Economics and Philosophy, **20** (2004) 307–330 Copyright © Cambridge University Press
DOI: 10.1017/S0266267104000227

INFINITE UTILITARIANISM: MORE IS ALWAYS BETTER

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We address the question of how finitely additive moral value theories (such as utilitarianism) should rank worlds when there are an infinite number of locations of value (people, times, etc.). In the finite case, finitely additive theories satisfy both Weak Pareto and a strong anonymity condition. In the infinite case, however, these two conditions are incompatible, and thus a question arises as to which of these two conditions should be rejected. In a recent contribution, Hamkins and Montero (2000) have argued in favor of an anonymity-like isomorphism principle and against Weak Pareto. After casting doubt on their criticism of Weak Pareto, we show how it, in combination with certain other plausible principles, generates a plausible and fairly strong principle for the infinite case. We further show that where locations are the same in all worlds, but have no natural order, this principle turns out to be equivalent to a strengthening of a principle defended by Vallentyne and Kagan (1997), and also to a weakened version of the catching-up criterion developed by Atsumi (1965) and by von Weizsäcker (1965).

1. INTRODUCTION

Call a theory of moral value *finitely additive* just in case it morally ranks worlds with finitely many locations of value on the basis of the sum of the values at each location. Utilitarianism is a paradigm finitely additive theory of value (with people as the locations of value, and welfare as the conception of value). There has been much debate about how finitely

For valuable comments, we would like to thank Marc Fleurbaey, Bart Capéau, Joel Hamkins, Barbara Montero, Tim Mulgan, and two anonymous referees for this journal.

https://doi.org/10.1017/S0266267104000227 Published online by Cambridge University Press

additive moral theories should rank worlds when there are *infinitely* many locations of value.[1] Everyone agrees that worlds should be ranked on the basis of the sums of their local values, whenever these sums exist and are finite. The problem is that when there are infinitely many locations of value, the sums may be infinite or not exist at all and thus may provide no guidance for ranking worlds. Many authors, however, believe that some worlds with infinite sums are better than others. Vallentyne and Kagan (1997), for example, have argued that a world with an infinite number of people each having two units of value (e.g., happiness) is morally better than a world with the same people but each having only one unit of value. The sums in both cases, however, are infinite.

The core idea that Vallentyne and Kagan – and others – appeal to and defend is a Pareto principle:

> **Weak Pareto:** *If each location has more goodness in world U than it does in world V, then U is better than V.*[2]

This principle gives the desired judgment in the above example (that everyone with 2 units is better than the same people with only 1 unit). Vallentyne and Kagan, and others, develop and defend various strengthenings of Weak Pareto. Unfortunately, Weak Pareto is incompatible with the following seemingly plausible principle:

> **Isomorphism:** *Any world is equally as good as any isomorphic copy.*

Worlds are *isomorphic* just in case they have the same structure (or patterns) with respect to value at locations. In other words, if there is no natural structure to locations (as in the case of people), then one world is isomorphic to another just in case there is a one-to-one mapping from the locations of one world onto the locations of the other such that the value at a given location in one world is the same as the value at its corresponding location in the other world. When worlds have the same locations and locations have no natural order, Isomorphism is equivalent to a well-known anonymity condition. On the other hand, if there is a natural structure to locations (as in the case of points of time), then the mapping must also respect that structure (e.g., if time t_1 is one year earlier

[1] In the 1920s, Frank Plumpton Ramsey discovered the problem of aggregating utilities over an infinite time horizon. This problem was further analyzed in growth theory and in social choice theory; see for instance, Koopmans (1960), Atsumi (1965), von Weizsäcker (1965), Lauwers (1998), and Fleurbaey and Michel (2003). In philosophy, the problem was independently discovered, and a solution proposed, by Segerberg (1976). More recently, the problem was rediscovered by Nelson (1991), and a solution was proposed by Vallentyne (1993).

[2] This is a special case of their Basic Idea.

https://doi.org/10.1017/S0266267104000227 Published online by Cambridge University Press

than time t_2, then the counterpart of t_1 must be one year earlier than the counterpart of t_2).

The incompatibility of Weak Pareto and Isomorphism is well known[3] and is illustrated by the following example:

Location:	...	a_{-2}	a_{-1}	a_0	a_1	a_2	...
A	...	–2	–1	0	1	2	...
A'	...	–1	0	1	2	3	...

TABLE 1.

Weak Pareto says that world A' is better than world A, since each location has more value in A'. Isomorphism, on the other hand, says that the two worlds are equally valuable, since they are isomorphic (same pattern). Thus, at least one of these principles must be rejected.

In this paper we endorse Weak Pareto and show how it, in combination with certain other plausible principles, generates a plausible and fairly strong principle for the infinite case. More specifically, where locations are the same in all worlds, but have no natural order, this principle turns out to be equivalent to a strengthening of a principle defended by Vallentyne and Kagan and also to a weakened version of the catching-up criterion developed by Atsumi and by von Weizsäcker. Thus, three apparently distinct ranking rules turn out to be equivalent. Before establishing these results, however, we provide some support for Weak Pareto by casting doubt on a criticism of Weak Pareto, and defense of Isomorphism, recently given by Hamkins and Montero (2000).

Throughout, it's important to keep in mind that our arguments are addressed to those who accept finite additivity of value (e.g., utilitarians). They are not meant to defend finite additivity against those who reject it (e.g., egalitarians). The issue concerns how finite additivity should be extended when there are infinitely many locations of value.

2. BACKGROUND

Throughout we focus on the ranking of worlds on the basis of the relation of being morally at least as good. Moreover, we restrict our attention to cases where the two worlds have the same locations. Things are much more

[3] See, for instance, Vallentyne (1995), Lauwers (1998), and Fishkind, Hamkins, and Montero (2002).

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complex where the locations are not the same, and we wish to focus on the simpler case. In what follows, we leave this very important assumption implicit.

We shall assume, without argument, each of the following conditions, which are also assumed by Hamkins and Montero (2000) and virtually everyone else in the debate. One condition is:

> **Transitivity:** *If a world U is at least as good as V, and V is at least as good as W, then U is at least as good as W.*

As usual, a world is *better* than a second just in case it is at least as good but not vice versa, and a world is *equally good* as a second just in case it is at least as good and vice versa. We do not assume completeness (i.e., that, for any two worlds, one of them is at least as good as the other). When there are an infinite number of locations of value, completeness is difficult to achieve in a plausible explicitly stated ranking.[4]

Although we shall be concerned with cases where the total value in a world is infinite, we limit our attention throughout (as do Hamkins and Montero) to cases where the value *at each location* is finite. We shall, that is, only be considering cases where infinite total value arises from the sum of infinitely many finite values. Moreover, we assume throughout that the number of locations is countably infinite. We also assume a very weak Pareto condition:

> **Loose Pareto:** *If each location has at least as much goodness in world U as it does in world V, then U is at least as good as V.*[5]

Compared with Weak Pareto, this principle is weaker in some respects and stronger in others. It is weaker in that when each location has more in U than it does in V, it only requires that U be at least as good as V (as opposed to being better, as required by Weak Pareto). Loose Pareto is, however, stronger than Weak Pareto in that it applies to all cases where each location has *at least as much* goodness in U as it does in V (and not merely to cases where each location has *more* goodness in U).

