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A NOTE ON SOCIAL SECURITY WELFARE WITH SELF-CONTROL PROBLEMS

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We develop an overlapping-generations model for a closed economy with uncertainty on labor income and mortality risk to show that unfunded social security programs may increase welfare in economies where agents are affected by self-control problems à la Gul and Pesendorfer (2001, *Econometrica* 69, 1403). We depart from the existing literature by setting the agent's preference parameters to match target levels of macro-variables observed in the real U.S. economy. In our approach, economies with tempted and nontempted agents are indistinguishable in terms of aggregate consumption, labor, and saving behavior when social security provides a replacement rate of 40% (as in the United States). This situation makes agents bear costly self-control problems over more years. Our simulations indicate that social security improves welfare with degrees of temptation equal to 11% or higher. A social security program with a replacement rate of 40% finds support for degrees of temptation not lower than 15%.

Keywords: Social Security, Temptation Disutility Model, Self-Control Problems, Overlapping-Generation Models

1. INTRODUCTION

Unfunded social security is the largest transfer program in most industrialized countries, and has an enormous impact on the main economic variables. Although this program serves several purposes (e.g., income redistribution, insurance against longevity risks), it is commonly believed that it exists primarily because some people lack the discipline needed to save for retirement [see, for instance, Diamond (1977)]. Despite this, a growing literature based on quantitative overlapping-generations models finds no welfare-improving role for social security, even after making behavioral assumptions on the agent's preferences [see Imrohoroglu et al. (2003); Kumru and Thanopoulos (2008); and the literature review in Fehr et al. (2008)]. The main reasons are that social security crowds out private savings

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[Auerbach and Kotlikoff (1987)], creates tight liquidity constraints to the youngest [Hubbard and Judd (1987)], and offers an implicit interest rate below the one available for private savings [Imrohoroglu et al. (2003)].

The aim of this paper is to study the welfare implications of social security when the economy is populated by agents affected by temptation and self-control problems à la Gul and Pesendorfer (2001). Temptation is a feature of human behavior consistent with psychological theories [DellaVigna (2009)] and observed empirically in different experiments [see Ameriks et al. (2007)]. Agents with temptation problems are rational, forward-looking, and sophisticated and have time-consistent preferences, but their preferences are biased toward alternatives that offer more immediate gratification. Their utility depends on actual consumption, on the one hand, and the self-control cost of resisting the temptation to consume all their assets, on the other. These agents may want to hold their savings in commitment devices to eliminate the most tempting options available for consumption, and therefore reduce the utility cost associated with the exertion of self-control.

Social security can be seen as a commitment device because it forces agents to make a mandatory saving, shielding them from the temptation to consume early [Akerlof (1998)]. Hence it is more valuable in an economy populated by tempted agents, because it reduces their self-control costs. Despite this, findings in Kumru and Thanopoulos (2008) suggest that accounting for temptation is not enough to generate welfare improvement with social security, unless unrealistically large preference parameters are taken into account. Their results are drawn from simulations of an overlapping-generations model where the discount factor is set endogenously to match the capital–output ratio of the U.S. economy, and the remaining preference parameters are set exogenously based on the literature with standard preferences. In such a setting, however, tempted agents are free to allocate their consumption and saving over the lifespan; in particular, they delay the accumulation of savings to limit their self-control costs. Although they benefit from the commitment brought by social security, the gain is small because it protects against small self-control costs. As a result, social security does not improve welfare it merely reduces welfare less than in an economy with nontempted agents.

In this work we analyze an environment similar to that in Kumru and Thanopoulos (2008), from which we depart in two directions. First, we take an agnostic perspective on the agents' preference parameters. For a range of reasonably small degrees of temptation we calibrate the remaining preference parameters—in our model, the discount factor, the utility curvature parameter, and the share of leisure in the utility function—to match three aggregate features of the U.S. data: the aggregate capital–output, labor–output, and consumption–output ratios. With this approach, economies with tempted and nontempted agents are indistinguishable in terms of steady-state aggregate consumption, labor, and savings behavior when social security provides a replacement rate of 40% (as currently in the United States). To generate the target levels of the economy, agents have less freedom in their saving allocation than in the setting of Kumru and Thanopoulos (2008); because they are forced to save from young ages, agents face higher self-control

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costs, and benefit more from the commitment provided by social security. We are thus interested in studying whether social security improves welfare in this framework. To emphasize that the issue of saving allocation is not negligible, a second departure from Kumru and Thanopoulos (2008) is the introduction into the model of endogenous labor supply decisions. Because social security has a distortionary effect on labor supply, the welfare gain it generates would be even stronger were labor supply decisions exogenous.

Our underlying model builds on the tradition of analyzing steady-state economies with unfunded social security programs. We simulate a quantitative overlapping-generations model for a closed economy populated by agents with self-control problems, and facing liquidity constraints, missing annuity markets, and idiosyncratic income and mortality risk similar to those in Imrohoroglu et al. (1995, 2003) and Kumru and Thanopoulos (2008). We calibrate the theoretical model to reproduce key features of the real U.S. economy, and simulate and compare steady states with different social security arrangements.

When the degree of temptation rises, all the other preference parameters change in our analysis. In particular, both the discount factor and the curvature parameter must increase (still remaining within reasonable values) to preserve target values of the macro variables. This environment indeed generates a welfare-improving role for social security. In our benchmark analysis, social security improves the welfare of agents with a degree of temptation equal to 11% or above. A program with a replacement rate of 40%, such as the one currently adopted in the United States, is considered optimal with degrees of temptation not below 15%.

