The Australian shuffle and two self-working card tricks

ANITA A. GAENKO, MICHAEL A. JONES, VANEESHA NATOGI, ALEX SWANSON, ANIRUDH S. VENKAT and KHOI M. VU

1. *Introduction*

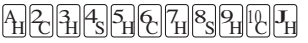
Self-working card tricks have been a staple for budding magicians and entire books have been written about them, e.g. Fulves [1]. Lately self-working card tricks have become common fodder for social media. In this note, we generalise two such card tricks (from [2] and [3]) that are based on the Australian shuffle.

2. *The Australian shuffle*

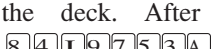
The Australian shuffle, which gets its name from the colloquial use of 'Down Under' to refer to Australia, is described online [4] and in the recent book on card shuffling [5]; as part of the shuffle, successive cards are placed down on the table and under the deck.

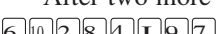
Definition: The Australian shuffle (Down-Under shuffle) is performed on a deck of cards by alternating between placing one card down on the table and the next card under the deck, until you are left with all the cards in a single pile on the table.

Example: The Australian shuffle reorders the following deck of 11 cards



arranged top to bottom, by sequentially placing cards down on the table and under the deck. The sequence begins by placing the first card down on the table, and the next card  under the deck. After the first pass (where each of the 11 cards is either put down or under) the cards, arranged top to bottom, on the table are  and the cards remaining in the hand are .

Because the last card from the original 11 cards  was placed down on the table, the next time through the cards, the first card, , is placed under the deck. After the second pass the cards on the table are  and the cards in the hand are .

After two more passes all cards are on the table in the following order: .

The two tricks generalised in this paper hinge on which card is on top of the deck after the Australian shuffle. For the previous example, the sixth

https://doi.org/10.1017/mag.2025.10080 Published online by Cambridge University Press

card in the deck became the top card after the Australian shuffle. The following theorem explains which card, based on its original position in the deck, ends up on top for a deck of any size.

Theorem: For a deck containing n cards with $2^k < n \leqslant 2^{k+1}$ and $k > 0$, the top card after the Australian shuffle is the card that was originally in position $2n - 2^{k+1}$.

Proof: Let the cards be numbered from top (card 1) to bottom (card n) in the deck. For the first pass through the deck under the Australian shuffle, the odd-positioned cards are placed down. For $k = 0$, $n = 2$ and the card in position $2 \cdot 2 - 2^1$ is on top after the Australian shuffle. For $k = 1$, $n = 3$ or 4. If $n = 3$, then the second card is placed on top (satisfying $2 \cdot 3 - 2^2 = 2$) because it was not placed down on the first pass. If $n = 4$, the second card is placed down first in the second pass, leaving card 4 on top and $2 \cdot 4 - 2^2 = 4$. We now proceed by induction on k and assume that, if there are n cards in the deck with $2^\ell < n \leqslant 2^{\ell+1}$, then the top card after the Australian shuffle started in position $2n - 2^{\ell+1}$ for $\ell = k - 1$ and k.

Our proof that the claim holds for a deck of n cards with $2^{k+1} < n \leqslant 2^{k+2}$ is separated into two cases, n even and n odd. If n is even, then after one pass through the deck there are $n/2$ even-positioned cards remaining in the hand. These are viewed as a new deck where each card's position in this half deck is its original position divided by 2. Because $2^k < n/2 \leqslant 2^{k+1}$ and, by our assumption, the top card after the Australian shuffle was in position $2 \cdot (n/2) - 2^{k+1} = n - 2^{k+1}$ in the half deck. By multiplying by 2, we get the position in the original deck: $2\left(n - 2^{k+1}\right) = 2n - 2^{k+2}$ as required.

If n is odd, one pass of the shuffle leaves $\frac{1}{2}(n-1)$ cards in the hand with $2^k \leqslant \frac{1}{2}(n-1) < 2^{k+1}$. After the first card (originally in position 2) is placed down at the beginning of the second pass, the cards in the hand form a near-half deck where the cards, numbered based on their original positions, are ordered $4, 6, \cdots (n-1), 2$. The next card is to be placed down. The card with original position 2ℓ ($\ell \neq 1$) is in position $\ell - 1$ in the near-half deck; the card with original position 2 is in position $\frac{1}{2}(n-1)$.