[4] Fleurbaey and Michel (2003: 794) conjecture that there does not exist an explicit description of a complete (and transitive) rule that satisfies Weak Pareto and the very weak requirement that permutations of value between a finite number of locations keep a world equally as good as the original. They conjecture that although such rules exist, proof of their existence involves some version of the axiom of choice. The rules that we defend below satisfy Weak Pareto and indifference to finite permutations, but are not complete.

[5] Hamkins and Montero calls this "The Fundamental Idea," but we relabel it to emphasize its Pareto character.

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Finally, we assume the following principle:

Sum: *If, for each of two worlds, the sum of the values at their locations exists and is finite, then the first world is at least as good as the second world if and only if its sum is at least as great.*[6]

This principle asserts that, even in the infinite case, worlds should be ranked on the basis of their summed goodness – if these sums exist and are finite. The whole problem with ranking infinite worlds (i.e., worlds with infinitely many locations of value) is that often the sums are infinite or do not exist. Where finite sums exist, there is no problem. The ranking should be done on the basis of the sums (as in the finite case). Thus, for example, if a world has an infinite number of people with value (e.g., happiness) levels (on some arbitrary ordering of the individuals) of 1/2, 1/4, 1/8, 1/16, etc., and a second world has an infinite number of people with value levels of 0.9, 0,0,0, etc., then the first world is better than the second world. It's a mathematical fact that the first sum is one and the second is 0.9.

In the infinite case, Sum must be applied with caution. If locations of value have a natural order[7] – as points in time do – then it can be applied straightforwardly using the natural order. If, however, locations have no natural order (e.g., as in the case of people), then the sum exists and is finite if and only if no matter how the values are ordered, the corresponding series converges to the same finite number (absolute convergence). For example, $1/2+1/4+1/8+1/16\ldots$ converges absolutely to 1, but $1/2-1/3+1/4-1/5\ldots$ converges only conditionally (to about 0.307). This latter convergence is conditional because rearranged as $(1/2+1/4)-1/3+(1/6+1/8+1/10)-1/5\ldots$ the series diverges to positive infinity. Hence, there is no sum for this series. Furthermore, even if the order is kept fixed, the sum exists only if the grouping of terms (by parentheses) does not affect the result. For example, $1+(-1+1)+(-1+1)\ldots$ converges conditionally to 1, but the convergence is not unconditional, since $(1-1)+(1-1)+(1-1)\ldots$ converges conditionally to 0. Thus, no sum exists for this series.

3. HAMKINS AND MONTERO ON ISOMORPHISM

Hamkins and Montero (2000) offer the following defense of Isomorphism and criticism of Weak Pareto. They ask us to consider the assessment of soccer teams in the game of "infinite soccer." Each team has an infinite number of players, all of whom play at any given time (there are no extras).

[6] In Fishkind, Hamkins, and Montero (2002), Sum is considered a rock-bottom principle for additive theories.

[7] See Vallentyne and Kagan (1997) for a discussion of the tricky notion of "natural" order.

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Furthermore, each player is assumed – solely for reasons of simplicity of illustration – to be equally good at all positions. (The fact that there are specialized positions on a soccer team is irrelevant to the point of the example.) Reconsider Table 1 and interpret locations as players. Row A then describes a team in which player a_k has a talent level equal to k units (on some scale of measurement).

The coach of the team wants to improve his team. There is a trainer able to raise the talent level of each player by one unit. Should the coach engage this trainer? According to Weak Pareto, the answer appears to be yes. Interpret the final row in Table 1 as Team A', this is Team A after the training. Applying Weak Pareto to the talent levels of the players, Team A' is better than team A. Each player has more skill and this makes the team better. Hamkins and Montero, however, believe this is the wrong answer. They believe that Team A' is equally as good as Team A on the ground that it is the pattern of value and not the specific locations of value that matters. In particular, in the case of the infinite soccer team, the team has the same pattern of value after training as it did before. For any given level of talent displayed by a member of the team after training (e.g., that of a_k), there was someone with that level of talent before training (namely a_{k+1}). If the distribution of talent is the same, there is, they claim, no reason to think that the team is any better. Isomorphism is, they claim, correct and Weak Pareto is thus mistaken.

We shall argue against Isomorphism. First, however, it's important to note that Weak Pareto cannot be so straightforwardly applied in the above example. When dealing with the infinite case (unlike the finite case), it is extremely important to specify what the *basic locations of value* are (people, points in time, etc.). This is because in the infinite case it is possible, for example, for one world to be better for every single person but worse at every single point in time.[8] Hence, it's crucial to know whether persons or points in time are the basic bearers of value. (In the case of moral theory, the most natural assumption is, of course, that people are the basic locations.) Throughout, the principles that we defend should be understood as applying to basic locations of value – and not to just any arbitrary locations.

In light of this, we need to reconsider the above assessments of the infinite soccer team. In applying Weak Pareto, it was implicitly assumed that individual players were the basic locations of values. This assumption, however, seems implausible. Suppose that each player improves by one unit and then takes on the position previously held by the player who previously had one unit more of talent. In this case, at each position, the

[8] For an ingenious example, see Cain (1995). For analyses emphasizing the importance of the prior determination of the basic locations of value, see Vallentyne (1995) and Mulgan (2002). We here appeal to Mulgan's analysis of the soccer game example.

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talent of the player is unchanged. We fully agree that in this case the team is not improved. Weak Pareto applied to positions (and not individual players) is silent in this case. The values at each position are unchanged, and hence Weak Pareto (which requires that the value be increased at each basic location) does not apply. Consider, however, a version of the above example in which the talent of the player *at each position* is improved by one unit. In this case, assuming – as we shall – that positions are the basic locations of value for assessing the value (talent) of a soccer team, and that positions have at most a merely ordinal order (e.g., no natural center position), Weak Pareto does indeed hold that the soccer team has improved. Isomorphism, on the other hand, holds that the team has not improved. Therefore, there is a genuine conflict between Weak Pareto and Isomorphism. In what follows, then, we shall focus on the "increase at each position" version of this example.