The paper is organized as follows. Section 2 describes the overlapping-generations model used in this analysis. In Section 3 we discuss our main findings in the benchmark economy and perform a sensitivity analysis on some of the model's assumptions. Section 4 concludes. Finally, the Appendix provides computational details.

2. THE MODEL

The economy is populated by overlapping generations of individuals with identical preferences. Time is discrete and the period is assumed to be one year. Each year a new generation is born, and the population grows steadily at a constant rate n. In the following notation we omit the time subscript because our goal is to compare steady states.

Agents in the economy are observed over their adult lives (starting at 25). They face long but random lives that last at most $D = 75$ years, with retirement at an exogenous age $R = 40$. There is a constant probability $1-\pi$ that a person will die at the end of each period. Under the stationary population assumption, the cohort shares $\{\mu_j\}_{j=1}^{D}$ are stable over time and given by the relations

$$\mu_j = \mu_{j-1}\frac{\pi}{1+n}, \quad j = 1, \ldots, D, \tag{1}$$

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where their sum is normalized to one. An implication of (1) is that the size of the cohort aged j is greater than the size of the cohort aged $j-1$, $\mu_j > \mu_{j-1}$.

2.1. Agents

The timing of the model is as follows. A worker enters any age $j < R$ with a given amount of assets a_j, and chooses a fraction of time to spend working, $l_j = l_j(a_j) \in [0, 1]$. He or she then receives a lump-sum government transfer b and an age-specific income w_j. The sum of assets, government transfers, and income defines the total resources available to the agent, or cash on hand, $x_j = a_j + b + w_j$. The agent then determines the optimal level of consumption, $c_j = c_j(x_j)$, and invests the remaining savings, $x_j - c_j$, in a market portfolio yielding the risk-free real return r. The timing for a retiree is identical, with the exception that there is no labor choice and $l_j = 0$ always.

Income after retirement stems from a certain social security benefit $\rho_s w \bar{l}$, where ρ_s is the social security replacement rate, w the wage rate of the economy, and $\bar{l}$ the average amount of time spent working in the economy; income prior to retirement is subject to a stochastic employment opportunity. We denote the employment state by $\varepsilon \in \{0, 1\}$, where 0 and 1 indicate unemployment and employment states, respectively. This random variable follows an i.i.d. binomial distribution with probability ψ. That is, in any given year the probability of employment is ψ, independent of the employment state in the previous year. An unemployed agent ($\varepsilon = 0$) aged $j < R$ makes no time allocation and receives an unemployment insurance benefit equal to a fraction ρ_u of the income of an employed agent with the same characteristics (age and assets), $\rho_u w l_j$. An employed agent ($\varepsilon = 1$) makes his or her own time allocation, earns an income proportional to the time spent working, $w l_j$, and pays contributions $\theta_s w l_j$ and $\theta_u w l_j$, respectively, to the social security and unemployment insurance programs. The disposable income of an agent aged j is thus summarized as

$$w_j = \begin{cases} (1 - \theta_s - \theta_u)\, w l_j & j < R \text{ and } \varepsilon = 1 \\ \rho_u w l_j & j < R \text{ and } \varepsilon = 0 \\ \rho_s w \bar{l} & j \geq R. \end{cases} \tag{2}$$

Preferences are described by a Gul and Pesendorfer (2001) temptation utility function,

$$U(c_j, l_j, x_j) = u(c_j, l_j) - [v(x_j, l_j) - v(c_j, l_j)], \tag{3}$$

where $u(\cdot)$ and $v(\cdot)$ are two von Neumann–Morgenstern utility functions. We assume as in Gul and Pesendorfer (2004) that agents are tempted by the opportunity to consume immediately as much as possible.[1] If they cannot die with debt, uncertainty in death age binds consumption at any time to be no larger than cash on hand, $c_j \leq x_j$.[2] Cash on hand, $x_j = \arg\max_{c_j}\{v(c_j, l_j)\}$, is thus the most tempting alternative from the set of possible consumption choices, and the

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difference $v(x_j, l_j) - v(c_j, l_j)$ represents the psychological cost of self-control faced when choosing c_j rather than x_j.

We characterize $u(c_j, l_j)$ with a Constant Relative Risk Averstion (CRRA) utility function with curvature parameter $\gamma > 0$ and argument given by a Cobb–Douglas aggregate of consumption, c_j, and leisure, $1 - l_j$, with the parameter $\phi \in [0, 1]$ denoting the share of leisure. The function $v(\cdot)$ is equal to $u(\cdot)$ rescaled by a factor $\lambda \geq 0$:

$$u(c_j, l_j) = \frac{\left[c_j^{1-\phi}(1-l_j)^{\phi}\right]^{1-\gamma} - 1}{1-\gamma}, \quad v(c_j, l_j) = \lambda u(c_j, l_j). \tag{4}$$

The parameter λ measures the agent's sensitivity to the tempting alternative. Agents with $\lambda = 0$ are the forward-looking decision-makers in standard life-cycle models. The larger λ is, the more temptation to consume takes control of the individual's behavior, and agents rationally choose to postpone the accumulation of wealth to limit the cost of self-control. The extreme case of $\lambda \to \infty$ describes completely myopic agents who accumulate no wealth and consume all their income in every period.

