Solar sails 10

A solar sail is propelled by the pressure of photons emitted by the Sun or other star. Unlike a rocket, a sail needs to carry neither propellant nor an energy source. Although photons are emitted radially away from the Sun, tilting a sail produces a continuous non-radial force, and thus permits non-Keplerian orbits. In this chapter we explore the basic properties of solar sails and their orbits. Photon pressure is low, meaning sails must be light, so we also look at the demanding requirements on sail materials. Solar sails have already been deployed and tested in space, although not yet used to achieve significant propulsion. We look briefly at some past and future missions.

10.1 Radiation pressure

Consider a single photon incident normally on a flat surface. The energy E and momentum p of the photon relative to the surface are related by $E = pc$, where c is the speed of light. When the photon is absorbed by the surface, then from Newton's second law the force imparted by the photon on the surface is

$$F = \dot{p} = \frac{\dot{E}}{c} = \frac{IA}{c} \quad \text{(photons absorbed)}, \tag{10.1}$$

where in the last step we have written the rate of energy absorption in terms of the intensity I (units $\mathrm{W\,m^{-2}}$) and cross-sectional area A. The expression above tells us that the force per unit power of an absorbed photon is very small, just $1/c = 3.3\,\mathrm{nN\,W^{-1}}$. Equation 10.1 also applies to an ensemble of photons of different energies incident at different rates, with total intensity I.

As the pressure P is force per unit area, it follows from equation 10.1 that

$$P = \frac{I}{c} \quad \text{(photons absorbed)}. \tag{10.2}$$

When the photons are perfectly reflected by the surface, their change of momentum is twice that given in equation 10.1, and so

$$P = 2\frac{I}{c} \quad \text{(photons reflected)}. \tag{10.3}$$

At 1 au from the Sun, the solar intensity is $I = 1361\,\mathrm{W\,m^{-2}}$, and so the pressure imparted when photons across all wavelengths are reflected is $P = 9.1\,\mu\mathrm{Pa}$.

Perfect reflection means that the reflected photons have the same energy, and therefore same magnitude of momentum, as the incoming photons. But a reflection changes the direction of the photons, so due to conservation of momentum they must have imparted some momentum to the surface. Perfect reflection can only occur if the surface has infinite mass. For a finite mass sail, the reflected photons must actually lose some of their energy, as we

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will see in section 11.1.1 when we consider laser sails. But for the small velocity changes of solar sails we can ignore this, and in practice we can get very close to perfect reflection. Later in this chapter we'll anyway generalize some expressions to the case of imperfect reflection.

10.2 Forces on a sail

So far we considered photons incident normal to a surface. The more general case is shown in figure 10.1 for a flat sail of area A. We can think of the incoming photons as being absorbed by the sail and the reflected photons as being emitted by the sail (which at a microphysical level is closer to what really happens). The incoming photons exert a force $\mathbf{F}_i$ on the sail in direction $\hat{\mathbf{u}}_i$ (hats denote unit vectors). If the intensity of the incoming photons is I, then their pressure is I/c (equation 10.2). The projected area of the sail is $A\cos\alpha$. Hence the force exerted by the incoming photons on the sail is

$$\mathbf{F}_i = \frac{IA}{c}\cos\alpha\,\hat{\mathbf{u}}_i. \tag{10.4}$$

By a similar argument the force on the sail due to the reflected photons is

$$\mathbf{F}_r = -\frac{IA}{c}\cos\alpha\,\hat{\mathbf{u}}_r. \tag{10.5}$$

From trigonometry we see that $\hat{\mathbf{u}}_i - \hat{\mathbf{u}}_r = 2\cos\alpha\,\hat{\mathbf{n}}$, where $\hat{\mathbf{n}}$ is the direction normal to the sail. Thus the net force exerted by photons reflecting perfectly off the sail is

$$\mathbf{F} = \mathbf{F}_i + \mathbf{F}_r = \frac{2IA}{c}\cos^2\alpha\,\hat{\mathbf{n}}. \tag{10.6}$$

Note the $\cos^2\alpha$ factor. We might have expected simply a $\cos\alpha$ factor on the basis of this being the projection of the forces along the normal, but there is an extra factor due to the projected area of the sail. There are also components of force along the surface of the sail (in the plane of the page) but these cancel out. In the limit of normal incidence, $\alpha = 0$, equation 10.6 reproduces equation 10.3 (as $F = PA$). The main thing to notice here is that the net force on the sail is in the direction $\hat{\mathbf{n}}$, which is not parallel to the incident radiation. This means that if

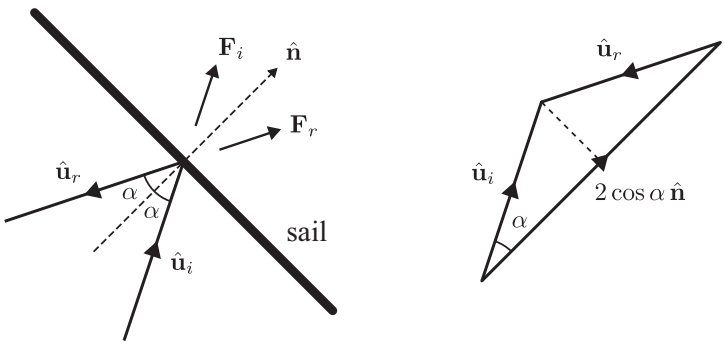


Fig. 10.1 Forces on a solar sail. Left: A photon wavefront with normal $\hat{\mathbf{u}}_i$ at an angle α to the normal $\hat{\mathbf{n}}$ of the sail is perfectly reflected into the direction $\hat{\mathbf{u}}_r$. Due to conservation of momentum, the incident and reflected wavefronts impart forces of $\mathbf{F}_i$ and $\mathbf{F}_r$, respectively, on the sail. Right: Vector triangle $\hat{\mathbf{u}}_i - \hat{\mathbf{u}}_r = 2\cos\alpha\,\hat{\mathbf{n}}$. Left panel adapted from McInnes (1999) with permission of Springer Nature BV.

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we tilt the normal of the sail away from the radial direction to the Sun, there is a non-central force on the sail. This can produce some interesting non-Keplerian orbits, which we will examine in section 10.3.2.

For a sail of mass m, the magnitude of its acceleration from the reflected photons is

$$a = \frac{2I\cos^2\alpha}{c\sigma} \tag{10.7}$$

where $\sigma = m/A$ is the mass per unit area of the sail, also known as the *loading parameter*. When the sail is 1 au from the Sun, $I = 1361\,\mathrm{W\,m^{-2}}$, and so the maximum acceleration (when $\alpha = 0$) is

$$a_{\mathrm{rad}} = \frac{2I}{c\sigma} = \frac{9.1\times10^{-6}}{\sigma}\,\mathrm{m\,s^{-2}}, \tag{10.8}$$

with σ in $\mathrm{kg\,m^{-2}}$. This compares to the acceleration of the sail due to the Sun's gravity at the same distance of $a_{\mathrm{grav}} = G\mathrm{M}_\odot/r^2 = 5.9\times10^{-3}\,\mathrm{m\,s^{-2}}$. As the Sun's gravity and radiation pressure are the two main forces acting on an deep space solar sail, a useful parameter is the ratio of these two accelerations. The Sun's intensity can be written as

$$I = \frac{\mathrm{L}_\odot}{4\pi r^2}, \tag{10.9}$$

where $\mathrm{L}_\odot$ is the solar luminosity (units W). The ratio of the accelerations is therefore

$$\lambda = \frac{a_{\mathrm{rad}}}{a_{\mathrm{grav}}} = \frac{\mathrm{L}_\odot}{2\pi Gc\mathrm{M}_\odot}\frac{1}{\sigma} = \frac{1.53\times10^{-3}}{\sigma}. \tag{10.10}$$

This ratio λ is known as the *lightness number*. It is independent of distance because both the solar radiation pressure and the Sun's gravity vary as $1/r^2$. Indeed, for a given star – $\mathrm{L}_\odot$ and $\mathrm{M}_\odot$ – the lightness number depends only on the mass per unit area of the sail, σ. The larger λ, the larger the acceleration from radiation compared to gravity, and so the 'lighter' the sail.

