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NOTES ON (PS.?) BALBUS GROMATICUS PP. 107.10–108.8 ED. LACHMANN

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This article subjects the final part of the treatise *Expositio et ratio omnium formarum* (*Systematic presentation of all figures*), included in the *Corpus Agrimensorum Romanorum* and commonly ascribed to Balbus, to fresh philological scrutiny (Balbus gromaticus pp. 107.10–108.8 ed. Lachmann). The passage deals with methods of constructing right angles for the purpose of drawing rectangular and rectilinear figures. On the basis of a reconsideration of the preserved textual witnesses and previous scholarship, the passage is critically and interpretatively studied in the form of a running commentary.

The standard Latin text of the treatise *Expositio et ratio omnium formarum* (*Systematic presentation of all figures*) (*c.* AD 100), commonly ascribed to the *agrimensor* Balbus, is still the one by Karl Lachmann, published in 1848 in his edition of the *Corpus Agrimensorum Romanorum* (*CAR*): *Gromatici veteres*, Berlin, pp. 91–108.[1] Two recently published contributions to our understanding of this treatise are the following works: Jean-Yves Guillaumin (1996) *Balbus. Présentation systématique de toutes les figures.* [. . .]. *Introduction, traduction et notes* [. . .], Naples, and Brian Campbell (2000) *The writings of the Roman land surveyors. Introduction, text, translation and commentary*, London. However, both Guillaumin and Campbell printed Lachmann's text (with some alterations of their own, either made in the text itself or suggested in their commenting notes).

According to Lachmann's edition, the final part of the treatise is spurious and should be excised: *Qualemcumque rectorum angulorum formam . . . normam facturas* (pp. 107.10–108.8 L). In this respect, however, neither Guillaumin nor Campbell followed Lachmann.[2] Nicolaus

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1 References to Lachmann's edition are henceforth made thus: p. 1.1 L (= page 1, line 1 ed. Lachmann).

2 Guillaumin (1996) 83 n. 94 states the following (with special regard to p. 107.12–16 L):

> Ce passage est un exercice d'*agrimensor*. Lachmann le considère comme interpolé, sans donner les raisons de son jugement. A-t-il pensé que cet exercice de construction n'avait pas sa place dans une *Expositio* géométrique? Mais le titre du traité comporte aussi le mot *ratio*, qui suggère la possibilité d'un exposé sur la manière de construire les figures.

Campbell (2000) 435 n. 24, for his part, argues as follows:

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Bubnov and Carl Thulin, on the other hand, held the *Expositio* to be incompletely preserved and the final part to be a fragment of some geometrical work by Sextus Iulius Frontinus; for in two manuscripts (J and V; see the section on the textual witnesses below) the transmitted explicit of the treatise runs *Explicit liber Frontonis* [read *Frontini*] *primus*, whereas the incipit in the oldest extant manuscript (*Codex Arcerianus*, B; see below) reads *Incipit liber Balbi ad Celsum Expositio et ratio omnium formarum*. Lachmann's excision was thus endorsed by Bubnov and Thulin.[3] Another view of the matter, which has received little notice among recent *CAR* scholars, has been that Frontinus, not Balbus, is the author of the whole treatise (pp. 91–108 L), for in yet another manuscript (P; see below) the incipit runs *Iulius Frontinus Celso*. In his doctoral dissertation on Frontinus, Gerhard Bendz discusses this view and concludes:[4]

> Dass die Schrift Expositio et ratio omnium formarum nicht mit Lachmann und Mommsen dem Balbus, sondern mit Goesius und Dederich dem Frontin zuzuschreiben ist, dürfte nach alledem freilich nicht unwiderruflich bewiesen, aber sehr wahrscheinlich sein.[5]

The final part of the treatise has also been studied by Anne Roth Congès in an article published in 1996: 'Modalités pratiques d'implantation des cadastres romains: quelques aspects [. . .]', *MEFRA* 108, 370–4. And one particular part of it (p. 107.12–16 L) has been treated by Jelle Bouma (1993), in his work *Marcus Iunius Nypsus: Fluminis varatio. Limitis repositio. Introduction, text, translation and commentary*, Frankfurt am Main, 109.

In this paper, the supposed fragment is subjected to renewed philological scrutiny. It focuses mainly on the text itself and its interpretation. The question of authorship, however, is not addressed to any further extent than pointing out some common features

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The text at C 213.42–215.14 [= Campbell's translation (page and line) of pp. 107.10–108.8 L] was omitted by MSS EGR, and Lachmann seemingly regarded it as an interpolation. But it is appropriate in its context because it was part of the surveyor's job to plot and draw angles in the measurement process.

The three manuscripts mentioned by Campbell are: E = Erfurt, Wissenschaftliche Allgemeinbibliothek, Amplon. 4° 362 (belongs to the 'mixed class'); G = Wolfenbüttel, Herzog-August-Bibliothek, 105 Gud. lat. 2° (belongs to the 'PG-recension'); R = Rostock, Universitätsbibliothek, philol. 18 (belongs to a group of manuscripts called X, which contains excerpts from the *CAR*; see e.g. Folkerts (1970) 95–7). That, as Campbell states, the passage is absent from the MSS EGR is no valid argument for claiming it to be spurious; for another manuscript of the 'mixed class', H (Folkerts' N; see the section on the textual witnesses below), has indeed preserved the passage, and the testimonies of the MSS G and R cannot, given their stemmatical positions, be adduced as evidence against its authenticity; see Folkerts (1970) 97.

