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Task-space regulation of robots with approximate actuator model

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SUMMARY

Though many research efforts have been devoted to developing robot controllers with uncertainties in the robot dynamics, most research so far has assumed that the model of actuator is known exactly. In implementing the robot controllers, calibration is necessary to identify the exact parameters of the actuator model. When there is an error in calibrating or modeling, it is unknown whether the stability of the system could still be guaranteed. In addition, the actuator model could change as the temperature varies due to overheating of the motor or changes in the ambient temperature. To overcome this problem, we propose in this paper, a task-space controller for the regulation of the robot with an approximate actuator model. We consider second-order dynamics of electric actuators with negligible armature inductances. Sufficient conditions for choosing the feedback gains are presented to guarantee stability.

KEYWORDS: Stability; Setpoint control; Uncertain Actuator Model

1. INTRODUCTION

In most applications of robot manipulators, a desired path for the end effector is specified in the task space, such as Cartesian space. Many research efforts have been devoted towards the development of control schemes for robotic manipulators. In most of the control methods,[1–11] the controllers are designed to move the robots along a desired joint angle. To move the end effector to the desired position, an inverse kinematics problem is solved to generate the desired position in joint coordinates.

A more effective control strategy without solving the inverse kinematics problem is the task-space control method.[1,12–22] In task-space control, feedback errors of the control laws are defined and computed directly in the task space, such as Cartesian space and visual space. Takegaki and Arimoto[1] proposed a transposed Jacobian controller for setpoint control in Cartesian coordinates. A local feedback control law with an imperfect Jacobian matrix from the Cartesian space to visual space is proposed by Miyazaki and Masutani.[16] In these controllers, an exact knowledge of the robot kinematics from the joint space to the Cartesian space is required. However, since the robot is interacting with its environment, its kinematics changes for different tasks when it picks up different objects. To overcome the problem of uncertain kinematics, Cheah et al.[17–22] proposed several task-space feedback laws with uncertain kinematics from the joint space to the task space.

Most research so far on robot control has assumed that the model of actuator is known exactly. However, since a model cannot be obtained precisely in practice, calibration is necessary to identify the exact parameters of the actuator in implementing robot controllers. In addition, the actuator model could change as the temperature varies due to the overheating of the motor or changes in the ambient temperature. Hence, in the presence of the modeling uncertainty or calibration error, the convergence of the position error may not be guaranteed.

To overcome this problem, we propose in this paper, a task-space controller for the regulation of the robot with an approximate or uncertain actuator model. To the best of our knowledge, such a problem has not been studied before. Therefore, it is unknown whether the stability of the robot's motion can still be guaranteed in the presence of uncertainty in the actuator model. We consider second-order dynamics of electric actuators with negligible armature inductances. This assumption is reasonable because the electrical time constant of the motor is much smaller than the mechanical time constant. Sufficient conditions for choosing the feedback gains are presented to guarantee stability. Simulation results are presented to illustrate the performances of the proposed controller.

The rest of this paper is organized as follows: Section 2 formulates the robot dynamic equations and kinematics; Section 3 presents a task-space controller with uncertain actuator model; Section 4 presents some simulation studies and Section 5 concludes this paper.

2. PROBLEM FORMULATION

In order to describe a task for the robot manipulator, a desired path for the end effector is usually specified in the task space. Let $X \in R^m$ represents a task-space vector.[14–18]

$$X = h(q), \tag{1}$$

where $m \leq n$ and $h(\cdot) \in R^n \rightarrow R^m$ is generally a nonlinear transformation describing the relation between the joint space and the task space. The task-space velocity $\dot{X}$ is related to joint-space velocity $\dot{q}$ as:[18]

$$\dot{X} = J(q)\dot{q}, \tag{2}$$

where $J(q)$ is a Jacobian matrix of the mapping from the joint space to the task space.

The equation of motion for the robotic manipulator with n degrees of freedom is given in the joint space as:[8]

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$$M(q)\ddot{q}+\left(\frac{1}{2}\dot{M}(q)+S(q,\dot{q})\right)\dot{q}+g(q)=\tau, \tag{3}$$

where $q\in R^n$ denotes joint angles, $M(q)\in R^{n\times n}$ denotes the inertia matrix which is positive definite and symmetric, $g(q)\in R^n$ denotes the gravitational force, $\tau\in R^n$ denotes control inputs, $S(q,\dot{q})$ is a skew-symmetric matrix expressed by

$$S(q,\dot{q})\dot{q}=\frac{1}{2}\dot{M}(q)\dot{q}-\frac{1}{2}\left\{\frac{\partial}{\partial q}\dot{q}^T M(q)\dot{q}\right\}^T, \tag{4}$$

and $P(q)$ is the potential energy due to the gravitational force.

The second-order dynamics of an actuator-controlled DC motor that drive the link is given by:[8,11]

$$J_{mi}\ddot{\theta}_i+B_{oi}\dot{\theta}_i+r_i\tau_i=K_{oi}v_i, \tag{5}$$

since the electrical time constant of the motor is much smaller than the mechanical time constant. In equation (5), θ_i denotes the angle of the ith motor rotor shaft and v_i is the voltage input, J_{mi} is the inertia, $B_{oi}=B_{mi}+K_{\tau i}K_{bi}/R_{ai}$, $K_{oi}=K_{\tau i}/R_{ai}$, B_{mi} is the rotor damping constant, $K_{\tau i}$ is the torque constant, R_{ai} is the armature resistance, K_{bi} is the back emf constant and r_i is the transmission gear ratio defined as:

$$q_i=r_i\theta_i \tag{6}$$

From equations (3), (5) and (6), the dynamics of the robot with actuators is given as:

$$(M_0+M(q))\ddot{q}+(B_0+\frac{1}{2}\dot{M}(q)+S(q,\dot{q}))\dot{q}+g(q)=Kv, \tag{7}$$

where

$$\begin{aligned} &M_o=diag(J_{m1}/r_1^2,\ldots,J_{mn}/r_n^2),\\ &B_o=diag(B_{o1}/r_1^2,\ldots,B_{on}/r_n^2)),\\ &K=diag(K_{\tau 1}/R_{a1}r_1,\ldots,K_{\tau n}/R_{an}r_n),\\ &v=(v_1,\ldots,v_n)^T. \end{aligned} \tag{8}$$

In actual implementations of the robot controllers, calibration is necessary to identify the exact parameters of matrix K in equation (7). However, since both $K_{\tau i}$ and R_{ai} are temperature sensitive, the actuator model K may change as the temperature varies due to overheating of the motor or changes in the ambient temperature. In the presence of modeling uncertainty or calibration error, a position error may result; it is also not sure whether the stability of the system could still be guaranteed. In this paper, we shall consider a setpoint control of a robot with an approximate or uncertain actuator model and show the asymptotic stability of the robot motion with the uncertainty.

