \times \cos\left(k_z z - ct \sqrt{\frac{4}{w_0^2} v + k_z^2} \right). \end{aligned} \quad (31)$$

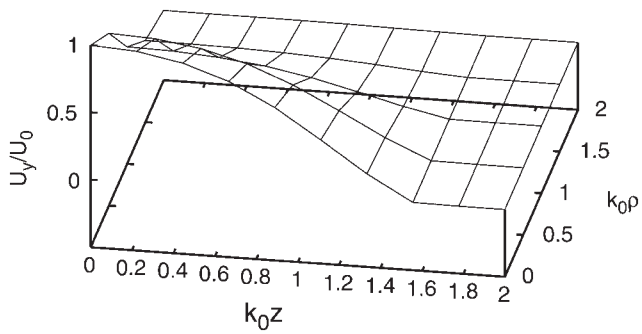


Fig. 3. The function U_y/U_0 with $U_0 = E_0/(k_0^2 c)$ as a function of $k_0 z$ and $k_0 \varrho$ for $k_0 ct = 0$ and $k_0 w_0 = 1$.

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These integrals cannot be solved analytically in general; they have to be handled numerically. In Figures 2, 3, 4, and 5, we show $(k_0^2 c/E_0)U_y$ as a function of the coordinates $k_0 z$ and $k_0 \varrho$ for fixed widths $k_0 w_0 = 1$ and 1000. These numerical calculations are time-consuming and presently not suitable for solving the equation of motion. It is seen from the figures that the wave with $k_0 w_0 = 1000$ runs like a plane wave whereas the wave with $k_0 w_0 = 1$ behaves quite differently.

6. APPROXIMATION OF FREQUENCY

In order to be able to solve the equation of motion for the electron with finite-ranged electromagnetic pulses, we approximate the frequency ω in Eq. (27). Assuming that the wave with the initial condition (29) proceeds essentially with the wave number k_0 in z-direction, we introduce the following expansion of the square root determining the frequency ω that is true for positive $k_z \approx k_0$

$$\begin{aligned}\frac{\omega}{c} &= \sqrt{k_x^2 + k_y^2 + (k_0 + (k_z - k_0))^2} \\ &= k_0 \sqrt{1 + \frac{k_x^2 + k_y^2}{k_0^2} + \frac{1}{k_0^2}(2k_0(k_z - k_0) + (k_z - k_0)^2)} \\ &\approx k_z + \frac{k_x^2 + k_y^2}{2k_0}.\end{aligned} \tag{32}$$

If k_z is negative and about $-k_z \approx k_0$ we can carry out a similar expansion setting $k_z = -k_0 + (k_z + k_0)$ and obtain for negative k_z the frequency

$$\frac{\omega}{c} \approx -k_z + \frac{k_x^2 + k_y^2}{2k_0}. \tag{33}$$

If these approximations and the Fourier transform (30) are inserted into Eq. (26), we can separate the integrals over

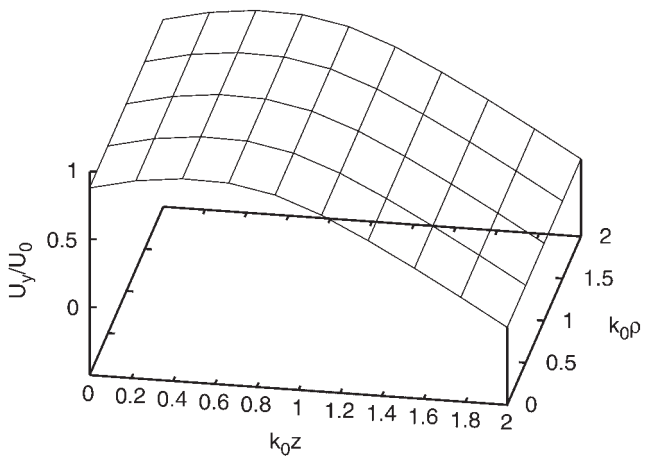


Fig. 4. The function U_y/U_0 with $U_0 = E_0/(k_0^2 c)$ as a function of $k_0 z$ and $k_0 \varrho$ for $k_0 ct = 0.5$ and $k_0 w_0 = 1000$.

