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ARTICLE

The Aboutness Problem for Mathematical Realism

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Abstract

According to mathematical realism, mathematics is the study of a necessary and unchanging reality that exists independently of human activities, beliefs, and capacities. It seems to follow that gaining mathematical knowledge consists merely in acquiring knowledge that accurately describes this mind-independent mathematical reality. Against this, I will argue that, even if there is a mind-independent mathematical reality, gaining mathematical knowledge requires more than merely acquiring knowledge that accurately describes it: it requires knowledge that employs specifically mathematical ways of thinking about mathematical entities – so-called *modes of presentation*. Hence, even if mathematical realism is true, a full understanding of the nature of mathematics calls for an understanding not only of the ontological status of mathematical objects – as has been the realists' focus of research so far – but also of the specifically mathematical ways of thinking about these objects. We will see that, for realists, providing such an understanding is by no means trivial.

Keywords: Mathematical realism; proper names; propositional attitude ascriptions; modes of presentation

1. Introduction

A relatively widespread (though not universal) view of the nature of mathematics is that it is the study of a necessary and unchanging reality that exists independently of human activities, beliefs, and capacities. Accordingly, humans do not invent or create mathematics but rather discover it. This is known as *mathematical realism*. Mathematical realists are mostly Platonists, that is, they usually maintain that mathematical objects are abstract entities that exist independently of intelligent agents and their language, thoughts, and practices (see, e.g., Burgess 1983; Colyvan 2001; Frege 1884; Gödel 1964; Hale 1987; Katz 1998; Marcus 2015; McEvoy 2005, 2012; Parsons 1965, 1971, 1994; Putnam 1971; Resnik 1981, 1997; Shapiro 1989, 1997; Steiner 1975; Wright 1983; Zalta 1983, 1999). Alternatively, so-called Aristotelian realists claim that mathematical objects can be physically realized (see, e.g., Gillies 2015; Franklin 2022; Maddy 1990; Newstead 2001).

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Regardless of the ontological status of mathematical objects, for realists the subject matter of mathematics is a mind-independent mathematical reality. It seems to follow that gaining mathematical knowledge consists merely in acquiring knowledge that accurately describes this reality. Against this, I will argue that, even if there is a mind-independent mathematical reality, gaining mathematical knowledge requires more than merely acquiring knowledge that accurately describes it: it requires knowledge that employs specifically mathematical ways of thinking about mathematical entities – so-called *modes of presentation*. Hence, even if mathematical realism is true, a full understanding of the nature of mathematics calls for an understanding not only of the ontological status of mathematical objects – as has been the realists' focus of research so far – but also of the specifically mathematical ways of thinking about these objects. We will see that, for realists, providing such an understanding is by no means trivial.

2. The aboutness problem

If mathematical knowledge is distinguished, as realists suggest, by its describing a mind-independent mathematical reality, then the question arises: what does it mean for knowledge to describe (parts of) such a reality?

Let us, for example, assume that Sarah's favorite number is eight. Then, although (1) ascribes to Harry knowledge that is in a certain sense about the number eight, (1) does not seem to ascribe *mathematical* knowledge about eight to Harry.

(1) Harry knows that Sarah's favorite number is the sum of five and three.

After all, Harry could understand (2) and know it to be true without knowing what Sarah's favorite number is – for example, if a reliable source had told him that (2) is true.

(2) Sarah's favorite number is the sum of five and three.

Together with the following highly plausible disquotational principle connecting knowledge of sentence-truth with knowledge simpliciter, it would follow that (1) is true:[1]

(DP_K) If a normal English speaker knows that 'p' is true, then they know that p.[2]

However, since Harry would not know what Sarah's favorite number is, he presumably would not know what the sum of five and three is either. In other words, Harry would lack the mathematical knowledge normally ascribed to him with an utterance of (3), leading to the question of what distinguishes the mathematical knowledge ascribed by (3) from the knowledge ascribed by (1), even though both are in a sense about the number eight.

[1]This disquotational principle can be compared to Kripke's (1979) disquotational principle connecting sincere assent and belief, where 'p' can be replaced, inside and outside quotation marks, by any standard English sentence lacking indexical or pronominal devices and ambiguity:

(*DP*) If a normal English speaker is disposed to sincerely and reflectively utter 'p', then they believe that p.

Arguably, Kripke's disquotational principle is based on the following two principles:

(DP_T) If a normal English speaker is disposed to sincerely and reflectively utter 'p', then they believe that 'p' is true.