Remark 1.
From equation (7), the dynamics of the robot with actuators can be formulated as:

$$K^{-1}(M_0+M(q))\ddot{q}+K^{-1}\left(B_0+\frac{1}{2}\dot{M}(q)+S(q,\dot{q})\right)\dot{q}+K^{-1}g(q)=v. \tag{9}$$

However, note that $K^{-1}(M_0+M(q))$ is not a symmetric matrix and hence the results in references 1 to 22 cannot be applied to analyze the stability of the system by treating the uncertainty in the actuator model K as an uncertainty in the robot dynamics. A transformation of coordinates using $p=K^Tq$ is also not feasible since K is uncertain. Hence p is also uncertain and cannot be used in the feedback control.

Remark 2.
If the DC motor is controlled directly by a torque mode signal, the second-order dynamics of the DC motor can be expressed as:[8]

$$J_{mi}\ddot{\theta}_i+B_{mi}\dot{\theta}_i+r_i\tau_i=K_{\tau i}I_{ai}, \tag{10}$$

where I_{ai} is the armature current of the ith motor rotor shaft. Hence, the dynamics of the robot with actuators is given as:

$$(M_0+M(q))\ddot{q}+\left(B_1+\frac{1}{2}\dot{M}(q)+S(q,\dot{q})\right)\dot{q}+g(q)=K_\tau I, \tag{11}$$

where $B_1=diag(B_{m1}/r_1^2,\ldots,B_{mn}/r_n^2))$, $K_\tau=diag(K_{\tau 1}/r_1,\ldots,K_{\tau n}/r_n)$ and $I=(I_{a1},\ldots,I_{an})^T$. The above equation is in a similar form as (7) and hence the result in this paper can also be applied to a robot with motors driven by current inputs. In this case, the uncertainty is due to the torque constants K_τ only.

3. FEEDBACK CONTROL OF ROBOT WITH APPROXIMATE ACTUATOR MODEL

Let us define a scalar potential function $S_i(\theta)$ and its derivative s_i as shown in Figure 1 and with the following properties:[8]

(i) $S_i(\theta)>0$ for $\theta\neq 0$ and $S_i(0)=0$.
(ii) $S_i(\theta)$ is twice continuously differentiable, and the derivative $s_i(\theta)=\dfrac{dS_i(\theta)}{d\theta}$ is strictly increasing in θ for $|\theta|<\gamma_i$ with some γ_i and saturated for $|\theta|\geq\gamma_i$, i.e. $s_i(\theta)=\pm s_i$ for $\theta\geq+\gamma_i$ and $\theta\leq-\gamma_i$ respectively where s_i is a positive constant.
(iii) There are constants $\bar{c}_i>0$, $d_i>0$, $\bar{d}_i(>d_i)>0$ such that for $\theta\neq 0$,

$$\bar{d}_i s_i^2(\theta)\geq\theta s_i(\theta)\geq d_i s_i^2(\theta)>0,\quad S_i(\theta)\geq\bar{c}_i s_i^2(\theta). \tag{12}$$

□

Let $\hat{K}$ be an approximate actuator model chosen so that

$$\| I-K\hat{K}^{-1}\|\leq\bar{p}, \tag{13}$$

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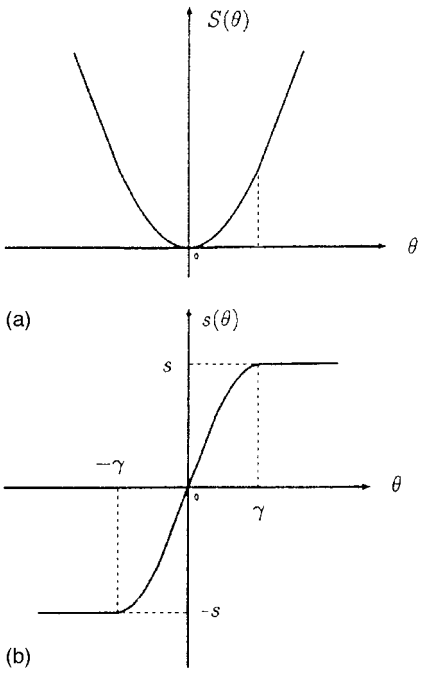


Fig. 1. (a) Quasi-natural potential: $S(\theta)$, (b) derivative of $S(\theta)$: $s(\theta)$.

where $\bar{p}$ is a positive constant to be defined later. Using the approximate actuator model, the control input is proposed as

$$v = -\hat{K}^{-1}(J^T(q)K_p s(e) + J^T(q)K_v\dot{X}) - \hat{K}^{-1}K_I\int_0^t y(\tau)d\tau, \tag{14}$$

where

$$y = \dot{q} + \alpha J^+(q)s(e), \tag{15}$$

and $e = X - X_d = (e_1, \ldots, e_m)^T$ is a positional deviation from a desired position $X_d \in R^m$, K_p, K_v and K_I are positive definite diagonal feedback gains for the position, velocity and integration of y, respectively, α is a positive constant, $s(e) = (s_1(e_1), \ldots, s_m(e_m))^T$ are saturated functions, and $J^+(q)$ is the pseudo-inverse of $J(q)$ such that $J(q)J^+(q) = I$. It is assumed in this paper that the Jacobian matrix $J(q)$ is known and the uncertainties are due to the second-order actuator dynamics and robot dynamics described in equations (5) and (3), respectively. In addition, we assume that the robot is operating in a finite workspace:

$$X \in \Omega, \tag{16}$$

such that the Jacobian matrix is of full rank and $J(q)J^T(q)$ is non-singular.

If we substitute equation (14) into equation (7), we have

$$(M_0 + M(q))\ddot{q} + \left(B_0 + \frac{1}{2}\dot{M}(q) + S(q, \dot{q})\right)\dot{q} + g(q) - g(q_d)$$
$$+ K\hat{K}^{-1}J^T(q)K_v\dot{X} + K\hat{K}^{-1}J^T(q)K_p s(e) + K\hat{K}^{-1}K_I z = 0, \tag{17}$$

where

$$z(t) = K_I^{-1}\hat{K}K^{-1}g(q_d) + \int_0^t y(\tau)d\tau, \tag{18}$$

where we note that $\dot{z} = y$ and $K\hat{K}^{-1}K_I$ is a diagonal matrix since K, $\hat{K}^{-1}$ and K_I are all diagonal matrices. Since manipulator has its own configuration at joint space, we define q_d as a desired joint position such that $h(q_d) = X_d$. However, note that, q_d, $g(q_d)$ are introduced for the purpose of analysis and are not required in the controller, as seen from equation (14).

