TRANSIENT RESPONSE OF FUNCTIONALLY GRADED MATERIAL CIRCULAR CYLINDRICAL SHELLS WITH MAGNETOSTRICTIVE LAYER

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ABSTRACT

The generalized differential quadrature (GDQ) method is used to investigate the transient response of magnetostrictive functionally graded material (FGM) circular cylindrical shells. The effects of control gain value, thermal load temperature and power-law index on transient responses of dominant normal displacement and thermal stress are analyzed. With velocity feedback and suitable product values of coil constant by control gain in the magnetostrictive FGM shells can reduce the transient amplitude of displacement into a smaller value.

Keywords: Transient responses, Magnetostrictive, FGM, Shell.

1. INTRODUCTION

There are many presentations in the analyses of functionally graded materials (FGMs). In 2012, Alibeigloo *et al*. used the differential quadrature method (DQM) to analyze the free vibration of a piezoelectric FGM cylindrical shell [1]. Some numerical results are presented for the effects of material gradient index and the piezoelectric thickness on the vibration of shell. In 2011, Ootao *et al*. presented the transient thermoelastic numerical solution for the magneto-electro-thermo-elastic FGM strip [2]. In 2010, Sepiani *et al*. used the finite element method (FEM) to investigate the free vibration of a two-layered FGM cylindrical shell [3]. They found that the transverse shear has the effect on the fundamental frequency of the FGM cylindrical shell. In 2010, Zahedinejad *et al*. used the DQM to examine the free vibration behavior of FGM curved panels [4]. In 2009, Yu and Wu presented a dynamic harmonic waves solution for the piezoelectric-piezomagnetic FGM cylindrical curved plate [5]. In 2009, Wang *et al*. presented micromechanics results for the piezoelectric and magnetostrictive FGM composites [6]. In 2010, there are also some 2000 ~ 2009 researches reviewed by the Qatu *et al*. on the dynamic analysis of composite shells [7]. On the past work as earlier as in 2000s on dynamic responses of FGMs cylinders, pioneer and consistent works were done by Liu's group [8-11]. Related works on generalized differential quadrature (GDQ) and magnetostrictive material are also done by Liu's group [12-19].

The author has used the GDQ method to compute and analyze some topics for the laminate, magnetostrictive and piezoelectric materials. In 2014, Hong considered the effect of varied shear correction coefficient on the linear, first-order transverse shear deformation to study the thermal vibration of FGM thick circular cylindrical shells [20]. In 2012, Hong calculated the rapid heating results of magnetostrictive FGM plates [21]. In 2010, Hong found that the displacement and stresses GDQ results are generally affected by the thickness and the length of laminated piezoelectric shells under the electromechanical loads [22]. In 2009, Hong presented the time response GDQ solutions of axial, circumferential and normal displacements in the thermal vibration of laminated shells subjected to rapid heating [23]. The main physical motivation of this magnetostrictive FGM circular cylindrical shell study might be applied to the fields of mechanical and automotive parts, intently find some new components of part to reduce the amplitude of vibration. It is interesting to study the amplitudes of displacement and stress with and without the effect of velocity feedback, respectively in the magnetostrictive FGM circular cylindrical shell under thermal transient response by using the GDQ method. Usually, the GDQ method has superiority for catch the best results by fewer grids with less computational time.

2. FORMULATION

2.1 FGM Properties

For the preliminary study in the power-law index function of two-material FGM circular cylindrical

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shells, the Young's modulus property E_{fgm} is focused in the main part of all properties. All the material properties can be expressed and assumed in terms of constituent material properties P_i as follows [21].

$$E_{fgm} = (E_2 - E_1)\left(\frac{z + h/2}{h}\right)^{R_n} + E_1 \tag{1}$$

$$P_i = P_0(P_{-1}T^{-1} + 1 + P_1T + P_2T^2 + P_3T^3) \tag{2}$$

in which R_n is the power-law index, z is the thickness coordinate, h is the thickness of FGMs shells. E_1 and E_2 is the Young's modulus, respectively of the constituent FGM material 1 and FGM material 2. P_0, P_{-1}, P_1, P_2 and P_3 are the temperature-dependent coefficients of FGMs can be found as listed in the referred paper [24]. T is the environmental temperature.

