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INTERACTION OF A SINGULAR SURFACE WITH A STRONG SHOCK IN THE INTERSTELLAR GAS CLOUDS

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Abstract

We investigate the interaction between a singular surface and a strong shock in the self-gravitating interstellar gas clouds with the assumption of spherical symmetry. Using the method of the Lie group of transformations, a particular solution of the flow variables and the cooling–heating function for an infinitely strong shock is obtained. This paper explores an application of the singular surface theory in the evolution of an acceleration wave front propagating through an unperturbed medium. We discuss the formation of an acceleration, considering the cases of compression and expansion waves. The influence of the cooling–heating function on a shock formation is explained. The results of a collision between a strong shock and an acceleration wave are discussed using the Lax evolutionary conditions.

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1. Introduction

Great strides in the study of the self-gravitational collapse of the interstellar gas clouds have opened up new opportunities to understand the astronomical phenomena of star formation. Applying a mathematical model to this core astronomical feature includes exploring effective solutions to a system of partial differential equations, and several methods have already been put forward by some eminent researchers.

A series of papers [6, 7, 22] in the literature provided an elaborate understanding of the methods adopted to study the details of self-gravitating collapse of the interstellar gas clouds, considering the loss of energy. In the work of Disney et al. [8], this was followed by their application on a series of cloud models to study the impact of the cooling–heating function and density distribution on the collapse. Penston [26] performed the numerical integration of the equations of hydrodynamics, including the

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effects of self-gravitation, to study the collapse of stars and galaxies. A concatenation of studies led to analytical solutions, considering the cases of either free falling gas or isothermal gas [27]. Hunter [12] constructed the theoretical models of spherical clouds in the initial phases, and discussed their collapse and the formation of stars. The details of the model describing the theories as well as derivations of equations related to dust and star formations along with the collapse of interstellar gas clouds were presented in [8–10]. Further, introducing rotation and magnetic field within the framework of spherical symmetry, numerical calculations were performed by Larson [18] and Mouschovias [25]. Delving deeper into the medium of interstellar gas clouds, Kazhdan et al. [17] inquired about the involvement of sphericity in the shock wave emergence at the stellar surface, and talked about self-similarity spherical accretion. Bisnovatyi-Kogan and Murzina [1, 2] dealt with the early stages of the relativistic fireball expansion, and discussed self-similar solutions of the burst propagation through the medium of interstellar gas.

We start with the use of the Lie group of transformations. The Lie group is imperative to mention when talking of continuous symmetry in the field of mathematics and theoretical physics. The expanded Lie group of transformations involves the expanded space of variables including independent and dependent variables of the equations together with the system parameters. The work is motivated by the thorough discussions and comprehensive treatment of the symmetry methods and the Lie group of transformations for solving partial differential equations [4]. Logan and Perez [20] investigated the self-similar solutions for a one-dimensional, time-dependent problem in reactive shock hydrodynamics that can model various processes in detonation physics. The work is done from an application point of view on various material media [15, 16, 31]. Bluman et al. [3] presented the concept of finding systematic symmetries and conservative laws, both local and nonlocal, for partial differential equations along with various applications. The recent work includes the study of the self-similar solutions and converging shocks in a nonideal dusty gas [5].

Next, we discuss the singular surface theory approach. The first-order singular surface theory deals with the flow variables having jump discontinuities in its first-order derivatives, while the variables are continuous. Varley [33] extensively studied the nonlinear theory of wave propagation. His work unfolded the general theory of how the weak discontinuity propagates. The equations governing the mode of propagation together with the relation in strengths across the wave fronts were derived. The first-order discontinuities at the wave front were emphasized to study the formation of shock. The work also involved the application of the general theory on gravity waves and sound waves on sloping beaches. In quasi-linear hyperbolic systems with discontinuous coefficients, Jeffrey [13] extended the study on propagation of weak discontinuities. The changes in the flow variables were investigated in detail; specifically, the evolution of jump discontinuity in the flow quantities and then the deduction of transport equations. Further, the breakdown of a weak wave under the impact of strong discontinuity was analysed, along with the derivation of the condition which governed the breakdown time. Also, considering elastic materials as the mode of

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propagation and using the theory of wave propagation and discontinuities, the results on shock formation, influenced by the elastic parameters, were presented and discussed by McCarthy [21].

Jeffrey [14] discussed at length the first-order quasi-linear systems, nonlinear equations with their matrix formulations and the Cauchy problem, considering the wave propagation, which by definition is a characteristic surface. The concepts of the Riemann invariants, simple waves and evolution of shock waves were discussed thoroughly. The framework of interaction of waves and the resulting transmitted and reflected waves was presented, and was further extended by various researchers in different mediums of materials. Radha et al. [28] investigated the general theory for problems of wave interaction. Specifically, their work dealt with the jump in shock acceleration caused by the interaction between a weak wave and a shock. The following reflected and transmitted waves with their amplitudes were analysed. Taking into account the plane and radially symmetric flows, self-similar solutions were examined, and further used in the study of the interaction between a blast wave and weak discontinuity. Mentrelli et al. [23] studied the interaction between a shock and an acceleration wave with varying shock strength for a perfect gas. The impact of change in the shock strength on the magnitude of transmitted and reflected waves in addition to the jump in shock acceleration was presented. Note that the numerical results obtained were in accordance with the applied theory. Also, it was established that the jump disappears in shock acceleration in the case of a weak shock, if the incident wave and shock belong to the same family. Shah and Singh [29] analysed the collision between a blast wave and a steepened wave in dusty real reacting gases, where the equation of state for the real gas is of van der Waals type. Their work examined the effects of various parameters of the dusty real reacting gas on the steepening of waves and on reflected and transmitted waves as a result of the interaction. Also, using the Lie group theory, these authors discussed the interaction with a strong shock in the reacting polytropic gases [30]. First, the singular surface approach was used to find the equation governing the amplitude of the singular surface, and then the results of interaction were given, taking into account the reacting polytropic gases. Recently, Mittal and Jena [24] have studied the interaction between a singular surface and characteristic shock in the relaxing gas with dust particles. The influence of the dust particles present in the system is the topic of emphasis. The effects were discussed on the steepening of waves and the evolution of a characteristic shock together with the amplitudes of the waves that reflect or transmit after the interaction.