The intertemporal maximization problem is

$$V_j = \max_{\{c_{j+s}, l_{j+s}\}_{s=0}^{D-j}} E_j \left[\sum_{s=0}^{D-j} (\beta\pi)^s U(c_{j+s}, l_{j+s}, x_{j+s}) \right], \tag{5}$$

with discount factor β, one-period survival probability π, and $x_{j+s} = a_{j+s} + b + w_{j+s}$, subject to constraints (2)–(4) and the intertemporal budget constraint

$$a_{j+s+1} \leq (1+r)(a_{j+s} - c_{j+s} + b + w_{j+s}), \quad a_{j+s+1} \geq 0, \tag{6}$$

with $a_1 = 0$ (no assets at birth).

Because we assume no bequest motives, optimal consumption at death age $j = D$ is $c_j = x_j$. At any age $j < D$ the optimal choice of consumption, c_j, and labor, l_j, is implicit in the first-order conditions

$$\begin{cases} \dfrac{\partial U}{\partial c_j} = (1+r)\,\beta\pi\, E_j \left[\dfrac{\partial U}{\partial c_{j+1}} + \dfrac{\partial U}{\partial a_{j+1}} \right] + \zeta_{j+1} \\ \dfrac{\partial U}{\partial l_j} = -\dfrac{\partial w_j}{\partial l_j} \dfrac{\partial U}{\partial c_j} \end{cases}, \tag{7}$$

where only the first equation in (7) holds after retirement, because there is no labor choice and $l_j = 0$ for any $j = R, \ldots, D$. The Lagrange multiplier ζ_{j+1} is positive if the liquidity constraint is binding ($c_j = x_j$, which implies $a_{j+1} = 0$) and 0 otherwise. If $\zeta_{j+1} > 0$, marginal utility of consumption is higher than the consumer would wish, and consumption is lower.

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2.2. Aggregate Technology

The production technology of the economy is given by a constant–returns to scale Cobb–Douglas function that uses capital K and labor L to produce the single good in the model,

$$Y = f(K, L) = BK^{1-\alpha}L^{\alpha}, \tag{8}$$

where Y is aggregate output and $\alpha \in (0, 1)$ is labor's share of output. B is a technological parameter growing at a deterministic, exogenously given rate σ. The firm solves the maximization problem

$$\max_{K,L} \{Y - (r + d) K - wL\} \tag{9}$$

subject to the constraint (8), with capital depreciation rate d. The first-order conditions require that

$$\begin{cases} r = (1 - \alpha) Y/K - d \\ w = \alpha Y/L. \end{cases} \tag{10}$$

2.3. Government

The government has a threefold role in this economy. It administers the social security and unemployment insurance programs after choosing the replacement rates ρ_s and ρ_u. The two programs are self-financed, and the contribution rates θ_s and θ_u are chosen to meet the conditions

$$\sum_{j=R+1}^{D} \sum_{a} \mu_j \xi_j (a) \rho_s w = \sum_{j=1}^{R-1} \sum_{a} \mu_j \xi_j (a, 1) \theta_s w l_j(a_j), \tag{11}$$

$$\sum_{j=1}^{R-1} \sum_{a} \mu_j \xi_j (a, 0) \rho_u w l_j(a_j) = \sum_{j=1}^{R-1} \sum_{a} \mu_j \xi_j (a, 1) \theta_u w l_j(a_j) \tag{12}$$

with $\xi_j(a) = \xi_j(a, 1) + \xi_j(a, 0)$ and $\xi_j(a, \varepsilon)$ a measure of agent types in age j, with assets a and employment state ε.

Additionally, the government distributes lump-sum transfers b equally among the members of all generations. Transfers are financed solely by accidental bequests, according to the following condition:

$$\sum_{j=1}^{D} \sum_{a} \sum_{\varepsilon} \mu_j \xi_j (a, \varepsilon) b = \frac{1}{1 + n} \sum_{j=2}^{D} \sum_{a} \sum_{\varepsilon} \mu_{j-1} \xi_{j-1} (a, \varepsilon) (1 - \pi) a_j. \tag{13}$$

2.4. Market Equilibrium

In equilibrium, all per capita quantities grow at a balanced growth rate $g = (1 + \sigma)^{1/\alpha} - 1$. Given the government policy $\{\rho_s, \rho_u\}$ and the individual preference parameters $\{\tau, \beta, \gamma, \phi\}$, a steady-state competitive equilibrium defines the

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allocation $\{c_j, l_j\}_{j=1}^{D}$, the *price system* $\{r, w\}$, and the lump-sum distribution of accidental bequests b such that (i) agents maximize their intertemporal utility function by solving (7), given the price system and the lump-sum transfers; (ii) the firm maximizes its profits solving the first-order conditions (10); (iii) the government administers social security and unemployment insurance programs and distributes transfers satisfying conditions (11)–(13); (iv) aggregate and individual behaviors are consistent,

$$\begin{cases} C = \sum_{j=1}^{D} \sum_{a} \sum_{\varepsilon} \mu_j \xi_j (a, \varepsilon)\, c_j(a_j) \\ L = \sum_{j=1}^{R-1} \sum_{a} \mu_j \xi_j (a, 1)\, l_j(a_j) \\ K = \dfrac{1}{1+n} \sum_{j=1}^{D} \sum_{\grave{a}} \sum_{\varepsilon} \mu_j \xi_j (\grave{a}\varepsilon)\, \grave{a}_j, \end{cases} \tag{14}$$

where $\{\grave{a}_j\}_{j=1}^{D}$ are previous-year asset holdings; (v) the capital market clears,

$$Y = C + K' - (1-d)K, \tag{15}$$

with $K' = \sum_{j=1}^{D} \sum_{a} \sum_{\varepsilon} \mu_j \xi_j(a, \varepsilon) a_j$ the next-year level of capital; and (vi) the measure of agent types satisfies the conditions

$$\xi_j(a', 1) = \sum_{a} \sum_{\varepsilon} \psi \xi_{j-1}(a, \varepsilon) \quad \xi_j(a', 0) = \sum_{a} \sum_{\varepsilon} (1-\psi)\, \xi_{j-1}(a, \varepsilon), \tag{16}$$

where ξ_1 is given.