2.2 Thermal Stress-Strain Equations

The thermal stress-strain relationship in the k^{th} layer of a thin generally orthotropic magnetostrictive multilayered FGM circular cylindrical shell as shown in Fig. 1 can be obtained in the following equations without considering the shear deformation effects [21,25,26].

$$\begin{Bmatrix} \sigma_x \\ \sigma_\theta \\ \sigma_{x\theta} \end{Bmatrix}_{(k)} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_{(k)} \begin{Bmatrix} \varepsilon_x - \alpha_x \Delta T \\ \varepsilon_\theta - \alpha_\theta \Delta T \\ \varepsilon_{x\theta} - \alpha_{x\theta} \Delta T \end{Bmatrix}_{(k)} - \begin{bmatrix} 0 & 0 & \bar{e}_{31} \\ 0 & 0 & \bar{e}_{32} \\ 0 & 0 & \bar{e}_{36} \end{bmatrix}_{(k)} \begin{Bmatrix} 0 \\ 0 \\ \tilde{H}_z \end{Bmatrix} \tag{3}$$

$$\varepsilon_x = \frac{\partial u}{\partial x} - \frac{\partial^2 w}{\partial x^2} z\,,\quad \varepsilon_\theta = \frac{1}{R}\left(\frac{\partial v}{\partial \theta} + w\right) - \frac{1}{R^2}\left(\frac{\partial^2 w}{\partial \theta^2} - \frac{\partial v}{\partial \theta}\right) z\,,$$
$$\varepsilon_{x\theta} = \frac{\partial v}{\partial x} + \frac{1}{R}\frac{\partial u}{\partial \theta} - \frac{2}{R}\left(\frac{\partial^2 w}{\partial x \partial \theta} - \frac{\partial v}{\partial x}\right) z \tag{4}$$

$$\tilde{H}_z = k_c c(t) \frac{\partial w}{\partial t} \tag{5}$$

where σ_x, σ_θ and $\sigma_{x\theta}$ are stresses. ε_x, ε_θ and $\varepsilon_{x\theta}$ are strains. $\bar{Q}_{ij}$ is the transformed reduced stiffness of FGM shell. α_x and α_θ are the coefficients of thermal expansion, $\alpha_{x\theta}$ is the coefficient of thermal shear. ΔT is the temperature difference between the laminate and curing area. u, v and w are the displacement components. x, θ and z are the axes direction. R is the mean radius of shell. $\bar{e}_{ij}$ is the transformed coupling modulus of magnetostrictive material. $\tilde{H}_z$ is the magnetic field intensity. k_c is the coil constant, $c(t)$ is the control gain. t is the time.

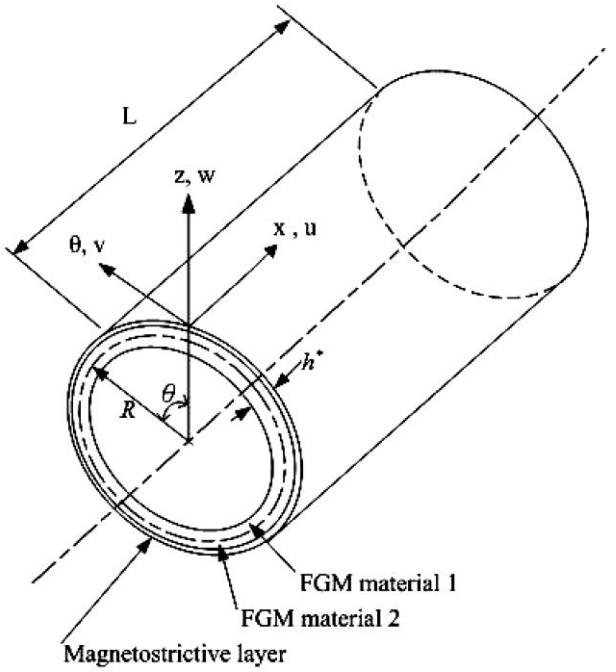


Fig. 1 Magnetostrictive FGM shells.

2.3 The System of Equation

The thermally dynamic equilibrium differential equations of Love's theory for thin multilayered circular cylindrical shell in terms of displacement components, the thermal loads and the magnetostrictive loads can be obtained as follows [23].

$$\begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \begin{Bmatrix} b_1 \\ b_2 \\ b_3 \end{Bmatrix} \tag{6}$$

where L_{ij} are in the partial derivative terms with stiffness coefficients A_{ij}, B_{ij} and D_{ij}, in which $(A_{ij}, B_{ij}, D_{ij}) = \int_{-h/2}^{h/2} \bar{Q}_{ij}(1, z, z^2)\, dz$. b_i are the partial derivative terms with thermal and magnetostrictive loads, in which subscript i = 1, 2, 3 and j = 1, 2, 3.