The key idea of this paper is to investigate the results of the interaction between a singular surface and a strong shock in the self-gravitating spherically symmetric interstellar gas clouds. The system of equations describing the fluid flow is discussed in detail in Section 2. Using the Lie group of transformations, the forms of the flow variables and the cooling–heating function are evaluated for an infinitely strong shock in Section 3. In Section 4 we use the singular surface theory to examine the evolution of an acceleration wave front corresponding to the particular solution obtained from similarity analysis. The interaction resulting in reflected and transmitted waves is

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studied in detail in Section 5. The impact of the cooling–heating function is inspected on the formation of shock. Lastly, in Section 6, we discuss the results on the amplitude of the reflected and transmitted waves (if any) along with the jump in the shock acceleration.

2. Problem formulation

We consider the system of partial differential equations representing the one-dimensional, inviscid, self-gravitating interstellar gas cloud with spherical symmetry. We assume the value of γ, the adiabatic index, to be $5/3$ for the interstellar gas where $\gamma = c_p/c_v$, where c_p is the specific heat of the gas at constant pressure, and c_v is the specific heat of the gas at constant volume. We consider the following system of governing equations, the details of the theories and derivations of which can be found in the literature [8–10]:

$$\begin{aligned}
&\rho_t + u\rho_r + \rho u_r + \frac{2\rho u}{r} = 0,\\
&u_t + uu_r + \frac{1}{\rho}p_r = g,\\
&p_t + up_r + \rho a^2\left(u_r + \frac{2u}{r}\right) = -\frac{2}{3}\rho L(p,\rho),\\
&g_t + ug_r + \frac{2gu}{r} = 0,
\end{aligned} \tag{2.1}$$

with equation of state $p = \rho RT$, where t is time, u is the velocity of the particle along r, ρ the density, g the acceleration due to gravity, p is the pressure, $L(p,\rho)$ is the cooling–heating function, $a = (\gamma p/\rho)^{1/2}$ is the isentropic speed of sound in gas, R the gas constant, and T the temperature. Unless otherwise stated, a variable as a subscript will indicate partial derivative with respect to that variable.

System of equations (2.1) can also be written in matrix form as

$$U_t + AU_r = f,$$

where

$$U = (\rho, u, p, g)^{tr}, \quad f = \left(\frac{-2\rho u}{r}, g, -\left(\frac{2}{3}\rho L(p,\rho) + \frac{5}{3}\frac{2pu}{r}\right), -\frac{2gu}{r}\right)^{tr},$$

$$A = \begin{pmatrix} u & \rho & 0 & 0\\ 0 & u & 1/\rho & 0\\ 0 & \rho a^2 & u & 0\\ 0 & 0 & 0 & u \end{pmatrix}, \tag{2.2}$$

in which tr denotes transposition.

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The shock front $r = R(t)$ propagates into the nonhomogeneous medium with velocity $V = dR/dt$ and the following boundary conditions:

$$\rho_0 = \rho_0(x), \quad u_0 = 0, \quad p_0 = \text{constant}, \quad g_0 = g_0(x).$$

System (2.1) in the conservation form behind and ahead of the shock is represented as

$$\begin{aligned} &G_t(r,t,U) + F_r(r,t,U) = H(r,t,U),\\ &G_{*t}(r,t,U_*) + F_{*r}(r,t,U_*) = H_*(r,t,U_*). \end{aligned} \tag{2.3}$$

Here, U and U_* characterize the solution vectors behind and ahead of the shock respectively, and

$$\begin{aligned} G(r,t,U) &= (\rho, \rho u, \tfrac{1}{2}(3p+\rho u^2), \rho g)^{tr},\\ F(r,t,U) &= (\rho u, \rho u^2 + p, \tfrac{1}{2}(5p+\rho u^2)u, \rho u g)^{tr},\\ H(r,t,U) &= \left(\frac{-2\rho u}{r}, \rho g - \left(\frac{2\rho u^2}{r}\right), \rho(ug - L) - \frac{u}{r}(5p+\rho u^2), -\frac{4\rho g u}{r}\right)^{tr}. \end{aligned} \tag{2.4}$$

Further, on using the Rankine–Hugoniot conditions $|[G]|V = |[F]|$, where V is the shock velocity and the symbol $|[N]| = N - N_0$ is the jump in the variable N across the wave front entailing N_0 and N as the value just ahead and just behind the shock [15], we obtain

$$\begin{aligned} \rho(R(t),t) &= \frac{4\rho_0^2(R(t))V^2}{\rho_0(R(t))V^2 + 5p_0}, \quad u(R(t),t) = \frac{3\rho_0(R(t))V^2 - 5p_0}{4\rho_0(R(t))V},\\ p(R(t),t) &= \frac{3\rho_0(R(t))V^2 - p_0}{4}, \quad g(R(t),t) = g_0(R(t)). \end{aligned} \tag{2.5}$$

