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Nonlinear 3D MHD code NFTC for simulations of plasma instabilities

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Abstract. The nonlinear three-dimensional magnetohydrodynamics (MHD) code NFTC and the basic algorithms included in the code are presented. NFTC code is used for simulations of nonlinear neoclassical instabilities (or neoclassical tearing modes (NTMs)) in tokamaks. An effective fully implicit numerical scheme allows the transport profile to evolve self-consistently with the nonlinear MHD instabilities and externally applied sources and sinks. It is shown that numerical code has enough accuracy for simulation of plasma instabilities in arbitrary three-dimensional geometry. The coefficients of a special finite-difference scheme in the radial direction are constructed in the form which gives the exact asymptotic dependence of the solution by the ratio of small parameters near high derivatives and the mesh step size. The reproduction of the nonlinear threshold effects for NTMs in conditions of anisotropic heat conductivity is presented.

1. Introduction

Numerical nonlinear MHD simulations of plasma instabilities are important for predictions of stable operating regimes in tokamaks. High plasma temperature and density in future devices could lead to the appearance of small parameters near high derivatives in the mathematical formulation of the problem and give the special requirements for numerical algorithms for the precise determination of stability parameters. Accurate numerical resolution of internal layers and magnetic islands is needed for stability prediction in the limit of high magnetic Reynolds number. In the present paper we present the nonlinear three-dimensional (3D) magnetohydrodynamics (MHD) code NFTC which satisfies these requirements. The definite set of the basic algorithms included in the code helps us to construct a special one-dimensional finite difference scheme which has uniform convergence properties by the small parameters of the problem. The NFTC code is used for simulations of nonlinear neoclassical instabilities (or neoclassical tearing modes (NTMs)) in tokamaks [1–3]. Simulation of the plasma behavior in future tokamaks is important to establish the right scaling dependencies on resistivity, viscosity and heat conductivity.

2. Basic equations in the NFTC model

The nonlinear 3D evolution of a tokamak plasma is described by the MHD system of equations which include plasma rotation, external helical magnetic fields, viscosity, resistivity and sources of heat and electric field. We seek the solution $\mathbf{Y} = \{\mathbf{V}, \mathbf{B}, P\}$ of the full MHD equations, with the velocity $\mathbf{V}$, magnetic field $\mathbf{B}$, and pressure P.

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The basic equations are

$$\overline{\rho}\frac{\partial \mathbf{V}}{\partial t} = -\overline{\rho}(\mathbf{V}\cdot\nabla)\mathbf{V} - \nabla P + [[\nabla\times\mathbf{B}]\times\mathbf{B}] + \nu\nabla^{\mathbf{2}}\mathbf{V}; \tag{1.1}$$

$$\frac{\partial \mathbf{B}}{\partial t} = [\nabla\times[\mathbf{V}\times\mathbf{B}]] - [\nabla\times(\eta[\nabla\times\mathbf{B}])] + [\nabla\times\mathbf{E}_{\mathbf{s}}]; \tag{1.2}$$

$$\frac{\partial P}{\partial t} = -\nabla\cdot(P\mathbf{V}) - (\Gamma-1)P\nabla\cdot\mathbf{V} + \nabla_{\parallel}\cdot(K_{\parallel}\nabla_{\parallel}P) + \nabla_{\perp}\cdot(K_{\perp}\nabla_{\perp}P) + Q \tag{1.3}$$

In these dynamic equations $K_{\perp}, K_{\parallel}, \eta, \nu$ and Q are dimensionless values of the perpendicular thermal conductivity, heat conductivity along perturbed magnetic surfaces, resistivity, the kinematic viscosity and heat source term. In the dimensionless units [1] the characteristic values of dissipative terms used in the calculations are: viscosity $\nu = 10^{-7}$, resistivity $\eta = 10^{-6}$–10^{-8}, $K_{\perp} = 10^{-5}$, $K_{\parallel} = 5\times10^{3}$. For the MHD model of instabilities, the concerned problem presents the problem with small parameters near high derivatives. Dissipative terms are important only in the narrow layers near resonance surfaces. For this reason, dissipative operators are included only in the simplest form. There are several sources of current density: the bootstrap current j_{BS}, the ECCD current j_{cd}, the current from neutral beam injection j_{NB}, and the Ohmic current density j_{Ω}. The total parallel current density is $j = j_{\Omega} + j_{\mathrm{BS}} + j_{\mathrm{cd}} + j_{\mathrm{NB}}$. Then $E = \eta j_{\Omega} = \eta j - E_{\mathrm{s}}$, where the source term is $E_{\mathrm{s}} = \eta(j_{\mathrm{BS}} + j_{\mathrm{cd}} + j_{\mathrm{NB}})$. The bootstrap current j_{BS} is included in the simplest form: $j_{\mathrm{BS}} = 1.46\sqrt{\varepsilon}[-(\partial P/\partial\rho)/B_{\mathrm{pol}}]$.