4. AGAINST ISOMORPHISM

Below we introduce and defend a principle that we label "Zero Independence." We show that those who accept Transitivity, Sum, Loose Pareto, and this third principle, must reject Isomorphism.

Consider another case involving a soccer team with infinitely many players. The players are all beginners and they each have a talent level equal to zero. A trainer shows up and improves the talent of one player (b_0) by one unit. *All players stay in the same position*. The talent levels of the initial team, Team *B*, and the team after training, Team *B′*, are thus as follows:

Position:	...	b_{-2}	b_{-1}	b_0	b_1	b_2	...
Team *B*	...	0	0	0	0	0	...
Team *B′*	...	0	0	1	0	0	...

TABLE 2.

Does this training improve the quality of the team? The intuitive answer, without any doubt, is yes. The player at one of the positions has become a better player and no player at any position has become a worse player. Furthermore, Sum makes this judgment as well. The sum of the values of the players at each position in Team *B′* is one, and the sum for Team *B* is zero. Given that the sums exist and are finite, Sum says that Team *B′* is better than Team *B*.

Consider now some training applied to Team *B′*. The talent level of each position other than b_0 (the only position whose talent improved from the first training program) is increased by one unit. This produces Team *B′′*:

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Position:	...	b_{-2}	b_{-1}	b_0	b_1	b_2	...
Team B	...	0	0	0	0	0	...
Team B'	...	0	0	1	0	0	...
Team B''	...	1	1	1	1	1	...

TABLE 3.

Intuitively it is plausible that Team B'' is better than Team B'. After all, one position's talent did not change, and everyone else's talent improved. Here, however, we only insist on the weaker claim that Team B'' is at least as good as (as opposed to better than) B'. This judgment is endorsed by Loose Pareto (which is endorsed by Hamkins and Montero).

We have, then, the following intermediate result: B'' is at least as good as B' (by Loose Pareto), and B' is better than B (by Sum). It follows (by Transitivity) that B'' is better than B. Thus, in at least *some* cases, the fact that one world has more value at every location than a second world does guarantee – as claimed by Weak Pareto – that it is a better world. This does not, however, establish Weak Pareto, which makes the much stronger claim that this is always so. Nor does it challenge Isomorphism, since there is no Isomorphism involved in this case. We shall argue, however, that if we add one plausible assumption, then Weak Pareto follows, and Isomorphism must be rejected.

The crucial point about the above example is that for both B and B' the sums of the values at the locations are finite and could thus be used to rank them. Let us extend the example to see the significance of this. Suppose that the trainer returns to B'' and once again improves the talent of position b_0 by one unit (just as he did in improving B to B') to produce Team B^*. Suppose further that the trainer returns later to B^* and improves the talent of all remaining positions by one unit (just as he did in improving B' to B'') to produce Team B^{**}. We have then:

Position:	...	b_{-2}	b_{-1}	b_0	b_1	b_2	...
Team B	...	0	0	0	0	0	...
Team B'	...	0	0	1	0	0	...
Team B''	...	1	1	1	1	1	...
Team B^*	...	1	1	2	1	1	...
Team B^{**}	...	2	2	2	2	2	...

TABLE 4.

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In comparing B'', B^*, and B^{**}, Loose Pareto yields the judgment that B^{**} is at least as good as B^*, which is at least as good as B'' – just as it did in comparing B, B', and B''. We cannot, however, here apply Sum to get the judgments that B^* and B^{**} are each better than B'' (as we did in the case of B, B', and B''). This is because the sums are infinite for these three worlds. Surely, however, we should be able to make the comparable judgments here as well. After all, the improvements made in moving from B'' to B^* to B^{**} are the same as the respective improvements made in moving from B to B' to B''. In both cases, the first improvement was to improve one position's talent by one unit, and the second improvement was to improve the talent of all remaining positions by one unit. So, if B' and B'' are better than B, then B^* and B^{**} are better than B''.

The following principle captures the idea that the ranking of two worlds is determined by the pattern of local *differences*. It appeals to the notion of a "zero world" and to the notion of world value "subtraction." A given world's *zero world* is a world with the same locations but zero value at each location. *World value subtraction* (and, for future reference, addition) is defined as follows. For two worlds, U and V, with the same locations, $U–V$ (respectively, $U + V$) is the world with the same locations, with the value at each location equal to value at U at that location minus (respectively, plus) the value at V at that location. For example, if U has 4 at each location, and V has 3, then $U–V$ has 1 at each location, and $U + V$ has 7 at each location. Here, then, is the principle:

> **Zero independence:** *World U is at least as good as world V if and only if U–V is at least as good as the zero world Z.*[9]

This principle has the effect of saying that the ranking of two worlds depends only on the pattern of *differences* in local value at the two worlds, and not on the particular values present at those locations. It says, for example, that, above, the ranking of B^* relative to B'' is the same as that of $B^*–B''$ relative to its zero world. $B^*–B''$ just is B' and its zero world just is B. Hence, B^* is better than B'' because B' is, according to Sum, better than B.

Zero Independence on its own has no implications for how any two worlds are ranked. It is rather a consistency condition that requires that

[9] An equivalent formulation of Zero Independence is: Let U, V, and W be three worlds. Then U is at least as good as V if and only if the world $U + W$ is at least as good as $V + W$. Hervé Moulin (1991: 36) uses this formulation to characterize finite utilitarianism. Zero Independence is equivalent to a condition in social choice theory (e.g., Bossert and Weymark forthcoming 2004) known as Translation Scale Invariance (which holds that interlocational comparisons of zero points are irrelevant to the ranking of worlds).

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judgments by other principles be made in a certain way.[10] The important point to note is that it allows judgments about a world ranking relative to its zero world to be applied to judgments about the ranking of two other worlds having the same relative differences of local value. It thus allows the power of Sum to be exported to cases in which finite sums do not exist. In the above example, it permits B^* to be judged better than B'' even though neither world has a finite sum. It does this because B^*–B'' and its zero world do have finite sums.[11]

Let us now apply Zero Independence to the original example given by Hamkins and Montero (where the talent level of each position is one unit greater in A' than in A). Team A'–A has the following local values:

Position:	...	a_{-2}	a_{-1}	a_0	a_1	a_2	...
Team A'–A	...	1	1	1	1	1	...

TABLE 5.

Above, we established that Team B'' was better than Team B (using Sum and Loose Pareto). We can here use the exact same logic to establish that A'–A is better than its zero world (indeed, if the locations are the same, A'–A just is B'' and the zero world of A'–A just is B). Applying Zero Independence we get the result that A' is better than A – *as claimed by Weak Pareto, and as denied by Isomorphism*.