3. Similarity analysis by invariance group

We assume the existence of self-similar solutions of differential equations (2.1) and hence use the following one-parameter Lie group of transformations:

$$\begin{aligned} x_j^* &= x_j + \epsilon\zeta_x^j(x_1,x_2,u_1,u_2,u_3,u_4), \quad j = 1,2,\\ u_i^* &= u_i + \epsilon\zeta_u^i(x_1,x_2,u_1,u_2,u_3,u_4), \quad i = 1,2,3,4, \end{aligned} \tag{3.1}$$

where $x_1 = t$, $x_2 = r$, $u_1 = \rho$, $u_2 = u$, $u_3 = p$, $u_4 = g$, ϵ is the parameter of the Lie group of transformations and ζ_x^j and ζ_u^i are the infinitesimals of the Lie group of transformations. Here, we use $p_j^i = \partial u_i/\partial x_j$, and rewrite the system of equations (2.1) in the form

$$F_k(x_j, u_i, p_j^i) = 0, \quad k = 1,2,3,4,$$

which are constantly, conformally invariant [11] under the infinitesimal group (3.1), if there exist constants $\alpha_{rs}(r,s = 1,2,3,4)$ such that for all smooth surfaces, $u_i = u_i(x_j)$,

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we have

$$SF_k = \alpha_{kr} F_r, \quad k = 1, 2, 3, 4,$$

where S denotes the Lie derivative in the direction of the extended field vector and is given as

$$S = \zeta_x^j \frac{\partial}{\partial x_j} + \zeta_u^i \frac{\partial}{\partial u_i} + \zeta_{p_j}^i \frac{\partial}{\partial p_j^i},$$

with

$$\zeta_{p_j}^i = \frac{\partial \zeta_u^i}{\partial x_j} + \frac{\partial \zeta_u^i}{\partial u_k} p_j^k - \frac{\partial \zeta_x^l}{\partial x_j} p_l^i - \frac{\partial \zeta_x^l}{\partial u_n} p_l^i p_j^n$$

being the infinitesimals of derivative of the transformation. We obtain the following forms of infinitesimals ζ_x^j and ζ_u^i from (2.1) by using the above procedure:

$$\begin{aligned}
&\zeta_x^1 = ax_1 + b, && \zeta_x^2 = (\alpha_{22} - \alpha_{11} + a)x_2,\\
&\zeta_u^1 = (\alpha_{11} + a)u_1, && \zeta_u^2 = (\alpha_{22} - \alpha_{11})u_2,\\
&\zeta_u^3 = (2\alpha_{22} - \alpha_{11} + a)u_3, && \zeta_u^4 = (\alpha_{22} - \alpha_{11} - a)u_4,\\
&L = u_1^{(2(\alpha_{22}-\alpha_{11})-a)/(\alpha_{11}+a)} L^*(\phi), && \phi = u_3 u_1^{-(2\alpha_{22}-\alpha_{11}+a)/(\alpha_{11}+a)},
\end{aligned} \tag{3.2}$$

where a, b, α_{11}, α_{22} are arbitrary constants, and L^* is an arbitrary function of its arguments. We consider the following cases based on the presence of arbitrary constants.

3.1. Case I When $a \neq 0$ and $\alpha_{22} - \alpha_{11} + a \neq 0$, the translational invariance of x_1 given by $\tilde{x_1} = x_1 + b/a$ on suppressing the tilde, yields from the first equation in (3.2) that $\zeta_x^1 = ax_1$, while the other equations in (3.2) remain unchanged. Using the invariant surface conditions, $u_i(x^*, t^*) = u_i^*(x, t)$, we have

$$\frac{dx_1}{\zeta_x^1} = \frac{dx_2}{\zeta_x^2} = \frac{du_1}{\zeta_u^1} = \frac{du_2}{\zeta_u^2} = \frac{du_3}{\zeta_u^3} = \frac{du_4}{\zeta_u^4}. \tag{3.3}$$

The parameters are nondimensionalized using

$$\begin{aligned}
&\tilde{r} = \frac{r}{r_c}, \quad \tilde{t} = \frac{t}{t_c}, \quad \tilde{u} = \frac{ut_c}{r_c}, \quad \tilde{\rho} = \frac{\rho}{\rho_c},\\
&\tilde{p} = \frac{pt_c^2}{\rho_c r_c^2}, \quad \tilde{g} = \frac{gt_c^2}{r_c}, \quad \tilde{a} = \frac{at_c}{r_c}, \quad \tilde{L}(\tilde{p}, \tilde{\rho}) = \frac{t_c^3}{r_c^2} L(p, \rho),
\end{aligned}$$

where r_c, t_c and ρ_c are reference values of r, t and ρ, respectively.

Further, using equations (3.2) along with the nondimensional form of the variables in equation (3.3), the following forms of flow variables ρ, u, p, g and the

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cooling–heating function L are obtained:

$$\rho = t^{\mu}S(\xi), \quad u = t^{(\delta-1)}U(\xi), \quad p = t^{(2(\delta-1)+\mu)}P(\xi),$$
$$g = t^{(\delta-2)}G(\xi), \quad L(p,\rho) = \rho^{(2\delta-3)/\mu}L^{*}(\phi), \tag{3.4}$$

where $\xi = rt^{-\delta}$, is the dimensionless similarity variable

$$\mu = (\alpha_{11}+a)/a, \quad \delta = (\alpha_{22}-\alpha_{11}+a)/a, \quad \phi = PS^{-(2(\delta-1)+\mu)/\mu},$$

and S, U, P, G and L^{*} are arbitrary functions of their arguments.