From equation 10.10 we see that the loading parameter required to balance gravity in the solar system (i.e. to achieve $\lambda = 1$) is $\sigma = 1.53\times10^{-3}\,\mathrm{kg\,m^{-2}}$, or 1.53 grammes per square metre. This is small. For comparison, plastic food wrap has a volume density of about $1000\,\mathrm{kg\,m^{-3}}$ (similar to water) and a thickness of about $10\,\mu\mathrm{m}$, and thus an area density of around $10^{-2}\,\mathrm{kg\,m^{-2}}$, around seven times larger. The sail material should also be highly reflective. Furthermore, σ is the average loading parameter of the entire spacecraft, including the supporting structures and payload. This makes it clear that a solar sail spacecraft will require both extremely light materials and very large sails to compensate for the payload and structural mass. We will examine possible materials in section 10.5.

It is evident from equation 10.10 that a given sail acquires a larger net acceleration when orbiting a star of larger luminosity and/or lower mass than the Sun. For stars on the main sequence, which produce energy in their cores by the fusion of hydrogen to helium, the luminosity is related to the mass approximately as $L \propto M^{3.9}$ for lower-mass stars. Equation 10.10 then tells us that $\lambda \propto M^{2.9}/\sigma$ for such stars. Thus a given sail orbiting a main sequence star that is twice as massive as the Sun would have a lightness number 7.4 times larger. Solar sailing would be much more favourable around such stars. Most stars are less massive than the Sun, and for the lowest-mass stars the luminosity drops very rapidly. For example, Proxima Cen has $L/M = 0.0017/0.12 = 0.014$ in solar units, and so the lightness number of a given sail is about 70 times lower at this star than it is at the Sun.

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10.3 Solar sail orbits

We can now write down the equation of motion for a solar sail of mass m in the gravitational and radiation field of a star of mass M. The gravitational force is $-(GMm/r^2)\,\hat{\mathbf{r}}$, where $\hat{\mathbf{r}}$ is the unit vector in the radial direction away from the star. Using the definition of the lightness number λ from above, the radiation force is $(GMm/r^2)\lambda\cos^2\alpha\,\hat{\mathbf{r}}$. From Newton's second law, the vectorial equation of motion of the sail relative to the star is

$$\ddot{\mathbf{r}} = -\frac{\mu}{r^2}\,\hat{\mathbf{r}} + \frac{\mu}{r^2}\lambda(\hat{\mathbf{r}}\cdot\hat{\mathbf{n}})^2\hat{\mathbf{n}} \quad (\lambda \geq 0), \tag{10.11}$$

where I have used $\cos\alpha = \hat{\mathbf{r}}\cdot\hat{\mathbf{n}}$ and $\mu = G(M+m) \simeq GM$. In general, $\hat{\mathbf{n}} \neq \hat{\mathbf{r}}$, so the acceleration is not in the radial direction, and the orbits will not be Keplerian.

In all of this we are implicitly assuming that the Sun is a point source of radiation. In reality the Sun has a finite extent, so the radiation pressure is not exactly radial, with a consequence that the inverse square law for the radiation pressure is only an approximation. This becomes more significant the closer the sail is to the Sun. We will ignore this here, but for accurate or practical calculations this must be taken into account.

10.3.1 The sail pointed at the Sun: Keplerian orbits

To solve the equation of motion, let us first consider the simpler case in which the sail is kept pointed at the Sun at all times, $\hat{\mathbf{n}} = \hat{\mathbf{r}}$. Equation 10.11 then becomes

$$\ddot{\mathbf{r}} = -\frac{\mu(1-\lambda)}{r^2}\,\hat{\mathbf{r}} \quad (\lambda \geq 0). \tag{10.12}$$

This is the same as the equation of motion for Keplerian orbits in which the mass of the central body has been decreased by a factor of $(1-\lambda)$. The sail acts to effectively decrease the gravity, and the lightness number introduces a new degree of freedom into the classification of orbits. We can simply replace μ with $\mu(1-\lambda)$ in some of the equations for Keplerian orbits. For a circular orbit, for example, the orbital speed is $\sqrt{\mu(1-\lambda)/r}$, and so will be smaller for a sail than for a non-sail body at the same distance. Whether or not a particular spacecraft is actually on a circular orbit or not depends also on the initial conditions, of course. When $\lambda = 1$ the net force is zero, so the sail moves on a straight line or remains at rest relative to the star. When $\lambda > 1$ the radiation force is larger than gravity, so the net force points outwards, and the sail moves either radially outwards (if starting at rest) or on a hyperbolic orbit.

We can understand the orbits of a solar sail better if we consider how we might use the sail to achieve interstellar travel. Consider a spacecraft in a circular orbit of radius r about a star. The orbital speed is $\sqrt{\mu/r}$, and the escape velocity is $\sqrt{2\mu/r}$ (section 4.1.4). If this spacecraft now unfurls a solar sail such that the sail plus spacecraft has a lightness number of λ, and points this sail at the Sun, the speed of the spacecraft does not change in that instant, but its escape velocity has been reduced to $\sqrt{2\mu(1-\lambda)/r}$. So if $\lambda = 1/2$, then the spacecraft's velocity is now equal to its escape velocity and the spacecraft will begin to move on a parabolic orbit. If $\lambda < 1/2$ the spacecraft will remain bound to the star on an elliptical orbit. For $\lambda > 1/2$ the spacecraft will move on a hyperbolic orbit, and will reach infinity with a non-zero velocity, paving the way for interstellar travel.

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For this last case, how fast will the spacecraft be when it gets very far from the Sun? We can address this using conservation of energy. Suppose the spacecraft is initially a distance r_i from the Sun moving with velocity v_i. The specific potential energy once the sail is open and pointed at the Sun is $-\mu(1-\lambda)/r$ and the specific kinetic energy is $v_i^2/2$. At infinity the potential energy is zero, and the spacecraft is moving with velocity v_∞. Equating the initial and final energies we get

$$v_\infty^2 = v_i^2 + \frac{2\mu(\lambda-1)}{r_i}\,. \tag{10.13}$$

When $\lambda > 1$, then the closer the spacecraft is to the Sun initially, the faster the spacecraft will be at infinity. Even though a spacecraft closer to the Sun has to overcome more gravity to escape, the higher radiation pressure more than compensates in this case. Even when $\lambda < 1$, a spacecraft initially on a Keplerian orbit that is closer to the Sun also has a larger velocity v_i, and so v_∞ could still be larger. For the specific case of a spacecraft initially on a circular orbit, we have $v_i^2 = \mu/r_i$ before it opens its sail, and so the above equation becomes

$$v_\infty^2 = \frac{2\mu}{r_i}\left(\lambda - \frac{1}{2}\right) \quad \text{(initial circular orbit)}. \tag{10.14}$$

This only gives a real value – the sail only reaches infinity – if $\lambda \geq 1/2$, but then a smaller r_i gives a larger v_∞. For $\lambda = 1$, equation 10.14 yields $v_\infty = \sqrt{\mu/r_i}$, which for $r_i = 1$ au from the Sun gives $v_\infty = 29.8\,\mathrm{km\,s^{-1}}$. But if we started on a smaller orbit of $r_i = 0.5$ au, then the sail achieves $v_\infty = 42.1\,\mathrm{km\,s^{-1}}$. For a general initial Keplerian orbit we can use the vis-viva equation (equation 4.3) to substitute for v_i in equation 10.13 to get

$$v_\infty^2 = \mu\left(\frac{2\lambda}{r_i} - \frac{1}{a}\right)\,. \tag{10.15}$$

This motivates the idea of the sundiver, which we will examine in section 10.7.