3 Bubnov (1899) 407, 437, 509. Thulin (1911a) 22–3; ibid. 23:

> Lachmann hat dieses Fragment eingeklammert dem Balbustext angehängt. Aber da die Überschrift des Balbus in B richtig erhalten ist, so kann die Unterschrift *Explicit liber Frontonis primus* sich nicht auf Balbus beziehen, weshalb Bubnov mit Recht den Text des Balbus La. 107,9 unvollendet und ohne Subscriptio ausgehen läßt und das folgende gemäß der Subscriptio als ein Frontinusfragment aussondert.

4 Bendz (1938) 30–3 with further references.

5 Bendz (1938) 31.

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in the supposed fragment and the *Expositio* – features which, in my view, seem to indicate that the supposed fragment was written by the same author as the *Expositio* and hence is a genuine part of this treatise, rather than the opposite. In any case, Bubnov and Thulin did not ground their view on any other form of evidence than the varying statements of the manuscript incipits and explicits.

Although the text poses several problems, to many of which convincing solutions seem far out of reach, it nonetheless deserves attention. For it is an important example – as is the whole *Expositio* – of how elementary Greek geometry was rendered in Latin and adopted to serve in a Roman instructional context. Menso Folkerts has described the supposed fragment thus: 'Der Text befindet sich in einem korrupten Zustand; obwohl der Name "Euklid" genannt wird, kann von exakter Mathematik keine Rede sein.'[6] In my view, however, Folkerts' statement can be regarded as somewhat unfair. If we look beyond the present state of the text, its content, consisting of fundamental geometrical constructions of right angles, seems to be compendiously but yet correctly treated by the author.

The textual witnesses[7]

The text, pp. 107.10–108.8 L, is partly or completely preserved in the following known manuscripts:[8]

J Jena, Thüringer Universitäts- und Landesbibliothek, Prov. f. 156 (sixteenth century):[9] ff. 69v–70v

V Vatican City, Biblioteca Apostolica Vaticana, Vat. lat. 3132 (sixteenth century):[10] f. 110rv (pp. 107.10–22; 107.24–108.8 L)

V_V Vatican City, Biblioteca Apostolica Vaticana, Barb. lat. 164 (sixteenth century):[11] f. 14rv

V_P Paris, Bibliothèque Nationale, lat. 7229 (sixteenth century):[12] f. 87rv

6 Folkerts (1992) 317–18.

7 For general overviews – with further references – of the transmission of the *CAR* texts, see e.g. Reeve (1983) 1–6, Toneatto (1994a) 1–88 (esp. 1–20), Bouma (1993) 11–13 and 19–41, Campbell (2000) pp. xx–xxii, and Guillaumin (2005) 45–50.

8 All references to folios are made according to Toneatto's (1994a and 1994b) foliations.

9 For descriptions, see esp. Thulin (1911b) 425–32 and Toneatto (1994b) 591–606; see also Bouma (1993) 29–31 and Guillaumin (2005) 47.

10 For descriptions, see esp. Thulin (1911b) 419–23 and Toneatto (1994b) 694–706; see also Bouma (1993) 29–31 and Guillaumin (2005) 47.

11 For descriptions, see Thulin (1911b) 423, 425, 441–3 and Toneatto (1994b) 679–93. The manuscript is here called V_V instead of Grillone's B_1, so that it is clearly distinguished from B, which is the generally accepted siglum of one part of the *Codex Arcerianus*.

12 For descriptions, see Thulin (1911b) 423–5 and Toneatto (1994b) 650–8. The manuscript is here called V_P instead of Grillone's P, since P is the generally accepted siglum for another *CAR* manuscript: Vatican City, Bibl. Apost. Vat., Pal. lat. 1564 (ninth century).

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H London, British Library, Add. 47679 (twelfth century):[13] ff. 46v–47r (pp. 107.10–19; 107.23–108.8 L)

H_L Leiden, Bibliotheek der Rijksuniversiteit, Scal. 6 (sixteenth–seventeenth century):[14] ff. 14v–15r

J and V are two independent copies of the so-called *Codex Arcerianus* (Wolfenbüttel, Herzog-August-Bibliothek, 36.23 Aug. 2° (late fifth century–early sixth century); in two parts: A and B), the earliest extant manuscript of the CAR, in which our text, originally located in B, has unfortunately been lost.[15] The *Codex Arcerianus* and its copies constitute one class of CAR manuscripts, the so-called 'AB-recension'. Although both J and V exhibit alterations – for the most part erroneous emendations – of the text of the *Codex Arcerianus*, J is generally considered to be more faithful to the original than V.[16] In J and V, moreover, our text is illustrated with figures. Thulin argued that they stem from a common source and were drawn by a later hand in the margin of B (whence they were copied into J and V). He also suggested that they were, in turn, either inserted into B from some older and more complete manuscript of the so-called 'PG-recension' – another class of CAR manuscripts in which we find P (Vatican City, Biblioteca Apostolica Vaticana, Pal. lat. 1564 (ninth century)) and G (Wolfenbüttel, Herzog-August-Bibliothek, 105 Gud. lat. 2° (ninth century)) – or invented by the drawer himself.[17] As this study shows, however, the latter alternative can now be dismissed: the figures were not created by the drawer.[18]

V_V and V_P are copies of V, and they are of no particular critical value.[19]