Since the shock must be a similarity curve, it may be normalized to be at $\xi = 1$ which then yields the shock path $R(t)$, the shock velocity $V(t)$ and the flow variables in the form

$$R(t) = t^{\delta}, \quad V(t) = \delta t^{(\delta-1)}, \qquad \rho|_{\xi=1} = t^{\mu}S(1), \quad u|_{\xi=1} = t^{(\delta-1)}U(1),$$
$$p|_{\xi=1} = t^{(2(\delta-1)+\mu)}P(1), \qquad eg|_{\xi=1} = t^{(\delta-2)}G(1). \tag{3.5}$$

From equation (3.5) and (2.5) we obtain

$$S(1) = \frac{4\rho_0^2(R(t))V^2}{\rho_0(R(t))V^2+5p_0}t^{-\mu}, \qquad U(1) = \frac{3\rho_0(R(t))V^2-5p_0}{4\rho_0(R(t))V^2}t^{(1-\delta)},$$
$$P(1) = \frac{3\rho_0(R(t))V^2-p_0}{4\rho_0(R(t))V^2}t^{-(2(\delta-1)+\mu)}, \qquad G(1) = g_0(R(t))t^{-(\delta-2)}. \tag{3.6}$$

In the case of a shock of arbitrary strength, the above relations from equation (3.6) imply that ρ_0V^2 must be a constant and further provide that ρ_0 and g_0 must satisfy the power law in r such that $\rho_0 = r^{\theta}$ and $g_0 = r^{\kappa}$. Also, the jump conditions (3.6) signify that $\kappa = (\delta-2)/\delta$ along with the condition that for a shock of arbitrary strength $\delta\theta + 2(\delta-1) = 0$ and for a strong shock $\delta\theta = \mu$. Hence, from equation (3.4) we get

$$\rho = \rho_0(R(t))S^{*}(\xi), \quad u = VU^{*}(\xi),$$
$$p = \rho_0(R(t))V^2P^{*}(\xi), \quad g = g_0(R(t))G^{*}(\xi), \tag{3.7}$$

where $S^{*}(\xi) = S(\xi)$, $U^{*}(\xi) = U(\xi)/\delta$, $P^{*}(\xi) = P(\xi)/\delta^2$ and $G^{*}(\xi) = G(\xi)$.

Using equations (3.4), (3.7) and (2.1), a system of ordinary differential equations in S^{*}, U^{*}, P^{*} and G^{*} is obtained, which on suppressing the asterisk becomes

$$(U-\xi)S' + \theta S + SU' + \frac{2US}{\xi} = 0,$$
$$(U-\xi)U' + \frac{\delta-1}{\delta}U + \frac{P'}{S} = \frac{G(\xi)}{\delta^2},$$
$$(U-\xi)P' + \left(\frac{2(\delta-1)}{\delta} + \theta + \frac{5}{3}\left(U' + \frac{2U}{\xi}\right)\right)P = -\frac{2}{3\delta}(S(\xi))^{(2\delta-3+\mu)/\mu}L^{*}(\phi),$$
$$(U-\xi)G' + \frac{(\delta-2)}{\delta}G = -\frac{2UG}{\xi}. \tag{3.8}$$

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Here, a primes denote differentiation with respect to the independent variable ξ and $\phi = \delta^2 PS^{-(2(\delta-1)+\mu)/\mu}$. The above system of equations (3.8) satisfy the following jump conditions for a shock of arbitrary strength:

$$S(1) = \frac{4\rho_0^2(R(t))V^2}{\rho_0(R(t))V^2 + 5p_0}, \quad U(1) = \frac{3\rho_0(R(t))V^2 - 5p_0}{4\rho_0(R(t))V^2},$$
$$P(1) = \frac{3\rho_0(R(t))V^2 - p_0}{4\rho_0(R(t))V^2}, \quad G(1) = 1, \tag{3.9}$$

and for an infinitely strong shock

$$S(1) = 4, \quad U(1) = \tfrac{3}{4}, \quad P(1) = \tfrac{3}{4}, \quad G(1) = 1. \tag{3.10}$$

3.2. Particular solution Self-similar solutions are of two types. In the solutions of the first type, the similarity exponent δ is determined either by dimensional considerations or from conservation laws. In the second type of solutions, the determination of the similarity exponent requires that the ordinary differential equations for the reduced functions be integrated, since δ cannot be found from dimensional considerations or from the conservation laws without solving the equations as in first type. In this paper we have considered the case of self-similarity of the second type. Since the system of equations (3.8) cannot be solved analytically to obtain an exact solution, any particular solution of the system is of practical importance to get an idea of the nature of the solutions. Equations (3.8) with shock conditions (3.10) admit particular solutions for U, S, P and G as

$$U = \tfrac{3}{4}\xi, \quad S = 4\xi^{4\theta+9}, \quad P = \tfrac{3}{4}\xi^{4\theta+11}, \quad G = \xi, \tag{3.11}$$

where the similarity exponent $\delta = 8/9$ and $\theta = -11/16$. The form of the cooling–heating function L is

$$L(p,\rho) = \rho^2 L^*(\phi), \quad L^*(\phi) = \phi^m,$$

with

$$\phi = \frac{4^{7/11}\xi^{-3/11}}{27}, \quad m = \frac{77}{2}. \tag{3.12}$$

Using relations (3.11)–(3.12), the flow variables are given by

$$\rho = \rho_0(R(t))S(\xi), \qquad u = VU(\xi),$$
$$p = \rho_0 V^2 P(\xi), \qquad g = g_0(R(t))G(\xi), \tag{3.13}$$

where

$$R(t) = t^{8/9}, \qquad V = \tfrac{8}{9}t^{-1/9}, \qquad \rho_0(R(t)) = t^{(8/9)\theta},$$
$$p_0(R(t)) = \text{constant}, \qquad g_0(R(t)) = t^{-80/81}, \qquad u_0(R(t)) = 0.$$

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3.3. Case II When $(\alpha_{22} - \alpha_{11}) \neq 0$ and $a = 0$, from equations (3.6) and (3.8) we get the following form of flow variables and cooling–heating function:

$$\begin{aligned} &\rho = \rho_0(R(t))S^*(\xi), \qquad u = VU^*(\xi), \qquad p = \rho_0(R(t))V^2P^*(\xi), \\ &g = g_0(R(t))G^*(\xi), \qquad L(p,\rho) = \rho^{2\delta/\mu}L^*(\phi), \qquad \phi = P^*S^{*-(2\delta+\mu)/\mu}, \end{aligned} \tag{3.14}$$

where the dimensionless similarity variable ξ, the shock location R at $\xi = 1$, the shock velocity V, $\rho_0(x)$ and $g_0(x)$ are given by

$$\xi = re^{-\delta t}, \quad R = e^{\delta t}, \quad V = \delta e^{\delta t}, \quad \rho_0(x) = x^{\theta}, \quad g_0 = r, \tag{3.15}$$

with $\delta = (\alpha_{22} - \alpha_{11})/b, \mu = \alpha_{11}/b = \theta\delta$. Note that this case leads to a class of similarity solutions with an exponential shock path given by (3.14).

