Advance Access publication November 12, 2009

Political Analysis (2010) 18:227–244
doi:10.1093/pan/mpp027

Strategic Voting in Plurality Elections

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This paper extends the *Calculus of Voting* of McKelvey and Ordeshook, providing the first direct derivation of the conditions under which voters will vote *strategically*: choose their second-most preferred candidate in order to prevent their least-preferred candidate from winning. Addressing this theoretical problem is important, as nearly all empirical research on strategic voting either implicitly or explicitly tests hypotheses which originate from this seminal model. The formal result allows us to *isolate the subset of voters to which strategic voting hypotheses properly apply* and in turn motivates a critical reevaluation of past empirical work. In making this argument, we develop a unified and parsimonious framework for understanding competing models of tactical voter choice. The typology helps to elucidate the methodological difficulties in studying tactical behavior when faced with heterogeneous explanatory models and suggests the need for both theoretical caution and more precise data instruments in future empirical work.

1 Introduction

Democratic institutions frequently provide electors the incentive to engage in *strategic voting*: choosing a candidate or policy alternative ranked lower than first in one's preference ordering due to the fact that this most preferred option has little to no chance of winning. Building on Duverger's famous ''wasted-vote'' hypothesis (Duverger 1954), a stream of research tackles the issue of strategic voting in single-member district plurality elections. Such work generally comes in one of two guises. The first uses *game theory* to demonstrate that, under most circumstances, strategic voting will restrict the effective number of candidates receiving positive vote shares in an electoral district (Palfrey 1989; Cox 1994).[1] Consider, for example, a single-member district election with three candidates. These authors demonstrate that, as long as voters have some shared information as to the identity of the two leading candidates, in equilibrium only these two ''viable''

Authors' note: We would like to thank John Aldrich, Gary Cox, Dean Lacy, Brendan Nyhan, Camber Warren, and anonymous reviewers for valuable feedback on past versions of the paper.
[1]For a related but distinct set of arguments see Myerson and Weber (1993).

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https://doi.org/10.1093/pan/mpp027 Published online by Cambridge University Press

candidates will receive positive vote shares (Duvergerian Equilibrium). On the other hand, if expectations are such that it is impossible for voters to identify exactly the first- and second-place candidates, in equilibrium all three candidates will receive votes (Non-Duvergerian Equilibrium). Such models thus generate aggregate level empirical predictions as to the situations under which Duvergerian and non-Duvergerian outcomes should obtain and as to the relationship between an electoral district's magnitude (i.e., the number of seats which it contains) and its effective number of vote-receiving parties.[2]

A second domain of research investigates strategic voting in mass elections from a *decision-theoretic* perspective. Since the 1970s, this largely empirical literature has become one of the most prominent in behavioral political science. Although its guiding intuition is similar to that in the above-reviewed game theoretic work, its predictions reside at the individual level. In particular, all such research either explicitly or implicitly tests hypotheses that originate in McKelvey and Ordeshook's seminal Calculus of Voting (COV) model, a decision-theoretic analysis of voter behavior in plurality elections (McKelvey and Ordeshook 1972). Interestingly, McKelvey and Ordeshook themselves only hinted that their model could be extended to study strategic voting, without ever presenting a full theoretical exposition. Thus, unlike the game theoretic research reviewed above, empirical studies in the decision-theoretic literature still suffer from the lack of a rigorous theoretical statement of their most basic hypotheses.

In part as a result of this theoretical lacuna, empirical studies have at times employed mutually exclusive methodologies, such that the correctness of one approach entails the incorrectness of others. One prominent such example involves identifying the proper initial "pool" of voters upon which to investigate strategic voting hypotheses. In this paper, we introduce a typology of voter choice in three-candidate elections (Section 2) that helps to elucidate different methods various authors have proposed for identifying this initial pool. We then present the first direct theoretical derivation of the conditions for strategic voting in the COV (Section 3). The result allows us to isolate the *subset of voters to which strategic voting hypotheses properly apply* and in turn identify which among the many proposed methodological options most accurately captures the model's comparative static implications. Section 4 then presents empirical evidence in favor of our methodological recommendations. Finally, the concluding discussion demonstrates our typology's strength as a general framework for understanding competing models of tactical voter behavior and offers a number of cautionary notes regarding future empirical studies of strategic voting. Taken together, the paper's theoretical and methodological contributions represent an important step in identifying the explanatory power of instrumental rationality as a paradigm for the study of voter behavior and more generally in our quest for rigorous empirical tests of formal theoretic models.

2 A Typology of Voter Choice

Downs (1957) presented the first informal specification of voting in *expected utility terms.* The expected utility model of voting in two-candidate elections, dubbed the COV, was formally specified and developed by Riker and Ordeshook (1968). This model yields unequivocal predictions as to voter choice in two-candidate plurality elections: a rational voter will always choose her most preferred candidate, denoted henceforth as "1" (a voter's second-most preferred candidate will be denoted "2," and so on). The following is the familiar condition for voter turnout in two-candidate contests:

[2]These predictions have been tested with encouraging success in a variety of empirical settings (Ordeshook and Shvetsova 1994, Cox 1997).

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$$E_i^1 - E_i^0 = \frac{1}{2}(q_{12}^1 + q_{12}^0)(U_1 - U_2) - C > 0, \quad (1)$$

where E_i^1 is voter i's expected utility given she votes for 1, and E_i^0 is the utility associated with abstaining. Further, q_{12}^1 is the probability that 1 and the second-most preferred candidate 2 tie given that i votes for 1; q_{12}^0 denotes the same probability if i abstains; U_j is voter i's utility payoff if j wins the election; and C represents the exogenous cost of voting.

Building on this framework, McKelvey and Ordeshook (1972) generalized the COV to elections with more than two candidates. Although, like original model of Riker and Ordeshook (1968), this generalization was undertaken first to study the question of voter turnout, the authors also illustrate the possibility of extending the framework to strategic voter choice, a task later taken up explicitly in papers by Black (1978), Cain (1978), and Gutkowski and Georges (1993). The gist of their theoretical results is intuitive: when a voter's most preferred candidate 1 has little chance of winning, and the same voter perceives little utility difference between 1 and 2 but a large difference between 2 and her least-preferred candidate 3, this voter may under some circumstances maximize expected utility by *choosing 2 over 1 to prevent 3 from assuming office*.