3. Numerical method

The equations are formulated in general toroidal geometry. Equilibrium terms are explicitly excluded from the system. One component of the magnetic field is excluded from the system using the solenoidal equation $\nabla\cdot\mathbf{B} = 0$.

Metric elements $g_{ik}(\rho,\theta)$ are calculated corresponding to the straight field line coordinate system ρ, θ, φ using equilibrium $(\mathbf{B}_{\mathrm{eq}}, P_{\mathrm{eq}})$, where $\rho = \sqrt{\psi_N}$ is a radial-like coordinate which labels the magnetic surface, θ is a poloidal-like angle, and φ is the toroidal angle. The contravariant components of the solution $\mathbf{Y} = \{V^1, V^2, V^3, Y^1, Y^2, Y^3, P\}$ and metric elements $g_{ik}(\rho,\theta), i,k = 1,2,3$ are represented as finite Fourier series in both poloidal and toroidal angles. We then have the unknown functions $Y_{mn}(\rho,t)$, which are Fourier coefficients, where m is a poloidal and n is a toroidal wave number, and a system of nonlinear one-dimensional equations for Fourier coefficients. The system of equations is separated by two connecting parts: for $n = 0$ and $n \neq 0$ For the linear operators we use an implicit finite difference scheme in the radial direction. The coupling of nonlinear toroidal modes are resolved by the Block–Seidel method. For the quasilinear operators we use an implicit scheme resolved iteratively, using the Newton method. One of the new features of the code is the parallel version realized on a shared memory multiprocessor computer IBM p-Series 690. The parallel library PESSL is used for the solution of the algebraic equations with the conservation of the structure of the inverse matrix for linear operators. The other new feature of the method is the use of the special fitted asymptotic parameters scheme (analog of the exponential fitted scheme for boundary layers) in the radial direction for each Fourier harmonic. The construction of such exponential fitting schemes can be found in [4]. Coefficients of the scheme near second-order derivatives are constructed in a such a way to have numerically the exact analytical dependence as in the asymptotic behavior of the solution. The main feature of the scheme is that we have the same ratio of mesh step size to

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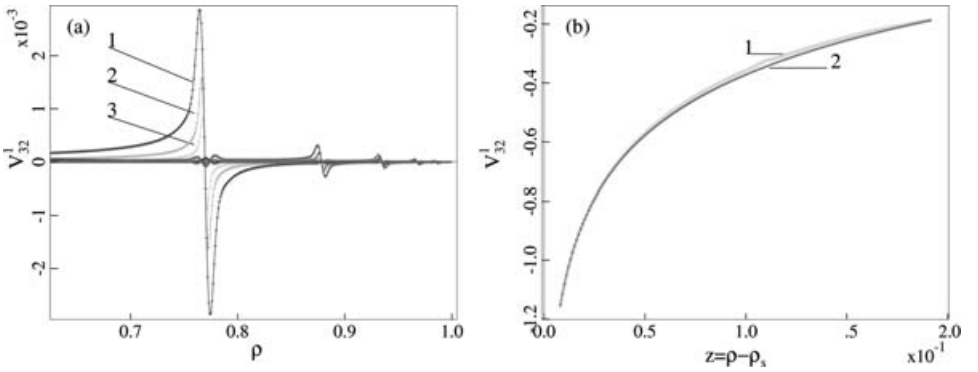


Figure 1. Radial profiles of radial velocity $V_{32}^{1}(\rho)$ for basic unstable harmonic $m/n = 3/2$: (a) different magnetic Reynolds number; (b) comparison of numerical solution with analytical asymptotic profile close to the rational surface.