We define a Lyapunov function candidate $V(s(e), \dot{q}, z)$ as:

$$V(s(e), \dot{q}, z) = \frac{1}{2}\dot{q}^T(M_0 + M(q))\dot{q} + \alpha\dot{q}^T(M_0 + M(q))J^+(q)s(e)$$
$$+ \sum_{i=1}^{m}(k_{pi} + \alpha k_{vi})S_i(e_i) + P(q) - P(q_d)$$
$$- \Delta q^T g(q_d) + \frac{1}{2}z^T K\hat{K}^{-1}K_I z, \tag{19}$$

where $\Delta q = q - q_d$, k_{pi}, k_{vi} denote the ith diagonal elements of K_p and K_v, respectively. Differentiating $V(s(e), \dot{q}, z)$ with respect to time, we have

$$\frac{d}{dt}V(s(e), \dot{q}, z) = \dot{q}^T(M_0 + M(q))\ddot{q} + \frac{1}{2}\dot{q}^T\dot{M}(q)\dot{q}$$
$$+ \alpha s(e)^T(J^+(q))^T(M_0 + M(q))\ddot{q}$$
$$+ \alpha s(e)^T(J^+(q))^T\dot{M}(q)\dot{q}$$
$$+ \alpha\dot{s}(e)^T(J^+(q))^T(M_0 + M(q))\dot{q}$$
$$+ \alpha s(e)^T(J^+(q))^T(M_0 + M(q))\dot{q}$$
$$+ \dot{X}^T(K_p + \alpha K_v)s(e) + \dot{q}^T(g(q) - g(q_d))$$
$$+ z^T K\hat{K}^{-1}K_I y. \tag{20}$$

From equation (17), we have

$$(M_0 + M(q))\ddot{q} = -\left\{\left(B_0 + \frac{1}{2}\dot{M}(q) + S(q, \dot{q})\right)\dot{q} + g(q) - g(q_d)\right.$$
$$+ K\hat{K}^{-1}J^T(q)K_v\dot{X} + K\hat{K}^{-1}J^T(q)K_p s(e)$$
$$\left. + K\hat{K}^{-1}K_I z\right\}, \tag{21}$$

Substituting equation (21) into equation (20), and noting that $S(q, \dot{q})$ is a skew-symmetric matrix, we have

$$\frac{d}{dt}V(s(e), \dot{q}, z) = -W(s(e), \dot{q}), \tag{22}$$

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where

$$W(s(e), \dot{q}) = \dot{q}^T(B_0 + K\hat{K}^{-1}J^T(q)K_vJ(q))\dot{q} - \dot{q}^T(J^T(q) - K\hat{K}^{-1}J^T(q))K_p s(e) + \alpha s^T(e)(J^+(q))^T(g(q) - g(q_d)) + \alpha s^T(e)(J^+(q))^T K\hat{K}^{-1}J^T(q)K_p s(e) - \alpha s^T(e)(I - (J^+(q))^T K\hat{K}^{-1}J^T(q))K_v J(q)\dot{q} + \alpha\left\{ s(e)^T(J^+(q))^T\left[B_0 - \frac{1}{2}\dot{M}(q) + S(q, \dot{q}) \right]\dot{q} - \dot{s}(e)^T(J^+(q))^T(M_0 + M(q))\dot{q} - s(e)^T(\dot{J}^+(q))^T(M_0 + M(q))\dot{q} \right\} \quad (23)$$

Since,

$$\frac{1}{4}\dot{q}^T(M_0 + M(q))\dot{q} + \alpha\dot{q}^T(M_0 + M(q))J^+(q)s(e) + \sum_{i=1}^{m} \alpha k_{vi} S_i(e_i) = \frac{1}{4}(\dot{q} + 2\alpha J^+(q)s(e))^T(M_0 + M(q))(\dot{q} + 2\alpha J^+(q)s(e)) - \alpha^2 s(e)^T(J^+(q))^T(M_0 + M(q))J^+(q)s(e) + \sum_{i=1}^{m} \alpha k_{vi} S_i(e_i) \geq \sum_{i=1}^{m} \{\alpha(k_{vi}\bar{c}_i - \alpha\lambda_m)\}s_i^2(e_i), \quad (24)$$

where α can be chosen small enough or k_{vi} can be chosen large enough to satisfy the inequality,

$$k_{vi}\bar{c}_i - \alpha\lambda_m > 0, \quad (25)$$

and $\lambda_m \triangleq \lambda_{max}[(J^+(q))^T(M_0 + M(q))J^+(q)]$, $\lambda_{max}[A]$ denotes the maximum eigenvalue of a matrix A. Note that α must be chosen sufficiently small or K_v must be chosen sufficiently large to guarantee the positive definiteness of $V(s(e), \dot{q}, z)$. Since $g(q) = \frac{\partial P(q)}{\partial q}$ and $P(\cdot)$ are trigonometric functions of q, it is possible to choose K_p large enough so that the following conditions are satisfied for a sufficiently small k.[8]

$$P(q) - P(q_d) - \Delta q^T g(q_d) + \sum_{i=1}^{m} k_{pi} S_i(e_i) \geq k\|s(e)\|^2 \quad (26)$$

$$s(e)^T(J^+(q))^T\{g(q) - g(q_d)\} + s(e)^T K_p s(e) \geq k\|s(e)\|^2 \quad (27)$$

If we substitute equations (24) and (26) into equation (19), we have,

$$V(s(e), \dot{q}, z) \geq \frac{1}{4}\dot{q}^T(M_0 + M(q))\dot{q} + \sum_{i=1}^{m} \{\alpha(k_{vi}\bar{c}_i - \alpha\lambda_m)\}s_i^2(e_i) + k\|s(e)\|^2 + \frac{1}{2}z^T K\hat{K}^{-1}K_I z > 0. \quad (28)$$