2.5. Welfare Evaluation

We define the certain equivalent consumption CEC as the constant consumption flow over the life cycle under perfect commitment technology that makes a newborn agent (with $j = 1$) as well off as with the intertemporal utility V_1,

$$\mathrm{CEC}(\rho_s) = \left(V_1 \frac{1}{\sum_{l=0}^{D-1} (\beta\pi)^l} \frac{(1-\gamma)}{(1-\tilde{l})^{\phi(1-\gamma)}} \right)^{\frac{1}{(1-\phi)(1-\gamma)}}, \tag{17}$$

where $\tilde{l}$ is the average time spent working over the lifetime,

$$\tilde{l} = \frac{L}{\sum_{l=1}^{R-1} \sum_{a} \mu_j \xi_j (a, 1) + \sum_{l=R}^{D} \sum_{a} \mu_j \xi_j (a)}. \tag{18}$$

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We follow the literature [see Imrohoroglu et al. (2003); Kumru and Thanopoulos (2008)] and quantify welfare gains and losses by computing compensating variations. These are defined as the percentage variation in CEC between two scenarios A and B differing in the social security replacement rate (ρ_s^A and ρ_s^B respectively):

$$\mathrm{CV}\left(\rho_s^A, \rho_s^B\right) = 100\frac{\mathrm{CEC}\left(\rho_s^A\right) - \mathrm{CEC}\left(\rho_s^B\right)}{\mathrm{CEC}\left(\rho_s^B\right)}. \tag{19}$$

In our analysis, scenario A always refers to a laissez-faire economy with no social security, $\rho_s^A = 0$. A positive $\mathrm{CV}_1(0, \rho_s^B)$ means that agents are worse off with social security, and $\mathrm{CV}_1(0, \rho_s^B)$ denotes the percentage by which consumption must be increased in B to compensate for the decrease in welfare generated by the presence of social security.

3. SIMULATION RESULTS

This section reports the main results from the simulation of the model described in Section 2. The model has eleven exogenous parameters, which we classify into two groups. The first group consists of seven independent parameters than can be pinned down independent of all the other parameters by one or more targets. The targets we consider are seven statistics of the economy when the social security replacement rate is $\rho_s = 0.4$[3]: the average length of adult life, the old-age dependency ratio, the unemployment rate, the unemployment insurance rate, labor's share of output, the market rate of return, and the balanced growth rate. The targets, shown in Table 1, are relatively standard in the macroeconomic literature [see, for instance, Imrohoroglu et al. (2003); Chatterjee et al. (2007)] and are meant to capture the main economic, demographic, and political aspects of the U.S. economy.

The second group of parameters consists of four preference parameters (the degree of temptation, λ, the discount factor, β, the curvature parameter, γ, and the share of leisure in the utility function, ϕ) that interact with each other to determine the actual behavior. In this case, we set a range of values exogenously for the normalized degree of temptation $\tau = \lambda/(1+\lambda) \in [0, 1]$ and determine the remaining parameters jointly to match target capital–output, labor–output, and consumption–output ratios;[4] when $\tau = 0$ we obtain $\beta = 0.9808$, $\gamma = 1.9943$, and $\phi = 0.5116$. Finally, for the sake of simplicity, we set the technology parameter B in the production function so that output $Y = 1$ when $\rho_s = 0.4$.

3.1. Benchmark Case: Closed Economy

When temptation τ increases in the range $\{0, 0.01, \ldots, 0.15\}$, both the discount factor β and the curvature parameter γ rise steadily, respectively from 0.9808 to 1.0001, and from 1.9943 to 2.7976. Intuitively, a higher level of temptation makes self-control costlier, and agents choose to save less and consume more.

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TABLE 1. Calibration of the exogenous parameters

Definition	Symbol	Value
	Targets when $\rho_s = 40\%$	
Average length of adult life	$\sum_{j=1}^{T} j\pi^{j-1}(1-\pi)$	55
Old age dependency ratio	$\sum_{j=R+1}^{T} \mu_j \Big/ \sum_{j=1}^{R} \mu_j$	0.25
Unemployment rate	$(1-\psi)$	0.06
Unemployment insurance rate	ρ_u	0.25
Labor's share of output	wL/Y	0.67
Market rate of return	r	0.06
Balanced growth rate	$(1+\sigma)^{\frac{1}{\alpha}} - 1$	0.02
Capital–output ratio	K/Y	2.50
Labor–output ratio	L/Y	0.26
Consumption–output ratio	C/Y	0.70
	Independent parameters	
Survival rate	π	0.9762
Population growth rate	n	0.0089
Employment probability	ψ	0.94
Unemployment insurance rate	ρ_u	0.25
Labor's share of output	α	0.67
Capital depreciation rate	d	0.072
Technological progress	σ	0.0133
	Preference parameters	
Degree of temptation	$\tau = \lambda/(1+\lambda)$	$\{0, 0.01, \ldots, 0.15\}$
Other parameters	β, γ, ϕ	Vary with τ 0.9808, 1.9943, 0.5116 if $\tau = 0$

The discount factor and the curvature parameter then rise to guarantee the same aggregate level of savings and consumption as when $\tau = 0$. The share of leisure in the utility function ϕ may instead rise or fall when τ increases. This parameter varies from 0.5116 when $\tau = 0$ to 0.5215 when $\tau = 0.14$.