Now using equations (3.14) and (3.15) in (2.1) and suppressing the asterisk, we obtain the following set of ordinary differential equations:

$$\begin{aligned} &(U-\xi)S' + \theta S + SU' + \frac{2US}{\xi} = 0, \\ &(U-\xi)U' + U + \frac{P'}{S} = \frac{G(\xi)}{\delta^2}, \\ &(U-\xi)P' + \left\{2 + \theta + \frac{5}{3}\left(U' + \frac{2U}{\xi}\right)\right\}P = -\frac{2}{3\delta}(S(\xi))^{(2\delta+\mu)/\mu}L^*(\phi), \\ &(U-\xi)G' + G = -\frac{2UG}{\xi}. \end{aligned} \tag{3.16}$$

In this case, $\phi = \delta^2 PS^{-(2\delta+\mu)/\mu}$. The system of equations (3.16) can be solved using the jump conditions (3.9) or (3.10), depending on a shock of arbitrary strength or an infinitely strong shock, respectively.

4. Acceleration wave front

We take the flow variables ρ, u, p, g to be continuous, but we may encounter discontinuities in their first-order derivatives across the associated propagating wave surface. The equation of the wave front Φ is expressed as $r = \chi(t)$, providing the flow parameters as functions of t only. The kinematic and geometric compatibility conditions of first and second order for a singular surface [32] are given by

$$[[J_x]] = B, \quad [[J_t]] = -ZB, \quad [[J_{xx}]] = \overline{B}, \quad [[J_{xt}]] = Z(B_x - \overline{B}),$$

where J represents any of the flow variables, and $Z = d\chi/dt$ denotes the speed at which Φ propagates.

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The matrix A in equation (2.2) has eigenvalues $\lambda^{(1)} = u + a$, $\lambda^{(2)} = u$ (a double root) and $\lambda^{(3)} = u - a$. The corresponding eigenvectors are:

$$\begin{aligned} L^{(1)} &= (0, \rho a, 1, 0), & R^{(1)} &= (1/(2a^2), 1/(2\rho a), 1/2, 0)^{tr}, \\ L^{(2,1)} &= (-a^2, 0, 1, 0), & R^{(2,1)} &= (-a^{-2}, 0, 0, -1/(ga^2))^{tr}, \\ L^{(2,2)} &= (0, 0, 0, 1), & R^{(2,2)} &= (u^{-2}, 0, 0, a^{-2})^{tr}, \\ L^{(3)} &= (0, -\rho a, 1, 0), & R^{(3)} &= (1/(2a^2), -1/(2\rho a), 1/2, 0)^{tr}. \end{aligned}$$

As per the assumption of spherical symmetry of cloud model, the wave travels towards the centre for which there arise two cases due to the presence of the unperturbed state, that is, on the left-hand side or the right-hand side of the moving wave front. If the unperturbed state is considered to be on the right-hand side or on the left-hand side, then $Z = u + a$ or $Z = u - a$, respectively. Using the following relations in the system of equations (2.1) on the inner boundary of Φ,

$$\begin{aligned} &|[\rho_r]| = \varrho, && |[u_r]| = \alpha^{(1)}, && |[p_r]| = \xi, && |[g_r]| = \eta, \\ &|[\rho_t]| = -Z\varrho, && |[u_t]| = -Z\alpha^{(1)}, && |[p_t]| = -Z\xi, && |[g_t]| = -Z\eta, \end{aligned} \tag{4.1}$$

and simplifying, we obtain

$$Z = u_0 \pm a_0, \quad \alpha^{(1)} = \pm\frac{1}{\rho_0 a_0}\xi = \pm\frac{a_0}{\rho_0}\varrho \quad \text{and} \quad \eta = 0.$$

Now we have the following set of compatibility conditions:

$$\begin{aligned} &|[\rho_{rr}]| = \overline{\varrho}, && |[u_{rr}]| = \overline{\alpha^{(1)}}, && |[p_{rr}]| = \overline{\xi}, && |[g_{rr}]| = \overline{\eta}, \\ &|[\rho_{rt}]| = Z(\varrho_r - \overline{\varrho}), && |[u_{rt}]| = Z(\alpha_r^{(1)} - \overline{\alpha^{(1)}}), && |[p_{rt}]| = Z(\xi_r - \overline{\xi}), && |[g_{rt}]| = Z(\eta_r - \overline{\eta}). \end{aligned} \tag{4.2}$$