(DP_B) If a normal English speaker believes that 'p' is true, then they believe that p.

[2]'p' can be replaced, inside and outside quotation marks, by any standard English sentence lacking indexical or pronominal devices and ambiguity.

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(3) Harry knows that eight is the sum of five and three.

I call this *the aboutness problem*.

Prima facie, the aboutness problem simply comes down to the exportation problem: the question of why (4) can be inferred from (3) but not from (1).[3]

(4) Eight is such that Harry knows that it is the sum of five and three.

This is a more general problem that extends beyond ascriptions of mathematical knowledge. In fact, the proposed solutions to the aboutness problem discussed in the next section also provide a solution to the exportation problem. However, as we will see in Section 4, the aboutness problem can be further exacerbated so that ultimately realists can no longer reduce it to the exportation problem.

3. Russellian content

In response to the aboutness problem, mathematical realists could point out that, although both (1) and (3) ascribe to Harry knowledge that is, in a sense, about the number eight, their embedded sentences – (2) and (5) – nevertheless have different truth-conditions.

(1) Harry knows that Sarah's favorite number is the sum of five and three.
(3) Harry knows that eight is the sum of five and three.
(2) Sarah's favorite number is the sum of five and three.
(5) Eight is the sum of five and three.

Unlike (2), (5) is true at a circumstance (possible world, situation, or the like) *C* if and only if, in *C*, eight is the sum of five and three. Accordingly, what distinguishes the mathematical knowledge ascribed by (3) from the knowledge ascribed by (1) is that the truth-conditions of the former involve only mathematical objects, properties, and relations. This can then be said to put Harry *en rapport* with these objects, thereby explaining why (4) can be inferred from (3) but not from (1).

The main problem for such a solution to the aboutness problem is that the embedded sentences of (6) and (7), respectively, share their truth-conditions with (5).

(6) Harry knows that Sarah's actual favorite number is the sum of five and three.
(7) Harry knows that the sum of five and three is the sum of five and three.

For instance, if truth-conditional circumstances are possible worlds, both the embedded sentence of (6) and the embedded sentence of (7) are true at a circumstance *C* if and only if, in *C*, eight is the sum of five and three. However, neither (6) nor (7) seems to ascribe mathematical knowledge about eight to Harry.

Mathematical realists could respond that truth-conditional circumstances include not only possible worlds but also (mathematically) impossible worlds, such as worlds in which

[3]Quine (1956, 182) famously argues that, in propositional attitude ascriptions such as (1) and (3), exportation "should doubtless be viewed in general as implicative." Thus, according to Quine, even from (1) it can be inferred that eight is known by Harry to be the sum of five and three. Today, however, few would follow Quine here. See also Quine (1960, 1970).

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the sum of five and three is not eight. Alternatively, realists could maintain that what distinguishes the knowledge ascribed by (3) from the knowledge ascribed by (6) and (7), respectively, is not its truth-conditional but its Russellian content. Russellian propositions are structured entities whose constituents are the objects, properties, and relations our thoughts and speech acts are about. Advocates of such a view usually maintain that proper names are directly referential terms, that is, that the propositional content of a proper name is just its referent (see, e.g., Braun 1998, 2006; Salmon 1986a, 1986b, 1989, 2006).[4] Accordingly, unlike the embedded sentences of (1), (6), and (7), (5) expresses the Russellian proposition consisting of the number eight and the property of being the sum of five and three. This, the explanation goes, is precisely what sets mathematical knowledge apart, namely, that its Russellian content consists only of mathematical objects, properties, and relations.

There are thus different ways for mathematical realists to make the claim that mathematical knowledge describes a (mind-independent) mathematical reality more precise. However, in the next section, we will see that mathematical realism nevertheless leads to a problem very similar to the aboutness problem.

4. The aboutness problem again

If, as mathematical realists maintain, mathematical objects, such as numbers, exist, then normal English speakers could simply introduce a new name for the number eight by saying something like, "Let's call eight 'Sally.' " In such a case, just like "eight," "Sally" would be a name designating eight.

Let us take the scenario a little further and assume that Harry, a normal English speaker, has been introduced to the name "Sally." However, Harry does not know that "Sally = eight" is true. He only knows that Sally is a number and that Sally is even. Nevertheless, with his uses of "Sally," Harry can refer to the number eight. In this regard, he can be compared to a speaker who associates with the name "Feynman" only the information that Feynman is a physicist. As Kripke (1980) argues, as long as the speaker has been appropriately introduced to the name "Feynman," this does not prevent them from referring to Feynman with their uses of the name.[5]

Now, as a normal English speaker, Harry could even come to know that (8) is true – for example, if a reliable source tells him so.