Social security has two opposite effects in this economy. On the one hand, it reduces wealth (and welfare), as it provides an implicit return on contributions lower than the market return available for private savings *(distortionary effect)*. On the other, it reduces the cost of self-control, thus increasing welfare *(commitment effect)*. This latter effect prevails if there is a substantial cost of self-control or the difference between implicit and market returns is small. Table 2 reports for each degree of temptation the compensating variation between a laissez-faire economy with $\rho_s = 0$ and programs respectively with a small replacement rate, $\rho_s = 0.1$, and a replacement rate similar to the one in the United States, $\rho_s = 0.4$. Positive values of the CV measure indicate that welfare is higher under the laissez-faire economy. For instance, $\text{CV}(0, 0.4) = 8.3418\%$ when $\tau = 0$

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TABLE 2. Preferred social security program by temptation

			Preferred scenario						
τ	CV(0, 0.1)	CV(0, 0.4)	ρ_s	r	w	Y	K	L	C
0.00	1.6939	8.3418	0	0.0435	2.7523	1.1844	3.3845	0.2883	0.7813
0.02	1.3821	7.6441	0	0.0419	2.7706	1.1963	3.4663	0.2893	0.7836
0.04	1.1981	6.9439	0	0.0409	2.7828	1.2076	3.5295	0.2907	0.7878
0.06	0.8598	5.9111	0	0.0395	2.8001	1.2188	3.6067	0.2916	0.7900
0.08	0.4177	4.6994	0	0.0382	2.8159	1.2319	3.6874	0.2931	0.7929
0.10	0.2063	3.4011	0	0.0370	2.8315	1.2432	3.7631	0.2942	0.7958
0.11	−0.0391	2.5846	0.05	0.0396	2.8004	1.2101	3.5792	0.2895	0.7848
0.12	−0.3869	1.5808	0.10	0.0355	2.8512	1.2629	3.8770	0.2968	0.8014
0.13	−0.6710	0.5145	0.10	0.0381	2.8172	1.2269	3.6760	0.2918	0.7887
0.14	−0.9274	−0.6703	0.15	0.0337	2.8774	1.2771	3.9876	0.2974	0.8022
0.15	−1.3965	−2.0614	0.40	0.0600	2.5769	1.0000	2.5000	0.2600	0.7000

means that nontempted agents have a welfare gain of 8.34% of their lifetime equivalent consumption if they move from an economy with $\rho_s = 0.4$ to one with $\rho_s = 0$. With greater temptation, the commitment effect becomes relatively more important. Both CV measures indeed decrease as temptation increases, and we observe welfare gains when $\rho_s = 0.1$ for $\tau \geq 0.11$.

Table 2 also shows for each level of temptation the preferred social security replacement rate, along with the main equilibrium variables generated by this program. We search for the preferred replacement rate ρ_s within the range $\{0, 0.05, \ldots, 1\}$. From the table we see that an economy without social security is always the preferred scenario as long as $\tau \leq 0.10$. When $\tau = 0.11$, the best scenario involves a program with $\rho_s = 0.05$. When τ increases further, the welfare improvement is greater and other replacement rates also provide higher welfare than is the case with $\rho_s = 0$. However, a replacement rate $\rho_s = 0.4$ is the preferred choice only when there is more severe temptation, $\tau = 0.15$.

Figure 1 shows the compensating variations CV(0, 0.1) (top panel) and CV(0, 0.4) (bottom panel) by level of temptation, assuming two different preference structures. The solid line refers to the environment of Table 2, where all three parameters $\{\beta, \gamma, \phi\}$ vary with temptation to match target levels of capital, labor, and consumption. The dashed line is instead based on the assumption that only β varies with temptation, to match the target level of capital; the remaining two parameters $\{\gamma, \phi\}$ are kept fixed to their values when $\tau = 0$. The latter environment is similar in spirit to previous studies, noticeably Imrohoroglu et al. (2003) and Kumru and Thanopoulos (2008). Although the CV measure declines with temptation under both preference structures, showing that the commitment effect of social security becomes more important than the distortionary effect, the measure on the dashed line is always higher and never finds support for a program with $\rho_s = 0.1$ in the range of selected degrees of temptation.

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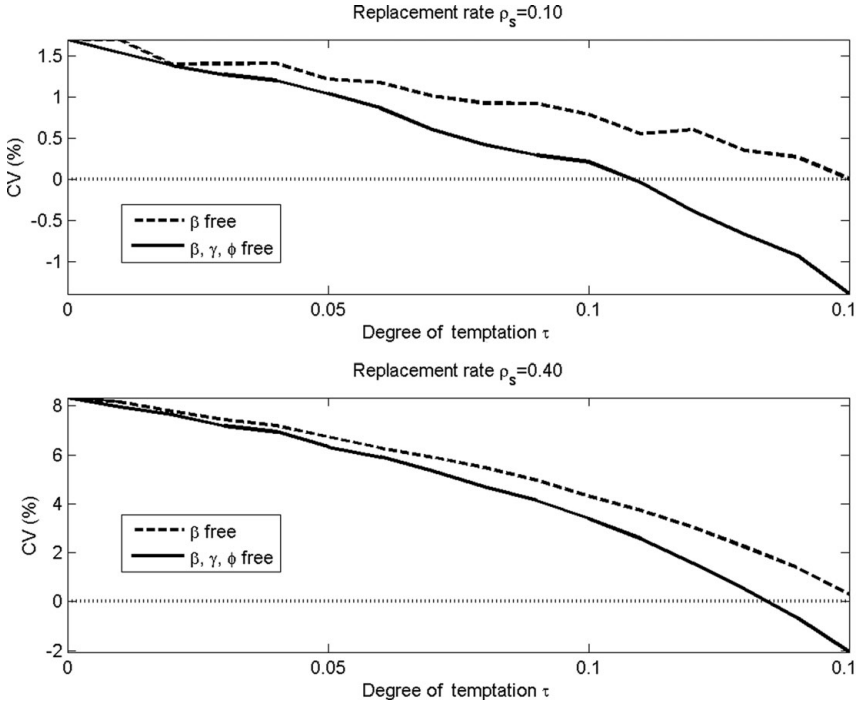