This initial theoretical work stimulated a stream of empirical research. Though a number of empirical methodologies have been employed to study strategic voting, including those based on aggregate vote returns and those based on voters' self-reported motivations, the most satisfying approach directly tests the predicted relationship between survey respondents' preferences, their expectations, and the likelihood of choosing their second-most preferred candidate. For illustrative purposes, we review a representative set of empirical papers adopting this latter approach: Cain (1978); Abramson et al. (1992); Blais and Nadeau (1996); Ordeshook and Zeng (1997); Alvarez and Nagler (2000); and Alvarez, Boehmke, and Nagler (2006).

Although distinguished by the electoral setting, statistical technology, and types of data they employ, all six papers uncover evidence that voters' expectations affect their eventual choice in exactly the manner posited by an expected utility approach: choosing one's second-most preferred candidate 2 becomes more likely when one's first choice 1 is not a viable contender and when one's preference for 1 over 2 (2 over 3) is small (large). However, behind these parallel findings lies an unresolved methodological issue: namely, that of *identifying the proper pool of survey respondents upon which to test strategic voting hypotheses*. To elucidate this issue, consider the following typology of voter choice as shown in Table 1.

Beginning with the table's top row, notice the set of column markers {C1, C2, . . . , C6}. Each of these markers is associated with a particular *expectation profile*, a ranking of 1, 2, and 3 according to their expected success in the election. For each profile, the candidate listed at the top is expected to place first, the candidate listed in the middle is expected to place second, and the candidate listed at the bottom is expected to place third. For example, voters have strategic profile C1 if they believe 1 will place first, 2 will place second, and 3 will place third: $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$. Similarly, voters have profile C5 if they believe 2 will place first, 3 will place second, and 1 will place third: $\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$. Moving to the table's far left-hand side, V_j denotes the choice to vote for one's jth preference; for example, any voter who chooses 1 will find herself somewhere in the row denoted V_1. Coupling these two dimensions yields a 24-cell typology that classifies voters according to their vote choice and

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Table 1 A typology of choice in three-candidate plurality elections preferences/viabilities

		C1 *1* *2* *3*	*C2* *1* *3* *2*	*C3* *2* *1* *3*	*C4* *3* *1* *2*	*C5* *2* *3* *1*	*C6* *3* *2* *1*
Vote choice	Vote for 1	Straightforward vote	Straightforward vote	Straightforward vote/sincere vote	Straightforward vote	Straightforward vote/sincere vote	Straightforward vote/sincere vote
	Vote for 2	Protest vote	Protest vote	Strategic vote/ protest vote	Protest vote/ coordination vote	Strategic vote	Strategic vote/ coordination vote
	Vote for 3	No current explanation	No current explanation	No current explanation	No current explanation	No current explanation	No current explanation
	Abstention	Abstention	Abstention	Abstention	Abstention	Abstention	Abstention

strategic profile. A voter with strategic profile C6 who chooses her second-most preferred candidate 2 would thus occupy the cell labeled ''Strategic Voting'' in the table's rightmost column, where C6 and row V_2 intersect; a voter with strategic expectations C1 who chooses her most preferred candidate 1 would fall into the cell labeled ''Straightforward'' in column C1.[3]

Most past studies (including Cain 1978; Abramson et al. 1992; and Alvarez and Nagler 2000 from the list above) have tested the COV's strategic hypotheses on the entire sample of voters, that is, voters from all six columns. However, consider a voter with expectation profile C1, who believes her most preferred candidate 1 will place first in the election, and her second and third preferences will place second and third in the election, respectively. As well, recall that strategic voting is defined as abandoning one's most preferred candidate when this candidate has little chance of winning, so as to prevent the victory of a genuinely disliked candidate. Thus, almost by definition voters in C1 should not cast strategic votes for their second-most preferred candidate 2 since 1 is a perfectly viable candidate and 3 is expected to place last. The following section's theoretical derivation provides a formal basis for this intuition, demonstrating that the necessary conditions for strategic voting in the COV are *never satisfied among voters from* C1.

What does this imply for empirical analysis? In short, it implies that empirical tests of strategic voting that include respondents with expectation profile C1 and furthermore any other expectation profile for which strategic voting is not possible are mis-specified. For example, the three studies listed above all test the hypothesis that the likelihood of choosing 2 should increase as 1's vote share goes down. Among potential strategic voters, this should in fact be true: as 1's chances of winning become lower and lower, its chances of competing with the remaining two candidates becomes lower, in turn making it easier to choose 2 for strategic reasons. However, among voters from C1 strategic voting should *never* be optimal according to the COV. To the extent that statistical results regarding strategic behavior include such voters, they will be affected by the choice for 2 among voters who are clearly not behaving strategically in the traditional sense. As such, it is not possible to interpret results from a large segment of empirical papers as irrefutable evidence for the presence of strategic voter behavior.

Recent contributions begin to address this problem, but without a direct theoretical derivation their analyses provide competing methodological prescriptions. For example, Ordeshook and Zeng (1997) and Alvarez, Boehmke, and Nagler (2006) argue that rigorously specified tests of strategic voting should be restricted to voters whose *most preferred candidate is expected to place last in the election*, that is, that correctly specified strategic voting hypotheses should be restricted to voters with expectation profiles C5 and C6 from Table 1. On the other hand, Blais and Nadeau (1996) argue that tests of strategic voting should be applied to any voters whose *most preferred candidate is expected to place lower than their second-most preferred candidate*, regardless of whether or not this most preferred candidate is expected to come in last place. In words, they assert that any voters with profile C3, C5, or C6 should be included in strategic voting regressions. To settle this difference, we must return to original model of McKelvey and Ordeshook (1972) to establish the precise formal condition for strategic voting in three-candidate elections. This will in turn allow us to identify the subset of voters to which strategic voting hypotheses should be restricted, a necessary step in establishing the importance of expected utility maximization as a model of voter choice.

[3]The individual behavioral labels in Table 1's various cells are discussed more extensively below.