As λ is defined as the ratio of the solar photon force to the gravitational force, we might think that $\lambda > 1$ is required for the spacecraft to escape to infinity. But this is only the case if the spacecraft is initially at rest. If it is initially moving, then it already has kinetic energy, so less solar photon energy is required for escape, which can therefore be achieved with a lower lightness number.

10.3.2 The sail inclined: non-Keplerian orbits

When the sail is not pointed at the Sun, the net force on the sail is not radial and so its orbit is not Keplerian. Equation 10.11 then has an extra degree of freedom, namely how the angle α varies over time, and so the equation of motion generally does not have a closed-form solution. A special case is when α is held constant. The orbit is then described by

$$r(\theta) = r_0 \exp(\theta \tan\gamma), \tag{10.16}$$

where γ is the angle between the direction of travel and the tangential direction at position (r, θ) (figure 10.2). The angle γ is a function of α and λ and must be solved for numerically. This orbit is called a 'logarithmic spiral' orbit. An example is shown in figure 10.3 for $\lambda = 0.3$ for $\alpha = \pm 10°$.

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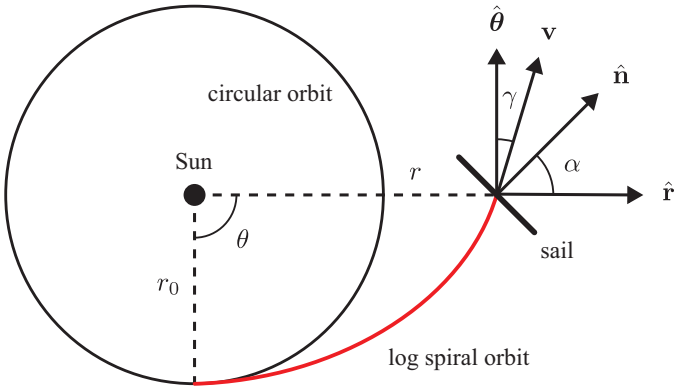


Fig. 10.2 The configuration of a solar sail in a logarithmic spiral orbit as described by equations 10.16 and 10.17. Definitions: $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\theta}}$ are the radial and tangential unit vectors respectively; $\hat{\mathbf{n}}$ is the normal to the sail; $\mathbf{v}$ is the direction of travel of the sail. The sail moves at an angle γ from $\hat{\boldsymbol{\theta}}$. The configuration shown here has a positive inclination angle α of $\hat{\mathbf{n}}$ relative to $\hat{\mathbf{r}}$, so the sail spirals outwards. Adapted from McInnes (1999) with permission of Springer Nature BV.

Denoting the radial and tangential components of the sail's velocity with v_r and v_θ respectively, then

$$v_r = v_{\text{tot}} \sin\gamma \,, \tag{10.17a}$$

$$v_\theta = v_{\text{tot}} \cos\gamma, \quad \text{where} \tag{10.17b}$$

$$v_{\text{tot}} = k\sqrt{\frac{\mu}{r}} \quad \text{and} \tag{10.17c}$$

$$k = \sqrt{1 - \lambda \cos^2\alpha(\cos\alpha - \sin\alpha \tan\gamma)}. \tag{10.17d}$$

Note that k is a positive constant depending only on λ and α. For positive values of α, a component of the solar radiation pressure acts to increase the speed of the sail in the direction of travel and the sail moves outwards. For negative values of α the opposite happens. The total velocity, as well as the radial and tangential components, are monotonically decreasing functions of the radius in both cases, as we see in figure 10.3.

As the force is no longer central, the potential energy of the sail is a more complicated function than for Keplerian orbits. Furthermore, the total energy is no longer a constant of the orbit, although it still only depends on r and not θ. When the sail moves outwards its total energy increases.

A particular property of the logarithmic spiral orbit is that when $\alpha > 0$, the sail will spiral outwards indefinitely. In the limit of infinite time it will reach infinite distance, albeit with zero velocity. Note, however, that the velocity of the logarithmic spiral orbit is not equal to the Keplerian velocity at that radius, neither in magnitude nor direction. So to move a spacecraft onto a logarithmic spiral orbit from a Keplerian one, we would need to apply additional Δv boosts.

10.4 Imperfectly reflecting sails

So far we have considered perfectly reflecting sails. In practice, some of the photons can be absorbed by the sail or, if it is thin enough, transmitted through the sail. Let $\tilde{r}$, $\tilde{a}$, and $\tilde{t}$ be the reflectance, absorptance, and transmittance of the sail, respectively, where

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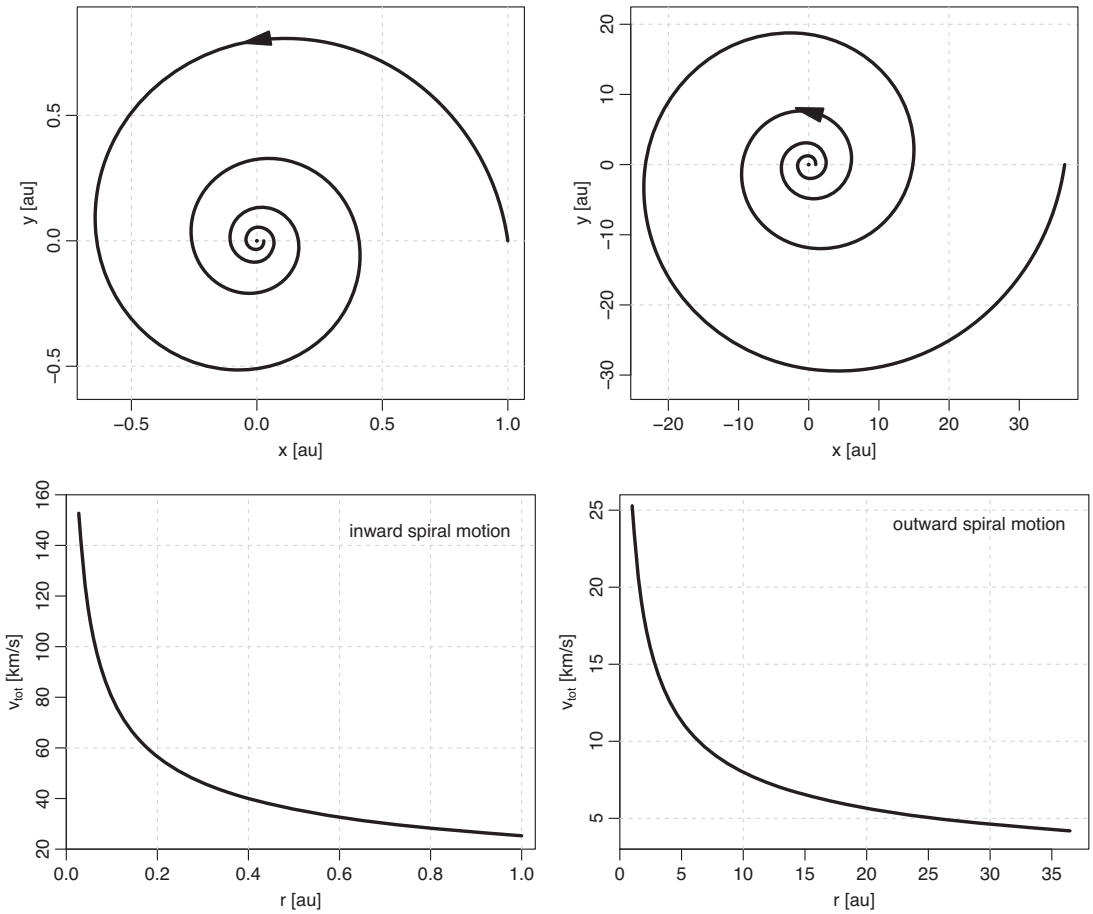