FIGURE 1. Social security compensating variation.

Table 3 reports the main variables in steady-state economies populated by agents with $\tau = 0.11$, with replacement rates ranging from 0 to 0.60. Here, $\beta = 0.9951$, $\gamma = 2.4747$, and $\phi = 0.5199$.[5] We see that social security has an enormous impact on the factor prices: the higher the replacement rate, the higher the market interest rate and the smaller labor income. When $\rho_s = 0.4$ output, capital, labor, and consumption are equal respectively to 1, 2.50, 0.26, and 0.70 by construction; they all reach their highest levels when the replacement rate is zero. Notice that agents in this economy are better off with some social security protection ($\rho_s \leq 0.1$), even though aggregate consumption falls. Welfare gains arise because only tempted agents face self-control costs, and they receive large welfare gains from the commitment implicit in social security.

Agents with the same level of temptation, $\tau = 0.11$, but with (γ, ϕ) fixed to the levels of nontempted agents would instead show lower levels of output and capital, and higher levels of labor and consumption. In particular, they would exhibit higher labor and consumption when $\rho_s = 0.4$ ($L = 0.2607$, $C = 0.7142$), as no restriction is imposed on such variables. This small difference in the variables has a marked impact on welfare. Figure 2 provides an intuition for this, comparing the average life-cycle asset holdings when $\rho_s = 0$ of agents with $\tau = 0$, $\tau = 0.11$ and free discount factor, $\tau = 0.11$ and free preference parameters (as in Table 3). In the

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TABLE 3. Equilibrium aggregate values, $\tau = 0.11$

ρ_s	r	w	Y	K	L	C	CV$(0, \rho_s)$
0.00	0.0364	2.8407	1.2488	3.8024	0.2945	0.7968	—
0.05	1.0396	2.8004	1.2101	3.5793	0.2895	0.7848	−0.1564
0.10	0.0425	2.7646	1.1729	3.3800	0.2843	0.7720	−0.0391
0.15	0.0455	2.7298	1.1555	3.2449	0.2836	0.7599	0.4327
0.20	0.0482	2.6999	1.1253	3.0902	0.2792	0.7479	0.9295
0.25	0.0504	2.6750	1.0809	2.9131	0.2707	0.7357	1.4281
0.30	0.0541	2.6363	1.0747	2.8122	0.2732	0.7230	2.0923
0.35	0.0563	2.6138	1.0124	2.6033	0.2595	0.7106	2.3545
0.40	0.0600	2.5769	1.0000	2.5000	0.2600	0.7000	2.5846
0.45	0.0627	2.5520	0.9732	2.4493	0.2555	0.6889	2.5484
0.50	0.0648	2.5327	0.9532	2.2990	0.2522	0.6807	2.4764
0.55	0.0662	2.5205	0.9447	2.2562	0.2511	0.6758	2.5570
0.60	0.0669	2.5141	0.9401	2.2339	0.2505	0.6736	2.9297

three cases the discount factor β is set respectively to 0.9808, 0.9926, and 0.9951; the curvature parameter γ is set to 1.9943, 1.9943, and 2.4747; the consumption–leisure substitution parameter ϕ is set to 0.5116, 0.5116, and 0.5199. A positive

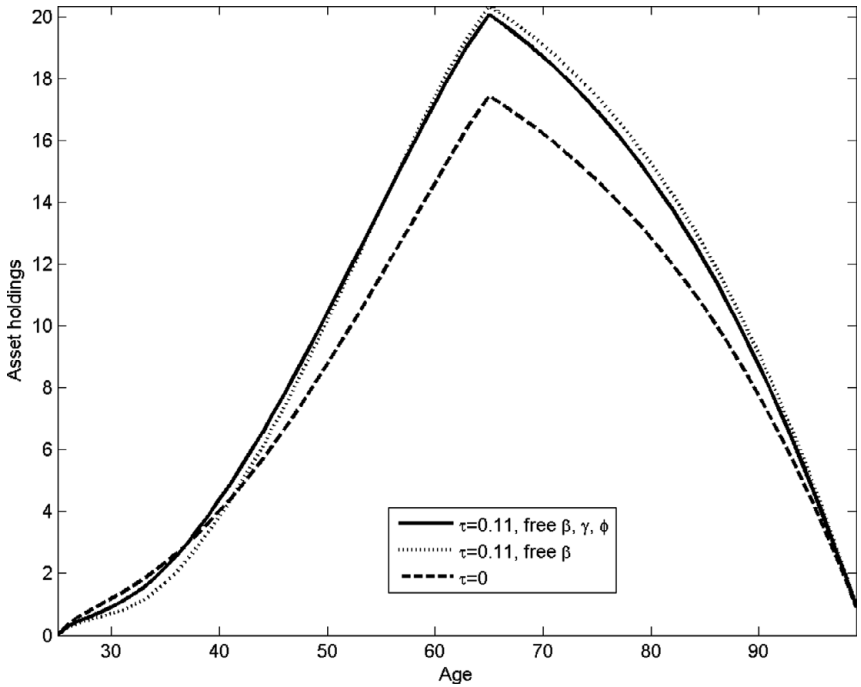


FIGURE 2. Life-cycle asset holdings, $\rho_s = 0$.