If $n = 2^{k+1} + 1$, then $\frac{1}{2}(n-1) = 2^k$ and $2^{k-1} < \frac{1}{2}(n-1) \leqslant 2^k$. By assumption, the card on top after the Australian shuffle is in position $2 \cdot \frac{1}{2}(n-1) - 2^k = (n-1) - 2^k = 2^k$ of the near-half deck. This is the last card of the deck, the card originally in position 2. And $2\left(2^{k+1} - 1\right) - 2^{k+2} = 2$, as desired.

If $n \neq 2^{k+1} + 1$, then $2^k < \frac{1}{2}(n-1) < 2^{k+1}$. By assumption, the top card after the Australian shuffle is from position $2 \cdot \frac{1}{2}(n-1) - 2^{k+1} = (n-1) - 2^{k+1}$ of the new deck. This top card's position in the original deck is determined by adding 1 and then multiplying by 2: $2 \cdot \left[(n-1) - 2^{k+1} + 1\right] = 2n - 2^{k+2}$. This completes the proof.

https://doi.org/10.1017/mag.2025.10080 Published online by Cambridge University Press

3. *The sixteen card trick and its generalisation for 2^n cards*

The following trick, which we call the sixteen card trick, depends on the Australian shuffle and appeared on social media [2]. We describe the trick and prove that the trick works for any deck of 2^n cards.

The sixteen card trick

1. Take 16 random cards from a deck. Have the audience look at the bottom card (which we call the target).
2. Move the top card to the bottom of the deck.
3. Perform the Australian shuffle.
4. Take the new top card (which we call the helper) and flip it over so that it faces up.
5. Have an audience member cut the deck as many times as desired.
6. Create two piles by repeatedly taking the top card and alternating between placing it down on a left pile and a right pile, until there are no cards left.
7. Take the pile with the helper card. Discard the other pile.
8. Repeat steps 6 and 7 until there is a pile with two cards. Flip over the non-helper card. It is the target card.

Proposition 1: The 2^n trick works for any n.

Proof: Let the cards be numbered from top (card 1) to bottom (card 2^n) in the deck. The bottom card is called the target. The first step of the trick is to move the top card to the bottom of the deck, resulting in the ordering: $2\ 3\ 4 \cdots 2^n - 1\ 2^n\ 1$. Under the Australian shuffle, the first 2^{n-1} cards placed down are the cards in the odd positions, which are the even-numbered cards for this ordering. This means that the target card (being in position $2^n - 1$ before shuffling) is the 2^{n-1}th card placed down on the table and will be in position $2^{n-1} + 1$ of the shuffled deck. By the Theorem, because there are 2^n cards in the deck, the top card after the Australian shuffle was in position $2 \cdot 2^n - 2^n = 2^n$, which was the card numbered 1. Call this card the helper and turn it face up.

The cards are now ordered as

$$1 \underbrace{\quad \ldots \quad}_{2^{n-1}-1 \text{ cards}} 2^n \quad \underbrace{2^n - 2 \ \ldots \ 4 \ 2}_{2^{n-1}-1 \text{ cards}};$$

for this trick what is important is that the helper card, 1, and the target card, 2^n, have $2^{n-1} - 1$ cards between them, so are in positions 1 and $2^{n-1} + 1$. A spectator now cuts the cards as many times as desired; successive cuts are equivalent to cutting the cards once. The cut can be between the helper and the target, which leaves the pile as

$$\ldots\ 2^n \ \underbrace{2^n - 2 \ \ldots \ 4 \ 2 \ 1}_{2^{n-1}-1 \text{ cards}} \ \ldots,$$

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or after the target, which results in an ordering

$$\ldots\ 2\ \ 1 \overbrace{\qquad\ldots\qquad}^{2^{n-1}-1 \text{ cards}} 2^n\ 2^n - 2\ \ldots .$$

In each case, there are still $2^{n-1} - 1$ cards between the helper and target, so that the positions of the cards are k_0 and $2^{n-1} + k_0$.