More generally, the above reasoning shows that Sum conjoined with Loose Pareto and Zero Independence entails Weak Pareto. Indeed, that conjunction entails the following stronger version of Pareto:

> **Strong Pareto:** *If every location has at least as much goodness in world U as it does in world V, and at least one location has more goodness in U than in V, then U is better than V*.

Weak Pareto judges a world as better than a second world if it is better at all locations, whereas strong Pareto makes this judgment as long as it is better at some locations and no worse anywhere else. Strong Pareto thus implies Weak Pareto but not vice versa.

[10] Of course, the imposition of Zero Independence rules out those rankings that do not satisfy the principle. Nevertheless it is clear that on the basis of Zero Independence alone, nothing can be said about how to rank two particular worlds.

[11] Zero worlds always have a finite sum of zero, but the "value subtraction" of one world from another does not always have a finite sum. B^{**}–B^*, for example, does not have a finite sum.

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Generalizing the arguments of the previous examples, one obtains the following theorem (proof omitted):

> **Theorem 1:** The conjunction of Transitivity, Sum, Loose Pareto, and Zero Independence entails Strong Pareto (and hence Weak Pareto).[12]

In the context of Transitivity, Sum, and Zero Independence, then, Loose Pareto – which is endorsed by Hamkins and Montero – leads to Strong Pareto (as well as Weak Pareto). Given that Isomorphism is incompatible with Weak Pareto, the issue here boils down to a conflict between Isomorphism and Zero Independence. We claim that in the context of finite additivity – which is assumed throughout this article – Zero Independence is more plausible than Isomorphism.

Although we shall not here attempt a full defense of Zero Independence, the following two points in its defense are worth noting. First, if there is no natural zero point that separates positive from negative value, then any particular zero point is arbitrary. Zero Independence is clearly plausible in this case. Second, Sum and Loose Pareto each respect Zero Independence: each on its own judges U at least as good as V if and only it judges $U–V$ at least as good as its zero world.

Isomorphism is not, we claim, as plausible as Zero Independence. In the finite case, Isomorphism is indeed plausible, but in the finite case it is not possible for there to be (1) a structure-respecting correspondence between the locations of two worlds such that the value at each location in one world is the *same* as the value at its counterpart location in the second world (i.e., an isomorphism), and also (2) a structure-respecting correspondence between the locations of two worlds such that the value at each location in one world is *greater than* the value at its counterpart location in the second world. In the infinite case, however, both kinds of correspondences may hold between two worlds (e.g., between the infinite soccer team before and after training). When both kinds of correspondences exist, it is implausible to hold that the two worlds are equally valuable.

We do not deny that some isomorphism-like principles are valid. We believe, for example, that the following principle is valid:

> **Isomorphism Invariance:** If (1) U is at least as good as V, and (2) U^* and V^* are, for a given isomorphism, the isomorphic counterparts of U and V respectively, then U^* is at least as good as V^*.

[12] It might seem strange that we prove Strong Pareto on the basis of Sum and other conditions, given that in finite contexts Sum trivially entails Strong Pareto. In infinite contexts, however, the two principles are independent. Sum, but not Strong Pareto, is silent when one world has an infinite sum and the second world is a Pareto improvement. In any case, the crucial point here is that Hamkins and Montero accept Sum, and we are trying to appeal to conditions that they accept.

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This principle (which is entailed by Isomorphism in the presence of Transitivity) is fully compatible with Strong (and hence Weak) Pareto. We mention it here merely to illustrate the rejection of Isomorphism does not require the rejection of all isomorphism principles.

Zero Independence is, we believe, more plausible for finitely additive value theories than Isomorphism. Given that they conflict in the context of Sum and Loose Pareto, Isomorphism should be rejected.

This concludes our defense of Weak (and Strong) Pareto and our criticism of Isomorphism. We shall now identify the ranking relation generated by the conjunction of Sum, Loose Pareto, and Zero Independence and show that it is slightly stronger than a principle defended by Vallentyne and Kagan and that it is equivalent to a weakened version of the catching-up criterion developed by Atsumi and by von Weizsäcker.

5. DIFFERENTIAL BETTERNESS AND INDIFFERENCE

The conjunction of Transitivity, Sum, Loose Pareto, and Zero Independence results in the following rule for ranking any two worlds U and V. In accordance with Zero Independence, this ranking can be done by comparing U–V with its zero world. To do this, select those locations in U–V at which there is a non-negative amount of goodness. If there are some such locations, set G equal to their sum. If there are no such locations, set G to zero. G is either a non-negative finite number or plus infinity. Next, select those locations with a negative amount. If there are some such locations, set L equal to their sum. If there are no such locations, set L equal to zero. L is either a non-positive finite number or minus infinity. Now, we distinguish the following cases:

1. If both G and L are finite numbers, then U is at least as good as V if and only if $G + L \geq 0$.
2. If G is infinity and L is a finite number, then U is better than V.
3. If G is a finite number and L is minus infinity, then V is better than U.
4. If both G and L are infinite, then no judgment is made.

Cases 1, 2, and 3 are captured by the following two principles, where infinity is understood to be greater than any finite number and negative infinity understood to be less than any finite number:

> **Differential Betterness:** For any two worlds U and V, if the sum of the non-negative local values of U–V is (finitely or infinitely) greater than the absolute value of the sum of the negative local values of U–V, then U is better than V.
>
> **Differential Indifference:** For any two worlds U and V, if the sum of the non-negative local values of U–V is finite and equal to the absolute

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value of the sum of the negative local values of U–V, then U is equally good as V.

It's worth noting that Differential Indifference is considerably stronger than the principle (entailed by Sum) that worlds with equal finite sums are equally valuable. To see this, consider the following example:

Locations:	l_1	l_2	l_3	l_4	l_5	...
World X	–1	1/2	1/4	1/8	1/16	...
World X'	1	1	1	1	1	...
World X''	0	3/2	5/4	9/8	17/16	...

TABLE 6.

Sum is silent about the ranking of X'' and X', for each sum is infinite. Differential Indifference, however, judges them equally good. It does this because $X'' - X'$ is simply X, and X is equally as valuable as its zero world (because for X the sum of its negative values is –1 and the sum of its positive values is 1). This is an example of the power of Zero Independence implicit in Differential Indifference.

It's also worth noting these two principles are silent about case 4 pairs of worlds (where G and L are each infinite). Thus, the two principles can be strengthened into different ranking rules by adding judgments about case 4 pairs.

The next theorem (proved in the appendix) makes this statement more precise. This theorem appeals to the concept of the ranking rule generated by a set of conditions. This concept is defined as the intersection of all rankings satisfying the specified conditions: The generated rule ranks one world as at least as good as another if and only if *all* the rankings satisfying the specified conditions do so. For example, the ranking rule generated by Differential Betterness and Differential Indifference is only able to rank the case 1, 2, and 3 pairs (as indicated above).