Usually, the thermal motion is coupled with the elastic vibration, the thermal load will cause the thermal waves. The vibrating motion mode in this thin-walled (elastic strain did not consider the shear deformation in radial direction z, but the thermal strain is considered due to ΔT in radial direction z) circular cylindrical shell vibration study is something like for the type of thermal sleeve with heated fluid flow through inner of pipe presented by Hong *et al*. in 2005 [27]. The thermal expansion strain distribution is considered independent of axial direction x and circumferential direction θ, but considered in an even function of z. For the simplification, it is assumed in a particular thermal motion under a thermal load. The attenuation resulted from the thermo-elastic coupling should influence the elastic

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response, it might be have an acceptable error to obtain a preliminary basic data. The thermally dynamic equilibrium differential equations are investigated in the vibration case of sinusoidal temperature difference ΔT variation of pure thermal load, displacement components in triangular function variations, and extensional stiffness A_{11} coefficient expressed directly from the integration of $\bar{Q}_{11} = E_{fgm}/(1-\upsilon_{fgm}^2)$, $\upsilon_{fgm} = (\upsilon_2+\upsilon_1)/2$ as follows.

$$\Delta T = \frac{A_{11}}{R^2} T_0 \, x \sin(\gamma t) \tag{7}$$

$$u = U(x)\cos(n\theta+\omega t),\ v = V(x)\sin(n\theta+\omega t),$$
$$w = W(x)\cos(n\theta+\omega t) \tag{8}$$

$$A_{11} = \frac{h}{1-\left(\frac{\upsilon_1+\upsilon_2}{2}\right)^2}\left(\frac{R_n E_1 + E_2}{R_n+1}\right) \tag{9}$$

where υ_1 and υ_2 are the Poisson's ratios. T_0 is thermal load temperature. γ is frequency of applied heat flux. ω (rad/sec) is the natural circular frequency. n is an integer for the circumferential wave number of the multilayered shells. $U(x)$, $V(x)$ and $W(x)$ are the amplitude values of displacement components.

The GDQ method is perused by Shu and Du in 1997; it is a numerical approximation to discretize the derivative of a smooth function into a weighting series form of the function [22,23,28]. The one-dimensional GDQ method is used to discretize the thermally dynamic equilibrium differential vibration equations under two clamped edges boundary condition, symmetric, orthotropic shell, the following non-dimensional parameters are introduced.

$$X = x/L,\ W = W(x)/h^*,\ Z = z/h^* \tag{10}$$

where L is the length of shell. h^* is the total thickness of shell.

Thus frequency parameter f^* can be obtained as follows.

$$f^* = \omega R\sqrt{\rho_t / A_{11}} \tag{11}$$

in which ρ_t is the density of magnetostrictive FGM shells.

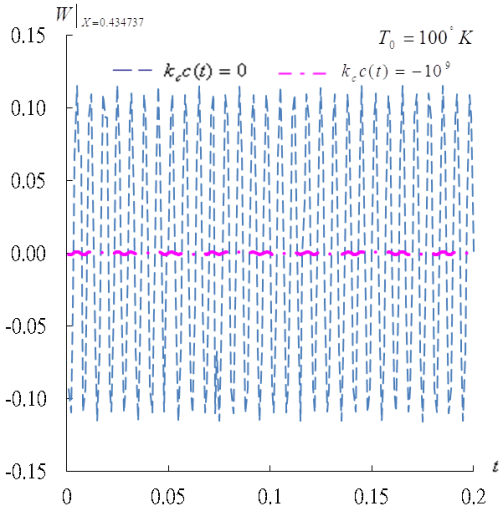
3. COMPUTATIONAL RESULTS AND DISCUSSIONS

In the GDQ computation, the following one-dimensional triangular function coordinate is used.

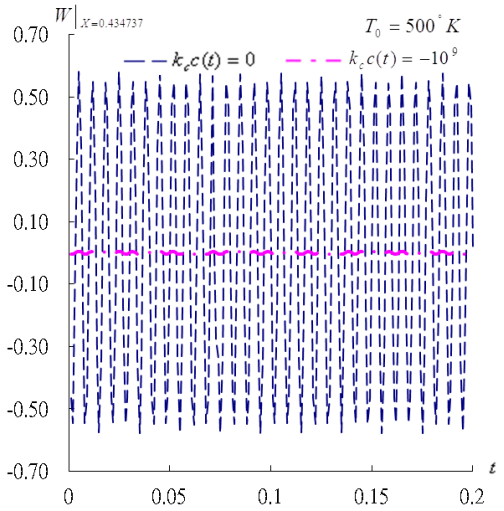
$$x_i = 0.5\left[1-\cos\left(\frac{i-1}{N-1}\pi\right)\right]L,\ i = 1, 2, ..., N \tag{12}$$

Total three layers of circular cylindrical shell is considered, the outer surface magnetostrictive material layer is Terfenol-D, the inner layer of FGM material 1 is SUS304 and the medium layer of FGM material 2 is Si_3N_4. The thickness of Terfenol-D is h_3, SUS304 thickness h_1 equal to Si_3N_4 thickness h_2. Thus $h_1 + h_2 + h_3 = h^*$. The $N = 49$ grid point has been found in the good convergence of displacement. The value of frequency parameter $f^* = 0.0128571$ and time step is 0.001 sec are used for the following transient response analyses. These calculated results are verified in the published open literature [29]. And to understand which one of displacements is dominant in the transient responses.