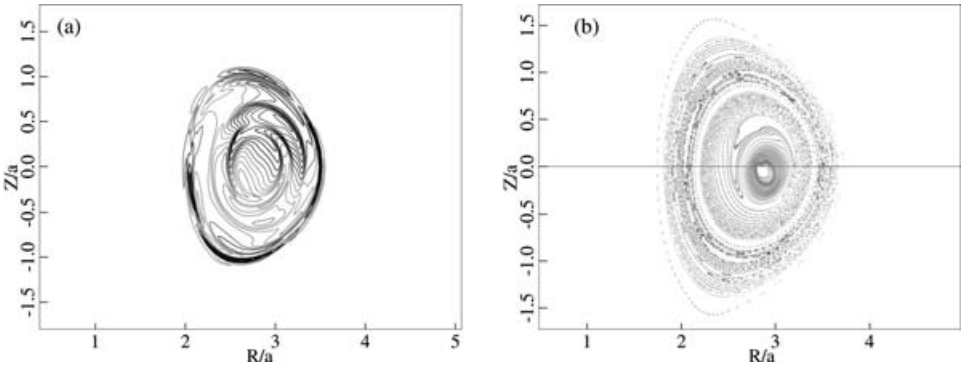


Figure 2. 3D characteristics of the numerical solution in simulation of seed island appearance during a period for ITER: (a) contour plots for pressure perturbation; (b) Poincaré map.

the cube root of the resistivity. So, when we change resistivity in the parametric study, then the dependence of the physical characteristic does not depend on mesh size, even when we have a few grid points inside magnetic island. Fig. 1a shows an accuracy study for ITER calculations. The numerical resolution of small islands in the limit of large different Reynolds numbers is shown in Fig. 1(a). Radial profiles of the radial component of 3/2 harmonic of the magnetic field shows regularized singularities near the basic rational surfaces. Calculations were done for different magnetic Reynolds number $S = 10^6$ (curve 1), 10^7 (curve 2), 10^8 (curve 3) with the same grid that contains 1000 points. Comparison of the numerical solution near the rational surface (curve 1) and exact asymptotic solution [4] (curve 2) are presented in Fig. 1(b) which shows the good correspondence between them.

The 3D characteristics of the numerical solution are used for the estimation of the sizes of magnetic islands and the distortion of magnetic surfaces. Fig. 2 shows (a) a contour plot of pressure perturbation and (b) a Poincaré plot of the magnetic field lines in these calculations.

4. Examples of NTM simulations

The nonlinear instabilities were studied for the DIII-D tokamak [1–3]. The formulation of the problem for neoclassical effects in the NFTC model is described in

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[2]. One of the key methodology steps in the code is the use of the fully implicit numerical scheme. The characteristic numerical parameters used in the calculations are: radial number of grid points $N = 10^3$ to 5×10^3, maximal number of dynamic poloidal harmonics $m_{\max} = 25$ and five toroidal harmonics $n = 0, 1, 2, 3, 4$. This allows us to give an explanation of the evolution of MHD instabilities in NCS discharges on DIII-D [1]. The transport profiles in this case evolve self-consistently with the nonlinear MHD instabilities and externally applied sources and sinks. This analysis elucidates the relationship between the early MHD bursts and the much later hard disruption. A special fitted scheme permits the highly anisotropic heat conductivity cases for NTM instabilities to be considered. A direct comparison of NTM evolution with experimental observation in different DIII-D discharges gives a good examples of the features of this code [2, 3].

As an example of the nonlinear resolution of the NFTC code, we provide the calculations done for ITER where the double threshold effect was obtained. For ITER parameters we need to have the following ratio of the heat conductivities $K_\perp / K_\parallel = 10^{-10}$. The nonlinear time evolution of NTM $n = 2$ islands width presents the main new result from 3D simulation. Two thresholds in the problem were found: the first is related to the introduced transport model (predicted by theory) and the second threshold can be called the nonlinear resistive MHD threshold and can be predicted only by numerical simulations. The problem of NTM control by ECCD was considered for ITER under conditions of strong nonlinear coupling of the NTM modes which cannot be done by simplified models.

5. Conclusions

It is shown that the numerical code NFTC has enough accuracy for the simulation of nonlinear plasma instabilities in arbitrary 3D geometry. Representation of the solution and Ohm's law formulation (which can include two fluids effects) permit one to consider the different physical models. Two new features of the code are emphasized in the present work. One is concerned with the special finite difference scheme fitted to the asymptotic solution which allows us to perform the simulations with high magnetic Reynolds number and anisotropic heat conductivities. The second is the parallel version of the code based on the conservation of the structure of the inverse matrix for the main linear part of operators.

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