Hence, $V(s(e), \dot{q}, z)$ is a Lyapunov function candidate for the setpoint control problem with an approximate actuator model. Next, equation (23) is expressed as:

$$W(s(e), \dot{q}) = \dot{q}^T(J^T(q)K_vJ(q) + B_0)\dot{q} - \dot{q}^T\Delta_k J^T(q)K_vJ(q)\dot{q} - \dot{q}^T\Delta_k J^T(q)K_p s(e) + \alpha s^T(e)(J^+(q))^T(g(q) - g(q_d)) + \alpha s^T(e)K_p s(e) - \alpha s^T(e)J^+(q))^T\Delta_k J^T(q)K_p s(e) - \alpha s^T(e)(J^+(q))^T\Delta_k J^T(q)K_v J(q)\dot{q} + \alpha\{s(e)^T(J^+(q))^T\left[B_0 - \frac{1}{2}\dot{M}(q) + S(q, \dot{q}) \right]\dot{q} - \dot{s}(e)^T(J^+(q))^T(M_0 + M(q))\dot{q} - s(e)^T(\dot{J}^+(q))^T(M_0 + M(q))\dot{q}\}, \quad (29)$$

where $\Delta_k = I - K\hat{K}^{-1}$. From the last term of equation (29), since $s(e)$ is bounded, there exist constants $c_0 > 0$ and $c_1 > 0$ so that,[8]

$$\alpha\left| s(e)^T(J^+(q))^T\left[B_0 - \frac{1}{2}\dot{M}(q) + S(q, \dot{q}) \right]\dot{q} - \dot{s}(e)^T(J^+(q))^T(M_0 + M(q))\dot{q} - s(e)^T(\dot{J}^+(q))^T(M_0 + M(q))\dot{q} \right| \geq -\alpha c_0\|\dot{q}\|^2 - \alpha c_1\|s(e)\|^2. \quad (30)$$

Substituting equations (27) and (30) into equation (29), we have

$$W(s(e), \dot{q}) \geq \dot{q}^T(J^T(q)K_vJ(q) + B_0 - c_0 I)\dot{q} + \alpha(k - c_1)s^T(e)s(e) - \alpha s^T(e)(J^+(q))^T\Delta_k J^T(q)K_p s(e) - \dot{q}^T\Delta_k J^T(q)K_v J(q)\dot{q} - \dot{q}^T\Delta_k J^T(q)K_p s(e) - \alpha s^T(e)(J^+(q))^T\Delta_k J^T(q)K_v J(q)\dot{q}, \geq \{\lambda_{min}[J^T(q)K_vJ(q) + B_0] - \bar{p}c_2\lambda_{max}[K_v] - \alpha c_0\}\|\dot{q}\|^2 + \alpha(k - \bar{p}c_3\lambda_{max}[K_p] - c_1)\|s(e)\|^2 - \bar{p}b_{JT}(\lambda_{max}[K_p] + \alpha c_4\lambda_{max}[K_v])\|s(e)\| \cdot \|\dot{q}\|,$$

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where $c_2=b_{JT}b_J$, $c_3=b_{J+}b_{JT}$, $c_4=b_{J+}b_J$, b_{JT}, b_J, b_{J+} are the norm bounds for $J^T(q)$, $J(q)$, $(J^+(q))^T$ respectively. Since,

$$-\|s(e)\|\cdot\|\dot{q}\|\geq-\frac{1}{2}(\|s(e)\|^2+\|\dot{q}\|^2), \tag{31}$$

we have

$$W(s(e),\dot{q})\geq(\lambda_{max}[K_v]l_a-\alpha c_0)\|\dot{q}\|^2 + (\lambda_{max}[K_v]l_b-\alpha c_1)\|s(e)\|^2, \tag{32}$$

where

$$l_a=(\hat{\lambda}_1-\frac{p}{2}(2c_2+b_{JT}(a_1+\alpha c_4)),$$

$$l_b=\alpha a_1\frac{k}{\lambda_{max}[K_p]}-\frac{p}{2}(2c_3\alpha a_1+b_{JT}(a_1+\alpha c_4)),$$

$$\hat{\lambda}_1=\frac{\lambda_{min}[J^T(q)K_vJ(q)+B_0]}{\lambda_{max}[K_v]},\quad a_1=\frac{\lambda_{max}[K_p]}{\lambda_{max}[K_v]} \tag{33}$$

Hence if

$$min\left\{\frac{2\hat{\lambda}_1}{b_{JT}a_1+2c_2+\alpha b_{JT}c_4},\frac{2a_1\alpha\frac{k}{\lambda_{max}[K_p]}}{(2\alpha c_3+b_{JT})a_1+\alpha b_{JT}c_4}\right\}>\bar{p}, \tag{34}$$

then $l_a>0$ and $l_b>0$ and hence K_v can be chosen large enough so that

$$l_a-\frac{\alpha c_0}{\lambda_{max}[K_v]}>0,\quad l_b-\frac{\alpha c_1}{\lambda_{max}[K_v]}>0, \tag{35}$$

and hence W is positive semidefinite.

Figure 2 shows the graphical illustration of condition (34). In order to guarantee the stability of the system, the allowable bound $\bar{p}$ of the uncertainty $I-K\hat{K}^{-1}$ should be less than the minimum of the two curves shown in the figure. Note that a larger range of a_1 can be chosen for a smaller $\bar{p}$, as seen in the figure. If $\bar{p}$ is large, a narrower range of a_1 is allowed.

We are now in a position to state the following Theorem:

Theorem. For $X\in\Omega$ *such that the Jacobian matrix is of full rank, the closed loop system described by equation (17) gives rise to the asymptotic convergence of the state* $(X, \dot{q}, z)$ *to* $(X_d, 0, 0)$, *as* $t\rightarrow\infty$, *if feedback gains* K_p *and* K_v *are chosen to satisfy the conditions (25), (26), (27), (34) and (35),* $\hat{K}$ *is chosen to satisfy the condition (13).*

Proof: *Since* $V(s(e), \dot{q}, z)$ *is positive definite in* z, $s(e)$ *and* $\dot{q}$, *and* $W(s(e), \dot{q})$ *is positive definite in* $s(e)$ *and* $\dot{q}$, *from equation (22), we have*

$$\frac{d}{dt}V(s(e),\dot{q},z)=-W(s(e),\dot{q})\leq 0.$$

Hence, V is a Lyapunov function whose time derivative is negative in $(s(e), \dot{q})$. *Since* $W(s(e), \dot{q})=0$ *means that* $\dot{q}=0$ *and* $e=X-X_d=0$, *substitution of* $\dot{q}=0$, $\ddot{q}=0$ *and* $X=X_d$ *into equation (17) leads to the conclusion that* $z=0$. *By LaSalle's invariance Theorem, the proof is completed.*

Remark 3.
The stability conditions (25), (26), (27), (34), (35), and (13), presented in the Theorem are qualitative results to guarantee the stability of the robot system in the presence of an uncertain actuator model. Since the conditions are sufficient conditions, the actual stability region may be larger. However, the conditions are simple conditions to achieve in practice. Conditions (25), (26), (27) and (35) simply mean that the feedback gains should be chosen sufficiently large. Conditions (34) and (13) imply that the feedback gain K_p should be small as compared to K_v. Hence tuning can be established easily in practice.