Differentiating system (2.1) with respect to r and evaluating behind the wave front Φ, we get

$$\begin{aligned} &|[\rho_{rt}]| + 2|[u_r\rho_r]| + u_0|[\rho_{rr}]| + \rho_0|[u_{rr}]| + \frac{2u_0}{r}|[\rho_r]| + \frac{2\rho_0}{r}|[u_r]| = 0, \\ &|[u_{rt}]| + u_0|[u_{rr}]| + |[u_r u_r]| + \frac{1}{\rho_0}|[p_{rr}]| - \frac{1}{\rho_0^2}|[p_r\rho_r]| - |[g_r]| = 0, \\ &|[p_{rt}]| + \frac{8}{3}|[u_r p_r]| + u_0|[p_{rr}]| + \frac{5}{3}p_0\left(|[u_{rr}]| + \frac{2}{r}|[u_r]|\right) + \frac{10}{3}\frac{u_0|[p_r]|}{r} \\ &\qquad = -\frac{2}{3}\{L_0|[\rho_r]| + \rho_0((L_\rho)_0|[\rho_r]| + (L_p)_0|[p_r]|)\}, \\ &|[g_{rt}]| + |[u_r g r]| + u_0|[g_{rr}]| + \frac{2}{r}u_0|[g_r]| + \frac{2}{r}g_0|[u_r]| = 0. \end{aligned} \tag{4.3}$$

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Using equations (4.1) and (4.2) in (4.3), we obtain

$$
\begin{aligned}
&Z(\varrho_r - \overline{\varrho}) + 2((\varrho + \rho_{0r})\alpha^{(1)} + \varrho u_{0r}) + u_0\overline{\varrho} + \rho_0\overline{\alpha^{(1)}} + \frac{2}{r}(u_0\varrho + \rho_0\alpha^{(1)}) = 0,\\
&Z(\alpha_r^{(1)} - \overline{\alpha^{(1)}}) + u_0\overline{\alpha^{(1)}} + (\alpha^{(1)})^2 + \frac{1}{\rho_0}\overline{\xi} - \frac{1}{\rho_0^2}(\varrho(\xi + p_{0x}) + \rho_{0r}\xi) + 2\alpha^{(1)}u_{0r} = 0,\\
&Z(\xi_r - \overline{\xi}) + \frac{8}{3}(\alpha^{(1)}(\xi + p_{0r}) + u_{0r}\xi) + u_0\overline{\xi} + \frac{5}{3}p_0\left(\overline{\alpha^{(1)}} + \frac{2\alpha^{(1)}}{r}\right) + \frac{10}{3}\frac{u_0\xi}{r}\\
&\quad = -\frac{2}{3}(\varrho L_0 + \rho_0((L_\rho)_0\varrho + (L_p)_0\xi)),\\
&\alpha^{(1)}g_{0r} - a_0\overline{\eta} + \frac{2}{r}g_0\alpha^{(1)} = 0.
\end{aligned}
\tag{4.4}
$$

Considering the unperturbed state to be on the right-hand side and eliminating $\overline{\varrho}$, $\overline{\alpha^{(1)}}$ and $\overline{\xi}$ from equation (4.4) and further using equation (3.13), we obtain the following Bernoulli type of transport equation [13] for $\alpha^{(1)}$:

$$
2\frac{d\alpha^{(1)}}{dt} + \frac{8}{3}(\alpha^{(1)})^2 \pm \Theta\alpha^{(1)} = 0, \tag{4.5}
$$

where

$$
\Theta = \frac{14u_{0r}}{3} - \frac{\rho_{0r}a_0}{\rho_0} + \frac{5p_{0r}}{3\rho_0 a_0} + \frac{10u_0}{3r} + \frac{2a_0}{r} + \frac{2}{3}\left(\frac{L_0}{a_0^2} + \frac{(L_\rho)_0\rho_0}{a_0^2} + (L_p)_0\rho_0\right).
$$

Integration of equation (4.5) yields the solution

$$
\alpha^{(1)}(t) = \alpha_0^{(1)}\eta(t)\left\{1 + \frac{4\alpha_0^{(1)}}{3}\int_{t_0}^{t}\eta(w)\,dw\right\}^{-1}, \tag{4.6}
$$

with $\eta(t) = \exp(-\frac{1}{2}\int_{t_0}^{t}\Theta(w)\,dw)$.

The results are depicted in Figure 1 for rarefaction waves ($\alpha_0^{(1)} > 0$) and in Figure 2 for compression waves ($\alpha_0^{(1)} < 0$). From Figure 1, it is observed that the amplitude of the jump in velocity gradient $\alpha^{(1)}$ decreases as time increases and dies out eventually. In contrast, from Figure 2 it is observed that the solution breaks down when the denominator in equation (4.6) (that is, $1 + (4\alpha_0^{(1)}/3)\int_{t_0}^{t}\eta(w)\,dw$) equals zero at some time $t = t_c$, implying the existence of a steepened wave in a finite time only. Further, the following cases arise from equation (4.6).

(1) The conditions $|\alpha_0^{(1)}|/\alpha_c^{(1)} < 1$ and $|\alpha_0^{(1)}|/\alpha_c^{(1)} = 1$, where $\alpha_c^{(1)} = 1/I\{\infty\}$ with $I(t) = (4/3)\int_{t_0}^{t}\eta(w)\,dw$ lead to $\alpha^{(1)} \to 0$, showing the existence of a flattened wave as time advances.

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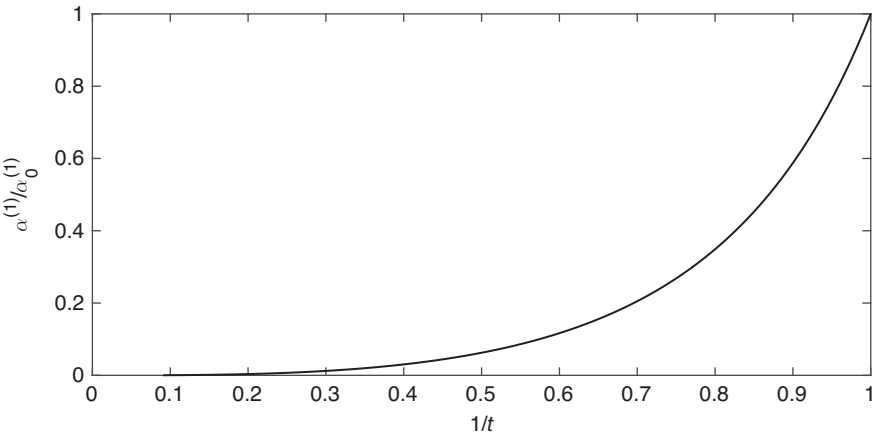


FIGURE 1. Jump in velocity gradient $\alpha^{(1)}$ varying with $1/t$ for rarefaction waves ($\alpha_0^{(1)} > 0$).