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3 Which Voters Can Be Strategic: A Direct Proof

To determine the conditions for strategic voting in the decision-theoretic framework, one simply compares the expected utility of choosing 1 to the expected utility for choosing 2 in order to discover the conditions under which strategically choosing 2 is a utility-maximizing choice. Cain (1978) undertakes this process, but his derivation does not adopt the precise mechanics made available by COV of McKelvey and Ordeshook (1972) and as such makes many assumptions not consistent with expected utility maximization: for example, ignoring the possibility of first-place ties in his expected utility statements and equating the probability of *breaking* a tie with the probability of *creating* a tie between two candidates. These simplifications render the expectation parameters in his derivation difficult both to interpret and operationalize and prevent him from deriving the precise result that must guide empirical tests.[4]

Black's (1978) derivation cleverly derives the conditions for strategic voting without engaging in an explicit comparison of the expected utility for choosing candidates 1 and 2. To do so, he adopts the turnout equations (see equation [1]) derived originally by McKelvey and Ordeshook (1972) and uses them to derive a result *indirectly*. Consider the following definitions:

$$E_i^1 - E_i^0 = \Delta^1, \quad \textit{and} \quad E_i^2 - E_i^0 = \Delta^2. \tag{2}$$

As above, E_i^j represents voter i's expected utility for choosing $j \in \{1, 2\}$, and Δ^j represents the expected utility differentials between choosing j and choosing abstention (again see equation (1) above). To generate an expected utility comparison for choosing 1 as opposed to 2, Black (1978) simply subtracts the latter equation from the former, producing a result of the following form: $E_i^1 - E_i^2 = \Delta^1 - \Delta^2$. Though not identical, this result contains a similar intuition to the one that we derive below. Our direct theoretical proof is nonetheless far from merely academic. Formal precision is a value in and of itself; and many of the methodological insights which motivate this paper and our more general theoretical project emerge from process of derivation itself.

Assume as above that a voter i strictly prefers $1 \succ 2 \succ 3$. If only $j \in \{1, 2, 3\}$ compete in a winner-take-all contest, there are seven possible outcomes in the set α:[5]

$$\alpha \in \begin{cases} \{1\} & 1_\text{wins} \\ \{2\} & 2_\text{wins} \\ \{3\} & 3_\text{wins} \\ \{12\} & 1,2_\text{tie} \\ \{13\} & 1,3_\text{tie} \\ \{23\} & 2,3_\text{tie} \\ \{123\} & 1,2,3_\text{tie} \end{cases}. \tag{3}$$

Define q_α^j as the probability the outcome α occurs given that i votes for candidate j, whereas q_α^0 is the probability that outcome α occurs if she abstains. Further define $U_1 > U_2 > U_3$ as

[4] Gutkowski and Georges (1993) also present a model of strategic choice which yields predictions somewhat similar to those below, but without undertaking an explicit derivation of the expected utility parameters that empirical work must operationalize.

[5] The model here can be extended to environments with more than three parties. In more complicated strategic environments, it may be the case that voters choose a party ranked *even lower* than second in one's preference ordering but for the same reasons as those derived below: because this candidate has a more viable chance of defeating an even less preferred alternative. As introducing additional candidates complicates the model without generating new qualitative insight, we study only the three-candidate case.

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the utility the voter i obtains from an electoral victory by 1, 2, or 3, respectively. These parameters are sufficient to state E_i^j, the expected utility for choosing any candidate $j \in \{1,2,3\}$:

$$E_i^j = \left\{ q_1^j U_1 + q_2^j U_2 + q_3^j U_3 + q_{12}^j U_{12} + q_{13}^j U_{13} + q_{23}^j U_{23} + q_{123}^j U_{123} \right\} - C. \tag{4}$$

Now write the expected utility comparison between 1 and 2 as follows:

$$E_i^1 - E_i^2 = \left\{ \begin{array}{l} \left(q_1^1 - q_1^2\right) \cdot U_1 + \left(q_2^1 - q_2^2\right) \cdot U_2 + \left(q_3^1 - q_3^2\right) \cdot U_3 \\ + \left(q_{12}^1 - q_{12}^2\right) \cdot U_{12} + \left(q_{13}^1 - q_{13}^2\right) \cdot U_{13} + \left(q_{23}^1 - q_{23}^2\right) \cdot U_{23} \\ + \left(q_{123}^1 - q_{123}^2\right) \cdot U_{123} \end{array} \right\}. \tag{5}$$

Each term in equation (5) represents the probability i is *pivotal* in securing a particular outcome multiplied by i's utility for the outcome. When $E_i^1 - E_i^2 > 0$ the voter should choose 1, whereas if $E_i^1 - E_i^2 < 0$, the voter should strategically choose 2. Note that the parameter C drops out of the expected utility comparison between choosing 1 and choosing 2. Thus, only a voter's utilities for candidates and expectations over outcomes appear in equation (5), supporting the idea that once in the voting booth, voters ought to choose the utility-maximizing candidate. To arrive at a more precise and parsimonious result, we now demonstrate the following Lemma, which provides explicit statements for the pivotal probabilities in equation (5):

Lemma 1.

(a) $q_1^1 - q_1^2 = q_{12}^0 + q_{12}^2 + q_{13}^2 + q_{123}^0 + q_{123}^2$;
(b) $q_2^1 - q_2^2 = -\left(q_{12}^0 + q_{12}^1 + q_{23}^1 + q_{123}^0 + q_{123}^1\right)$;
(c) $q_3^1 - q_3^2 = q_{23}^2 - q_{13}^1$

The full proof of Lemma 1 is contained in Appendix A. In short, to derive explicit statements for the pivotal probabilities contained in equation (5), we undertake an exhaustive case-by-case analysis of the strategic conditions under which a voter can affect the outcome of an election by altering his or her vote choice. This involves cataloguing all situations in which a voter is effective in either creating or breaking electoral ties. Substituting into equation (5) using Lemma 1 yields the following expression:

$$\begin{aligned} E_i^1 - E_i^2 = \{ & \left(q_{12}^0 + q_{12}^2 + q_{13}^2 + q_{123}^0 + q_{123}^2\right) U_1 - \left(q_{12}^0 + q_{12}^1 + q_{23}^1 + q_{123}^0 + q_{123}^1\right) U_2 \\ & + \left(q_{23}^2 - q_{13}^1\right) U_3 + \left(q_{12}^1 - q_{12}^2\right) U_{12} + \left(q_{13}^1 - q_{13}^2\right) U_{13} + \left(q_{23}^1 - q_{23}^2\right) U_{23} \\ & + \left(q_{123}^1 - q_{123}^2\right) U_{123} \}. \end{aligned} \tag{6}$$

Rearranging terms and assuming two-way and three-way ties are decided by chance, such that $U_{jk} = 1/2\left(U_j + U_k\right)$ and $U_{ijk} = 1/3\left(U_i + U_j + U_k\right)$, now restate equation (6) in the following form:

$$\begin{aligned} E_i^1 - E_i^2 = \Big\{ & \left(q_{12}^0 + \frac{1}{2} q_{12}^1 + \frac{1}{2} q_{12}^2 + q_{123}^0 + \frac{1}{3} q_{123}^1 + \frac{1}{3} q_{123}^2\right)\left(U_1 - U_2\right) \\ & + \left(\frac{1}{2} q_{13}^1 + \frac{1}{2} q_{13}^2 + \frac{1}{3} q_{123}^2\right)\left(U_1 - U_3\right) - \left(\frac{1}{2} q_{23}^1 + \frac{1}{2} q_{23}^2 + \frac{1}{3} q_{123}^1\right)\left(U_2 - U_3\right) \Big\}. \end{aligned} \tag{7}$$