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degree of temptation τ creates a desire to consume immediately, disregarding any forward-looking planning. This desire is offset by the additional precautionary saving motive induced by a larger discount factor. However, savings tend to be delayed and concentrated at older ages because holding wealth is costly (it creates self-control problems). As a consequence, because the individuals in the economy are identical, the level of aggregate consumption is higher than in an economy populated by nontempted agents. (Young cohorts consume more and old cohorts consume less, but in the economy there are more young people than old people.) Only when we allow (γ, ϕ) to vary do savings start earlier, so that the wealth profile at young ages is closer to the one without temptation. This happens because agents with these preferences are forced to have the same aggregate consumption as nontempted agents.[6] Because these agents save regularly and hold more wealth over a large number of years, they face larger self-control costs than agents with free discount factor, and give more value to the commitment offered by social security.

3.2. Alternative Case: Small Open Economy

Social security crowds out private savings and reduces labor supply. In a closed economy this affects labor income and the market interest rate; specifically, income decreases and the interest rate increases when the replacement rate is higher (see for instance Table 3).

The consequence of assuming a small open economy, and keeping the factor prices fixed to their level when $\rho_s = 0.4$ (that is, $r = 0.06$ and $w = 2.5769$), is nontrivial. On one hand, agents now earn greater returns from saving privately in a laissez-faire economy (6% rather than about 4%); this increases the crowding-out effect of social security. On the other, agents benefit more from the commitment offered by social security, as they would otherwise face more severe self-control costs, by saving more to earn higher returns. Which effect prevails depends on the degree of temptation and the size of the program. If the replacement rate is low, the first effect prevails and social security becomes more detrimental to welfare. In contrast, if the replacement rate is high, the second effect dominates and social security is seen more favorably (whether it improves welfare or not). In this setting, one may thus expect to find support only for programs with relatively high replacement rates.

Table 4 confirms this argument. In this framework, social security improves welfare for any degree of temptation $\tau \geq 0.08$.[7] However, only high replacement rates are preferred to the laissez-faire scenario ($\rho_s \geq 0.5$ when $\tau = 0.08$). A replacement rate $\rho_s = 0.4$ enhances welfare when $\tau \geq 0.11$, although agents would always opt for higher replacement rates. For degrees of temptation $\tau \geq 0.13$ a scenario with a replacement rate $\rho_s = 1$ is also preferred to a laissez-faire scenario with $\rho_s = 0$, but it is never the optimal scenario. What restrains agents from considering optimal a replacement rate $\rho_s = 1$ is the high payroll tax they would bear, which is painful at young ages given liquidity constraints.

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TABLE 4. Partial equilibrium

			Preferred scenario						
τ	CV(0, 0.1)	CV(0, 0.4)	ρ_s	r	w	Y	K	L	C
0.00	1.4265	6.1530	0	0.0600	2.5769	1.5533	5.2431	0.2755	0.7650
0.02	1.3051	5.2932	0	0.0600	2.5769	1.5708	5.4110	0.2757	0.7668
0.04	1.0637	4.2289	0	0.0600	2.5769	1.5914	5.6125	0.2761	0.7696
0.06	1.0343	3.1359	0	0.0600	2.5769	1.6073	5.7729	0.2755	0.7694
0.08	0.9443	1.7540	0.50	0.0600	2.5769	0.8503	0.9757	0.2567	0.6859
0.10	0.7506	0.2311	0.55	0.0600	2.5769	0.8493	0.9719	0.2572	0.6873
0.11	0.6534	−0.5179	0.55	0.0600	2.5769	0.8657	0.9938	0.2554	0.6832
0.12	0.6644	−1.5386	0.55	0.0600	2.5769	0.8552	0.9901	0.2561	0.6851
0.13	0.6674	−2.4326	0.55	0.0600	2.5769	0.8569	0.9948	0.2562	0.6852
0.14	0.6102	−3.1756	0.55	0.0600	2.5769	0.8583	0.9975	0.2550	0.6823
0.15	0.5649	−4.5568	0.60	0.0600	2.5769	0.8411	0.9392	0.2557	0.6833

An economy with these factor prices still generates our target levels of capital, labor, and consumption when $\rho_s = 0.4$. Notice, however, that in the preferred situation agents accumulate very little capital when $\rho_s > 0$ (1 or less). This happens because they make only limited private savings and rely almost exclusively on social security, to avoid self-control costs.