Fig. 10.3

Logarithmic spiral orbits of a solar sail with $\lambda = 0.3$ for $\alpha = -10°$ ($\gamma = -8.14°$, left), and $\alpha = +10°$ ($\gamma = +8.14°$, right) starting at $(r, \theta) = (1\text{ au}, 0°)$ and computed over four revolutions ($\Delta\theta = 4 \times 360°$). Both cases have $k = 0.840$ (equation 10.17). The top panels show the orbit in Cartesian coordinates; both plots have 1:1 aspect ratios but different scales. The bottom panels show how the total velocity varies with radial distance.

$$\tilde{r} + \tilde{a} + \tilde{t} = 1. \tag{10.18}$$

I use the tilde symbols for several quantities in this chapter to avoid duplication of notation. If we return to equation 10.4 for the force imparted by the incident photons, then as a fraction $\tilde{t}$ now pass through the sail, we need to include a factor $1 - \tilde{t}$ on the right side of the equation for the incident force. Similarly, only a fraction $\tilde{r}$ are reflected, so we need to include a factor $\tilde{r}$ on the right side of equation 10.5 for the reflected force. In the case of normal incidence ($\alpha = 0$), the magnitude of the pressure on the sail becomes

$$P = (1 - \tilde{t} + \tilde{r})\frac{I}{c} = (2\tilde{r} + \tilde{a})\frac{I}{c}. \tag{10.19}$$

This generalizes equations 10.2 and 10.3.

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Equation 10.19 tells us that imperfect reflection reduces the radiation pressure. Furthermore, in the general case of non-normal incidence shown in figure 10.1, now that the magnitudes of the forces in the $\hat{\mathbf{u}}_i$ and $\hat{\mathbf{u}}_r$ directions are different, the net force is no longer parallel to $\hat{\mathbf{n}}$. In other words, when a tilted sail reflects imperfectly, the sail does not accelerate along its normal.

The above generalization is a bit simplified, because the photons that are absorbed by the sail will, once the sail reaches thermal equilibrium, be re-emitted. In the above, I implicitly assumed that the emissivities of the sail are the same on both sides of the sail. If they are not, perhaps because the sail has a coating, this changes not only the force normal to the sail, but also the force tangential to the sail, so it also changes the direction of the net force. A further generalization of the material properties considers different types of reflection. I have assumed specular reflection (like a mirror), but some materials also exhibit diffuse reflection, like from a matte surface (Lambertian reflectance), and/or back reflection, whereby some photons are reflected directly back to the source.

With imperfect reflection (but ignoring unequal emissivity and assuming just specular reflection again) we need to redefine the lightness number. In equation 10.8 the factor of 2 is replaced with $2\tilde{r} + \tilde{a}$ and equation 10.10 becomes

$$\lambda = \frac{2\tilde{r} + \tilde{a}}{\sigma} \frac{\mathrm{L}_\odot}{4\pi G c \mathrm{M}_\odot} = \frac{2\tilde{r} + \tilde{a}}{2} \frac{\sigma_c}{\sigma}, \tag{10.20}$$

where $\sigma_c = 1.53 \times 10^{-3}\,\mathrm{kg\,m^{-2}}$.

10.5 Sail materials

The acceleration for photons incident normally on an imperfectly reflecting sail is, from equation 10.19,

$$a = \frac{P}{\sigma} = (2\tilde{r} + \tilde{a}) \frac{I}{c\sigma}\,. \tag{10.21}$$

To maximize this we should maximize the reflectance $\tilde{r}$ and minimize the mass per unit area σ of the sail material. The reflectance of materials varies with wavelength, because how a photon interacts with the free or bound electrons in the material depends on the energy of the photon. As most of the Sun's flux is in the optical part of the spectrum, we want materials that have large optical reflectance, such as metals. Aluminium has a high reflectivity[1] across a broad part of the optical spectrum. Silver has one of the highest reflectivities, even higher than aluminium, and for this reason is sometimes used in reflecting telescopes, although its reflectivity is poorer in the blue and ultraviolet, so would be less suitable in a solar sail for reflecting the broad solar spectrum. Gold is high too, but only in the red part of the spectrum (and it is sometimes used for mirror coatings in infrared telescopes).

To achieve a small mass per unit area, we should use the thinnest layer of reflecting material possible. There are at least two practical limits to this. First, if arbitrarily thin, the sail will be mechanically weak. It needs to be kept in a specific shape and must be attached to the

[1] The *reflectance* of a material measures how much light is reflected from a material of a given thickness. The *reflectivity*, in contrast, measures how much light is reflected off the surface of the material. They are equal for infinitely thick materials, but for thin materials, multiple internal reflections can result in variable reflectance for a given reflectivity.

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payload, so the sail will have to tolerate some tension. In practice one can attach a very thin layer of reflecting material to a stronger substrate, although that then contributes mass.

The second limit to the sail thickness is that very thin conductors become increasingly transparent. This can be understood when we consider the behaviour of electromagnetic waves incident on a thin film of conducting material. By solving the wave equation, we find that the amplitude of the electric and magnetic fields that penetrate a distance d through the material is proportional to $\exp(-d/L)$, so the degree of transmission is determined by the value of d/L, where L is a property of the material. Low transmittance – and so high reflectance – is achieved when $d >> L$, and so L is a characteristic minimum thickness of the material known as the *skin depth* (or penetration depth).

The value of the skin depth depends on the conducting properties of the material. A good electrical conductor has $\tilde{\sigma} \gg \tilde{\epsilon}\omega$, where $\tilde{\sigma}$ is its electrical conductivity, $\tilde{\epsilon}$ is its permittivity, and ω is the angular frequency of the light. For good conductors it turns out that

$$L = \sqrt{\frac{\tilde{\lambda}}{\pi c \tilde{\mu} \tilde{\sigma}}}, \tag{10.22}$$

where $\tilde{\lambda}$ is the wavelength of the light ($\omega = 2\pi/\tilde{\lambda}$) and $\tilde{\mu}$ is the magnetic permeability of the material. The latter is often listed in terms of the dimensionless relative permeability $\tilde{\mu}_r$, where $\tilde{\mu} = \tilde{\mu}_r \mu_0$ and $\mu_0 \simeq 1.26 \times 10^{-6}\,\mathrm{NA}^{-2}$ (newtons per square ampere) is the vacuum permeability. We see that the longer the wavelength, the thicker a given material must be to keep the reflectance high.

To minimize the mass of the sail, L should be small. Thus for a given wavelength, the larger the conductivity and permeability, the thinner the sail can be. Non-ferromagnetic metals like copper, aluminium, silver, and gold have relative permeabilities near to 1.0. Aluminium has a conductivity of $\tilde{\sigma} = 3.9 \times 10^7\,\mathrm{S\,m^{-1}}$ at room temperature (S is siemens, the inverse of the ohm); at $\tilde{\lambda} = 550\,\mathrm{nm}$, the peak of the solar spectrum, it has a skin depth of $L = 3.5\,\mathrm{nm}$.[2] This skin depth is much less than the wavelength of the electromagnetic radiation, which is typical for conductors. Copper, silver, and gold have slightly higher conductivities than aluminium of $4.5\text{–}6.3 \times 10^7\,\mathrm{S\,m^{-1}}$, but much higher densities (3–7 times as large). Iron has a very high relative permeability – over 100 at 99% purity, and up to tens or hundreds of thousands at very high purities – as well as a reasonably high conductivity ($\tilde{\sigma} = 1 \times 10^7\,\mathrm{Sm^{-1}}$), and so a much smaller skin depth than the non-ferromagnetic metals just mentioned. As it is only three times denser than aluminium, iron appears to be a good candidate sail material. But it is not considered because it absorbs in the bluer part of the spectrum and so would heat up. Being magnetic is also undesirable, given the presence of magnetic fields in space, plus it could interfere with instruments.