4. SIMULATION RESULTS

To illustrate the results, let us consider a 2-link manipulator as shown in Figure 3. The manipulator Jacobian matrix of the mapping from joint space to Cartesian space $r=[x, y]^T$ for this manipulator is given by:

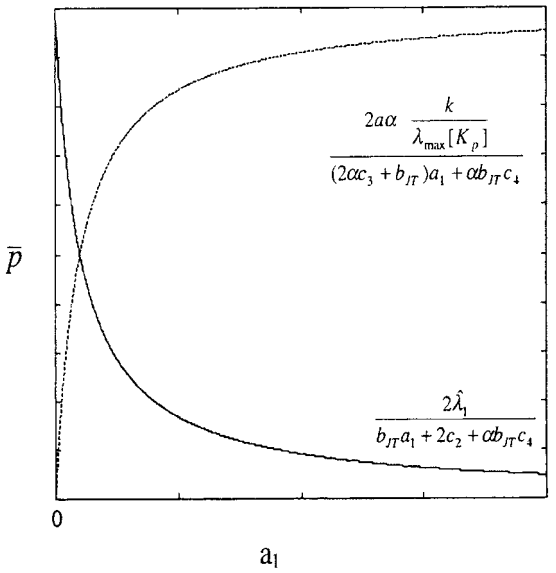


Fig. 2. Variation of $\bar{p}$ with a_1.

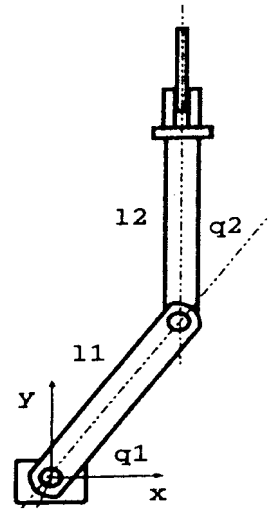


Fig. 3. A two link planar robot.

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$$J(q)=\begin{bmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} \end{bmatrix} \tag{36}$$

where $s_1=\sin(q_1)$, $c_1=\cos(q_1)$, $s_{12}=\sin(q_1+q_2)$, $c_{12}=\cos(q_1+q_2)$ and l_1, l_2, denote the lengths of the first, second links, respectively.

In the simulation, the masses m_1 and m_2 of the links were set to 2 kg, the lengths l_1 and l_2 of the links were set to 1 meter. The exact parameters of the actuator were set as $K_{\tau 1}=27$, $K_{\tau 2}=21$, $R_{a1}=1.5\Omega$, $R_{a2}=1.5\Omega$, $r_1=r_2=1$. Hence, $K=diag(K_{\tau 1}/R_{a1}r_1, K_{\tau 2}/R_{a2}r_2)=diag(18, 14)$. The robot was required to move from an initial position of $[x(0), y(0)]=[0, 1.6]$ meters to a desired position of $[x_d, y_d]=[1, 1]$ meters.

First, the position and velocity feedback gains were set as $K_p=80I$, $K_v=8I$ so that the feedback gain ratio $a_1=\dfrac{\lambda_{max}[K_p]}{\lambda_{max}[K_v]}=10$ was very large, the integration gain was set as $K_I=60I$ and α was set 1. With the actuator model poorly estimated as $\hat{K}=diag(80, 4)$, the system became oscillatory with a large ratio a_1 as shown in Figure 4 and Figure 5. However, with a better estimation of actuator model as $\hat{K}=diag(80, 4)$, the system was stable with the same gain setting (See Figure 6 and Figure 7). This agrees with the Theorem (see also Figure 2) which implies that a wider range of a_1 can be chosen only if the uncertainty is smaller. If the uncertainty is large, a_1 should be kept smaller.

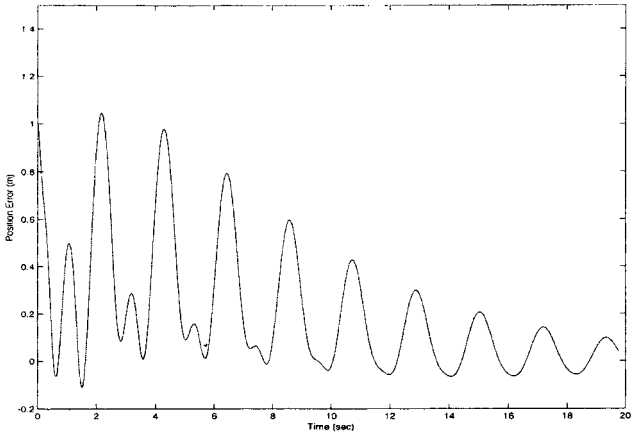


Fig. 4. Position error $x_d - x$ with an approximate actuator model $\hat{K}=diag(80, 4)$ $(a_1=10)$.

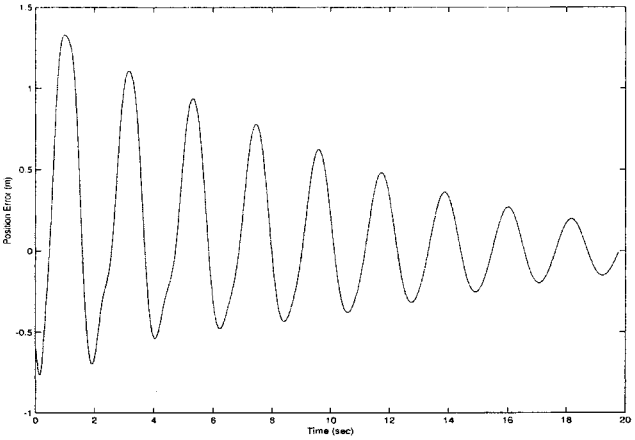


Fig. 5. Position error $y_d - y$ with an approximate actuator model $\hat{K}=diag(80, 4)$ $(a_1=10)$.

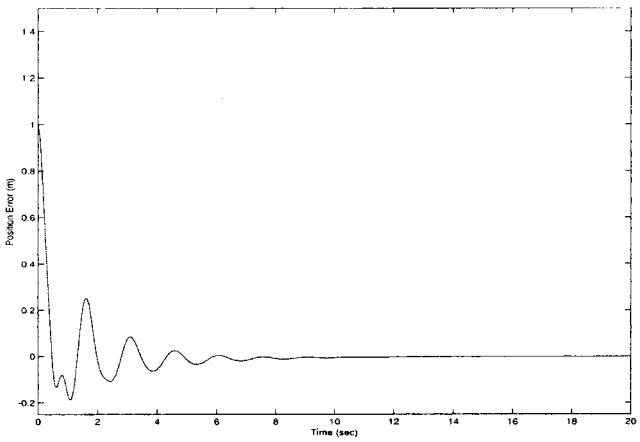


Fig. 6. Position error $x_d - x$ with an approximate actuator model $\hat{K}=diag(40, 8)$ $(a_1=10)$.