(8) Sally is the sum of five and three.

Together with (DP_K), it would follow that (9) is true.

(9) Harry knows that Sally is the sum of five and three.

However, since Harry does not know that "Sally = eight" is true, he presumably does not know that (5) is true either.[6]

[4]Direct reference goes back to Mill (1843). A similar view was advocated by Russell (1905, 1910–1911, 1912). However, according to Russell, the only directly referential terms are so-called logically proper names, that is, demonstratives, such as "this" and "that," which refer to sense-data or other objects of immediate acquaintance.

[5]According to Kripke, among other things, this requires that the speaker intends to use the name with the same reference as the speaker from whom they heard it. We can assume that this is true of Harry.

[6]To avoid the objection that a normal speaker of "eight," "five," and "three" cannot doubt that (5) is true, we could instead use a more complex arithmetical statement.

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(5) Eight is the sum of five and three.

We would then again say that Harry lacks the mathematical knowledge normally ascribed to him with an utterance of (3), namely, knowledge of what the sum of five and three is.

(3) Harry knows that eight is the sum of five and three.

For example, if in a math exam Harry answered the question of what the sum of five and three is by uttering (8), then, even if the teacher knew that "Sally" designates the number eight, they would not ascribe to Harry the mathematical knowledge in question if, at the same time, he were not disposed to accept (5) and, when asked to specify Sally, could only reply that Sally is an even number.[7]

This suggests that the solution to the aboutness problem cannot be that what sets mathematical knowledge apart is its truth-conditional or its Russellian content. After all, (3) and (9) ascribe to Harry knowledge with the same truth-conditional and the same Russellian content.[8]

(3) Harry knows that eight is the sum of five and three.
(9) Harry knows that Sally is the sum of five and three.

Nevertheless, unlike (3), (9) does not seem to ascribe to Harry mathematical knowledge of what the sum of five and three is. The problem extends beyond mathematics to also cover measurement statements. For example, if Harry does not know that "Sally = eight" is true, he may lack knowledge of how many planets there are even if he comes to know that "Sally is the number of planets" is true.

One might object that (9) does ascribe some mathematical knowledge to Harry, since it implies that he knows that five and three have a sum. However, this falls short of the kind of knowledge at issue here, namely, knowledge of what the sum of five and three is. The fact that (9) can be true without Harry knowing what the sum of five and three is also why realists ultimately cannot reduce the aboutness problem to the exportation problem. After all, realists seem committed to holding that from (9) we can nevertheless infer (4).

(4) Eight is such that Harry knows that it is the sum of five and three.

[7]If some readers doubt that, in the above example, "Sally" is a name designating the number eight, a similar problem arises with "π." For example, as a normal English speaker, Harry may know that (1′) is true without knowing that so is (2′) – namely, if he does not know that "$\pi = 3.141$" is true.

(1′) The ratio of a circle's circumference to its diameter is π.

(2′) The ratio of a circle's circumference to its diameter is 3.141.

In this case, we would say that Harry does not know what the ratio of a circle's circumference to its diameter is; that is, he lacks the mathematical knowledge normally ascribed to him with an utterance of (3′).

(3′) Harry knows that the ratio of a circle's circumference to its diameter is 3.141.

[8]It might be argued that ordinary proper names stand for definite descriptions, while numerals, such as "eight," are directly referential terms. Accordingly, the Russellian content of (5) and (8) would differ. However, following Kripke (1980), the received view is that both the Russellian content of (5) and the Russellian content of (8) consist of the number eight and the property of being the sum of five and three. Gómez-Torrente (2019), for instance, persuasively argues that verbal and Arabic numerals are special directly referential names of cardinality properties.

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Accordingly, the aboutness problem comes down to the question of how (4) can be true without Harry knowing what the sum of five and three is.

Realists may doubt that Harry lacks the mathematical knowledge in question, precisely because (4) can be inferred from (9). On this view, the truth of (4), and thus of (9), suffices for attributing to Harry mathematical knowledge of what the sum of five and three is. However, there is good reason to believe that Harry lacks knowledge of what the sum of five and three is in a mathematically relevant sense. For example, merely knowing that (8) is true – in addition to knowing how to multiply – does not put him in a position to solve the following calculation task:

(8) Sally is the sum of five and three.
(10) What is four multiplied by the sum of five and three?