From equation (7) we can state the criteria that must obtain in order for a voter i to strategically abandon 1 for 2:

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Proposition 1. The conditions for strategic voting in three-candidate elections are

$$E_i^2 - E_i^1 > 0 \text{ iff:} \tag{8}$$

$$\left(\frac{1}{2}q_{23}^1 + \frac{1}{2}q_{23}^2 + \frac{1}{3}q_{123}^1\right)(U_2 - U_3) > \left(q_{12}^0 + \frac{1}{2}q_{12}^1 + \frac{1}{2}q_{12}^2 + q_{123}^0 + \frac{1}{3}q_{123}^1 + \frac{1}{3}q_{123}^2\right)(U_1 - U_2) + \left(\frac{1}{2}q_{13}^1 + \frac{1}{2}q_{13}^2 + \frac{1}{3}q_{123}^2\right)(U_1 - U_3).$$

If, as in original analysis of McKelvey and Ordeshook (1972), one assumes that voters discount three-way ties (i.e., $q_{123}^j = 0$), then Proposition 1 can be stated in a simplified form:

$$E_i^2 - E_i^1 > 0 \text{ iff :} \tag{9}$$

$$\left(q_{23}^1 + q_{23}^2\right)(U_2 - U_3) > \left(2q_{12}^0 + q_{12}^1 + q_{12}^2\right)(U_1 - U_2) + \left(q_{13}^1 + q_{13}^2\right)(U_1 - U_3).$$

It is from this condition that empirical hypotheses regarding the likelihood of strategic voting must be derived. Proposition 1's comparative static implications are straightforward and well known: choosing 2 over 1 becomes more likely as the probability of a first-place tie between 2 and 3 increases and as the probability of first-place ties between 1 and $j \in \{2, 3\}$ decreases. Furthermore, choosing 2 over 1 becomes more likely as the utility differential between 1 and 2 decreases and as the utility differential between 2 and 3 increases.

A less straightforward implication that arises from Proposition 1 is that *not all voters can be strategic*. Consider once again a voter with profile C1 who believes that her first-preference 1 will win the election, her second-preference 2 will place second, and her least-preferred candidate will finish last. Among such voters, Proposition 1 can never be satisfied, as the *inequality's right-hand side will always be greater than its left-hand side* (see Appendix A). As argued above, including voters for whom the inequality in Proposition 1 cannot be satisfied in empirical analysis leads to mis-specified statistical regressions. Among which voters may this inequality be satisfied? Most obviously, under certain conditions, it may be satisfied for voters with expectation profiles C5 and C6, *who believe 1 will place last*, as the probability terms q_{12} and q_{13} will tend to be low among such voters. Indeed, Alvarez, Boehmke, and Nagler (2006) and Ordeshook and Zeng (1997) from above argue that properly specified strategic voting hypotheses should be restricted to such voters.

However, careful examination reveals that *voters with profile C3 may also under some conditions cast strategic votes*.[6] Voters in column C3 believe their second-preference 2 will win, their first-preference 1 will place second, and their least-preferred candidate 3 will finish last. Consider someone who sees very little difference between 1 and 2 and who believes that 2's lead over both 1 and 3 is small (e.g., that 2 will win with probability 35%, 1 with probability 33%, and 3 with probability 32%). In such circumstances, it is perfectly possible that strategically choosing 2 is a utility-maximizing choice. In other words, such voters choose the leading candidate 2 because this candidate has the best chance of defeating one's least-preferred candidate 3, who is in fact quite viable.

[6]We thank an anonymous reviewer for encouraging us to flesh out the following argument.

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Restricting empirical analysis to voters with strategic profiles C5 and C6 would thus neglect a subset of voters whose incentives may be perfectly strategic (i.e., those with profile C3). Finally, it is straightforward to show that among voters with profiles C2 and C4, like those with profile C1, Proposition 1 cannot be satisfied. These results, all derived formally in Appendix A, are summarized in the following Proposition:

Proposition 2. A *necessary condition* for strategic voting is that the voter's second-most preferred candidate has a better chance of winning than does his/her most preferred candidate. That is, strategic voting is only applicable to voters with profiles C3, C5, and C6 in Table 1.

This Proposition formally establishes that voters with profiles C3, C5, and C6, those who believe 2 will finish ahead of 1, constitute the set of *potential strategic voters*.[7] Studies of strategic voting that include voters from C1, C2, and C4 would thus be incorrectly specified. Furthermore, it provides the theoretical underpinning necessary to support Blais and Nadeau's (1996) approach to identifying the pool of potentially strategic voters as against the competing methodology proposed by Ordeshook and Zeng (1997) and Alvarez, Boehmke, and Nagler (2006). Restricting empirical analysis to voters with strategic profiles C5 and C6 neglects a subset of voters whose incentives may be perfectly strategic (i.e. those with profile C3), thus generating an incomplete sample.

4 Some Aggregate Data

We now present a series of aggregate data to demonstrate tangibly the importance of our methodological prescriptions and more particularly to demonstrate that there are indeed many voters who choose their second-most preferred candidate for reasons having little to do with strategic voting in the traditional sense. Tables 2 and 3 classifies survey respondents from the 1988 *Canadian Election Study* survey according to their preferences, expectations, and vote choice as in the typology above.[8,9]

To measure voters' relative preferences over candidates, we follow most previous work in using survey respondents' Feeling Thermometer ratings. In this survey, respondents were asked to assign each candidate competing in their district a ''Probability of Winning,'' which allows us to rank the candidates according to their expected finish. Finally, we used respondents' preelection vote ''intention'' to capture their vote choice, so as to avoid problems of misreporting in postelection surveys (Alvarez and Nagler 2000).

[7]By potential strategic voters, we mean those who *could possibly* satisfy the inequality in Proposition 1. In an innovative contribution to the game theoretic literature on strategic voting, Myatt (2007) studies the likely behavior of voters who are ''at risk'' of voting strategically. This paper uncovers a counterintuitive equilibrium property, namely that strategic voting should in fact *decrease* in competitive elections.