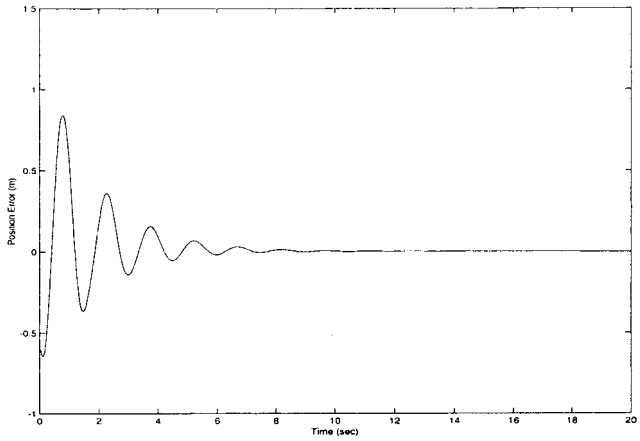


Fig. 7. Position error $y_d - y$ with an approximate actuator model $\hat{K}=diag(80, 4)$ $(a_1=10)$.

Second, to examine the effects of a_1 on the stability of the system with an uncertain actuator model, a_1 was varied with the actuator model poorly estimated as $\hat{K}=diag(80, 4)$. With $K_p=80I$, $K_v=40I$ $(a_1=2)$, $K_I=60$ and $\alpha=1$, the system became stable as shown in Figure 8 and Figure 9. Similarly, with a smaller $a_1=1$ by setting $K_p=80I$, $K_v=80I$, $K_I=60$ and $\alpha=1$, the system was also stable, as shown in Figure 10 and Figure 11.

Next, $\hat{K}$ as varied to examine the effects of actuator model on the stability of the robot's motion. Simulation results with $\hat{K}=diag(24, 24)$, $K_p=80I$, $K_v=80I$ $(a_1=1)$, $K_I=60$ and $\alpha=1$ are shown in Figure 12 and Figure 13. With $\hat{K}=diag(10, 10)$, $K_p=80I$, $K_v=80I$ $(a_1=1)$, $K_I=60$ and $\alpha=1$, the position errors were plotted as shown in Figure 14 and Figure 15.

In this paper, we assume that the robot is operating in a workspace $X\in\Omega$ such that the Jacobian matrix is of full rank. To illustrate the effects of singularity on the proposed controller, the robot was required to move from an initial position near singularity at $[q_1(0), q_2(0)]=[90, 0.1]$ degrees or $[x(0), y(0)]=[1.99985, -0.01745]$ meters to the desired

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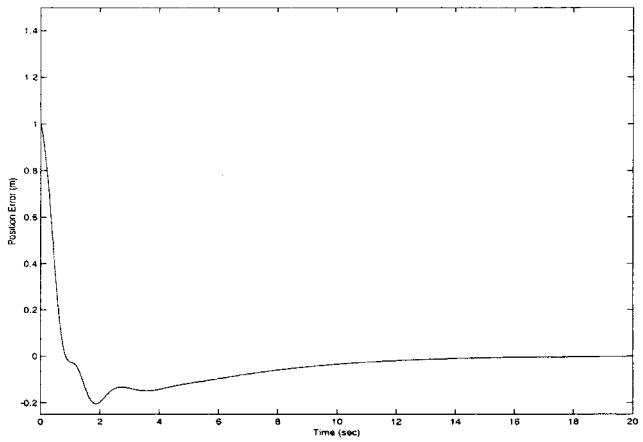


Fig. 8. Position error $x_d - x$ with an approximate actuator model $\hat{K} = diag(80, 4)$ $(a_1 = 2)$.

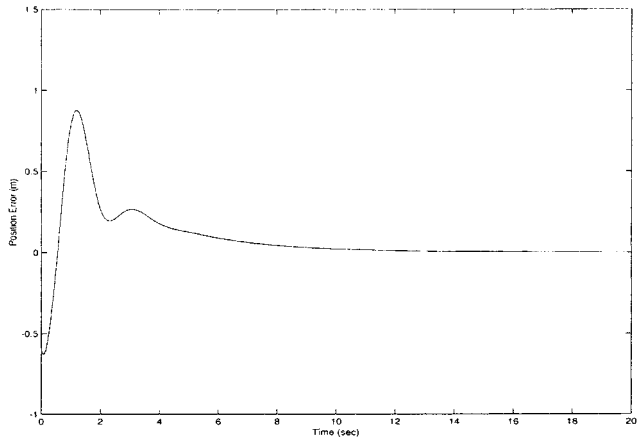


Fig. 9. Position error $y_d - y$ with an approximate actuator model $\hat{K} = diag(80, 4)$ $(a_1 = 2)$.

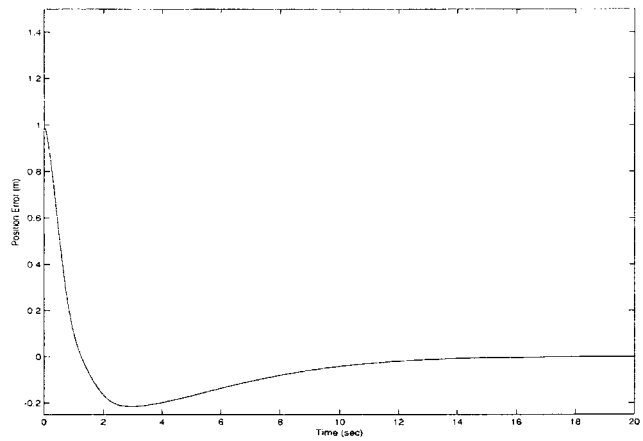


Fig. 10. Position error $x_d - x$ with an approximate actuator model $\hat{K} = diag(80, 4)$ $(a_1 = 1)$.

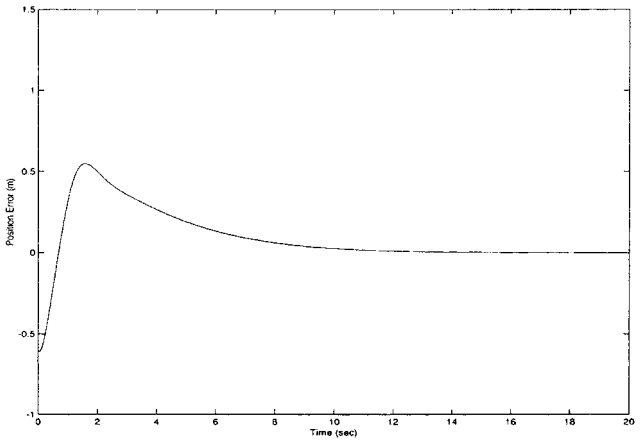


Fig. 11. Position error $y_d - y$ with an approximate actuator model $\hat{K} = diag(80, 4)$ $(a_1 = 1)$.