4. CONCLUDING REMARKS

In this paper we have shown that social security may improve welfare in economies where agents are affected by self-control problems à la Gul and Pesendorfer (2001), and preference parameters adjust to match some target variables. We have developed a quantitative overlapping-generation model for a closed economy, with endogenous labor supply and saving decisions, and uncertainty in employment and survival probabilities. In our exercise we have considered a range of possible values for the degree of temptation, and we have set the remaining preference parameters of the model (the discount factor, the utility curvature parameter, and the consumption–leisure substitution parameter) to match target levels of aggregate capital, labor, and consumption. Given our methodology, economies with tempted and nontempted agents are indistinguishable in terms of aggregate consumption, labor, and saving behavior when social security provides a replacement rate of 40%. Allowing all the preference parameters to vary with temptation, our simulations calibrated to the U.S. economy show a welfare-improving role of social security for degrees of temptation equal to 11% or higher.

Although our results provide justification for mandatory social security programs, the question of whether social security is the most efficient commitment device available in the economy goes beyond the purpose of this paper. The model does not consider alternative sources of commitment, such as retirement assets

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and real wealth, that are widespread in reality and can in principle provide higher returns than unfunded social security. Future research will address this issue.

NOTES

1. Although this case is not discussed in the literature, individuals might in principle exhibit temptation toward leisure as well as consumption. Compared to nontempted agents, those featuring temptation toward leisure find it costlier to work, and indulge in leisure at higher rates. This situation is, however, not relevant for our purpose, as it acknowledges no additional value to the saving commitment implicit in social security.

2. We make this assumption for sake of simplicity and consistently with Gul and Pesendorfer (2004) and Kumru and Thanopoulos (2008). If we removed it, and allowed agents to borrow against future income up to a certain amount, the most tempting alternative would be larger, raising the demand for commitment devices. Hence we would observe larger support for social security in our economy of tempted agents.

3. This value roughly corresponds to the current replacement rate in the United States, and generates a payroll tax rate of 10.64% as opposed to the 10.6% of full Old Age and Survivors Insurance (OASI).

4. As regards the labor–output ratio, the goal is to induce agents to work, on average across all cohorts, around 34% of their available time (38 hours in a week of 112 available hours—7 days times 16 hours per day). Summing over the working population currently employed, $L = 0.34 \times 0.8 \times 0.94 = 0.2557$.

5. We focus on this degree of temptation, as it is the lowest for which social security improves welfare. The qualitative results do not change if we focus on different degrees.

6. To reduce aggregate consumption while preserving the aggregate capital, agents must anticipate the accumulation of life-cycle savings. For them it is more convenient to increase savings at young ages, as the size of the increase is smaller and can be absorbed more easily over a long horizon. The increase needed to reduce aggregate consumption is smaller because the size of young cohorts is greater than the size of older cohorts, and therefore changes in the behavior of young cohorts weigh relatively more in the aggregate.

7. DeJong and Ripoll (2007) estimate that $\lambda = 0.0787$, which implies that $\tau = 0.073$, to reconcile the theory with data on stock price volatility, risk-free rates, and equity premiums.

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APPENDIX: MODEL COMPUTATION

Equilibrium is obtained using a computational method conditional on the degree of temptation τ. When $\rho_s = 0.4$ the algorithm consists of three steps: (i) An *initialization step*, where we set the target variables and compute the factor prices $\{r, w\}$ as well as the first group of independent parameters $\{\rho, n, \psi, \rho_u, \alpha, d, \sigma\}$. We also make a guess on the preference parameters $\{\beta, \gamma, \phi\}$. (ii) A *demand step*, where the agent's decision problem is solved given factor prices and independent and preference parameters. (iii) An *equilibrium step*, where the market-clearing conditions in (14) are solved to find the preference parameters. When $\rho_s \neq 0.4$ the algorithm also consists of three steps: (i) An *initialization step*, where we use the preference parameters and the independent parameters obtained above, and make a guess on the factor prices. (ii) A *demand step*, identical to the above. (iii) An *equilibrium step*, where the market-clearing conditions for capital and labor in (14) are solved to find the factor prices.

We solve the agent's decision problem recursively by backward induction using value function iteration. Following Carroll's (2006) "method of endogenous gridpoints," we discretize the state space in 100 points along savings s_i, $i = 1, \ldots, 100$, defined as the difference between cash on hand and consumption, using a grid with triple exponential growth to be more accurate at small values. To compute optimal leisure we also discretize the labor space in 50 points, l_k, $k = 1, \ldots, 50$. For each age j from terminal age backward to the first retirement age $j = D, D-1, \ldots, R$, the labor choice $l_j = 0$ and we get for any saving gridpoint the optimal consumption solution $\hat{c}_{i,j} = c(s_i, j)$ from the first equation in (7).

For each age j from the last working age backward to the starting age $j = R-1, R-2, \ldots, 1$, we solve for consumption as a function of savings and labor, $\hat{c}_{i,k,j} = c(s_i, l_k, j)$, and we determine the optimal labor choice $\hat{l}_{i,j} = l(s_i, j)$ from the second equation in (7). It follows that optimal consumption is $\hat{c}_{i,j} = c(s_i, \hat{l}_{i,j}, j)$. Cash on hand and wealth are then endogenously derived respectively as $\hat{x}_{i,j} = s_i + \hat{c}_{i,j}$ and $\hat{a}_{i,j} = \hat{x}_{i,j} - (1 - \theta_s - \theta_u) w \hat{l}_{i,j}$ before retirement, and $\hat{a}_{i,j} = \hat{x}_{i,j} - \rho_s w \bar{l}$ after retirement. These values are used as a state space grid to determine the optimal consumption and labor functions, $c(x, j)$ and $l(a, j)$. For points that lie outside the state space grid, we evaluate the functions using linear interpolation. We use these numerical solutions to generate 1,000 simulations of the optimal behavior over random realizations of unemployment spells and death age.

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