[8]Fieldwork for the study was undertaken by the Institute for Social Research at York University. The 1988 Canadian Election Study had three distinct modules: a pre-election telephone survey with a representative sample of 3609 Canadians; a post-election telephone survey with 2922 of the respondents who responded to the pre-election survey; and a mail-return questionnaire with 2115 of the total post-election survey respondents. We use here only data from the pre-election survey. See Johnston *et al*.(1992) for further information on the survey.

[9]To remain consistent with the three-party example developed throughout the paper, we exclude respondents from the four-party Quebecois districts. What follows is not intended to be an exhaustive empirical analysis of strategic voting in the 1988 Canadian election, but merely to suggest the importance of excluding voters with profiles C1, C2, and C4 from tests of strategic voting, while including those with profile C3. Investigating voters from Quebecois districts would neither upset nor strengthen the following argument but would require a rather lengthy theoretical proof of Proposition 1's equivalent conditions in the case of four-party competition. We reserve this and other extensions for future work.

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Table 2 Distribution of voters with strict preferences in the 1988 Canadian National Election Study (respondents with strict expectations)

	C1	*C2*	*C3*	*C4*	*C5*	*C6*	
	1	*1*	*2*	*3*	*2*	*3*	
	2	*3*	*1*	*1*	*3*	*2*	
	3	*2*	*3*	*2*	*1*	*1*	*Total*
$V{=}P_1$	427	231	103	81	28	34	904
$V{=}P_2$	20	8	28	5	13	8	82
$V{=}P_3$	7	5	0	2	1	7	22
A	20	10	6	4	4	5	49
Total	474	254	137	92	46	54	$N = 1057$

The data are complicated by the fact that many respondents assigned two or more candidates an equal probability of winning, thus falling into Table 3, which groups them according to their proper strategic profile. However, such respondents can in fact be assigned *strategically equivalent* profiles to those in Table 2, which in turn match the typological categories from Table 1.[10] For example, voters with profile C12 in Table 3 believe that their most preferred candidate 1 will place last and assign their second-most preferred candidate 2 and least-preferred candidate 3 an equal probability of winning. These voters find themselves in a situation which is qualitatively identical to those of columns C5 and C6 from Table 2, who also believe that 1 will place last whereas 2 and 3 are competing for first place. Proposition 2 applies as well to voters in Table 3 such that, regardless of expected ties, it is among voters who believe 2 will place ahead of 1 that Proposition 1 may be satisfied (C3, C5, C6, C10, and C12).

All told 84.9% of respondents choose their most preferred candidate. These voters come in two basic ''types.'' Within the context of the traditional COV studied here, many *straightforward* respondents see no conflict between expectations and preferences: if their first-preference 1 is expected to place ahead of their second-preference 2, choosing 1 will always be a utility-maximizing choice. The respondents directly above potential strategic voters in columns C3, C5, C6, C10, and C12 are either straightforward or *sincere*: some voters in this group may choose 1 because it is a straightforward utility-maximizing choice (i.e., the condition for choosing 2 in Proposition 1 is not satisfied); on the other hand, some voters may ''sincerely'' choose 1 for expressive reasons despite the fact that casting a strategic vote for 2 would in fact be the utility-maximizing choice.

Roughly 7.9% of respondents chose candidate 2 (121 of 1525). Among these, only 54.5% (66 of 121) of respondents (those from C3, C5, C6, C10, and C12) choose their second preference when this candidate is expected to place ahead of 1 and should be analyzed in empirical tests of strategic voting. Thus, in empirical tests that included voters from all columns, *nearly half* of the respondents that choose candidate 2 must be doing so for ''nonstrategic'' reasons. Statistical results from regressions that included such voters

[10] A significant number of voters also reported being indifferent between one or more of the election's candidates. Each indifference profile could similarly be assigned a strategically equivalent profile from the fundamental typology in Table 1. Our sole purpose in studying the Canadian data is to demonstrate that some non-negligible subset of respondents chose their second-most preferred candidate despite the fact that this candidate was perfectly viable. We are able to do so without including any voters who express indifference. Including respondents with weak-preference orderings would multiply combinatorially the number of categories in Table 2 without affecting the basic empirical argument and is thus not worth its costs in terms of parsimony.

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Table 3 Distribution of voters with strict preferences in the 1988 Canadian National Election Study (respondents with weak expectations[a])

	C7 *12* *3*	*C8* *13* *2*	*C9* *1* *23*	*C10* *2* *13*	*C11* *23* *1*	*C12* *23* *1*	*C13* *123*	*Total*
$V=P_1$	93	67	133	33	37	15	13	391
$V=P_2$	11	1	6	11	4	6	0	39
$V=P_3$	1	2	2	0	3	0	0	8
A	10	1	7	5	3	2	2	30
Total	115	71	148	49	47	23	15	$N = 468$

[a]Respondents assign two or more candidates an equal probability of winning. A respondent in column C7 assigns 1 and 2 an equal likelihood of winning, and assigns 3 a lesser likelihood. A respondent in C10 believes 2 to be the front runner and believes that the two trailing candidates 1 and 3 have the same probability of winning. A respondent in C13 believes all three candidates have an equal likelihood of winning; and so on.

would thus be inaccurate: their regression coefficients regarding the likelihood of strategic voting would be influenced by the choice for 2 among voters who are by definition ''non-strategic.'' Furthermore, among the 54.5% of voters who may indeed be choosing 2 for strategic reasons, over 40% come from column C3 (28 of 66). Excluding such voters from empirical analyses would thus neglect an important subset of potentially strategic voters and generate regression coefficients based on a nonrandomly incomplete sample.

For reasons addressed in the Conclusion, we reserve regression-based data analysis of this paper's prescriptions for future research, to be grounded in a more exhaustive theoretical understanding of tactical voter choice in its many guises. However, the relevance of our methodological contribution emerges even in a more basic analysis of strategic voting as an aggregate phenomenon. Indeed, beyond testing the COV's specific comparative static predictions, past research has also aimed to identify the total ''aggregate presence'' of strategic voting in a particular setting and in turn its ultimate consequence (or lack thereof) for electoral outcomes. A quick review of procedures used to estimate this aggregate presence demonstrates a heterogeneity of approaches, all of which can be understood according to their position in this paper's theoretical scheme.