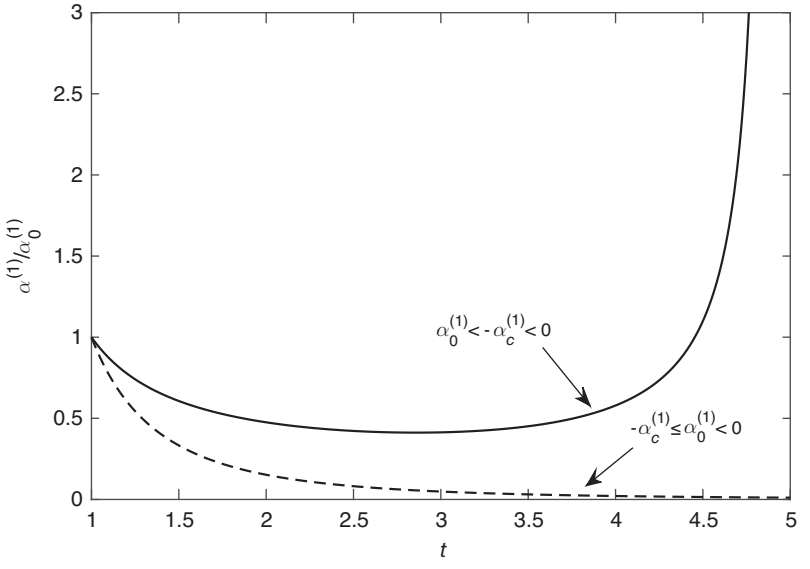


FIGURE 2. Jump in velocity gradient $\alpha^{(1)}$ varying with $1/t$ for compression waves ($\alpha_0^{(1)} < 0$).

(2) For $|\alpha_0^{(1)}|/\alpha_c^{(1)} > 1$, there exists a finite time t_c such that $I(t_c) = 1/|\alpha_0^{(1)}|$ and $|\alpha^{(1)}| \to \infty$ as $t \to t_c$. This yields the existence of a shock wave in a finite time only at an instant t_c.

The results depicted are for the case where the unperturbed state is on the right-hand side. Similar results have been obtained for the case where the unperturbed state is on the left-hand side, and hence omitted.

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5. Amplitudes of reflected and transmitted waves

The concepts of reflected and transmitted waves come into existence by the phenomena of interaction between nonlinear waves. The amplitudes of these reflected and transmitted waves are imperative to study and establish the results of collision. In this paper, the conservation form of the system in equation (2.3) behind and ahead of the shock front makes it viable to study the amplitudes. We consider the interaction of the strong shock and the fastest moving discontinuity which originates from the point (r_0, t_0), and propagates along the characteristic $dx/dt = \lambda^{(1)}$, represented as $\phi(x, t) = 0$. Let $M(x_m, t_m)$ be the point of intersection of the moving wave with the shock path.

For a strong shock, $p_0 \ll V$, and hence the Rankine–Hugoniot jump conditions in equation (2.5) yield

$$\rho(R(t), t) = 4\rho_0, \quad u(R(t), t) = \frac{3V}{4}, \quad p(R(t), t) = \frac{3\rho_0 V^2}{4},$$
$$g(R(t), t) = g_0(R(t)). \tag{5.1}$$

Further, in the case of strong shock, we obtain the speed of sound as $a = \sqrt{5}V/4$ and the eigenvalues as

$$\begin{aligned} \lambda^{(1)} &= \frac{3V}{4} + \frac{\sqrt{5}V}{4}, & \lambda_*^{(1)} &= \left(\frac{5p_0}{3\rho_0}\right)^{1/2}, \\ \lambda^{(2)} &= \frac{3V}{4}, & \lambda_*^{(2)} &= 0, \\ \lambda^{(3)} &= \frac{3V}{4} - \frac{\sqrt{5}V}{4}, & \lambda_*^{(3)} &= -\left(\frac{5p_0}{3\rho_0}\right)^{1/2}. \end{aligned} \tag{5.2}$$

The evolutionary conditions of the reflected and transmitted waves given by Lax [19] are formulated as

$$\lambda^{(p)} < \lambda^{(p-1)} < \cdots < \lambda^{(l+1)} < V < \lambda^{(l)} < \cdots < \lambda^{(1)},$$
$$\lambda_*^{(p)} < \lambda_*^{(p-1)} < \cdots < \lambda_*^{(l)} < V < \lambda_*^{(l-1)} < \cdots < \lambda_*^{(1)}.$$

In the present problem, we have the inequalities

$$\lambda^{(3)} < \lambda^{(2)} < V < \lambda^{(1)} \quad \text{and} \quad \lambda_*^{(3)} < \lambda_*^{(2)} < \lambda_*^{(1)} < V,$$

which imply that in this case of collision between a weak discontinuity travelling with velocity $\lambda^{(1)}$ at time $t = t_m$ and strong shock, there are two reflected waves moving with velocity $\lambda^{(2)}$ and $\lambda^{(3)}$ and no transmitted waves. The new wavefront from point M after interaction propagates through the characteristic $dr/dt = \lambda_*^{(1)}$, and is represented as $\phi_*(r, t) = 0$. Then, following the above discussion, the jumps in U_r across the incident and reflected waves at M are given by

$$\wedge_1(M) = \sum_{k=1}^{n_1} \alpha_k^{(1)}(t_m) R_s^{(1,k)}, \qquad \wedge_i^{(R)}(M) = \sum_{k=1}^{n_i} \alpha_k^{(i)}(t_m) R_s^{(i,k)},$$