Arguably, in order to solve this task, Harry must first find out what the sum of five and three is. Since merely knowing that (8) is true does not put Harry in a position to solve the task, even when combined with his knowledge of how to multiply, we can conclude that knowledge that (8) is true does not amount to knowing what the sum of five and three is.

The variant of the aboutness problem presented in this section might be taken simply to show that we need a more fine-grained notion of truth-conditions in order to distinguish the truth-conditional content of (5) and (8).[9]

(5) Eight is the sum of five and three.
(8) Sally is the sum of five and three.

Independent of how we conceive of truth-conditional circumstances (as a set of possible worlds, a set of situations, or the like), the only way for there to be a circumstance *C* in which (5) and (8) have different truth values is that "eight" and "Sally" have different extensions at *C*. Building on this, we could then say that the relevant truth-conditional circumstances for mathematical knowledge are such that, unlike (8), (5) is true at such a circumstance *C* if and only if, in *C*, eight is the sum of five and three. This would again explain why the knowledge ascribed by (3) puts Harry *en rapport* with eight, while the knowledge ascribed by (9) does not.

Such a conception of the truth-conditions of the embedded sentences of (3) and (9) is akin to Chalmers' (2011a/b) primary intensions, which assign extensions to epistemically possible worlds (scenarios).[10] In fact, according to Chalmers (2011a), unlike ordinary proper names, numerals, such as "eight," are epistemically rigid, that is, they designate the same object with respect to every scenario. However, Chalmers (2011b) also points out that the content of the propositional attitudes ascribed by sentences such as (3) and (9) nevertheless consists of both primary intensions and their respective Russellian content. Otherwise, we would have difficulty explaining that, for instance, an utterance of "Fred believes that I am hungry" can be true. Accordingly, the content of the knowledge ascribed by (3) would be something along the lines of ⟨⟨PI("eight"), eight⟩, ⟨PI("being the sum of five and three"), being the sum of five and three⟩⟩, and the content of the knowledge ascribed by (9) would be something along the lines of ⟨⟨PI("Sally"), eight⟩, ⟨PI("being the sum of five and three"), being the sum of five and three⟩⟩, where

[9]I would like to thank an anonymous reviewer for pointing this objection out to me.
[10]For possible alternatives to Chalmers' view, see Clarke-Doane (2019, 2022).

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PI(*e*) stands for the primary intension of the expression *e*, or some appropriately related intension. Hence, what sets mathematical knowledge apart, on this view, is no longer its Russellian but its primary content.

Such a solution to the aboutness problem will be discussed in more detail in the next section, where I introduce mathematical modes of presentation. After all, primary intensions are intended to play many of the roles that are normally associated with modes of presentation.

5. Mathematical modes of presentation

Mathematical realists could argue that the variant of the aboutness problem from the previous section is just a version of Frege's puzzle for propositional attitude ascriptions, and that therefore, the solutions to Frege's puzzle also provide a solution to this variant of the aboutness problem. For example, it has been pointed out that we have strong intuitions that there can be circumstances in which (11) is true and (12) is false (and vice versa), namely, if Ralph does not know that "Karol Wojtyła" and "John Paul II" are names of the same person.[11]

(11) Ralph knows that Karol Wojtyła is the third-longest-serving pope in history.
(12) Ralph knows that John Paul II is the third-longest-serving pope in history.

Since the embedded sentences of (11) and (12) share both their truth-conditional and their Russellian content, this has led philosophers to introduce so-called *Fregean* content, that is, structured entities whose constituents are the ways the objects, properties, and relations our thoughts and speech acts are about are presented to the speaker or agent (see, e.g., Frege 1892, 1918–19). Accordingly, what distinguishes the knowledge ascribed by (11) from the knowledge ascribed by (12) is the way its truth-conditional (or its Russellian) content is presented to Ralph.

Building on this, mathematical realists could suggest that something similar holds for (3) and (9), namely, that what distinguishes the mathematical knowledge ascribed by (3) from the knowledge ascribed by (9) is the way its truth-conditional (or its Russellian) content is presented to Harry.

(3) Harry knows that eight is the sum of five and three.
(9) Harry knows that Sally is the sum of five and three.