For example, Cain (1978) defines the aggregate presence of strategic voting as, quite simply, the percentage of *all survey respondents* who reported choosing their second-most preferred candidate, yielding an aggregate strategic vote of 14.6%. For reasons already discussed, this is likely an overestimate. Abramson et al. (1992) generate a similar figure (13.9%) using the following definition of strategic voting in multicandidate U.S. primary elections: any voter who ''... supports a more viable candidate over one who is better liked'' (p. 59). Although eliminating an important subset of problems associated with Cain's (1978) conceptualization, this approach is similarly subject to problems of overestimation.[11]

[11]As compared to Cain (1978), the Abramson et al. (1992) conceptualization does eliminate some nonstrategic voters from their aggregate tally. For example, whereas Cain's (1978) categorization would incorrectly include voters with expectation profile C1 who chose 2 in its tally of aggregate strategic behavior, Abramson et al.'s (1992) would not. However, the latter's definition presents its own overestimation problems. For example, included in their tally of 13.9% would be voters with expectation profile C6 who choose their least-preferred candidate. These voters satisfy the proposed criteria: they choose a more viable candidate over one who is better liked. However, they are clearly not behaving strategically in the intended sense.

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Alvarez and Nagler (2000) employ a novel technique in constructing their aggregate estimate of strategic choice in recent British elections. They first generate a respondent's predicted probability for choosing each of the three main British parties based on ''preferences'' alone, using predictor variables such as social class, geographic location, policy preference, etc. They then add ''viability-based'' independent variables (i.e., expected district-level vote shares) to this baseline model and define a strategic voter as any respondent whose ''most likely'' vote choice switches once viability-based independent variables are introduced. Though a novel technique, the 7.2% aggregate estimate that emerges is, as those above, made imprecise by using as a statistical sample the entire range of voters, rather than only those whose expectation profile makes strategic voting a feasible consideration.

Alvarez, Boehmke, and Nagler (2006) employ this same technique in their later piece, but aim to improve their estimate by distinguishing between voters whose first preference is expected to place last in the election (i.e., respondents in columns C5 and C6) and those whose first preference is expected to place first or second. In restricting their sample, they uncover a much higher rate of strategic voting measured as a percentage of all voters with expectation profiles C5 and C6: when combining data from 1987 and 1997, the most likely vote choice switches among roughly 53% of such voters when introducing viability-based parameters to their baseline preference model (yielding roughly 11% strategic voters in the entire sample). That said, this tally represents in fact only a partial improvement over that of Alvarez and Nagler (2000) for the following reasons: (1) voters with strategic profile C3 are excluded and (2) the baseline model from which this estimate is generated continues to include the entire sample of voters.[12]

The methodological approach outlined above allows us, as a first cut, to precisely estimate a particular electoral setting's aggregate level of strategic behavior, both as a percentage of the entire sample and as a percentage of voters in feasibly strategic situations. As noted above, those respondents with expectation profiles C3, C5, C6, C10, and C12 comprise the set of feasible strategic voters. Among this set, 30% of all respondents (66 of 218) do in fact report the intention to choose their second-most preferred candidate. As a percentage of the electorate as a whole, this yields an aggregate tally of 4.3% (66 of 1525).[13] Without distinguishing between potentially strategic voters and those for whom strategic voting is impossible, we would thus *overestimate* the aggregate tally (since 7.9% > 4.3%) but at the same time *underestimate* the importance of strategic voting among those respondents for whom it in fact represents a viable option (since 30% > 7.9%). Furthermore, were we to confine our estimates to respondents whose first preference was expected to place last (C5, C6, and C12), we would overestimate that 35% (27 of 77) of feasibly strategic voters do in fact choose their second preference but underestimate that only 1.8% (27 of 1525) of the entire sample behaves strategically.

[12]Put otherwise, although the authors confine their ''probability-switching'' analysis to voters with profiles C5 and C6, the parameter estimates used on these potentially strategic voters are generated by a regression whose sample remains the entire population.

[13]These estimates are quite similar to those of Blais and Nadeau (1996) who, using the same Canadian National Election Study, find that 28% of feasibly strategic voters and 6% of the electorate as a whole voted strategically. The slight difference between our tally and theirs arises due to (1) the absence in our data of voters from four-party districts (see footnote 8) and (2) the absence in our data of voters who expressed ''indifference'' between one of the two parties (see footnote 9). For these reasons, their sample contains roughly 1200 additional observations as compared to ours. As argued in footnotes 8 and 9, the current paper's conclusions are unaffected by the exclusion of such respondents. However, in the future a more exhaustive statistical analysis of strategic voting in the 1988 Canadian election would necessarily include such data.

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The distinction between strategic voting's aggregate presence in the electorate as a whole and its prevalence among voters in strategically feasible situations is important in uncovering the impact of tactical choice on democratic outcomes. For example, Although aggregate tallies that do not isolate the set of feasibly strategic voters tend to overestimate the presence of strategic voting as a percentage of the electorate, they fail to recognize the strong prevalence of tactical choice among those voters for whom it is in fact a viable alternative. As a result, our lower estimate of the aggregate phenomenon (4.3%) should not be taken as an indicator that strategic voting has little ultimate consequence. On the contrary, the elevated rate of tactical choice (30%) among those whose circumstances so permit suggests that strategic voting may have a profound impact in a subset of highly competitive electoral districts. In many electoral settings, it is exactly these competitive races that determine the eventual balance of parliamentary power.

In summary, the heterogeneous strategic profiles of respondents who choose 2 reinforce the importance for future empirical studies of isolating all voters whose most preferred candidate is trailing her second most preferred. Not doing so leads to one or the other specification problems described above, which in turn prevents an accurate empirical appraisal of strategic voter behavior. The existence of voters with profiles C1, C2, or C4 who choose their second preference suggests the potential presence of alternative tactical incentives. Such respondents, and furthermore those with profiles C3, C5, and C6, may in fact be responding to one of a variety of behavioral motivations when abandoning their most preferred candidate. A complete picture of strategic choice during the 1988 Canadian election will require empirical techniques capable of distinguishing between various forms of tactical behavior. The concluding discussion presents a theoretical framework for this more general endeavor.

5 Concluding Discussion

This paper extends COV of McKelvey and Ordeshook (1972) by providing a direct proof of the conditions under which voters should strategically abandon their most preferred candidate, 1, and vote for their second-most preferred candidate, 2. A useful theoretical implication of our model is that only voters who believe 2 has a better chance of winning than does 1, that is, voters with profiles C3, C5, and C6 in the typology above might choose their second-most preferred candidate. These theoretical results allow us to identify a recurring methodological problem in past empirical studies, namely that of identifying the initial pool of voters to which strategic voting hypotheses properly apply, and to provide a solution applicable to future empirical work in the field.