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with subscript s specifying the values calculated at the point M. The evolutionary equations deducing the amplitudes $\alpha_k^{(i)}$ of reflected waves along with the jump in the shock acceleration $|[\dot{V}]|$ at time $t = t_m$ in the absence of transmitted waves are obtained by the system of algebraic equations given as

$$|[\dot{V}]|(G - G_*)_s + (\nabla G)_s \sum_{i=p-q+1}^{p} \left\{ \sum_{k=1}^{m_i} \alpha_k^{(i)} (V - \lambda^{(i)})^2 R_s^{(i,k)} \right\} = -(\nabla G)_s \sum_{k=1}^{m_1} \alpha_k^{(1)} (V - \lambda^{(1)})^2 R_s^{(1,k)},$$

which simplifies to the following form in our problem:

$$(G - G_*)_s |[\dot{V}]| + (\nabla G)_s R_s^{(3)} (V - \lambda_s^{(3)})^2 \alpha^{(3)} + (\nabla G)_s R_s^{(2,1)} (V - \lambda_s^{(2)})^2 \alpha_2^{(1)} + (\nabla G)_s R_s^{(2,2)} (V - \lambda_s^{(2)})^2 \alpha_2^{(2)} = -(\nabla G)_s R_s^{(1)} (V - \lambda_s^{(1)})^2 \alpha^{(1)}.$$

Using (2.3), (2.4), (5.1) and (5.2), we get the system of algebraic equations

$$\begin{aligned}
&3\rho_0 |[\dot{V}]| + \tfrac{1}{10}(1 + \sqrt{5})^2 \alpha^{(3)} - \tfrac{1}{5}\alpha_2^{(1)} + \tfrac{1}{9}\alpha_2^{(2)} = -\tfrac{1}{10}(1 - \sqrt{5})^2 \alpha^{(1)}, \\
&3\rho_0 |[\dot{V}]| + \frac{1}{16}\left(\frac{6}{5} - \frac{2}{\sqrt{5}}\right)(1 + \sqrt{5})^2 \alpha^{(3)} - \frac{3}{20}\alpha_2^{(1)} + \frac{1}{12}\alpha_2^{(2)} \\
&\qquad = -\frac{1}{16}\left(\frac{6}{5} + \frac{2}{\sqrt{5}}\right)(1 - \sqrt{5})^2 \alpha^{(1)}, \\
&\frac{9\rho_0}{4} |[\dot{V}]| + \frac{1}{16}\left(\frac{6}{5} - \frac{3}{2\sqrt{5}}\right)(1 + \sqrt{5})^2 \alpha^{(3)} - \frac{9}{80}\alpha_2^{(1)} + \frac{1}{32}\alpha_2^{(2)} \\
&\qquad = -\frac{1}{16}\left(\frac{6}{5} + \frac{3}{2\sqrt{5}}\right)(1 - \sqrt{5})^2 \alpha^{(1)}, \\
&3\rho_0 g_0 |[\dot{V}]| + \frac{g_0}{10}(1 + \sqrt{5})^2 \alpha^{(3)} - \left(\frac{g_0}{5} + \frac{4\rho_0}{5g_0}\right)\alpha_2^{(1)} + \left(\frac{g_0}{9} + \frac{4\rho_0}{5}\right)\alpha_2^{(2)} \\
&\qquad = -\frac{g_0}{10}(1 - \sqrt{5})^2 \alpha^{(1)}.
\end{aligned} \tag{5.3}$$

Solving this system of equations yields the amplitudes of the reflected waves and the jump in the shock acceleration in the following form:

$$\begin{aligned}
\alpha_2^{(1)} &= \frac{-3g_0(1 - \sqrt{5})^2}{20\sqrt{5}A} \alpha^{(1)}, \\
\alpha_2^{(2)} &= \frac{-3(1 - \sqrt{5})^2}{20\sqrt{5}A} \alpha^{(1)}, \\
\alpha^{(3)} &= \frac{(1 - \sqrt{5})^2 \left(\dfrac{-3g_0}{100} - \dfrac{1}{8\sqrt{5}} + \dfrac{1}{24}\right)}{(1 + \sqrt{5})^2 A} \alpha^{(1)}, \\
|[\dot{V}]| &= \frac{(1 - \sqrt{5})^2}{30\sqrt{5}\rho_0 A}\left(\frac{-3g_0}{10} + \frac{5}{12}\right)\alpha^{(1)},
\end{aligned}$$

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where

$$A = \left(\frac{3g_0}{100} - \frac{1}{8\sqrt{5}} - \frac{1}{24}\right).$$

6. Results and conclusion

This paper considers the system of partial differential equations modelling the self-gravitating interstellar gas clouds in spherical symmetry. Using the Lie group of transformations, the governing system of partial differential equations reduces to a system of ordinary differential equations for obtaining self-similar solutions. Particular solutions are obtained for the flow variables and the cooling–heating function in the case of an infinitely strong shock. Further, singular surface theory is used to study the acceleration wave front, taking into account the particular solution so obtained. The Bernoulli type equation [15] for velocity gradient is obtained, which is solved numerically to analyse the variation of the velocity gradient with time. The numerical results are depicted, and we observe that the jump in velocity gradient decreases with increase in time and vanishes in the long run in the case of rarefaction waves. However, for compression waves, there exists a critical time at which the jump in velocity gradient becomes unbounded. The interaction between the strong shock and the singular surface is studied. We observe that there are no transmitted waves, and hence the amplitudes of reflected waves are obtained, which are directly proportional to the amplitude of the jump in velocity gradient.

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