> **Theorem 2:** The conjunction of Transitivity, Sum, Loose Pareto, and Zero Independence entails the conjunction of Differential Betterness and Differential Indifference, but not vice versa. Nonetheless, they generate the same ranking rule.

According to this theorem, the ranking rule generated by our four principles (Transitivity, Sum, Loose Pareto, and Zero Independence) is, like the conjunction of Differential Betterness and Differential Indifference, only able to rank case 1, 2, and 3 pairs (and is silent about case 4 pairs). In addition, both sets of principles agree upon how these pairs are ranked.

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Nevertheless, the combination of our four principles is stronger than the combination of Differential Betterness and Differential Indifference. To understand this, it is sufficient to see that Differential Betterness and Differential Indifference give no indication on how to rank case 4 pairs. Transitivity and Zero Independence, on the other hand, each impose some consistency requirements upon how case 4 pairs are ranked. Put differently, there exist ranking rules that satisfy Differential Betterness and Differential Indifference that (when applied upon case 4 pairs) violate Transitivity and/or Zero Independence.[13]

Finally, it's worth noting that it follows from Theorem 2 that, in the presence of Transitivity and Zero Independence, the conjunction of Differential Indifference and Differential Betterness is equivalent to the conjunction of Sum and Loose Pareto.

Let us now compare the latter principles with some other principles in the literature.

6. COMPARISONS WITH VALLENTYNE AND KAGAN'S SBI1

Differential Betterness, rather interestingly, turns out to be equivalent to Vallentyne and Kagan's SBI1:

> **SBI1 (Strengthened Basic Idea One):** *For any two worlds U and V, if for each finite set of locations there is a finite expansion (i.e., superset) and some positive number, k, such that relative to all further finite expansions U is k-better* [defined below] *than V, then U is better than V.*

The basic idea here is that worlds can be compared by comparing the values of *finite subsets* of their locations, which, given finite additivity, are evaluated by adding values together. Roughly, if, relative to some finite set of locations, one world is judged better than a second, and this judgment is also true no matter how one finitely expands the selected set of locations, then the first world is better than the second. More exactly, the principle states that if, no matter what finite set of locations you start with, it is possible to expand this set by adding finitely more locations so that at some point, no matter how one further finitely expands the set, relative to

[13] To see that our four principles have some extra strength in comparison with the two differential principles, consider three worlds *X*, *Y*, and *Z* for which each pair is a case 4 pair. If we suppose that *X* is better than *Y* and *Y* is better than *Z*, then our four principles (via Transitivity) entail that *X* is better than *Z*. The two differential principles, on the other hand, are silent about this (saying that *Z* is better than *X* is not in conflict with the differential principles). This example (and a similar one for Zero Independence) shows that the extra strength that comes from our four principles (and Transitivity and Zero Independence in particular) is "conditional": they generate certain additional results when certain other assumptions are added. On their own, they are equivalent to the conjunction of the two differential principles.

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the finite number of locations selected, the first world is *k*-better than the second – that is, has a total that, on some specified scale, is at least *k* units higher, for some fixed real number *k* – then the first world is better than the second (considering all locations). (The notion of *k*-betterness is introduced to handle complications that arise when sums of infinitely many values have a finite value.)

Consider, for example, the case illustrated above in Table 3 (in which *B* has 0 at all locations, *B*′ has 0 at all but one locations, and *B*′′ has 1 at all locations).

SBI1 agrees with Differential Betterness in judging *B*′′ as better than B′, and *B*′ as better than *B*. *B*′′ is judged better than *B*′ because, for any set of at least two locations (positions), the total goodness in *B*′′ in the selected locations is at least one unit greater than that in *B*′. *B*′ is judged better than *B* because, for any set that includes at least location b_0, the total goodness in *B*′ in the selected locations is at least one unit greater than that in *B*.

SBI1, it turns out, is equivalent to Differential Betterness. In the appendix, we prove:

> **Theorem 3:** Differential Betterness and SBI1 are equivalent; that is, Differential Betterness entails SBI1 and vice versa.

Thus, Transitivity, Sum, Loose Pareto, and Zero Independence entail Differential Indifference and Differential Betterness, the latter of which is equivalent to SBI1. Given that Differential Betterness is more intuitive and easier to understand than SBI1, all future discussion should, we suggest, focus on Differential Betterness. The complex and arcane language of SBI1 no longer needs to be considered.

7. COMPARISONS WITH THE CATCHING-UP CRITERION

In the economics literature, one of the best known finitely additive criteria for ranking worlds when there is an infinite number of locations comes from the work of Atsumi (1965) and von Weizsäcker (1965):

> **Catching-Up:** *One world, U, is at least as good as a second world V, if and only if the lower limit, as T approaches infinity, of the sum of the values of U–V at locations* 1 *to T is at least as great as zero (that is, if and only if* $[lim_{T\to\infty} inf \sum_{t=1,..,T} (U_t - V_t)] \geq 0$*).*[14]

This condition holds that (as with Zero Independence) the ranking of two worlds can be determined by considering the world that results from subtracting, at each location, the values of one world from those of the

[14] The notion of a lower limit is as follows: An infinite sequence of real numbers may have several limit points. For example, the sequence <1, –1, 1, –1, 1, –1, . . .> has two limit points, –1 and 1. The lower limit of a sequence is the smallest of the limit points. The lower limit in the above sequence is –1.

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other and comparing this world with the value of its zero world. Roughly, the idea is that if, when enough locations are considered, the sum of these differences is at least as great as zero, and this remains so no matter how many additional locations are added in, then the first world is at least as good as the second world.

There are two ways in which Catching-Up may be too strong. First, it states necessary and sufficient conditions for being at least as good. Any two worlds that do not satisfy these conditions are deemed incomparable. A weaker view would state the conditions only as sufficient for betterness and leave open whether they may be other grounds as well. Although we shall endorse the stronger view below, we shall start by focusing on this weaker view.