When a sail is made of a thin conducting material, there can be an optimal thickness that provides maximum acceleration. A large thickness increases the reflectance, but it also increases the mass. As the power of an electromagnetic wave is proportional to the square of its field amplitude, the intensity of the radiation that is reflected back from the surface is $1 - \exp(-2d/L)$, which is plotted in the left panel of figure 10.4. For a material with volume density ρ, then from equation 10.21 with $\sigma = \rho d$ the acceleration is

[2] Electrical conductivity generally decreases with increasing temperature. As the sail will inevitably absorb some radiation, it will heat up, thereby increasing the skin depth.

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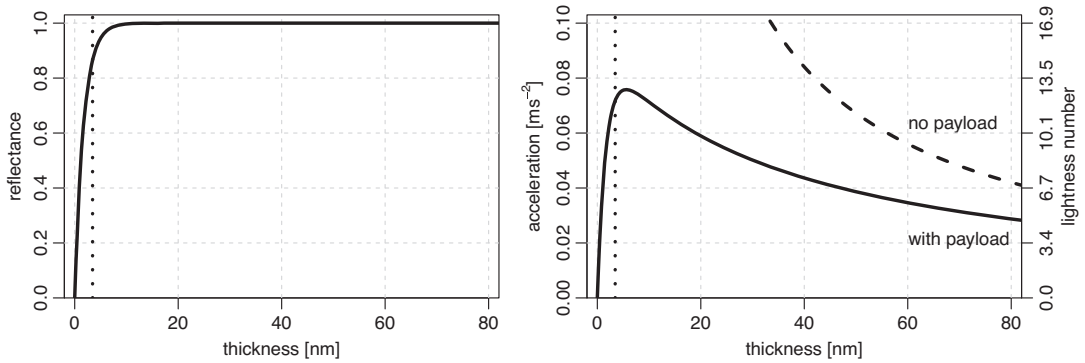


Fig. 10.4 Left: The variation of reflectance of a sheet of aluminium under normal incidence as a function of its thickness. Right: The corresponding variation of acceleration of a thin aluminium solar sail 1 au from the Sun at normal incidence for two cases. The dashed line shows just the sail (no payload), according to equation 10.23. This curve increases towards infinity as $d \to 0$ and drops to zero as $d \to \infty$. The solid line shows the case for a sail of 10 m^2 area carrying a payload of 1 g, as given by equation 10.24. The properties adopted for the aluminium in both cases are density $\rho = 2700\,\mathrm{kg\,m^{-3}}$, conductivity $\tilde{\sigma} = 3.9 \times 10^7\,\mathrm{S\,m^{-1}}$, and relative permeability $\tilde{\mu}_r = 1.0$. The intensity of the solar radiation is $I = 1361\,\mathrm{W\,m^{-2}}$ and is assumed for simplicity to be entirely at a wavelength of 550 nm, so that we can compute the skin depth to be $L = 3.5$ nm (the vertical dotted line).

$$a = \frac{2I}{c\rho d}[1 - \exp(-2d/L)]\,. \tag{10.23}$$

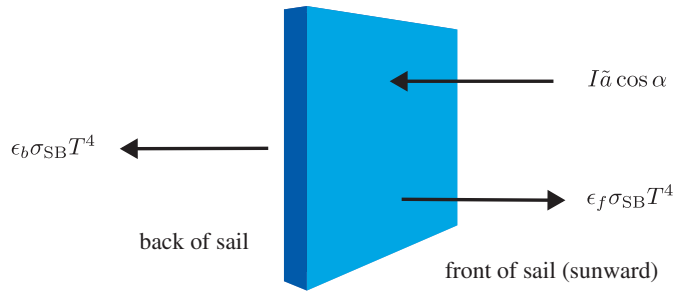
In the limit $d \to 0$ we get $a \to \infty$: Despite the trade-off between reflectance and mass with thickness, the decreasing mass wins, and so the thinner the material, the higher the acceleration. But this limit is useless in practice because the sail needs a finite thickness for structural integrity, plus it must carry some kind of payload. Let us assume the sail carries an additional non-reflecting and non-absorbing mass m, which for conciseness I refer to as 'payload'. We must now replace the mass per unit area σ with $\sigma + m/A$, where A is the area of the sail, so equation 10.23 becomes

$$a = \frac{2I}{c}\left[\frac{1 - \exp(-2d/L)}{\rho d + m/A}\right]\,. \tag{10.24}$$

This expression has a maximum at an intermediate value of d. This is plotted as the solid line in the right panel of figure 10.4 for a 10 m^2 aluminium sail carrying a 1 g payload. The maximum acceleration is achieved when the sail is just a few nm thick.

The above example is given to show how there is a trade-off in principle. As we will see below, current solar sails are made by depositing a thin film of reflecting metal on a sturdier substrate, often a polymer. These substrates are much thicker than the metal film, so we can make the metal film thick enough to maximize reflectance without adding much additional mass. When the substrate has a mass per unit area of $10^{-4}\,\mathrm{kg\,m^{-2}}$, for example a polymer around 100 nm thick, and the aluminium is 20 nm thick, then the substrate is almost twice as massive as the aluminium. A 10 m^2 sail out of this composite material has a total mass of 1.54 g (1 g for the susbtrate, 0.54 g for the aluminium). When carrying a 1 g payload and assuming unit reflectance, this sail has a lightness number of 6.0 and would achieve an acceleration at 1 au of $0.036\,\mathrm{m\,s^{-2}}$. This adopted substrate area density is much lower than is currently achievable, however. Current sails use polymers such as polyethylene

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A schematic of the radiation balance of a solar sail at equilibrium temperature T. Fig. 10.5

terephthalate (PET; density of $1400\,\mathrm{kg\,m^{-3}}$) 2500 nm thick, which corresponds to an area density of $35 \times 10^{-4}\,\mathrm{kg\,m^{-2}}$. This lowers the lightness number in the above scenario to 0.42. The total mass of the substrate is 35 g, so is much more than the payload too. Clearly, in order to use a metallic film for a solar sail, we need to reduce the thickness or density of the substrate as much as possible.

Several other factors beyond thickness affect a sail's reflectance. One example is the flatness of the metal film, at both a microscopic and macroscopic level. For the sail that has already been made for the Solar Cruiser spacecraft, for example (see section 10.6), the non-flatness is expected to reduce the overall reflectance by around 5%.

The above discussion is a simplified treatment. Real metals absorb some of the incoming radiation, so the reflectance is not unity even for large thicknesses. Furthermore, this absorption will heat the sail, which could eventually melt the sail. The sail will also radiate the heat, which can accelerate the sail.