However, even studies which followed this paper's basic methodological prescription would be confronted with a number of additional complications in attempting to interpret their empirical results. The decision-theoretic approach embodied in the COV is only one of many possible models that explain why voters choose their second-most preferred candidate 2. Voters in columns C3, C5, and C6 might choose 2 for any number of reasons not associated with the COV; similarly, it would be hasty to label voters who choose 2 from columns C1, C2, and C4 as ''nonrational'' simply because their behavior is not compatible with decision-theoretic expected utility maximization.

For example, Niou (2001) studies the incentives for strategic coordination when a three-candidate election's plurality winner is also the election's Condorcet loser. This situation arises when one large party competes against two smaller parties, and the 2 smaller parties' supporters rank the largest party last in their preference profile. This is possible for voters with strategic profiles C4 and C6, who expect their least-preferred candidate 3 to win the election and their most preferred and second-most preferred candidates 1 and 2 to come in

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second and third place (for voters in C4, 1 is in second place and 2 is in third, and vice versa for voters in C6). The analysis highlights that under these circumstances, elites from the two smaller parties find themselves in a *Battle of the Sexes coordination game*: proponents of both smaller parties prefer a victory by either small party over the least-preferred plurality winner; but each would also prefer that, from among the two small parties, their particular party be the eventual coordination choice.

This model generates insights not captured in the COV. Although voters in C4 never cast strategic votes of the type modeled in Proposition 1, they may yet choose 2 when this second preference becomes the *coordination choice of a Battle of the Sexes* game played by political elites. Similarly, we should avoid rushing to the conclusion that voters with profile C6 are behaving ''noninstrumentally'' if statistical analysis fails to support the COV's strategic hypotheses; their behavior may simply be the result of a distinct instrumental calculation based on the outcome of elite coordination. Thus, without adequately accounting for the predictions of competing explanatory models, one risks making incomplete inferences about the motivations that determine voter choice.

This point becomes especially salient as the diversity of explanatory approaches to tactical voter choice moves beyond the traditional strategic voting framework. Kselman and Niou (2008) model the conditions under which voters with profiles C1–C4, who believe their most preferred candidate 1 will place no lower than second in the election, will cast *protest votes* for 2: votes which register a *signal of dissatisfaction* with their most preferred party. In Britain, for example, traditional Labour voters who feel Labour has moved too far to the ideological left may choose their second-most preferred party, the centrist Liberals, in the hope of inducing Labour leaders back toward the political center. This becomes an optimal strategy when the short-term instrumental costs of defecting to one's second-preference 2 are low and/or when the expected impact of protest voting on 1's future behavior is high.

As well, though not included in Table 1 (which is explicitly designed to catalog forms of instrumental or ''outcome-oriented'' behavior), scholars of electoral ''tides'' and voter band-wagoning might argue that respondents from C3 and C5, whose second-preference 2 is the expected plurality winner, choose this second preference not to effectuate some tangible political outcome but simply out of a desire to ''choose the winning team'' (Schuessler 2000). Furthermore, consider voters with profile C3, for whom 2 is the expected plurality winner. Among such voters, Proposition 1 above and the logic of electoral band-wagoning generate overlapping predictions: they should become more likely to choose 2 as her lead over 1 increases. Since both arguments make the same prediction, evidence to this effect is insufficient to precisely identify voters' basic behavioral motivations.

The typology developed in Table 1 thus provides a general analytic framework within which to both classify and compare alternative theoretical explanations of tactical voter choice. Future empirical analysis should be grounded in theoretical advances that identify statistical procedures to account for this explanatory heterogeneity and that address the individual, institutional, and contextual factors that may impel voters to adopt one behavioral pattern over another. Where models generate parallel predictions, analysts must find additional survey items that help to identify the precise motivations that guide tactical choice. Indeed, theoretical precision is only a first step toward improved empirical analysis. Deepening our theoretical understanding of tactical behavior will in turn help us in developing survey instruments suited to identifying novel behavioral forms such as coordinated voting, voter signaling, and voter band-wagoning. As a whole, this research program will allow us a more nuanced and accurate appraisal of instrumental rationality as a paradigm for the study of voter behavior.

https://doi.org/10.1093/pan/mpp027 Published online by Cambridge University Press

Funding

Duke University Graduate School.

Theoretical Appendix A

A1. Proof of Lemma 1 and Proposition 1

Begin with the proof of Lemma 1 part (a). $q_1^1 - q_1^2$ is the probability that "1 wins" given a vote for 1 as opposed to 2, that is, the probability that choosing 1 as opposed to 2 is *pivotal* for 1's victory. The choice between 1 and 2 is pivotal for the outcome 1 wins if a vote for 2 as opposed to 1 either *creates* or *upsets* this outcome. In moving his or her choice from 1 to 2, a voter i can obviously not *create* the outcome 1 wins ($\alpha = 1$). We are thus looking for the competitive situations in which choosing 2 as opposed to 1 upsets the outcome $\alpha = 1$, a victory by the voter's first preference. The following demonstration identifies the conditions under which switching a vote from 1 to 2 changes the outcome from 1 wins ($\alpha = 1$) to one of the other six outcomes in α.

Case 1. 1 wins to "2 wins"

Shifting a vote from 1 to 2 changes the outcome from $\alpha = 1$ to $\alpha = 2$ only if

(a) all three candidates are tied given the voter's abstention, denoted by q_{123}^0 or
(b) 1 and 2 are tied given the voter's abstention, denoted by q_{12}^0

For example, in a 10-person electorate, if the vote shares among 1, 2, and 3 are (3,3,3) or (4,4,1) with the voter abstaining from voting, then the voter is pivotal in deciding whether 1 or 2 would win.

Case 2. 1 wins to "3 wins"

Under no circumstances will shifting a vote from 1 to 2 change the outcome from $\alpha = 1$ to $\alpha = 3$. By assumption, 1 is the plurality winner given a vote for 1, which implies that 1 must have at least one more vote than 3. By switching her vote to 2, the voter can thus at best put 3 in a first-place tie with 1, but under no circumstances will this switch lead to an outright victory by 3.

Case 3. 1 wins to "1 and 2 tie"

Under what circumstances will shifting a vote from 1 to 2 change the outcome from $\alpha = 1$ to $\alpha = 12$? Intuitively, this is the case only if a vote for 2 creates a tie between 1 and 2, q_{12}^2. For example, in a 10 personal electorate, the following profile would imply that switching from 1 to 2 changed the outcome from $\alpha = 1$ to $\alpha = 12$: (5,3,2). Switching from 1 to 2 thus creates a 4-4 tie between 1 and 2.