A second weakening of the criterion is needed to accommodate cases where locations have no natural structure. Atsumi and von Weizsäcker developed their criteria for the case where the locations were future times (assumed to be discrete for simplicity). They thus assume a natural structure for locations including a first location (the first future time relative to a time of evaluation). Once this assumption is dropped, their appeal to a specific numbering of locations (a specific way of ordering them) becomes arbitrary.[15]

The other main way of responding to the absence of a natural order is to reformulate Catching-Up so that it applies only where the original condition is satisfied by *all possible* enumerations of locations. In what follows, we shall invoke this weakening of the original criterion.[16]

Taking into account the above three weakenings of Catching-Up leads to the following:

> **Weak Catching-Up:** *For any two worlds, U and V, if, for all possible enumerations of locations, the lower limit, as T approaches infinity, of the sum of the values of U–V at locations* 1 *to T is at least as great as zero (i.e., $lim_{T\to\infty}\, inf \sum_{t=1,..,T} (U_t - V_t) \geq 0$), then U is at least as good as V.*

[15] One cannot simply reformulate this criterion so that it says that if there exists *at least one way* of numbering locations for which his original condition holds, then the first world is at least as good as the second. This approach is incoherent. Consider <1,1,–1,1,1,–1,1,1,–1 . . . .> and compare it to its zero world <0,0,0, . . . >. Relative to the order in which the locations are listed, Catching-Up judges the first world as better (the partial sums of the differences are <1,2,1,2,3,2,3,4,3, ...>, and thus are always greater than zero). If, however, the locations are listed in a different order, the opposite result can be obtained. For example, if locations have no natural structure, then the first world can also be specified by <–1,–1,1,–1,–1,1,–1,–1,1, . . . >, and, relative to this enumeration, Catching-Up says that the zero world is better. Hence, the mere existence of a particular enumeration that ensures that the original criterion is satisfied is not sufficient for being at least as good.

[16] We are indebted to an anonymous referee for suggesting that we pursue the connection between our conditions and the von Weizsäcker condition weakened in the way just identified.

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In the appendix we prove:

Theorem 4: Weak Catching-Up is equivalent to the conjunction of Differential Betterness and Differential Indifference.

Thus, in light of Theorem 2, it follows that (1) the conjunction of Transitivity, Sum, Loose Pareto, and Zero Independence entails Weak Catching-Up, but not vice-versa, but nonetheless (2) the two sets of conditions generate the same ranking rule.

8. NECESSARY CONDITIONS FOR BEING AT LEAST AS GOOD

The conjunction of Transitivity, Sum, Loose Pareto, and Zero Independence implies each of the following (equivalent) *sufficient* conditions for being at least as good: Weak Catching-Up, the conjunction of Differential Betterness and Differential Indifference, and the conjunction of SBI1 and Differential Indifference. Each of these is silent about the ranking of worlds that do not satisfy the imposed condition. More specifically, as discussed in Section 5, each is silent about the ranking of a world U and a world V if and only if (1) the sum of the non-negative values of $U–V$ is infinite, and (2) the sum of the negative values of $U–V$ is infinitely negative. Call two worlds that satisfy these two conditions *a double infinity pair*. Worlds $< \ldots 1,1,1,1,1, \ldots >$ and $< \ldots 2,0,2,0,2,0,2, \ldots >$ are, for example, a double infinity pair.

The above conditions are silent for double infinity pairs. This means that they are compatible with the two worlds being incomparable, but also compatible with one of them being at least as good as the other. We believe, however, that, *if locations have no natural structure*, then double infinity pairs should be judged incomparable. Hence, we believe, if locations have no natural structure, then the above conditions should be strengthened to necessary and sufficient conditions.

We shall not, however, attempt to defend this view here. Instead, we shall simply show that it follows from the above conditions if one final seeming plausible condition is added. The condition appeals to the notion of *a restricted transfer*, which is (1) a transfer of a positive amount of value from a location with positive value to a location with negative value such that (2) after the transfer, the donor location still has non-negative value and the recipient location still has non-positive value. For example, the move from $<–1, 3>$ to $<0,2>$ is a restricted transfer, but the move from $<–1, 3>$ to $<1,1>$ is not.

Consider then:

Restricted Transfers: *If locations have no natural structure, then, for any three worlds, U, U*, and V, if (1) U is better than V, and (2) U* is obtainable from U by some (possibly infinite) number of restricted transfers, then U* is better than V.*

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This condition is, we believe, plausible. Note first that the conjunction of Transitivity, Sum, Loose Pareto, and Differences entails that a *finite* number of transfers (restricted or not) always preserve value rankings. The only issue, then, concerns infinite numbers of transfers. Clearly, many kinds of infinite transfers do *not* preserve value rankings (and are not covered by Restricted Transfers). For example, an infinite number of transfers from one positively valued location to another *positively* valued location does not always preserve value rankings. World $< \ldots 1,1;1,1,1, \ldots >$ is (by Strong Pareto) better than $< \ldots 1,1;0,1,1, \ldots >$, but the latter is not better than itself – even though it can be obtained from the former by transferring, for all locations to the right of the semi-colon (here used only to identify a specific location), one unit of value one location to the right. An unrestricted transfer principle is for these reasons implausible.

Restricted transfers are uni-directional: They always have the effect of moving both the donor and recipient locations closer to zero. The effect of one restricted transfer on a given location cannot be reversed by some other restricted transfer. We shall not, however, here insist on the plausibility of Restricted Transfers. We wish merely to note that, in the presence of the other four conditions (Transitivity, Sum, Loose Pareto, and Zero Independence), it requires incomparability in all those cases where the four principles are jointly silent. For this purpose, consider the following condition:

> **Full Weak Catching-Up:** *If locations have no natural structure, then a world U is at least as good as world V if and only if, for all possible enumerations of locations, the lower limit, as T approaches infinity, of the sum of the values of U–V at locations* 1 *to T is at least as great as zero* (*i.e.*, $lim_{T\to\infty}\, inf \sum_{t=1,..,T} (U_t - V_t) \geq 0$).

This is exactly like Weak Catching-Up, except that (1) it asserts that satisfaction of the limit condition for all possible enumerations is necessary and sufficient (and not merely sufficient) for being at least as good, and (2) it is conditional on locations not having any natural structure. The second qualification is necessary, since where locations have a natural structure it is not necessary that the limit condition hold for all possible enumerations. It only needs to hold for certain kinds of (e.g., isometric) structure-preserving enumerations.

We can now characterize the Full Weak Catching-Up (proof in appendix):

> **Theorem 5:** Full Weak Catching-Up rule is the only ranking rule that satisfies Transitivity, Sum, Loose Pareto, Zero Independence, and Restricted Transfers.

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Where locations have no natural structure, satisfaction of the von Weizsäcker limit condition for all possible enumerations – which is equivalent to the conjunction of the conditions imposed by Differential Betterness and Differential Indifference – provides necessary and sufficient conditions for being at least as good. We believe that Restricted Transfers, and hence Full Weak Catching-Up, is plausible, but cannot here undertake a defense of Restricted Transfers.

9. CONCLUSION

Where there are infinitely many locations of value, Weak Pareto and Isomorphism-like principles can conflict. We have suggested (*pace* Hamkins and Montero) that Weak Pareto should be endorsed and Isomorphism rejected. We did this by appealing to Zero Independence, which ranks two worlds U and V on the basis of their differences in local value. The conjunction of Zero Independence with Sum and Loose Pareto (which Hamkins and Montero endorse) entails Weak Pareto (indeed Strong Pareto) and is incompatible with Isomorphism.