For his editing of the text in question, Lachmann used J and V, whereas H was held to be lost. In 1969, however, Folkerts brought H to light and described it.[20] H belongs to a

13 For descriptions of this manuscript, which is also known as *Codex Nansianus* or *Codex Scriverianus/Scriverii*, see esp. Folkerts (1969) and Toneatto (1994a) 465–90; see also Bouma (1993) 39–40 and Guillaumin (2005) 49–50. I have chosen to follow Reeve (1983) 3 and 3 n. 14 in calling the manuscript H, since he has pointed out that the siglum N used by Folkerts is already taken by another manuscript. Moreover, Lachmann's siglum S does not refer directly to the manuscript, but to Pieter Schrijver's (Petrus Scriverius') edition: 'La sigla dell'edizione lachmanniana del "CAR" si riferisce non direttamente al ms., ma all'edizione di Vegezio e di altri autori *de re militari* (Leida, 1607) curata dall'olandese Pieter Schrijver, che vi pubblicò alcuni opuscoli gromatici, attingendoli anche dal n[o]s[tro] ms.' (Toneatto (1994a) 487).

14 For a description of the manuscript, see Toneatto (1994b) 778–83. I call it H_L.

15 The *Codex Arcerianus* consists of two parts, A and B, of which B is generally considered to be a little older than A; see e.g. Reeve (1983) 1–2 and Toneatto (1994a) 151 and 160–1 with references. For detailed descriptions of and general information on the *Codex Arcerianus*, see Thulin (1911a) 10–39, Josephson (1950) 72–100, Butzmann (1970) 7–44, Bouma (1993) 22–9, Guillaumin (2005) 46–7, and esp. Toneatto (1994a) 139–63 with further references.

16 See Thulin (1911b) 418, 420–1; ibid. 420: 'V hat keine getreue Kopie des Arcerianus gegeben, sondern hat Kritik geübt und sehr oft, aber selten richtig, auf eigene Faust den Text geändert, sehr oft auch die Vorlage unrichtig gelesen.' But cf. Bouma (1993) 29–31 and Guillaumin (2005) 47.

17 Thulin (1911a) 23–4. For descriptions of the 'PG-recension' and its manuscripts, see Thulin (1911a) 41–72, Josephson (1950) 100–106, Bouma (1993) 31–6, Toneatto (1994a) 218–49 (P), Toneatto (1994a) 250–71 (G), and Guillaumin (2005) 47–8.

18 See the notes on ***Qualemcumque*** … ***comprehendere*** (p. 107.10–11 L) and ***quorum*** (p. 107.15 L) below.

19 See Thulin (1911b) 418.

20 Folkerts (1969).

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third class of CAR manuscripts which originate from a compilation for which a manuscript related to the *Codex Arcerianus* and a manuscript related to P and G were used; hence the name the 'mixed class'.[21] Although the group of manuscripts to which H belongs suffers from severe displacements and corruptions, one may nevertheless find readings of our text in H which are less corrupt than in J and V. And if our passage was included in the manuscript related to P and G (which we unfortunately do not know), readings of H could represent the 'PG-recension'. As in J and V, moreover, in H our text is illustrated with figures.

H_L, finally, is a copy of H by the hand of Joseph Justus Scaliger (1540–1609).

In the following section of this paper, Lachmann's text is presented together with a revised and extended critical apparatus, in which the readings of J, V and H and Lachmann's emendations are recorded. The readings of V_V, V_P and H_L have been excluded with one single exception (see the critical apparatus at p. 107.16 L). Abbreviations etc. are usually expanded, unless they are likely to be of critical significance.

The text according to Lachmann with a new critical apparatus

[p. 107.10] *Qualemcumque rectorum angulorum formam rectis | lineis comprehendere. |*

Ex data recta linea ducere posito signo <> | relato in utramque partem circino, aequali | punctorum diastemate circulos scribere opor- |* **[15]** *tet, per quorum conexionem recta linea transeat factura | normales in data linea angulos. sed quo in rec- | tarum linearum forma circularis linea non interueniat rec- | tis, a circumferentiarum parte chiasmi cuiusdam ratione | utamur. |*

[20] *Quod si ab eadem recta linea ducenda fuerit quae | rectum angulum faciat, ex quolibet puncto qui per caput | recta linea transeat rectam lineam eicere, per cuius signum | quod est in circumferentem lineam a capite rectae lineae | recta linea transeat factura in data linea rectum angulum. |* **[p. 108.1]** *in hanc autem rationem sublata circumferentia | chiasmis utendum est. nam quod ad extremam lineae | normationem pertinet, uulgaris consuetudinis est sex octo | et decem: haec de qua supra disputauimus cir- |* **[5]** *culi ratio magis artificialis est, quae numeros non prae- | finit: habemus enim apud Eucliden, quocumque loco ad | circumferentem lineam ex signis dimensionis duae lineae | concurrerint, normam facturas.*

p. 107.10 *formam* VH *forma* J | **12** *ex* JV *et* H | *ducere* JV *ducere uolueris* H | post *signo* lacunam indicavit Lachm. | **13** *relato* J *lato* V *et relato* H | *utramque partem* H et Lachm. ex *utraque* (*-roque* V) *parte* JV | *circino* JH *circinio* V | **14** *punctorum* Lachm. *iunctorum* JV *iuncturarum* H | *circulos* VH *circulus* J | **15** *quorum* $J^{p.c.}$VH *quattuor quorum* $J^{a.c.}$ | *conexionem* VH (*conn-* perperam legit Lachm. in V) *connexione* J | **16** *normales in data* Lachm. *normalis in recta* JV *normationis in recto* H | *linea* JV om. H | *angulos* V *angulus* J *angulo* H | *quo in* Lachm. *quoniam* J *qm̄* V *qm̄* *in* H *quando in* H_L | **17** *non interueniat* V

21 See Folkerts (1969) 53–62. On the 'mixed class' and its manuscripts, see e.g. Thulin (1911a) 72–101, Bouma (1993) 36–40, and Guillaumin (2005) 48–50.