However, from the point of view of mathematical realism, this is surprising. After all, if the subject matter of mathematics is a mind-independent mathematical reality, then one would expect that gaining mathematical knowledge consists merely in acquiring knowledge that accurately describes this reality. Against this, the results of the present paper suggest that, even if there is a mind-independent mathematical reality, gaining mathematical knowledge requires more than merely acquiring knowledge that accurately describes it: it requires knowledge that employs specific (mathematical) ways of thinking about mathematical entities.

One might think that this poses a challenge for realist views in general and, thus, cannot be considered a problem specific to mathematical realism. After all, just like there

[11]Frege's puzzle is usually discussed in connection with belief ascriptions. However, the problem also arises with other attitude ascriptions, such as knowledge ascriptions.

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are circumstances in which (9) is true and (3) is false, there are circumstances in which (11) is true and (12) is false (and vice versa). However, the problem for mathematical realists is not just that there are circumstances in which (9) is true and (3) is false. The problem for mathematical realists is that, unlike (3), (9) does not seem to ascribe mathematical knowledge to Harry of what the sum of five and three is, even though the embedded sentences of (3) and (9) appear to describe the same mind-independent mathematical reality. This cannot be compared to the differences between (11) and (12). Both (11) and (12) ascribe empirical knowledge to Ralph of who the third-longest-serving pope is. For instance, a normal speaker who comes to know that "Karol Wojtyła is the third-longest-serving pope" is true comes to know who the third-longest-serving pope is, independent of whether they know that "Karol Wojtyła = John Paul II" is true. The same holds for "John Paul II is the third-longest-serving pope."

It could be argued that the aboutness problem nevertheless arises once we distinguish between different kinds of empirical knowledge. For example, let "Bert" refer to the kind electron, and suppose that Cindy knows that "Bert" denotes a kind of entity but does not know which kind of entity it refers to. Suppose further that Cindy does not know that electrons are negatively charged but does know that Berts are negatively charged (because she has been told so). It might then seem that Cindy lacks knowledge of physics, in much the same way that Harry lacks mathematical knowledge. But, the objection continues, there is clearly no problem here for realism about electrons, and so the aboutness problem raised in this paper does not pose a serious challenge to realism about numbers.[12]

I believe that what Cindy lacks is not knowledge of physics but the vocabulary typically used in physics to express this very knowledge. Given my own limited knowledge of physics, I would not presume to know anything more about electrons than Cindy, except that I can use the standard term "electron" to express this knowledge. The same cannot be said about Harry and me when it comes to knowing what the sum of five and three is. While I know what the sum of five and three is, Harry does not.[13] Thus, what Harry lacks is not merely the canonical name for the number eight. Suppose the numeral "eight" had been transmitted to Harry in much the same way as "Sally," such that the only information he associates with "eight" is that it is an even number. Even if he were then told that (5) is true, he might still lack the mathematical knowledge of what the sum of five and three is.

(5) Eight is the sum of five and three.

Among other things, he might not be in a position to solve the following calculation task, even if he were fully competent with all the expressions involved and knew, in principle, how to multiply:

(10) What is four multiplied by the sum of five and three?

The case of Cindy is different: if she had been introduced to "electron" in the same way as to "Bert," we would no longer say that she lacks physical knowledge that electrons are negatively charged once she is told that "Electrons are negatively charged" is true.

[12]I would like to thank an anonymous reviewer for pointing this objection out to me.

[13]Similarly, Harry does not know how many planets there are if he only knows that Sally is an even number and that "Sally is the number of planets" is true.

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I therefore conclude that there is a crucial difference between Harry and Cindy and, thus, between mathematical and empirical knowledge. Whereas gaining empirical knowledge seems to require only the acquisition of knowledge that accurately describes empirical reality, gaining mathematical knowledge seems to require more: namely, knowledge that employs specifically mathematical ways of thinking about mathematical entities.

We can reach the same conclusion from a different angle. While Cindy can be successfully introduced to the kind term "electron" in the same way as to "Bert," this does not seem to apply to "eight" and "Sally." If "eight" were transmitted to Harry in the same way as "Sally," he would not count as fully competent with respect to it. After all, from the fact that Harry knows that (5) is true, we would still not infer that he has the mathematical knowledge typically ascribed by (3).

(3) Harry knows that eight is the sum of five and three.
(5) Eight is the sum of five and three.

Moreover, in the case at hand, Harry would not be in a position to solve the following calculation task, even if we assumed that he is fully competent with the remaining expressions and knows how to multiply:

(11) What is four multiplied by eight?