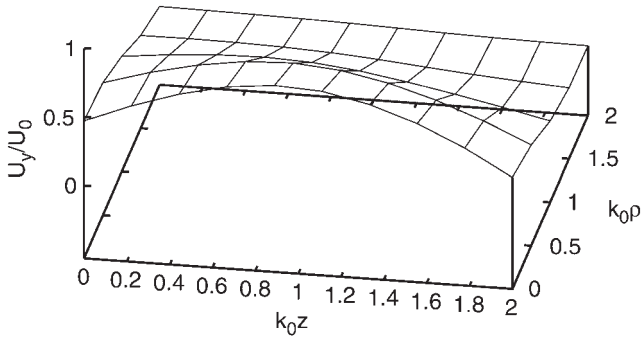


Fig. 5. The function U_y/U_0 with $U_0 = E_0/(k_0^2 c)$ as a function of $k_0 z$ and $k_0 \varrho$ for $k_0 ct = 0.5$ and $k_0 w_0 = 1$.

the coordinates:

$$\begin{aligned}U_y = \frac{E_0}{k_0^2 c}\frac{1}{2\pi}\Big\{ & I(k_0 w_0, k_0 x, k_0 ct) I(k_0 w_0, k_0 y, k_0 ct) \\ & \times \int_0^\infty dk \exp(i(k_0 z - k_0 ct)k) \\ & \times \left(\frac{\sin((1-k)\frac{\pi}{2})}{1-k} + \frac{\sin((1+k)\frac{\pi}{2k})}{1+k}\right) \\ & + I(k_0 w_0, k_0 x, -k_0 ct) I(k_0 w_0, k_0 y, -k_0 ct) \\ & \times \int_0^\infty dk \exp(-i(k_0 z - k_0 ct)k) \\ & \times \left(\frac{\sin((1-k)\frac{\pi}{2})}{1-k} + \frac{\sin((1+k)\frac{\pi}{2k})}{1+k}\right)\Big\}.\end{aligned} \tag{34}$$

Here, the dimensionless integral I depends on three real dimensionless parameters a, b, and c. It is given by

$$\begin{aligned}I(a,b,c) &= \frac{a}{2\sqrt{\pi}} \int_{-\infty}^{\infty} dv \exp(-(av/2)^2) \exp(i(bv - cv^2/2)) \\ &= \frac{1}{\sqrt{1 + i2c/a^2}} \exp\left(-\frac{b^2/a^2}{(1 + i2c/a^2)}\right).\end{aligned} \tag{35}$$

The complex integrals over k_z in (34) have to be solved numerically. For further simplification, we expand the integral (35) for $a^2 \gg c$, respectively, $w_0^2 \gg |ct/k_0|$, and obtain

$$I(a,b,c) \approx \exp\left(-\frac{b^2}{a^2}\right) \exp\left(-i\frac{c}{a^2}\left(1 - \frac{2b^2}{a^2}\right)\right). \tag{36}$$

If this approximate expression is inserted in (34), it results

$$\begin{aligned}U_y \approx \frac{E_0}{k_0^2 c}\frac{1}{\pi} \exp\left(-\frac{x^2 + y^2}{w_0^2}\right) & \int_0^\infty dk \\ & \times \cos\left((k_0 z - k_0 ct)k - \frac{k_0 ct}{(k_0 w_0)^2}\left(1 - 2\frac{x^2 + y^2}{w_0^2}\right)\right) \\ & \times \left(\frac{\sin((1-k)\frac{\pi}{2})}{1-k} + \frac{\sin((1+k)\frac{\pi}{2})}{1+k}\right).\end{aligned} \tag{37}$$

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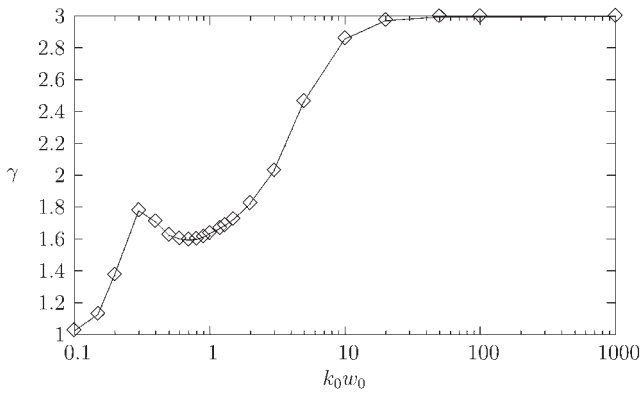


Fig. 6. The Lorentz factor γ for $a_0 = 1$, calculated with the approximation for U_y given in Eq. (34), as a function of k_0w_0. The diamonds are calculated values.