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non interuenit J *normaliter uenit* H | *rectis a* JV *recisa* H | **18** *circumferentiarum* JV *circumferentia linearum* H | *parte chiasmi* Lachm. *parte chiasmis* J *partem chiasinis* V om. H | *cuiusdam ratione* Lachm. *eiusdem rationis* JH *eiusdem rationem* V | **19** *utamur* JV *norma utamur* H | **20–23** *Quod si . . . quod est* om. H | **21** *puncto* Lachm. *iuncto* JV | **22** *transeat* J *transceat* V | **22–24** *rectam lineam . . . transeat* om. V | **23** *a capite* J *inclusio* H | **24** *recta linea* J om. H | **p. 108.1** *in hanc autem rationem* JV *in agratione* H | **2** *chiasmis* J *chiasinis* V *thomis* H | *extremam* JV *extremae* H | **3** *sex octo* JV *VI VIII* H | **4** *et decem* JV *uel X numeros ponere* H | *haec* JV *haec autem* H | *circuli* JV *circularis* H | **5** *quae* J *quā* V *quia* H | *numeros* JV *numerum* H | *non* JV om. H | **6** *Eucliden* (quare?) Lachm. *Euclidem* JV *Euclȳdem* H | **7** *ex* JV *intra semicirculum* H | *signis* JV *signi* H | *dimensionis* V *demensionis* J *dimensionem* H | *duae* JV *in qua si duae* H | **8** *concurrerint* JH *concurrunt* V | *normam* Lachm. *norma* V *nonma* J *normalis* H | *facturas* JV *factura est* H | post *facturas* leguntur haec: *Explicit liber Frontonis primus* J *Explicit liber Frontonis Primus. Amen.* V om. H

Notes[22]

p. 107.10–11 ***Qualemcumque rectorum angulorum formam rectis lineis comprehendere:*** In accordance with Guillaumin and Campbell, I translate thus:[23] 'To contain by straight lines whatever rectangular figure.'

This sentence, which begins our text, is obviously some type of heading, indicating the purpose of the geometrical methods subsequently described. In the text that follows three different methods of constructing right angles are presented. In order to draw exact rectangular and rectilinear figures, it is of course necessary to be able to construct right angles. In this respect, hence, the sentence makes good sense.[24]

Perhaps the sentence will seem oddly expressed: what is the reason for pointing out that a rectangular figure shall be *contained* by straight lines, not simply *consist* thereof? The answer is that in Euclidean geometry a figure is defined thus: Σχῆμά ἐστι τὸ ὑπό τινος ἤ τινων ὅρων περιεχόμενον (Euc. *Elementa* I def. 14), 'A figure is that which is contained by any boundary or boundaries.'[25] Cf. the Latin translation of this definition in the *Expositio* p. 104.1–2 L: *Forma est quae sub aliquo aut aliquibus finibus continetur.* The figure square, for example, is defined not as its boundaries, that is, the four straight lines, but as the plane surface contained by them.[26]

22 The English translations of Latin and Greek passages are my own, unless otherwise indicated. Moreover, the following three translations of the studied text are regularly quoted: Guillaumin (1996) 83, 85, 87, 89; Roth Congès (1996) 371–3; and Campbell (2000) 213, 215. For the sake of brevity, mere page references to these translations are henceforth omitted.

23 Guillaumin's (1996) 83 translation: 'Comprendre par des lignes droites une figure à angles droits quelconque.' Campbell (2000) 213 interprets thus: '. . . to enclose any figure containing right angles with straight lines.'

24 Guillaumin (1996) 83 n. 92, on the other hand, states: 'Cet énoncé ne va pas clairement avec les procédés qui vont être exposés dans la suite du texte pour la construction d'un angle droit.'

25 Heath (1926a) 153.

26 On Euclid's definition of a figure (σχῆμα), see esp. Heath (1926a) 182–3.

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It may seem unnecessary to say that straight lines are to contain a rectangular figure. In fact, it is not unnecessary. For from the account of angles in the *Expositio* pp. 100.5–103.21 L it follows that rectangular figures are not necessarily contained by straight lines; there are different types of right angles depending on the type(s) of the two lines that form the angle: right angle between two straight lines, right angle between two circular lines, right angle between one straight and one circular line. From the *Expositio* p. 101.13–16 L, for example, we infer that a semicircle is considered a rectangular figure. The semicircle is contained by a circular line (half a circumference) and a straight line; where the lines meet, they form right angles; consequently the two angles of the semicircle are right angles, which renders the figure rectangular.[27]