We also showed that the conjunction of Sum, Loose Pareto, and Zero Independence entails the conjunction of Differential Betterness and Differential Indifferences (each of which ranks two worlds U and V on the basis of U–V). We further showed that Differential Betterness is equivalent to Vallentyne and Kagan's SBI1 and that the conjunction of Differential Betterness and Differential Indifference is equivalent to a weakened version of Catching-Up developed by Atsumi and by von Weizsäcker. This general convergence of results from different sources thus provides some indirect support for each. In particular, it provides indirect support for Zero Independence, since, in the framework of finitely additive theories, Transitivity, Sum, and Loose Pareto are not controversial.

Finally, we suggested that, where locations have no natural order, the above principles exhaust the plausible judgments. The conditions that they hold are sufficient for one world being at least as good as another are also necessary. We did not defend this suggestion, but we did show that it follows if one further accepts the Restricted Transfers principle that we formulated. Defense of this principle must await another occasion.

APPENDIX

Theorem 1. The conjunction of Sum, Loose Pareto, and Difference entails Strong Pareto (and hence Weak Pareto).

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Proof: Omitted.

Theorem 2: The conjunction of Transitivity (T), Sum (S), Loose Pareto (LP), and Zero Independence (ZI) entails the conjunction of Differential Betterness (DB) and Differential Indifference (DI), but not vice versa. Nonetheless, they generate the same ranking rule.

Proof: The proof is split up into two parts. Part A shows that the conjunction of T, S, LP, and ZI entails DB and DI. The '*not vice versa*' part is explained in Section 5 (the conjunction of DI and DB does not entail T or ZI). Part B shows that both sets of axioms generate the same ranking rule. Throughout, U and V denote two worlds with the same (countable infinitely many) locations, G is the sum of the non-negative local values (set at zero if there are none), and L the sum of the negative local values, of U–V (set at zero if there are none).

A. The conjunction of T, S, LP, and ZI entails DB and DI.

A1. The conjunction of T, S, LP, and ZI entails DB.

Suppose that U and V satisfy the antecedent clause of DB. We have to show that (according to the conjunction of T, S, LP and ZI) U is better than V. The hypothesis implies that L is finite and G is (finitely or infinitely) greater than $-L \geq 0$.

If G is finite, then $G + L > 0$ and S implies that U–V is better than its zero world. Apply ZI and conclude that U is better than V. If G is infinite, let G^* be like U–V except with 0 wherever U–V has a negative value, and let L^* be like U–V except that it has 0 wherever U–V has positive value and has the absolute (hence positive) value where the values are negative. This ensures that U–$V = G^*$–L^*, and thus that U and V will be ranked as G^* and L^* are (by ZI). Given that L is finite, L^* is equally as valuable, by S, as any other world with the same finite (positive) total $|L|$. Let W be such a world, with (a) $|L|$ spread out over only locations which have positive value in G^*, and (b) so that the values in W are always non-negative and less than those in G^*. Such a construction is always possible, since G is infinite and L is finite. By S, W is equally good as L^*, and by Strong Pareto (which holds in virtue of Theorem 1), G^* is better than W. Hence, G^* is better than L^*. Consequently, U is better than V.

A.2. The conjunction of T, S, LP, and ZI entails DI.

Suppose that U and V satisfy the antecedent clause of DI. It follows that G and L are finite and $G + L = 0$. According to S, we have that U–V is equally good as its zero world. Apply ZI and conclude that U is equally as good as V.

B. Here we prove that the domain of the rule generated by T, S, LP, and ZI coincides with the domain of the conjunction of DB and DI. From Part A it follows that the domain of the rule generated by T, S, LP, and ZI includes the domain of the conjunction of DB and DI. From their definitions it is clear that DB and DI are silent on a pair if and only if it is a double infinity pair (i.e., a pair, U and V, for which the sum of the non-negative values (G) and the sum of the negative values (L) are each infinite). Hence, it is sufficient to show that the rule generated by T, S, LP, and ZI is also silent on double infinity pairs.

We define a collection of rules that satisfy T, S, LP, ZI, and completeness. The definition is as follows: Since we consider infinite worlds with the same countably many locations, the locations can be indexed by the set $N = \{0,1,2,\ldots,n,\ldots\}$ of natural numbers (of course, this numbering is not unique).

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Next, let F be a free ultrafilter on the set of natural numbers, i.e. F is a collection of subsets of N that satisfy:

U1. Finite sets do not belong to F,
U2. If both A and B are in F, then the intersection $A \cap B$ is in F,
U3. If A is in F and B is a superset of A (i.e. B includes A), then B is in F,
U4. For each set of natural numbers A, either A or its complement $N \setminus A$ is in F.

The existence of a free ultrafilter follows from the axiom of choice (Zorn's lemma). Each free ultrafilter defines a limit operator. (See, for instance, Dugundji (1990) or Edwards (1965)). Let $a = (a_0, a_1, \ldots, a_n, \ldots)$ be a sequence of numbers, then $\lim_F a$ selects the limit point of a subsequence of a indexed by a set that belongs to F. For example, let F contain the set $\{1,3,5, \ldots\}$ of odd numbers and let $a = (1,0,2,0,3,0,4, \ldots)$, then $\lim_F a = 0$. Indeed, the subsequence of the odd indexed locations reads $0,0, \ldots, 0, \ldots$ and has a limit equal to 0. Similarly, in case F' is a free ultrafilter that contains the set of even numbers, then $\lim_{F'} a = +\infty$. Indeed, now we can restrict a to the subsequence $(1,2,3,4 \ldots)$ of the even indexed locations. The limit operator $\lim_F$ is well defined in the sense that each sequence obtains a unique value. We define the following ranking rule:

> *Let U and V be two worlds with the same (countable infinite set of) locations. Then U is at least as good as V if and only if* $\lim_F \Sigma(U_k - V_k) \geq 0$.

This rule satisfies reflexivity, T, S, LP, and ZI (and hence, by Part A, also DB and DI). That this rule satisfies T and S is immediate: (*i*) the intersection property U2 of an ultrafilter implies that Transitivity is fulfilled; (*ii*) the fact that each subsequence of a converging sequence converges to the same limit implies that Sum is fulfilled. In addition, this rule is complete, i.e. this rule is able to judge any pair U, V of worlds. The intersection of all these $\lim_F$ principles includes the ranking rule generated by T, S, LP, and ZI. Furthermore, in defining a $\lim_F$ principle there are two moments of choice (or arbitrariness): First, we number the set of locations, and secondly, we select an ultrafilter.