Case 4. 1 wins to "1 and 3 tie"

As in Case 3, shifting from 1 to 2 shifts the outcome from $\alpha = 1$ to $\alpha = 13$ only if 1 and 3 tie given a vote for 2: q_{13}^2. In a 10-person electorate, consider the following profile: (5,1,4). Switching from 1 to 2 thus creates a 4-4 tie between 1 and 3.

Case 5. 1 wins to "2 and 3 tie"

By assumption, a vote for 1 accompanies a plurality victory for 1. Shifting the vote to 2 can at best put 3 in a first-place tie with 1 but can never put 3 ahead of 1. Thus, under no

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circumstances will shifting a vote from 1 to 2 give 2 and 3 sole possession of first place (implying 3 receives more votes than 1, which is a contradiction).

Case 6. 1 wins to "1, 2 and 3 tie"

Shifting one's vote from 1 to 2 changes the outcome from $\alpha = 1$ to $\alpha = 123$, if choosing 2 creates a three-way first-place ties: q^2_{123}. Consider the following profile: (5,3,4). 1 would lose its winning position and a three-way tie would be created if the voter switcher her vote from 1 to 2.

Adding up all the possible situations in which shifting a vote from 1 to 2 is pivotal in changing the outcome from 1 wins to some other outcome in α (from Cases 1, 3, 4, and 6 above) yields the following statement of Lemma 1 part (a): $q^1_1 - q^2_1 = q^0_{12} + q^2_{12} + q^2_{13} + q^0_{123} + q^2_{123}$. ■

To prove part (b) of Lemma 1, note that $q^1_2 - q^2_2 = -\left(q^2_2 - q^1_2\right) < 0$. In other words, 2 is always more likely to win when the voter chooses 2 rather than 1. The proof that $q^2_2 - q^1_2 = q^0_{12} + q^1_{12} + q^1_{23} + q^0_{123} + q^1_{123}$, which yields part (b) follows the same logic as in part (a) of Lemma 1 and is thus omitted. ■

To prove part (c) of Lemma 1, note again that $q^1_3 - q^2_3$ denotes the probability that voter i either creates or upsets the outcome 3 wins ($\alpha = 3$) by switching her vote from 1 to 2. Among the seven possible outcomes, if 1 wins ($\alpha = 1$) given a choice for 1, then by switching her vote from 1 to 2 voter i could at best move 3 into a tie for the first place with 1, but could never produce the outcome of interest 3 wins. If $\alpha = 2$, 12, 23, or 123 given a choice for 1, then a switch from 1 to 2 would either make or leave 2 the winner. Thus, in the cases $\alpha = 1$, 2, 12, 23, or 123, voter i cannot create the outcome $\alpha = 3$ by choosing 2 over 1; 3 has no chance of winning regardless of whether i chooses 1 or 2.

Only if 1 and 3 tie ($\alpha = 13$) or 3 wins ($\alpha = 3$) when i chooses 1 can i possibly affect the outcome 3 wins by switching his or her vote from 1 to 2. In the case $\alpha = 13$ given a choice for 1 (which occurs with probability q^1_{13}), in switching her vote from 1 to 2 she creates the outcome in question 3 wins. For example, if the vote distribution is (5,3,5) with the voter voting for 1, 3 would emerge as the winner if the voter switches her vote to 2 because the vote distribution would now become (4,4,5). In the case $\alpha = 3$ given a choice for 1, in switching her vote from 1 to 2 voter i can *upset* this outcome by moving 2 into a first-place tie with 3 (which occurs with probability q^2_{23}). For example, if the vote distribution is (4,4,5) when the voter votes for 1, then the voter would create a tie between 2 and 3 after switching her vote from 1 to 2 because the vote distribution would now become (3,5,5).

The combined wisdom of these individual cases indicates that a vote for 1 as opposed to 2 makes the outcome 3 wins more likely to the extent that voting for 2 upsets 3's outright victory over 2, creating the outcome 2 and 3 tie; but it makes the outcome 3 wins less likely to the extent that choosing 1 eliminates 3's outright victory over 1, creating the outcome 1 and 3 tie. We can thus state that $q^1_3 - q^2_3 = q^2_{23} - q^1_{13}$, establishing the final part (c) of Lemma 1. ■

A2. Proof of Proposition 2

Proposition 1 allows us to identify the subsets of voters for whom strategic choice is in fact a viable option. In particular, we will identify the subsets of voter expectation profiles from Table 1 for whom the condition expressed in Proposition 1 can in fact be satisfied. The proof is based on the following simple insight: if candidate 2 is expected to place ahead of

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candidate 1 (expectation profiles C3, C5, and C6), then candidate 2 will be more likely than candidate 1 to find itself in a first-place tie with candidate 3: $q^j_{23} > q^j_{13}$. If 1 is expected to place ahead of 2 (C1, C2, and C4), the reverse is true. Furthermore, if candidate 2 is expected to place ahead of 1, then $q^1_{123} > q^2_{123}$: the likelihood of a three-way tie decreases when choosing the election's expected plurality winner, thus further distancing her from the remaining electoral competitors. Again, the reverse is true when candidate 1 is expected to place ahead of candidate 2.

These relationships allow us to establish Proposition 2. To do so, we consider the case in which strategic voting is most likely, namely that in which $U_1 \cong U_2$ (such that $U_1 - U_2 \cong 0$): when voters perceive little difference between their most- and second-most preferred candidates, the choice for 2 becomes all the more acceptable. In such circumstances, it is straightforward to show that, among voters for whom 2 is expected to place ahead of 1, the condition for strategic voting expressed in Proposition 1 can be satisfied: since $q^j_{23} > q^j_{13}$ and $q^1_{123} > q^2_{123}$, as long as the difference between $(U_1 - U_3)$ and $(U_2 - U_3)$ is not excessive, the inequality's left-hand side will be greater than its right-hand side and choosing 2 will be the utility-maximizing option. On the other hand, if 1 is expected to place ahead of 2, we know that $q^j_{23} < q^j_{13}$ and $q^1_{123} < q^2_{123}$. Furthermore, by definition $(U_1 - U_3) > (U_2 - U_3)$. As such, even in the most favorable circumstances for strategic voting (i.e., when $U_1 \cong U_2$), if 1 is expected to place ahead of 2, the right-hand side of Proposition 1 will always be greater than its left-hand side and as such voters with expectation profiles C1, C2, and C4 cannot be strategic voters. ■

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