The equilibrium temperature of the sail can be calculated assuming it to be a very thin black body with thermal emissivity ϵ_f on the Sun-facing side (front) and ϵ_b on the other side (figure 10.5). From the Stefan–Boltzmann law, the total power emitted per unit area from a black body of emissivity ϵ and temperature T is $\epsilon\sigma_{\rm SB}T^4$, where $\sigma_{\rm SB}$ is the Stefan–Boltzmann constant. In thermal equilibrium, the energy radiated by the sail equals the energy it absorbs. When the sail is inclined at an angle α to the Sun, then the incident intensity is $I\cos\alpha$, and so

$$I\tilde{a}\cos\alpha = (\epsilon_f + \epsilon_b)\sigma_{\rm SB}T^4. \tag{10.25a}$$

$$T = \left[\frac{I\tilde{a}\cos\alpha}{(\epsilon_f + \epsilon_b)\sigma_{\rm SB}}\right]^{1/4}. \tag{10.25b}$$

For a sail coated on one side with aluminium, we can adopt $\tilde{a} = 0.1$ and $\epsilon_f = 0.1$ as reasonable averages for our purposes. Ideally the back side of the sail (the substrate) has $\epsilon_b \simeq 0$, because any photons emitted in this direction would decelerate the sail. Adopting this, then for a sail 1 au from the Sun pointing at it, the above expression gives an equilibrium temperature of 394 K. Figure 10.6 shows how this temperature varies with distance from the Sun (solid line). Once this sail approaches within 0.18 au of the Sun, its temperature reaches the melting point of aluminium (933 K). Thus if we want to operate close to the Sun (for example with the sundiver – section 10.7), we need a material with a lower absorptance, a higher emissivity, or a higher melting temperature.

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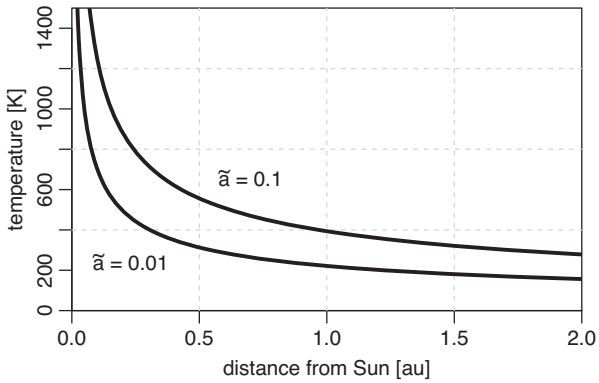


Fig. 10.6 The variation of the equilibrium temperature of a sail with distance from the Sun for two values of the absorptance $\tilde{a}$ according to equation 10.25b. The sail has emissivities $\epsilon_f = 0.1$ and $\epsilon_b = 0.0$ and is pointed at the Sun ($\alpha = 0$).

If $\epsilon_f \neq \epsilon_b$, then the rate of emission of photons from the two sides of the sail is not equal. This generates a net force on the sail. This was not taken into account in section 10.4, where the pressure in equation 10.19 implicitly assumed that any absorbed photons (i.e. when $\tilde{a} > 0$) were emitted isotropically.

The above treatment of the sail has been entirely classical. One quantum mechanical effect of some relevance is the photoelectric effect. Photons of high enough energies eject free electrons from a metallic sail. The photon energy required is given by the work function of the material, and is of order a few electronvolts. For aluminium it is 4.3 eV, which corresponds to photons with wavelength below 290 nm. Not only does the ejection of electrons represent lost energy (used to overcome the work potential), but it also leads to charging the sail. As the solar spectrum has a non-negligible intensity in the ultraviolet, this is unavoidable.

We have only considered the use of good conductors – metals – as the primary sail material. Poor conductors – dielectrics – generally have lower reflectances, so at first sight appear to be less interesting for solar sail applications. However, when it comes to the much higher intensity light from laser sails, non-conductors are interesting on account of their potentially higher melting temperatures (see section 11.4.1). They can also be made very thin and yet strong, and this reduced mass can compensate for their lower reflectances, also as a solar sail.

A solar sail cannot launch from the Earth's surface, of course. The sail material therefore has to be packaged into a rocket's fairing for launching. It must be strong enough to withstand folding or rolling and the stresses of launch. Once in space it must be deployed. Various methods have been developed and tested for doing this, including using motors to pull it along rigid booms, and rotating the spacecraft to open it via centrifugal forces. As we shall see in the next section, sails built so far have thicknesses no less than 2.5 μm. Thinner sails are at risk of tearing during fabrication, launch, or deployment. If sails can be manufactured in space, then thinner sails may be possible with current materials and technology.

In summary, we have the following requirements for a sail material: high reflectance; low mass per unit area (thin or low density); high thermal emissivity or melting point; foldable; sufficient tensile strength so as not to tear. These requirements cannot be optimized independently, and so the choice of material will be a compromise between different factors.

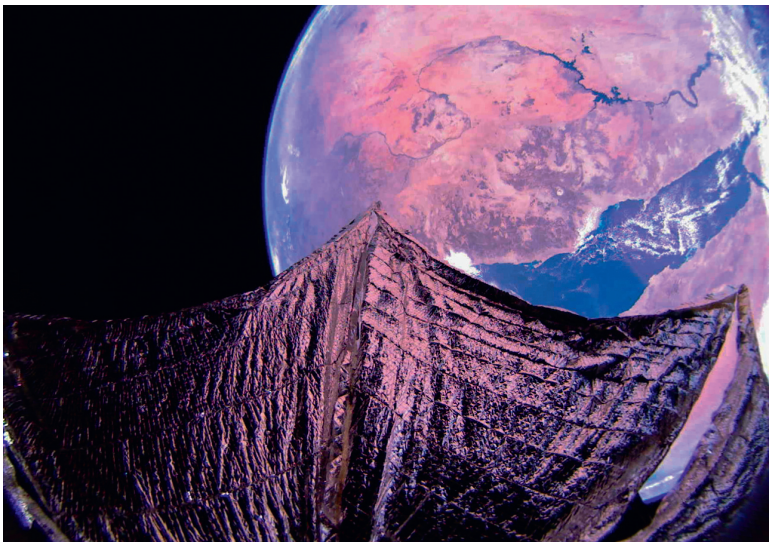
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10.6 Missions

Several solar sail missions have already flown. Several more were planned and partially developed but never flown. The German space agency DLR, for example, produced and deployed several sail materials in the late 1990s, and had a technology programme through the first decade of this century.

Of those missions that have successfully flown (as of mid 2025), all were tests and technology demonstrators. NASA, for example, has mapped out a series of missions of increasing size and sophistication, leading also into laser sails that we will discuss in the next chapter. Not all of the missions were successful, though. Nanosail-D2 (a replacement of Nanosail-D that was lost due to rocket failure) was a small mission primarily intended to test the deployment of its 10 m^2 sail, which it did successfully in 2010. NASA's NEA Scout was a cubesat with a deployed sail area of 85 m^2 of 2.5 μm-thick aluminized polyimide. It was meant to last for two years and encounter a small asteroid. It was launched in November 2022 on Artemis 1, but its sail failed to deploy, and communication was lost just after launch. The similar-sized Advanced Composite Solar Sail System (ACS3), launched in April 2024, did deploy successfully, but started to tumble in October of the same year, possibly after one of the sail support arms was bent. Solar Cruiser is a much larger sailcraft, comprising four sails each with an area of 440 m^2 made from a 2.5 μm-thick metal-coated polyimide. Sail deployment has been successfully tested on the Earth. Funding permitting, the spacecraft hopes to launch in 2029 to study the Sun.

A non-governmental organization, The Planetary Society, built and launched two solar sails into low Earth orbit in 2015 and 2019, called LightSail 1 and 2 (figure 10.7). Both were miniature cubesats, in which the 5 kg sail was folded for launch into a rectangular volume



LightSail 2 in orbit above the Red Sea. The Nile is clearly visible. The sail appears curved due to the wide angle lens. Credit: The Planetary Society, reproduced with permission. **Fig. 10.7**

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of just 10 litres. Once deployed, the square sail had an area of $32\,\mathrm{m}^2$. This was made out of a thin polyester (boPET) film and an aluminium coating with a total thickness of $4.6\,\mu\mathrm{m}$, and providing a reflectance of around 0.99. Together these data imply a lightness number of 0.01. Solar panels provided an average power of 8.5 W. After sail deployment, the sail was reoriented several times between facing the Sun and being edge-on to the Sun, which could be used to determine the acceleration due to the solar radiation pressure.