Figures 6, 7, and 8 show results obtained with the approximate U_y given in Eq. (34). If the parameter k_0w_0 is large (for example, $k_0w_0 = 1000$), then the laser pulse behaves like a plane wave and the Lorentz factor is $\gamma = 1 + 2a_0^2 = 3$ for $a_0 = 1$ as used in all numerical calculations. For $k_0w_0 = 1$, the Lorentz factor is smaller, namely $\gamma = 1.63$, since the electron leaves fast the region of high intensity of the laser pulse. The same behavior one can learn from Figures 7 and 8.

7. SUMMARY AND CONCLUSIONS

We showed that it is possible to use very short and intense laser pulses of a length of about one half-wavelength for the acceleration of the electrons. The laser pulse overruns the electron and lets the electron back in an accelerated state. A half-wavelength laser pulse with a strength parameter $a_0 = |eE_0|/(mc^2k_0)$ gives the electron an acceleration of $\gamma = 1 + 2a_0^2$ inside a longitudinal length of $(3\pi a_0^2)/(4k_0)$ and a transverse deviation of $-\pi a_0/k_0$. For $\gamma = 2 \times 10^6 (\sim 1$ TeV electrons) we obtain the longitudinal and transverse lengths $(1/k_0)0.75 \times \pi \times 10^6$ and $(1/k_0)\pi \times 10^3$ ($=0.75 \times \pi$ m and $\pi \times 10^{-3}$ m for $k_0 = 10^6$/m), respectively. This demonstrates the advantage of this acceleration that the transverse deviation of the electrons is relatively small.

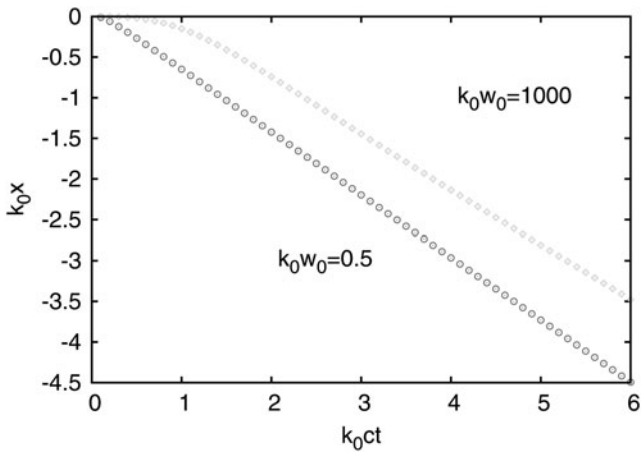


Fig. 7. The coordinate k_0x for $a_0 = 1$, calculated with the approximation for U_γ given in Eq. (34), as a function of time k_0ct, for $k_0w_0 = 0.5$ and 1000.

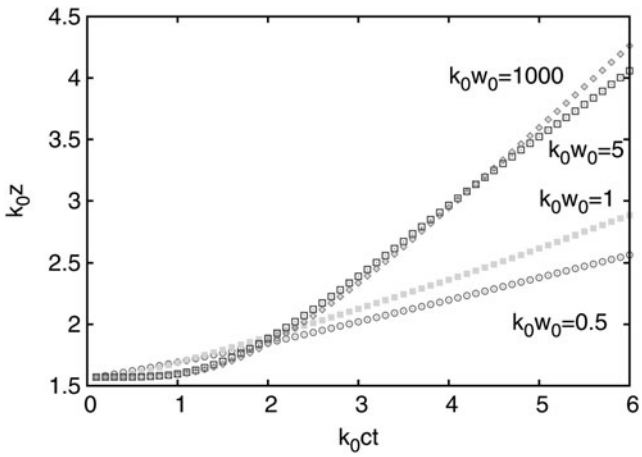


Fig. 8. The coordinate k_0z for $a_0 = 1$, calculated with the approximation for U_y given in Eq. (34), as a function of time k_0ct, for k_0w_0.

In contrast to several investigations in literature (see for example (Bahari & Taranukhin, 2003), our wave packets consist of waves with different frequencies. This makes the procedure more complex. We proposed an expansion of the frequency into the Cartesian components of the wave vector. It is clear that a direct integration of the three-dimensional integral over the three Cartesian components can be done, but presently our personal computers are too small for such a duty because a high precision for the electromagnetic fields is needed.

Further investigations for the construction of a practical accelerator have to be carried out. It is already evident that very high energy accelerators restricted to smaller length dimensions can probably only be developed with the help of intense laser pulses. More experimental facts are necessary for the near future of this research field.

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