In his translation Campbell indicates that something is lost before *qualemcumque*: '… to enclose' etc. Presumably the lack of a finite verb governing *comprehendere* was his reason. However, neither Lachmann nor Guillaumin was troubled by this. In my view, the transmitted text (MSS JVH (*forma* in MS J being, of course, a banal error)) is most likely sound. The infinitive can be satisfactorily explained as an example of the well-attested usage of infinitives in headings, especially common in technical literature; see Löfstedt (1936) 189, Hofmann and Szantyr (1965) 367, and Svennung (1935) 418–24. Cf. e.g. the initial enunciation of the first proposition in Euclid's *Elements*: Ἐπὶ τῆς δοθείσης εὐθείας πεπερασμένης τρίγωνον ἰσόπλευρον συστήσασθαι (Euc. *Elementa* 1 prop. 1), 'On a given finite straight line to construct an equilateral triangle.'[28]

According to Guillaumin, the three figures preserved in MS J f. 70r, appended to Lachmann's edition as figure 122, have nothing to do with the context.[29] It is true that these figures make no sense at the location given to them in MS J: between the words *diastemate* and *circulos* (p. 107.14 L). But taken with *Qualemcumque … comprehendere*, they aptly serve as illustrations of rectangular and rectilinear figures.

The figure at the bottom of MS H f. 46v is most likely a variant of one such figure.[30] Furthermore, since the figures in MS H are obviously related to those found in MSS JV, and since MS B itself was not used for the original compilation of the 'mixed class' (to which MS H belongs), the figures cannot have been invented by the drawer of the figures in MS B; see the section 'The textual witnesses' above and the note on ***quorum*** (p. 107.15 L) below.

27 *quaecumque autem linea in dimensione medium secans circulum per punctum transiens ad circumferentem lineam pares alternos secundum suam speciem rectos angulos faciet* (*Expositio* p. 101.13–16 L), 'And any line which as diameter cuts a circle in the middle and passes through the centre to the circumference will on each side make equal and, according to their type, right angles.' I translate *in dimensione* 'as diameter', understanding the preposition *in* in the sense of '(serving) as', 'in the capacity of', etc.; see Hofmann and Szantyr (1965) 275. (Perhaps one could even consider emending to *in dimensione<m>*, as *in* + the accusative would also be possible in this type of sense and *m* could easily have been lost in the transmission.)

28 Heath (1926a) 241.

29 Guillaumin (1996) 83 n. 93: 'Les figures qui s'interposent n'ont rien à faire dans le contexte.' Cf. Roth Congès (1996) 371 n. 260.

30 In his description of MS H, Toneatto (1994a) 473 erroneously indicates that figure 122 is missing and that figure 121 is located at the bottom of f. 46v; obviously it should be the other way around.

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No diagram illustrating rectangular and rectilinear figures has been preserved in MS V.

p. 107.12–16 *Ex data recta linea* … *in data linea angulos*: If – which seems quite reasonable – the portions of text before and after Lachmann's suggested lacuna (indicated in the text thus: <*>) are to be taken together, this passage describes how to draw a straight line at right angles to a given straight line, that is to say, to drop a perpendicular, which will pass through a point on or beside the given straight line.

Figures 1 and 2 are modern illustrations of the construction, to which I shall be referring in the following notes. Figure 1 illustrates the construction if the point A is placed *on* the given straight line l, whereas Figure 2 displays the construction if the point A is placed *beside* the given straight line l.

Guillaumin translates the passage thus:

> D'une droite donnée, mener, en un point que l'on a placé, <une droite perpendiculaire à la première, puis,> portant le compas de part et d'autre, décrire des cercles de rayon égal à la distance entre leurs centres, par les intersections desquels passera une droite qui fera des angles droits sur la droite donnée.

Roth Congès' translation runs as follows:

> Depuis une ligne droite donnée, élever, après avoir posé un jalon, [une ligne droite perpendiculaire à la première]: ayant tracé des repères au compas de part et d'autre, il faut décrire des cercles de rayon égal à la distance des centres, par l'intersection desquels une ligne droite passe, pour faire des angles droits sur la ligne donnée.

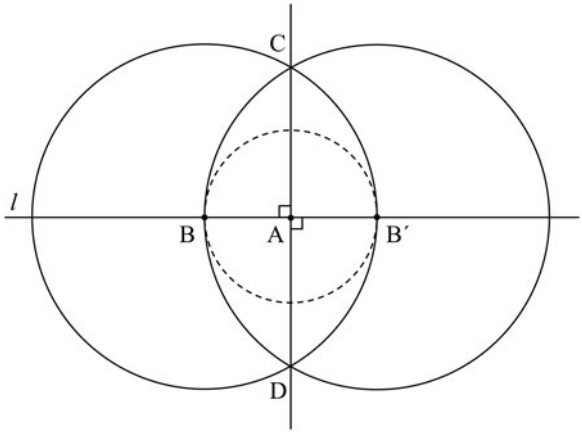


Fig. 1

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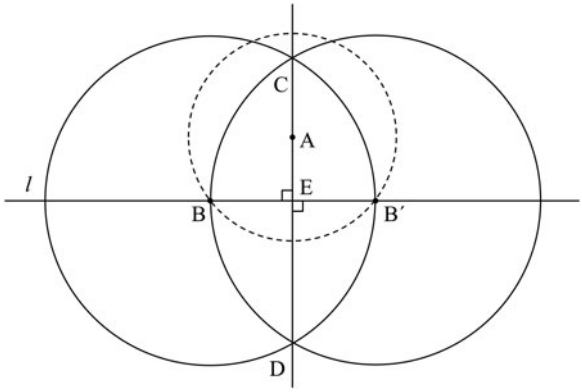


Fig. 2

Campbell, finally, renders the passage thus:

> To draw from a given straight line when a marker has been positioned … when the compass has been moved to both sides, it is appropriate to draw circles with a radius equal to the distance between their two centres, and where they intersect, a straight line should pass through, and will make right angles at a given line.