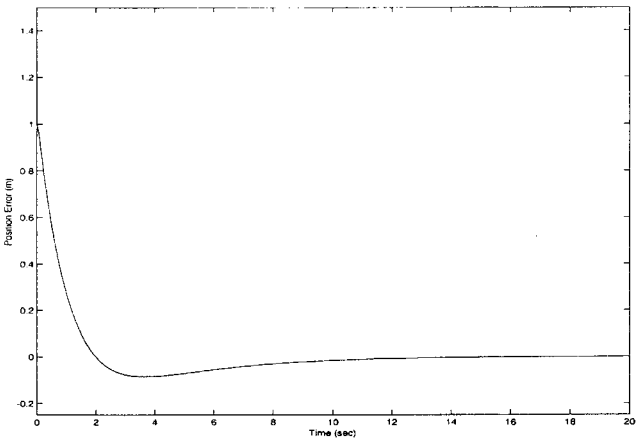


Fig. 12. Position error $x_d - x$ with an approximate actuator model $\hat{K} = diag(24, 24)$ $(a_1 = 1)$.

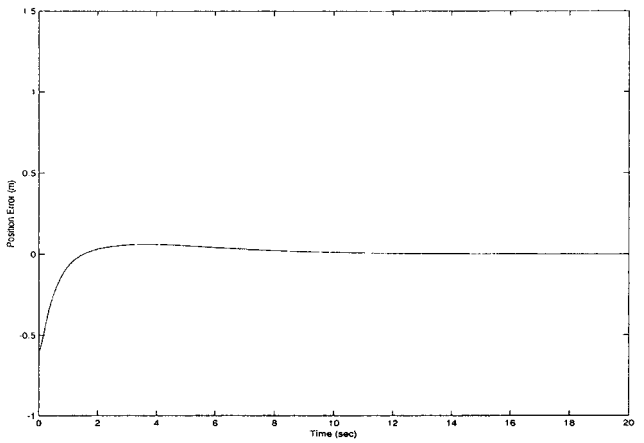


Fig. 13. Position error $y_d - y$ with an approximate actuator model $\hat{K} = diag(24, 24)$ $(a_1 = 1)$.

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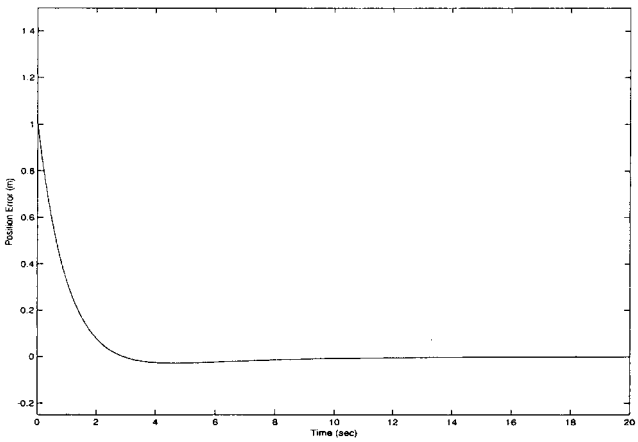


Fig. 14. Position error $x_d - x$ with an approximate actuator model $\hat{K}=diag(10, 10)$ $(a_1=1)$.

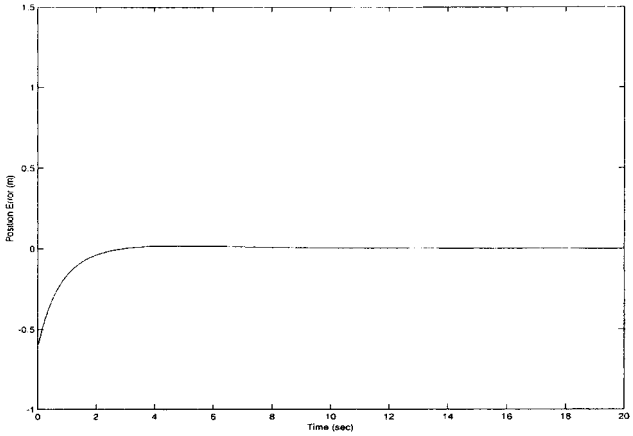


Fig. 15. Position error $y_d - y$ with an approximate actuator model $\hat{K}=diag(10, 10)$ $(a_1=1)$.

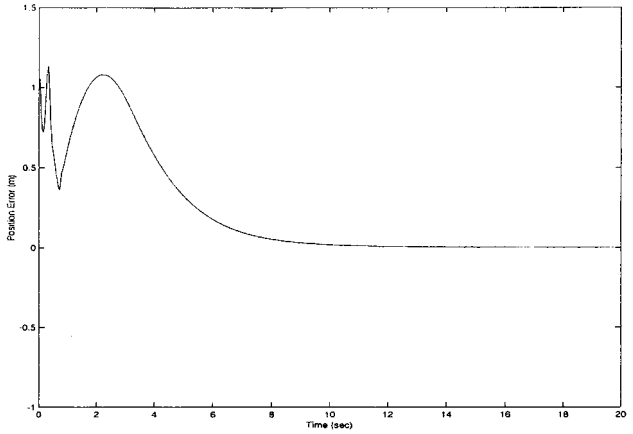


Fig. 16. Position error $x_d - x$ starting near a singular point and with $\hat{K}=diag(10, 10)$ $(a_1=1)$.