B1. Now consider a double infinity pair U and V (i.e., both G and L are infinite). In this case there are infinitely many locations at which U–V is positive and infinitely many locations at which U–V is negative. Select a finite set S of locations such that $\Sigma_S(U_k - V_k) > 1$ (this is possible because G is infinite). In numbering the locations, we start with this set S. Expanding S, we number the locations in such a way that at infinitely many locations the partial sum becomes greater than 1. Let F be a free ultrafilter that contains the set of these locations (where the partial sum is greater than 1). Then, according to the rule $\lim_F$, world U is better than V. If we repeat this construction with U and V interchanged, then we end up with a rule (that satisfies T, S, LP, and ZI) that considers V better than U (i.e., the opposite ranking). Hence, the conjunction of T, S, LP, and ZI is silent when G and L are each infinite.

Conclusion: The rule generated by S, LP, and ZI is silent if and only if G and L are both infinite, which is precisely when the conjunction of DB and DI is silent.

Theorem 3: Differential Betterness (DB) and SBI1 are equivalent; that is, DB entails SBI1 and vice versa.

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Proof: Let U and V be any two worlds, and let G be the sum of the non-negative local values of U–V (set at zero if there are none) and let L be the sum of the negative local values (set at zero if there are none), of U–V.

A. DB entails SBI1.
Suppose that the antecedent condition of SBI1 (concerning finite expansions) is satisfied. By definition, L is negative, and there are two cases to consider:

(1) Suppose first that L is finite. In this case, it is sufficient to show that $G+L$ is positive (since DB will then judge U as better than V, as required). We do this by deriving a contradiction from the supposition that G is less than or equal to the absolute value of L. If so, then G and L are both finite and the sum of the values in U–V absolutely converges to $G+L$, which is less than or equal to zero. If so, then for each positive k there exists a finite expansion such that (*i*) the restriction of U–V to this expansion adds up to a number less than k and (*ii*) all further expansions generate a total less than k. This is in conflict with the assumption that the antecedent conditions of SBI1 are satisfied. Hence, $G+L$ is positive, and thus DB judges that U is better than V, as required by SBI1.
(2) Suppose next that L converges to minus infinity. If so, then there are infinitely many locations with a negative utility. Take any finite set S of locations such that the restriction of U–V to this set adds up to a positive number (given that antecedent conditions of SBI1 hold, such a set S must exist). Then add enough locations with a negative amount of utility and this superset will end up with a negative total. This, however, contradicts the assumption that the antecedent conditions of SBI1 hold. Hence, this case is impossible.

Conclusion: Only the first case is possible and in it DB ensures that U is judged better than V. Hence, the consequent clause of SBI1 holds.

B. SBI1 entails DB.
Suppose that the antecedent of DB is true, that is, $G+L$ is (finitely or infinitely) positive, say $G+L > \varepsilon > 0$. For this to be so, L must be finite, and G is either finite (and greater than $-L$) or infinite. In either case, let $k = \varepsilon/2 > 0$. Then, no matter what finite set of locations one starts with, one can finitely expand it so that, relative to any further finite expansion, the sum of the values in U in this set is k-greater than the sum of the values in V in this set. Hence SBI1 ensures that U is better than V, as required by DB.

Theorem 4: Weak Catching-Up (WC) is equivalent to the conjunction of Differential Betterness (DB) and Differential Indifference (DI).

The proof of Theorem 4 uses the following lemma.

Lemma 1: The conjunction of Transitivity (T), Sum (S), Loose Pareto (LP), and Zero Independence (ZI) entails Weak Catching-Up (WC), but not vice versa. Nonetheless, they generate the same ranking rule.

Proof: First, observe that WC does not entail T (cf. proof of Theorem 2, Part A). Next, from the definition it follows that when applied to a pair of U and V with G, or L, or both, finite, WC agrees with the conjunction of T, S, LP, and ZI. Indeed, in this case $\lim_{T\to\infty} \inf \sum_{t=1,\ldots,T} (U_t - V_t)$ does not depend upon the particular enumeration of

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the locations and is equal to $\lim_{T\to\infty}$ inf $\sum_{t=1,\ldots,T}(U_t - V_t)$. Hence, it is sufficient to show that WC is silent for double infinity pairs. Now, if U and V is a double infinity pair, then one can define a $\lim_F$ principle (cf. Part B1 in the proof of Theorem 2) such that $\lim_F \sum(U_t - V_t) < 0$. Therefore, with respect to the corresponding enumeration, one has that lim inf $\sum(U_t - V_t) \leq \lim_F \sum (U_t - V_t) < 0$. Repeat this construction with U and V interchanged, and obtain another enumeration for which lim inf $\sum(V_t - U_t) < 0$. Conclude that WC is silent for double infinity pairs and that WC coincides with the rule generated by T, S, LP, and ZI.

Proof of Theorem 4: Both WC and the conjunction of DB and DI are silent on a pair if and only if it is a double infinity pair. Therefore, the statement is implied by the lemma in combination with Theorem 2.

Theorem 5: Full Weak Catching-Up (FWC) is the only ranking rule that satisfies Transitivity (T), Sum (S), Loose Pareto (LP), Zero Independence (ZI), and Restricted Transfers (RT).

Proof: In view of Lemma 1, it is sufficient to show that (*i*) FWC satisfies RT, and (*ii*) in case of a double infinity pair, RT combined with the conjunction of T, S, LP, and ZI entails incomparability.

(*i*) Let world U be judged better than V on the basis of FWC. Lemma 1 implies that L is finite and G is (finitely or infinitely) greater than $-L$. The total goodness that RT allows to be transferred does not exceed $-L$. After a restricted transfer of K units, U is changed into U^*, G decreases to $G-K$, and L increases to $L+K$ with $-L > K \geq 0$. The inequality $G > -L$ implies $G-K > -L-K \geq 0$. Therefore, U^* is better than V. In this case, RT is entailed by the conjunction of T, S, LP, and ZI. Hence, FWC satisfies RT.

(*ii*) Consider a double infinity pair U and V, that is, for which G and L are both infinite. According to FWC such a pair is incomparable. We show that the conjunction of T, S, LP, ZI, and RT entails incomparability. This is done by contradiction: Assume U is not worse than V. Fix a location l for which $n = U-V$ is negative. By RT, the local differences at all other locations can (with an infinite number of restricted transfers) be brought to zero without affecting the ranking of U and V. Hence, after these transfers, the local gains add up to 0 and the local losses to n. By S and ZI, world V should be better than U. This contradicts the initial assumption.

Conclusion: The conjunction of T, S, LP, ZI, and RT entails that double infinity pairs are incomparable, and elsewhere it coincides with FWC.

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https://doi.org/10.1017/S0266267104000227 Published online by Cambridge University Press