Full competence with "eight" thus appears to require more than the ability to refer to eight as part of a Kripkean chain of communication; it also requires grasping eight in a distinctive mathematical way – for example, as the successor of seven. By contrast, no analogous requirement seems to govern competence with empirical kind terms such as "electron," in line with the broadly Kripkean-Putnamian view, on which successful reference to natural kinds does not require such modes of presentation (see, e.g., Putnam 1975; Kripke 1980).

I take these considerations to suggest that, even if mathematical realism is true and mathematical objects exist independently of human activities, beliefs, and capacities, a full understanding of mathematics (and of measurement statements) requires not only an understanding of the ontological status of mathematical objects, such as numbers, but also of the specifically mathematical ways of thinking about them. Importantly, this notion of a mathematical way of thinking (or mode of presentation) must be independent of notation. Different notational systems can thus express the same mathematical content without thereby altering the underlying way of thinking involved. For realists, providing an account of such mathematical ways of thinking is by no means trivial. I illustrate this in what follows using the example of mathematical ways of thinking about numbers.

Standardly, the mathematical way of thinking that speakers are said to associate with "eight" consists in a mathematical property *F* that uniquely picks out eight, such as the property of being the successor of seven. Since, according to mathematical realists, "Eight is *F*" accurately describes the mind-independent mathematical reality, it would follow that, in order to associate such a way of thinking with "eight," an agent must first acquire mathematical knowledge of that reality – namely, the mathematical knowledge expressed by an utterance of "Eight is *F*." But, as the results of this section suggest, the agent must then already be in a position to think about the number eight in a specifically

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mathematical way, and thus, the proposal is circular: in order to acquire a mathematical mode of presentation of eight, one already needs such a mode of presentation. After all, the aboutness problem can be constructed for any sentence of the form "Eight is *F*," at least, it is not clear why mathematical realism should allow for exceptions here.[14]

A similar problem arises if mathematical realists conceive of mathematical modes of presentation as inferential roles. For instance, according to mathematical realists, "eight" has its inferential role because of certain facts about the mind-independent mathematical reality. Hence, in order for normal speakers to associate this inferential role with "eight," it seems that they must already be in a position to acquire mathematical knowledge about eight.

An alternative to mathematical realism that appears to avoid these problems is traditional constructivism. According to this view, mathematical objects must be constructed in order to exist. Although such a conception of mathematical objects likely gives rise to problems very similar to the aboutness problem, constructivists could maintain that mathematical ways of thinking arise as a byproduct of the construction of mathematical objects, thereby providing privileged access to the constructed mathematical reality. Suppose, for example, that a mathematical way of thinking about the number eight again consists in a mathematical property *F* that uniquely picks it out, such as the property of being the successor of seven. Then, according to traditional constructivists, an agent need not first acquire knowledge about the number eight in order to grasp such a way of thinking. They need only be able to construct the number eight. As a result, the agent may come to know that eight is the successor of seven. At this point, however, they have already acquired a mathematical way of thinking about the number eight.[15]

6. Conclusions

According to mathematical realism, mathematics is the study of a mind-independent mathematical reality. From this, it seems to follow that gaining mathematical knowledge consists merely in acquiring knowledge that accurately describes this reality. Against this, the results of the present paper suggest that, even if there is a mind-independent mathematical reality, gaining mathematical knowledge requires more: it requires knowledge that employs specifically mathematical ways of thinking about mathematical entities. Prima facie, this is better accounted for by constructivist conceptions of mathematics than by realist views. However, a more detailed discussion of the aboutness problem and its implications for the debate between realists and constructivists goes beyond the scope of this paper. The primary aim here has been to show (i) that, even if there is a mind-independent mathematical reality, a full understanding of mathematics (and of measurement statements) requires not only an understanding of the ontological status of mathematical objects – as has been the main focus of research so far – but also of the specifically mathematical ways of thinking about these objects, and (ii) that, for realists, providing such an understanding is by no means trivial.

[14]Note that, for mathematical realists, it is not an option to say that "Eight is *F*" expresses a definitional truth. If we assume that there is a mind-independent mathematical reality, then being, for instance, the successor of seven is not a property eight has by definition. Rather, it seems to be a property eight has because the mind-independent mathematical reality is constituted in a certain way.

[15]Alternatively, constructivists could maintain that "Eight is *F*" expresses a definitional truth, since constructing eight itself provides knowledge of the property *F*.

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