Several other organizations have developed solar sails, for example the French company GAMA, which launched a small sail into Earth orbit in January 2023. It was able to adjust its orbit using solar radiation pressure.

We now look in a bit more detail at one past successful mission, and one proposed interstellar mission.

10.6.1 IKAROS

One of the first solar sail missions, and so far the only interplanetary one, was IKAROS (Interplanetary Kite-craft Accelerated by Radiation Of the Sun), launched by the Japanese space agency JAXA in 2010. The sail itself had an area of $200\,\mathrm{m}^2$ and was made from a polyimide substrate $7.5\,\mu\mathrm{m}$ thick with an 80 nm-thick coating of aluminium. This had a mass per unit area of around $0.01\,\mathrm{kg\,m}^{-2}$, corresponding to a lightness number of 0.14 from equation 10.10. However, extra supports and tensioning masses were required, increasing the total mass from 2.1 kg to 16 kg, and so reducing its mass per unit area to $0.08\,\mathrm{kg\,m}^{-2}$. More significant, though, was the payload mass of 55 kg, which lowered the mass per unit area to $0.4\,\mathrm{kg\,m}^{-2}$, to give an overall lightness number of 0.0043.

The square sail was deployed by rotating at a low rate, several revolutions per minute (rpm). It continued to rotate at around 1 rpm after deployment to help stabilize the sail. IKAROS was not designed as a high-performance sail to achieve maximum acceleration, but rather as a technology demonstrator and test of interplanetary solar sailing, testing not only propulsion, but also power generation, guidance, navigation, and control.

For its power, IKAROS was equipped with thin-film photovoltaic silicon cells just $25\,\mu\mathrm{m}$ thick. These covered 5% of the sail area (i.e. $10\,\mathrm{m}^2$) and produced 500 W of power. Successfully generating electricity from these was a major mission goal. The instruments onboard included a number of cameras, a polarimeter to observe gamma-ray bursts, and a dust detector.

Attitude control is an important task for any spacecraft. As sailcraft must be light, it would be counter-productive to equip them with either traditional cold gas thrusters or massive gyroscopes. Instead, IKAROS used a number of liquid crystal panels on its sails to control its orientation. When a voltage is applied to the liquid crystals, they align and reflect light in a specular fashion, like the rest of the sail. When the voltage is removed, the crystals no longer point in a common direction and then reflect diffusely, which lowers their net reflectance. When this is done for panels on just one side of the sail, the solar radiation exerts a torque on the sail, thus changing its orientation. In this way the attitude of the sail can be controlled without requiring propellant, but just a small amount of energy.

IKAROS was launched in May 2010 and spent about two weeks carefully deploying the sail. A thrust of 1.12 mN due to solar radiation pressure was measured from the Earth by Doppler radio. For a total mass of 71 kg this implies an acceleration of $1.7 \times 10^{-5}\,\mathrm{m\,s}^{-2}$, or a lightness number of 0.0027. This is only slightly less than the (simplified) expected

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value based on the sail area and spacecraft mass computed above. Over the next six months IKAROS cruised to Venus, and during this period the sail provided a total Δv of about $100\,\mathrm{m\,s^{-1}}$. Clearly this Δv was not enough to get the spacecraft from Earth to Venus; most of the thrust was provided in advance by chemical rockets. IKAROS passed Venus at a distance of 80 000 km on 8 December 2010, and since 2015 has been in hibernation mode.

10.6.2 Interstellar mission concepts

The the late 1990s, NASA/JPL studied a concept for an interstellar probe propelled by a solar sail. The nominal design is a circular boPET/aluminium sail 400 m in diameter with a mass of 100 kg. Combined with a payload mass of 150 kg, this corresponds to $\sigma = 2.0 \times 10^{-3}\,\mathrm{kg\,m^{-2}}$ and thus a lightness number of 0.77. When this sail is opened from an initial 1 au circular orbit about the Sun, it would escape from the solar system, achieving an asymptotic velocity of $22\,\mathrm{km\,s^{-1}}$ (equation 10.14). In practice the mission design foresaw the use of chemical rockets, planetary gravity assists, and/or initially diving closer to the Sun (see section 10.7) to achieve a total Δv of $70\,\mathrm{km\,s^{-1}}$. The mission is designed to reach 200 au 15 years after launch (an average velocity of $63\,\mathrm{km\,s^{-1}}$), a distance that is well beyond the orbit of Neptune. Its main science objectives are to study the outer solar system and the interstellar medium and their interaction. The sail would be ejected once 5 au from the Sun, as the benefit of the small additional acceleration beyond this distance was considered inferior to the requirement to avoid interference of the sail with onboard instruments.

One of the big challenges of such a deep space mission is the data rate. Using a 2.7 m radio dish on the spacecraft with signals picked up by NASA's Deep Space Network, an average data rate of just $30\,\mathrm{b\,s^{-1}}$ from a distance of 200 au was predicted. We shall examine the challenge of communication in chapter 15.

There have been several other concepts for using solar sails to reach either the outer solar system or interstellar space. An ESA study considered a $60\,000\,\mathrm{m^2}$ sail to take a payload out to 200 au within 20 years in order to study the heliopause; this is similar in size and scope to the JPL concept just mentioned. Several authors have proposed solar sails as a fast way to get to Mars or to an asteroid, or to chase high-speed interstellar objects (ISOs) that pass through the solar system on hyperbolic orbits (see section 12.3.2). With velocities of many tens of kilometres per second in the inner solar system, ISOs are hard to catch. One concept therefore involves using solar sails with a lightness number close to unity that can hover, waiting, before tilting their sails to follow a particular target. Some solar system missions also envisage use of the sundiver to help the sail get to the outer solar system more quickly.

10.7 The sundiver

We saw earlier from equation 10.15 that provided λ is large enough, then the closer a sailcraft is to the Sun when it opens its sail, the larger its velocity at infinity. For fast interstellar travel it therefore makes sense to manoeuvre a sailcraft as close to the Sun as possible before starting the outward journey. How can we achieve this?

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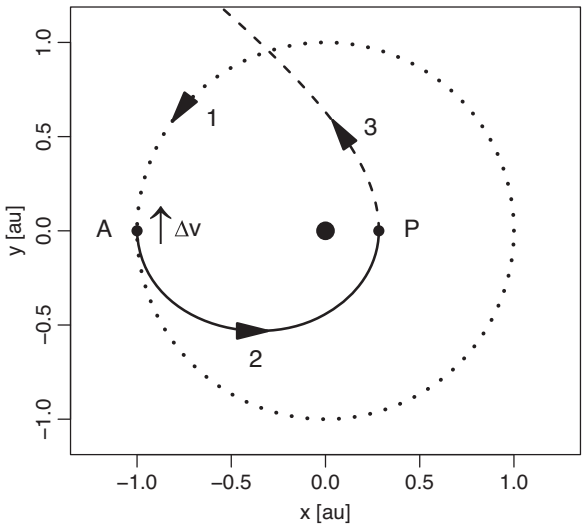


Fig. 10.8 A simple sundiver. A spacecraft initially on a circular orbit (1) about the Sun at (0, 0) performs a retrograde boost at point A to reduce its orbital velocity by Δv. This puts it on an elliptical orbit (2) with aphelion A and perihelion P. At perihelion the spacecraft jettisons its rocket and deploys its solar sail, keeping it pointed at the Sun. The solar radiation pressure accelerates the spacecraft onto a hyperbolic orbit (3), which achieves a velocity at infinity given by equation 10.26. The plot is drawn to scale with $r_a = 1$ au, $r_p = 0.28$ au (achieved using $\Delta v = 10$ km s^{-1}), and $\lambda = 0.3$. The velocity at infinity is 22.3 km s^{-1}. Adapted from figure 1 of Bailer-Jones (2021b).