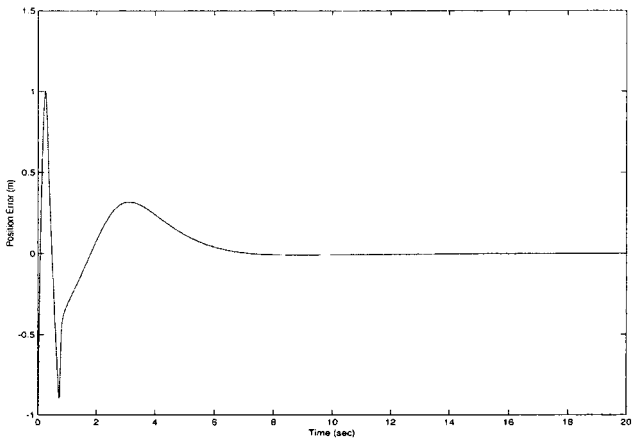


Fig. 17. Position error $y_d - y$ starting near a singular point and with $\hat{K}=diag(10, 10)$ $(a_1=1)$.

position of $[x_d, y_d] = [1,1]$ meters. With $\hat{K}=diag(10, 10)$, $K_p=80I$, $K_v=80I$ $(a_1=1)$, $K_I=60$ and $\alpha=1$, the results in Figure 16 and Figure 17 show stable responses with initial position near singular point. Note that the initial motion near the singular point was oscillating because the determinant of the Jacobian matrix was very small and hence the inverse of the Jacobian matrix was very large. As seen from equation (15), since $y=\dot{q}+\alpha J^{+}(q)s(e)$, the inverse of the

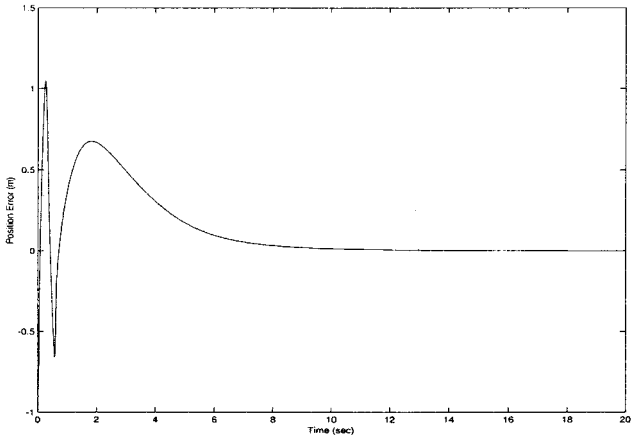


Fig. 18. Position error $x_d - x$ starting near a singular point and with $\hat{K}=diag(10, 10)$ $(a_1=1)$.

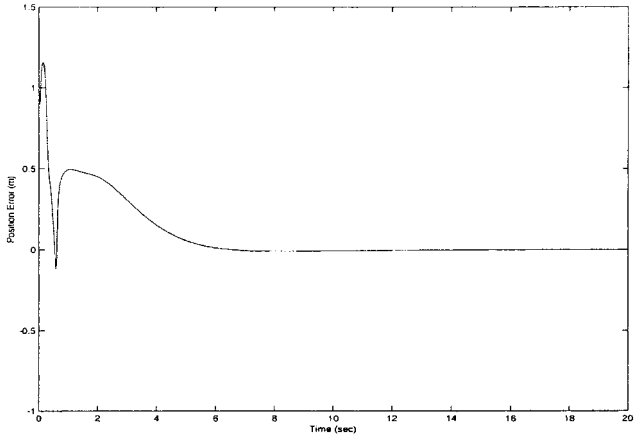


Fig. 19. Position error $y_d - y$ starting near a singular point and with $\hat{K}=diag(10, 10)$ $(a_1=1)$.

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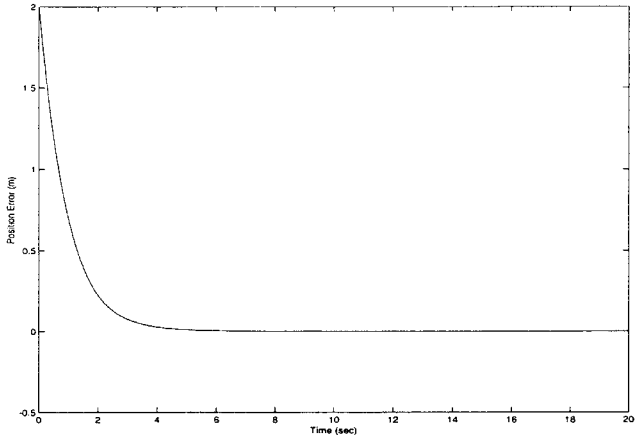


Fig. 20. Position error $x_d - x$ ending near a singular point and with $\hat{K} = diag(10, 10)$ $(a_1 = 1)$.

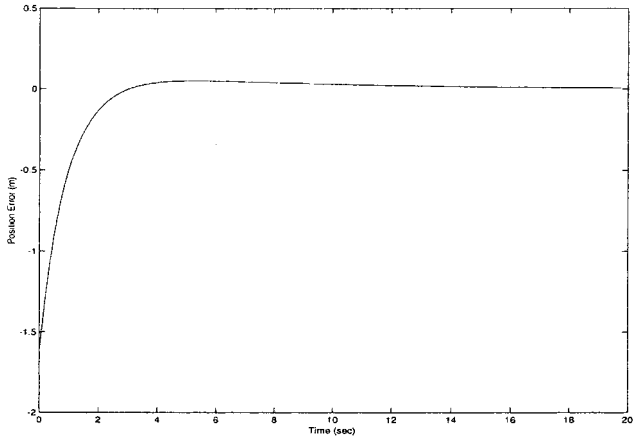


Fig. 21. Position error $y_d - y$ ending near a singular point and with $\hat{K} = diag(10, 10)$ $(a_1 = 1)$.

Jacobian matrix multiplied by a large initial error $e(0)$ would result in a very large control action. Figure 18 and Figure 19 show a similar response with the initial point near another singular point at $[q_1(0), q_2(0)] = [0, 0.1]$ degrees or $[x(0), y(0)] = [0.0017, 1.999998]$.

Finally, the robot was required to move from the initial position of $[x(0), y(0)] = [0, 1.6]$ meters to a desired position near singularity at $[x_d, y_d] = [1.99999, 0]$ meters. The results in Figure 20 and Figure 21 show a stable response. As compared with the results that started near singular points (see Figure 16 to Figure 19), we noted that the result that ended near singular did not oscillate (see Figure 20 and Figure 21). This was because when the position error e and joint velocity $\dot{q}$ converged to zero, the variable $y = \dot{q} + \alpha J^{+}(q)s(e)$ in equation (15) was finite though the inverse of the Jacobian matrix was very large. That is, the inverse of the Jacobian matrix multiplied by $e \approx 0$ would result in a finite control action. However, the system may become unstable if the integration of y continues.

5. CONCLUSIONS

In this paper, we have proposed a task-space controller for the setpoint control of a robotic manipulator with an uncertain actuator model. The main advantage of the proposed control law is that an exact knowledge of the actuator model is not required. We have shown that asymptotic stability can be guaranteed even with an uncertain actuator model. Sufficient conditions for choosing the feedback gains are presented to guarantee stability. Simulation results illustrate the performance of the proposed controller. Future works would be devoted to extending the result to deal with uncertainties in both the Jacobian matrix and actuator model. It is also interesting to derive a quantitative result for the bounds of uncertainties.

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