As the illustrating figures in Guillaumin and Roth Congès show, they describe only the case when the point A is placed on the given straight line l.[31] Bouma, for his part, offers a different description of the construction, which has been adopted by Campbell in his commentary;[32] it will be discussed in the excursus 'Bouma's explanation of *Ex data recta linea … in data linea angulos* (p. 107.12–16 L)' below.

p. 107.12 *Ex*: Giving sense to *data recta linea* (the straight line l in Figs. 1 and 2), the preposition *ex* (MSS JV) can safely be preferred to *et* (MS H).

p. 107.12 *ducere*: The infinitive evidently lacks an object; this indicates that the passage suffers from a lacuna; see further the separate note on the lacuna below.

The infinitive *ducere* is probably a so-called *infinitivus imperativus*, which is not uncommon in technical literature; see e.g. Hofmann and Szantyr (1965) 366–7, Löfstedt (1936) 186–90, and Svennung (1935) 436–38. It is best rendered in English with the imperative: 'draw'. Another instance of this type of infinitive in our text is *eicere*; see the note on it below (p. 107.22 L).

31 Guillaumin (1996) 83 n. 94; Roth Congès (1996) 371 with Fig. 40a.

32 Bouma (1993) 109; Campbell (2000) 435 n. 25.

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The *uolueris* which MS H has added to *ducere* is most likely an erroneous scribal emendation; it has been added in order to supply *ducere* (and *comprehendere* (p. 107.11 L)) with a governing finite verb.

p. 107.12 *posito signo*: Most likely an ablative absolute, meaning literally 'a point having been placed'. As Figures 1 and 2 show, the point, which is called A, can be placed arbitrarily (wherever the perpendicular is intended to be drawn).

Guillaumin translates *posito signo* thus: 'D'une droite donnée, mener, **en un point que l'on a placé**, <une droite perpendiculaire à la première, [. . .] > ' (my emphasis). To be precise, however, the Latin text as preserved does not explicitly state that the straight line to be drawn (the straight line CD in Figs. 1 and 2) will pass through the point referred to in *posito signo* (the point A in Figs. 1 and 2); but as a consequence of the construction, this will of course be the case.

p. 107.12 *signo*: I think that 'point' is the appropriate translation of the word *signum* (so Guillaumin), since the description of the construction seems to be a theoretical demonstration, not a direct account of a procedure performed in the field. Roth Congès, however, renders *signum* as 'jalon', and Campbell uses the word 'marker'. In our text *signum* also occurs at p. 107.22 L and p. 108.7 L in the sense of 'point'.

In its geometrical sense of 'point' the Latin word *signum* corresponds to the Greek σημεῖον; see e.g. Mugler (1958–9) s.v. σημεῖον and *OLD* s.v. *signum* 1b. Cf. e.g. *Expositio* p. 97.15 L *signum est cuius pars nulla est* = Euc. *Elementa* 1 def. 1 σημεῖόν ἐστιν οὗ μέρος οὐθέν; *Expositio* p. 98.15–16 L [. . .] *lineae autem fines signa* = Euc. *Elementa* 1 def. 3 γραμμῆς δὲ πέρατα σημεῖα.

p. 107.12 <*>: As Lachmann indicated in his edition, the passage evidently suffers from a lacuna; see the note on ***ducere*** (p. 107.12 L) above. Exactly what and how much has been lost is in my view not completely clear; nor is the location of the lacuna itself certain.

In his translation, Guillaumin supplied this: 'D'une droite donnée, mener, en un point que l'on a placé, <une droite perpendiculaire à la première, puis,> [. . .].' As he mentions in a footnote, he was using an earlier draft of Roth Congès' article.[33] In Roth Congès' published article, then, the lacuna is filled similarly: 'Depuis une ligne droite donnée, élever, après avoir posé un jalon, [une ligne droite perpendiculaire à la première] [. . .].' In my view, Guillaumin and Roth Congès may well be on the right track in principle.

On the one hand, Lachmann's suggested location of the lacuna is plausible; it seems reasonable that *posito signo* and *relato . . . circino* were originally not asyndetically coordinated (MSS JV) but separated by some portion of text. If, on the other hand, the reading *et* in MS H is genuine, not a banal scribal emendation, the lacuna could be located either thus: *Ex data recta linea <*> ducere posito signo et relato in utramque partem circino* etc. Or thus: *Ex data recta linea ducere <*> posito signo et relato in utramque partem circino* etc.

33 Guillaumin (1996) 83 n. 94.

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p. 107.13 *relato in utramque partem circino*: As regards this phrase, MS H offers the correct reading. However, the minor errors in MSS JV were easily corrected by Lachmann.

Guillaumin, Roth Congès and Campbell take *relato … circino* to be an ablative absolute. Guillaumin translates 'portant le compas de part et d'autre'; Roth Congès translates literally 'après avoir envoyé le compas de part et d'autre',[34] and then interprets 'ayant tracé des repères au compas de part et d'autre'; finally, Campbell: 'when the compass has been moved to both sides'.