Suppose our spacecraft has been launched from the Earth and has escaped Earth's gravity, and is initially on the Earth's orbit about the Sun (orbit 1 in figure 10.8). The most obvious way to drop close to the Sun is to use a rocket to apply a retrograde boost (at A in the figure) almost equal to the orbital velocity. We of course do not want to drop into the centre of the Sun, and let us assume the spacecraft cannot approach closer than a distance r_p to the Sun's centre for thermal reasons. After this boost, the spacecraft is on an elliptical orbit (orbit 2) with perihelion r_p and aphelion r_a, which in this case is 1 au. At perihelion (point P), the spacecraft ejects its engines and fuel tanks, opens its sail and points it at the Sun. If λ is large enough, the sailcraft will move on a hyperbolic orbit to infinity (orbit 3) where it will have a velocity given by equation 10.15 with $r_i = r_p$. The semi-major axis of the initial elliptical orbit is $a = (r_a + r_p)/2$, so we can write the velocity equation as

$$v_\infty^2 = \frac{2\mu}{r_p}\left(\lambda - \frac{r_p}{r_a + r_p}\right). \tag{10.26}$$

Adopting $r_p = 10\,\mathrm{R}_\odot = 0.047$ au (the closest approach of the Parker Solar Probe), the perihelion velocity at the moment of opening the sails is $v_p = 191$ km s^{-1}(from the vis-viva equation, equation 4.3). With $\lambda = 1/2$ the velocity at infinity is $v_\infty = 132$ km s^{-1}. For $\lambda = 1$ we get $v_\infty = v_p$ (from equation 10.13).

We found earlier that a sailcraft on an initial circular orbit can only reach infinity if $\lambda \geq 1/2$. For an initial elliptical orbit, a smaller λ is permissible. By setting $v_\infty = 0$ in equation 10.26, we see that the sail can reach infinity if $\lambda \geq 1/(1 + r_a/r_p)$. For example, with

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$r_a/r_p = 2$ the minimum λ is $1/3$. Once λ is large enough for the sailcraft to reach infinity, a lower r_p for given r_a increases v_∞.

For the spacecraft to dive down to 10 solar radii with a single retrograde boost, we need a large Δv. The velocity at aphelion for this transfer orbit can be found from the vis-viva equation (or via conservation of angular momentum), and the difference between this and the circular velocity is the required Δv. In the above case with $r_a = 1\,\text{au}$ and $r_p = 10\,\text{R}_\odot$ this is $20.9\,\text{km}\,\text{s}^{-1}$. We saw in section 4.2.2, however, that we can dive just as close with a lower total Δv by using a bielliptic Hohmann transfer orbit. This involves first boosting to a larger aphelion distance and then applying the retrograde boost (figure 4.7). In this case the perihelion velocity will be larger (for the same perihelion distance), and so the final velocity at infinity will also be larger. This gain is very small, however. If we now adopt $r_a = 2\,\text{au}$, then the perihelion velocity increases by just $2.2\,\text{km}\,\text{s}^{-1}$, to $193\,\text{km}\,\text{s}^{-1}$. For $\lambda = 1$ this will increase the velocity at infinity by the same amount, just 1%.

Assuming one has a rocket engine to boost a sailcraft, an interesting question is whether, before opening the sails, it is better to use the available Δv to make a retrograde boost to drop down to the Sun, or to use it instead to make a prograde boost from the initial orbit. After all, having dropped to the Sun to gain kinetic energy (an application of the Oberth effect; section 4.2.3), the sailcraft would then have to use more of this energy to climb out of its now deeper gravitational potential well. It turns out that the strategy that maximizes the final velocity depends on the lightness number of the sail and the ratio of Δv to the initial velocity. For a given velocity ratio, when the lightness number is large, it is better to use a retrograde boost to dive to the Sun. But when the lightness number is low, it is better to use a prograde boost and avoid the dive. For $\lambda > 1/2$ it is always better to dive. Curiously, it is never optimal to use a combination of retrograde and prograde boosts. This is explored in detail in Bailer-Jones (2021b).

Instead of using a rocket to bring the sailcraft close to the Sun, we could use the sail itself by tilting it to spiral into the Sun, as outlined in section 10.3.2. This would take longer, perhaps a lot longer, but as we see from the bottom left panel of figure 10.3, the sail can still achieve quite large velocities close to the Sun.

In section 11.4.1 we will discuss carbon-based materials such as graphene and aerographite with extremely low densities, which, while not very reflective, are absorptive and so could act as solar sail materials. They will therefore heat up, but they have high enough emissivities and melting points that they could survive a very close encounter with the Sun. A sail made only from graphene could theoretically have a lightness number as high as 500, albeit without any payload. If this is released by a spacecraft that has dived from the Earth's orbit to a perihelion distance of 10 solar radii, then it will achieve a velocity at infinity of $4370\,\text{km}\,\text{s}^{-1}$, or 1.5% the speed of light, and so gets to Proxima Cen in 290 years. If it could dive to five solar radii (four from the Sun's surface), then the velocity at infinity would increase to $6180\,\text{km}\,\text{s}^{-1}$, a 210-year journey. Naturally, any payload would significantly lower the lightness number of the sail. If we could achieve a high but more modest lightness of 20, and then dive to within two solar radii (one from the Sun's surface), it would take 650 years to get to Proxima Cen. This is optimistic, but it shows that if we can design a very light sail to get very close to the Sun, and are patient enough for travel to the stars to take hundreds, rather than tens, of years, then a solar sail might become an option for interstellar travel.

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10.8 Summary

A solar sail is a spacecraft propelled by solar radiation pressure. As this is low, the sail must be light. A mass density of $1.53 \times 10^{-3}\,\mathrm{kg\,m^{-2}}$ is required for a perfectly reflecting material to balance the force of gravity. The ratio of the radiation to gravitation force is called the lightness number λ (equation 10.10). As both the Sun's gravity and luminosity vary as $1/r^2$, λ is independent of the distance from the Sun.

The net force on a sail kept pointed at the Sun is radial, so it still moves on a Keplerian orbit, but effectively with a lower-mass star (for $\lambda < 1$). When the sail is tilted, the net force is no longer radial, allowing for a wide set of non-Keplerian orbits. For $\lambda > 1$, the net force from the Sun is outwards.

Very thin metallic sails become transparent, so there is a limit to how light a metal sail can be. In current sails, a thin layer (tens of nanometres) of highly reflective metallic foil is deposited on a supporting substrate; the latter usually dominates the mass budget.

A solar sail with a sufficiently large lightness number can escape the solar system, depending on its initial orbit. By first bringing the sailcraft close to the Sun, more kinetic energy can be transferred to the sail, thereby raising its velocity at infinity. This is known as the sundiver. If the sailcraft can be brought to within a few solar radii of the Sun's surface, a velocity at infinity of order $1000\,\mathrm{km\,s^{-1}}$ can be achieved with a large lightness number of 10 (equation 10.26). This optimistic scenario still results in a long journey time to the nearest stars – over a thousand years to Proxima Cen – so a solar sail is probably not a viable solution for fast interstellar travel.

Solar sails have already been successfully tested and deployed in space, although none have yet achieved significant propulsion. Within the coming decades they are likely to be used for interplanetary missions, and for exploring the outer reaches of the solar system.

10.9 Exercises

1. What is an appropriate figure-of-merit for a solar sail? Is a single mathematical expression in terms of the sail's properties appropriate or sufficient?
2. In the Table 10.1, fill in the type of orbit a solar sail will move on for the two different initial conditions specified in the columns, for the various values of the lightness number in the rows. The sail is always kept pointed at the Sun.

Table 10.1

λ	Initially at rest	Initially on a circular orbit
0		
0 to 1/2		
1/2		
1/2 to 1		
1		
> 1		

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