The effects of T_0 on transient responses of dominant normal displacement W and thermal stress $\bar{\sigma}_{x\theta} = \sigma_{x\theta}/E_2$ are investigated. Figure 2 shows the dominant W at $X = 0.434737$ vs. t with the product of coil constant k_c by control gain $c(t)$, values $k_c\, c(t) = 0$ and -10^9, under $T_0 = 100°K$ and $500°K$ cases, respectively, $h_3/h^* = 0.1$, $T = 500°K$, $R_n = 1$, $\theta = 1$ radian, $R/h^* = 500$, $L/R = 10$, $n = 4$. The amplitude of displacement can be reduced to smaller value by using the value $k_c\, c(t) = -10^9$. The values of displacement in



(a) W vs. t for $T_0 = 100°K$



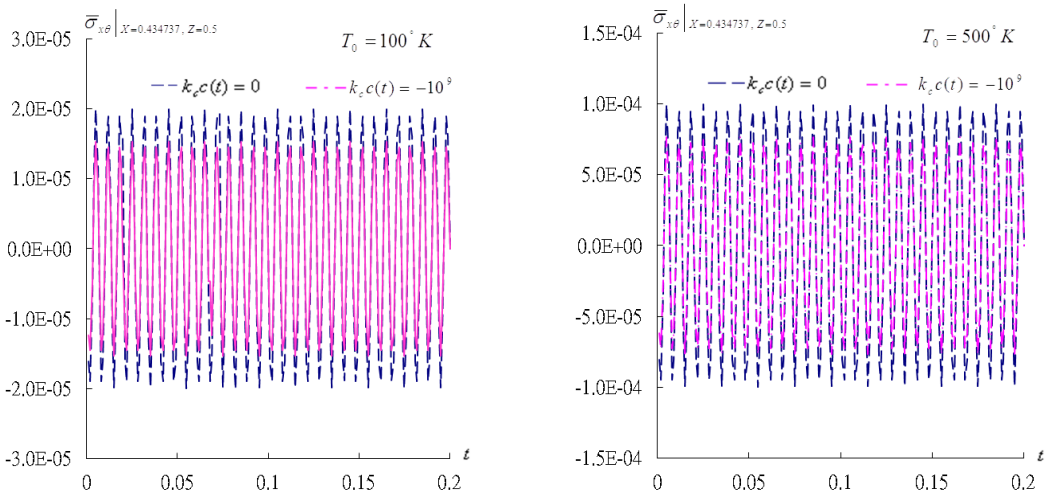
(b) W vs. t for $T_0 = 500°K$

Fig. 2 W vs. t for $T_0 = 100°K$ and $500°K$.