Roth Congès explains the purpose of the movement of the compass thus: '[…] pour déterminer sur la ligne droite les deux centres des cercles à décrire'.[35] With Roth Congès' explanation, the idea is that by using the compass one point on each side of and at equal distance to the point A is obtained on the given straight line l (see Figs. 1 and 2). In other words, a circle (or at least two circular arcs) is drawn with its centre at point A and with an arbitrarily taken radius AB, so that the circle intersects the straight line l at two points B and B′ (hence, AB = AB′). These two points, B and B′, will then constitute the centres of two circles that are to be drawn next.

Admittedly, the phrase *relato … circino* is rather compressed. As it describes such an important part of the construction, one would have wished it to be somewhat more elaborate.

p. 107.13–14 *aequali punctorum diastemate*: A condensed expression – due to the twofold meaning of the word *diastema*: 'distance' and 'radius' (see the separate note on ***diastemate*** (p. 107.14 L) below) – which goes with the subsequent *circulos scribere oportet*. It is most likely to be understood as if it said *diastemate diastemati punctorum aequali scribere oportet circulos*;[36] that is to say, the radius (*diastemate*) of the circles is equal (*aequali*) to the distance (*diastemati*) between the centres (*punctorum*; see the separate note on ***punctorum*** (p. 107.14 L) below) of the circles (BB′ = B′B; see Figs. 1 and 2). Cf. these two passages in the *Expositio:*

> *quotiens* ***ex uno duorum punctorum diastemate*** *duo circuli pares exeunt, ad conexionem circumferentiarum interiores rectos angulos facient.* (*Expositio* p. 102.13–16 L)
>
> Whenever two equal circles are drawn from one and the same radius and distance between the two centres, the circles will make the interior angles at the junction (= intersections) of the circumferences right angles.[37]
>
> *circumferentiarum linearum rectos angulos ideo quod si tres circuli pares inter se fuerint* ***aequali diastemate*** [sc. *punctorum*] *conexi* […]. (*Expositio* p. 102.18–20 L)

34 Roth Congès (1996) 371 n. 262.

35 Roth Congès (1996) 371 n. 262.

36 Cf. Guillaumin (1996) 83, Roth Congès (1996) 371 and Campbell (2000) 213. The full stop after *diastemate* in Guillaumin's (1996) 82 Latin text is obviously a misprint.

37 My highly modified version of Campbell's (2000) 211 translation.

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> (We also have) right angles of circular lines for the reason that, if three equal circles with the same radius/distance (between their centres) are joined together [. . .].[38]

As there are no other attestations in ancient Latin of *diastema* used in its two senses at the same time, I think that this feature, common to the supposed fragment and the *Expositio*, can be adduced as one piece of evidence favouring the view that the supposed fragment rightly belongs to the *Expositio*.

p. 107.14 *punctorum*: The word *punctum* is used in the sense of 'centre', thus corresponding to the Greek κέντρον; see e.g. Mugler (1958–9) s.v. κέντρον, *OLD* s.v. *punctum* 3b, *TLL* s.v. *punctum* 10:2.2648.27 ff.

Lachmann rightly, I think, corrected the transmitted *iunctorum* (MSS JV) to *punctorum*; MS H has *iuncturarum*, which could be an erroneous scribal emendation of *iunctorum*. In our text the word *punctum* also occurs at p. 107.21 L, where Lachmann corrected the transmitted *iuncto* (MSS JV) to *puncto*; see the note on ***ex quolibet puncto* . . . *rectam lineam eicere*** (p. 107.21–2 L) below. The attestations of the word *punctum* 'centre' in the *Expositio* are: pp. 101.15; 102.1; 102.7; 102.14 L.

In a footnote to her translation Roth Congès tries to defend *iunctorum* by suggesting that the participle *iunctus* could be used as a noun in the sense of 'ouverture de compas' or 'écartement des branches': *aequali iunctorum diastemate* 'à une distance égale à celle des (deux) ouvertures de compas'.[39] But since there is no support whatsoever to be found in ancient Latin literature for this sense of *iunctus*, and since the attestation of *punctorum diastemate* in the *Expositio* p. 102.14 L – quoted in the note above on ***aequali punctorum diastemate*** (p. 107.13–14 L) – is critically certain, Lachmann's correction is in my view as right as an emendation can be.

p. 107.14 *diastemate*: In geometrical contexts the Greek neuter noun διάστημα usually appears in one of two senses: 'distance' or 'radius' (that is, in the specific sense of the distance between centre and perimeter); see Mugler (1958–9) s.v. διάστημα 1. In Roman literature the Greek term διάστημα in the sense of 'radius' is also found translated as e.g. *interuallum* (Ps. Censorinus *fr.* 8.1), *interstitium* (Martianus Capella 6.722), and *spatium* (Euc. *elem. vers. M* l. 51 ed. Folkerts (1970)).

p. 107.14–15 *circulos scribere oportet*: Two circles are drawn. As their radii (BB′ and B′B; see Figs. 1 and 2) are equal to the distance between the centres (BB′ = B′B), each circumference will pass through the centre of the other circle.

On the meaning of the word *circulus*, see the note on ***conexionem*** (p. 107.15) below.

38 My highly modified version of Campbell's (2000) 211 translation.

39 Roth Congès (1996) 371 n. 263.

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p. 107.15 *quorum*: The antecedent is *circulos*.