Repeatedly take the top card and alternate between placing it down on a left pile and a right pile. One pile consists of the cards in the odd position and the other pile consists of the cards in the even positions. Because the target and helper cards are in positions k_0 and $2^{n-1} + k_0$, they are in the same pile because they have the same parity. They are now in positions k_1 and $2^{n-2} + k_1$.

Pick up the deck with the helper card and repeat the process, dividing it into two decks by placing the top card alternately into left and right piles. The helper and target again are in the same pile, but now are in positions k_2 and $2^{n-3} + k_2$. Repeat this process until there are two piles with two cards in them. The helper and target are in positions $k_{n-1} = 1$ and $1 + k_{n-1} = 2$ of the same pile. In the last step, the face down card paired with the helper is revealed to be the target card.

4. *The Elon Musk card trick and its generalisation*

A recent social media post [3] describes a trick attributed to Elon Musk that uses the 'Tesla shuffle'. The Tesla shuffle is really just the Australian shuffle with an Elon Musk-related name. Here is a description of the trick, along with an example.

The Elon Musk card trick

1. Deal out between four and eight cards; e.g. $\boxed{A_S}\boxed{10_H}\boxed{3_D}\boxed{7_D}\boxed{3_C}\boxed{2_S}$.
2. Shuffle the cards and look at the bottom card; call this card the target. e.g. the shuffled cards, $\boxed{3_D}\boxed{7_D}\boxed{3_C}\boxed{A_S}\boxed{2_S}\boxed{10_H}$, have $\boxed{10_H}$ as the target.
3. Take one card from the top of the deck and place it on the bottom of the deck for each letter in "E-L-O-N-M-U-S-K"; this reorders the cards as: $\boxed{3_C}\boxed{A_S}\boxed{2_S}\boxed{10_H}\boxed{3_D}\boxed{7_D}$.
4. Next, do the Australian shuffle, which results in $\boxed{10_H}\boxed{7_D}\boxed{A_S}\boxed{3_D}\boxed{2_S}\boxed{3_C}$.
5. The top card of the shuffled deck is the target card.

The Elon Musk card trick starts with four to eight cards, and places eight cards under because of the eight letters in "Elon Musk". We generalise the trick for n cards with $2^k \leqslant n \leqslant 2^{k+1}$ and place 2^{k+1} cards under.

Proposition 2: The Elon Musk trick works for any deck of n cards such that $2^k \leqslant n \leqslant 2^{k+1}$ and when 2^{k+1} cards are placed under in step 3.

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Proof: Let the cards be numbered from top (card 1) to bottom (card n) in the deck, with the card in the nth position being denoted as our target. When the top card is put under the deck 2^{k+1} times in step 3 of the trick, the first n of these repetitions simply cycle the deck around back to its original order, leaving $2^{k+1} - n$ repetitions to go. There are two cases, either $n \neq 2^k$ or $n = 2^k$.

If $n \neq 2^k$, then each of the remaining $2^{k+1} - n$ repetitions will put one card under the target and move it one position towards the front of the deck, meaning the target's new position after the whole operation will be $n - (2^{k+1} - n) = 2n - 2^{k+1}$. By the Theorem, the top card of this deck of n cards where $2^k < n \leqslant 2^{k+1}$ after the Australian shuffle will be the one originally from position $2n - 2^{k+1}$, which was just shown to be the target card.

If $n = 2^k$, then $2^{k+1} - n = 2^{k+1} - 2^k = 2^k = n$. The last $2^{k+1} - n$ repetitions of putting the top card under the deck will cycle through the deck once just like the first n repetitions. Hence, this operation preserves the order of the deck and keeps the target in position n. By the Theorem, the top card of this deck of n cards where $2^{k-1} < n \leqslant 2^k$ after the Australian shuffle is from position $2n - 2^k = 2n - n = n$, which was just shown to be the target card.

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ANITA A. GAENKO, VANEESHA NATOGI, ALEX SWANSON, ANIRUDH S. VENKAT, KHOI M. VU
Ann Arbor Math Olympiad Club
Ann Arbor, MI 48103 USA
MICHAEL A. JONES
Mathematical Reviews
Ann Arbor, MI 48103 USA
e-mail: *maj@ams.org*

10.1017/mag.2025.10080 © The Authors, 2025
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