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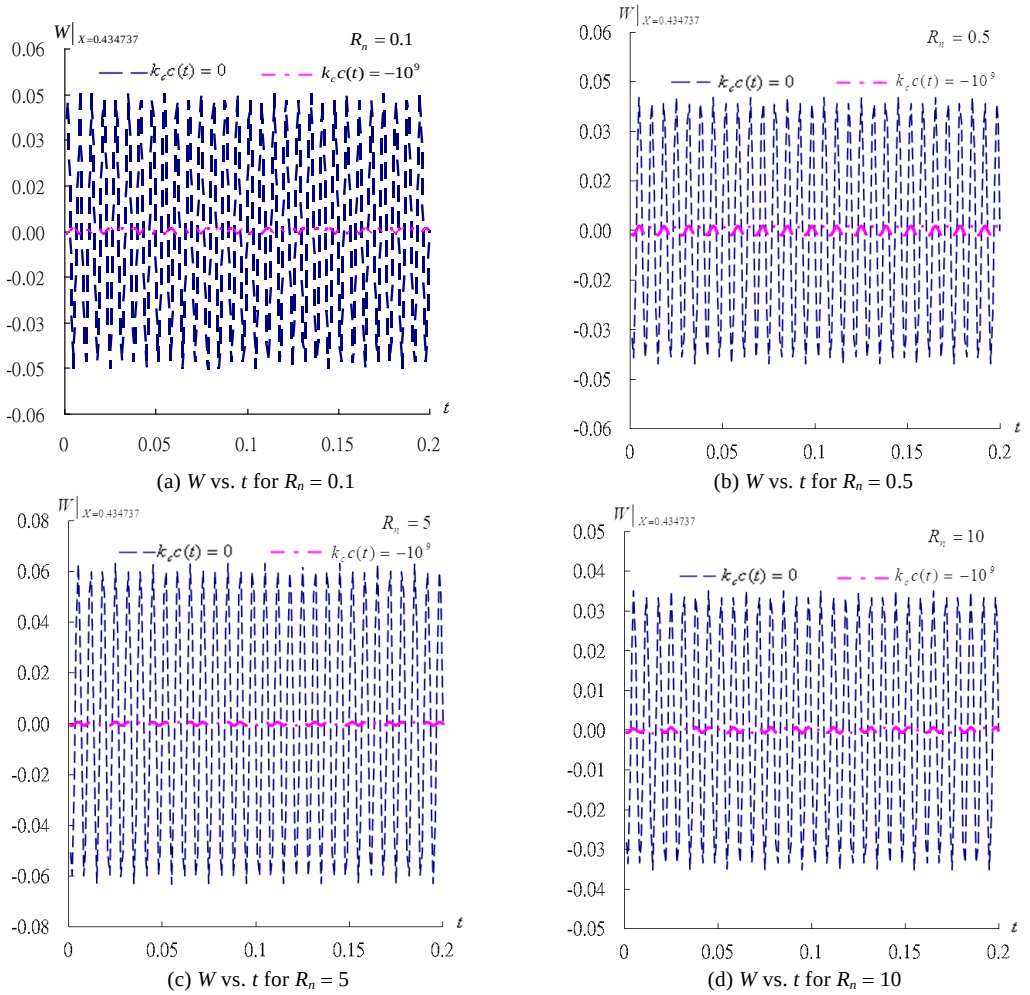
$T_0 = 500°\text{K}$ case are greater than that in $T_0 = 100°\text{K}$ case. Figure 3 shows the dominant $\bar{\sigma}_{x\theta}$ at $X = 0.434737$, $Z = 0.5$ vs. t under the $T_0 = 100°\text{K}$ and $500°\text{K}$ cases, respectively as in Fig. 2. The amplitude of thermal stress is in a smaller value. The values of thermal stress in $T_0 = 500°\text{K}$ case are greater than that in $T_0 = 100°\text{K}$ case.

The effects of R_n in FGM on transient responses of dominant normal displacement W and thermal stress $\bar{\sigma}_{x\theta}$ are investigated. Figure 4 shows the dominant W at $X = 0.434737$ vs. t with the values $k_c\, c(t) = 0$ and -10^9, under $R_n = 0.1$, 0.5, 5 and 10 cases, respectively, $T_0 = 100°\text{K}$, $h_3 / h^* = 0.1$, $T = 500°\text{K}$, $\theta = 1$ radian,



(a) $\bar{\sigma}_{x\theta}$ vs. t for $T_0 = 100°\text{K}$ (b) $\bar{\sigma}_{x\theta}$ vs. t for $T_0 = 500°\text{K}$

Fig. 3 $\bar{\sigma}_{x\theta}$ vs. t for $T_0 = 100°\text{K}$ and $500°\text{K}$.



(a) W vs. t for $R_n = 0.1$ (b) W vs. t for $R_n = 0.5$

(c) W vs. t for $R_n = 5$ (d) W vs. t for $R_n = 10$

Fig. 4 W vs. t for $R_n = 0.1$, 0.5, 5 and 10.

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$R/h^* = 500$, $L/R = 10$, $n = 4$. The amplitude of displacement can be reduced to smaller value by using the value $k_c\, c(t) = -10^9$. The uncontrolled ($k_c\, c(t) = 0$) values of displacement in the case of $R_n = 5$ are greater than that in the cases of 0.1, 0.5 and 10.

Extensive discussion on the potential application of the results might be as a basic data and applied to rapid heating induced thermal vibration of composited FGM shells with the transverse shear deformation effects.

4. CONCLUSIONS

The GDQ method can be used to compute the transient displacement and stresses in the magnetostrictive FGM circular cylindrical shell subjected to thermal vibration. With velocity feedback and suitable product value coil constant k_c by control gain $c(t)$ in the magnetostrictive FGM shells can reduce the transient amplitude of displacement into a smaller value, under the effects of temperature on deflection and power law index.

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