The erroneous reading *quattuor quorum* in MS J[a.c.] may well have something to do with the fact that two practically identical pairs of intersecting circles are drawn in MS J f. 7or between *angulos* (p. 107.16 L) and *sed* (p. 107.16 L); they are reproduced in Lachmann's edition as figure 123. These four circles, which, as Thulin suggests, were drawn in the margin of MS B and then copied into MS J,[40] may have led the scribe of MS J to commit the error.

On the one hand, the two pairs of intersecting circles could be the result of duplication of one original pair, which, if so, probably had already occurred in some ancestor to the drawings of MS B (see below in this note). On the other hand, it is also possible that both pairs are genuine and originally illustrated different stages of the construction, but, having lost their distinguishing details in the process of copying, now appear identical in MS J.

The four circles found in MS H f. 46v are most likely related to the circles found in MS J, but they appear in a more corrupt state. Their existence in MS H proves that the drawer of the figures in MS B did not draw at his own discretion; see the section 'The textual witnesses' and the note on ***Qualemcumque*** … ***comprehendere*** (p. 107.10–11 L) above.

In MS V f. 11or one pair of intersecting circles is found in the margin. As both MS J and MS H have four circles, I think that MS B had two pairs of intersecting circles and only one such pair was copied into MS V.

p. 107.15 *conexionem*: Literally *conexio* means 'junction', etc.; see e.g. *OLD* s.v. *conexio* 1. But here 'intersections' is quite an apt translation. Since the circumferences intersect at two points (C and D; see Figs. 1 and 2), one would perhaps have expected the plural *conexiones*; but cf. *Expositio* p. 102.13–16 L, where *conexio* in the phrase *ad conexionem circumferentiarum* also occurs in the singular (this passage is quoted in the note on ***aequali punctorum diastemate*** (p. 107.13–14 L) above). Hence, both intersections are included in the singular expression.

In Euclid κύκλος 'circle' usually means the plane figure, the surface, if you will, which is contained by the circumference, that is, 'disk', not the circumference of the circle;[41] see e.g. the Euclidean definition of the circle, which also appears translated into Latin in the *Expositio*:

> Euc. *Elementa* 1 def. 15 Κύκλος ἐστὶ σχῆμα ἐπίπεδον ὑπὸ μιᾶς γραμμῆς περιεχόμενον, πρὸς ἣν ἀφ' ἑνὸς σημείου τῶν ἐντὸς τοῦ σχήματος κειμένων πᾶσαι αἱ προσπίπτουσαι εὐθεῖαι ἴσαι ἀλλήλαις εἰσίν.
>
> *circulus autem est plana forma ab una linea conprehensa, ad quam ab uno signo intra formam posito omnes accidentes rectae lineae sunt inter se pares.* (*Expositio* pp. 104.17–105.1 L)[42]

40 Thulin (1911a) 23–4.

41 See e.g. Heath (1926a) 184–5.

42 Lachmann printed *accedentes*; I prefer *accidentes*, which is closer to Euclid's προσπίπτουσαι. Cf. the Latin translation of Euclid ascribed to Boethius, where αἱ προσπίπτουσαι is rendered as *quae incidunt*; see Euc. *elem. vers. M* l. 21 ed. Folkerts (1970).

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> A circle is a plane figure contained by one line such that all straight lines falling upon it from one point lying within the figure are equal to one another.[43]

Sometimes, however, Euclid evidently uses κύκλος in the sense of 'circumference of a circle'; see e.g. Euc. *Elementa* 3 prop. 10 Κύκλος κύκλον οὐ τέμνει κατὰ πλείονα σημεῖα ἢ δύο. 'A circle does not cut a circle at more points than two.'[44]

Now, as the antecedent to the genitive *quorum* which qualifies *conexionem* is *circulos*, this suggests that our author is not using the word *circulus* here in the usual sense of 'disk', but in the sense of 'circumference of a circle'.

p. 107.16 *normales in data linea angulos*: The right angles are attained at the intersection of the straight line CD and the given straight line l (see Figs. 1 and 2); that is, either at A (if point A was placed on l) or at E (if point A was placed beside l).

MSS JV read: *normalis in recta linea angulos* (*angulus* MS J). MS H reads: *normationis in recto angulo*. Most likely MSS JV exhibit a more genuine text (Lachmann's minor change of *normalis* to *normales* seems to be in order, since there are no indications in the *Expositio* of a regular use of the ending *–is* in the accusative plural). Even if *normationis* were changed to *normationem* or *normationes*, the reading of MS H remains awkward and obscure. Perhaps *normationis in recto angulo* is the futile result of scribal emendation after loss of *linea*: *normalis/–es in recta linea angulos* > **normalis/–es in recta angulos* > *normationis in recto angulo*. On the adjective *normalis*, see the next note.

Lachmann's emendation *data* for the transmitted *recta* seems to be supported by *factura in data linea rectum angulum* at p. 107.24 L; but by comparing *ex data recta linea* at p. 107.12 L one could also suggest that *data* should be supplied before *recta*: *in <data> recta linea*. In addition, the reading *recto* in MS H may be adduced in support of maintaining *recta*. Although these conjectures make the meaning clearer to the reader, I tend to think that they are unnecessary; the author may well have been somewhat vague, and for an attentive reader it is evident that the *recta linea* in question is the given straight line l, not the straight line CD (see Figs. 1 and 2).

p. 107.16 *normales*: At first glance the adjective *normalis* (from the noun *norma* 'set square'; see the note on ***normam*** (p. 108.8 L) below) seems to be used here as a synonym of *rectus*: *normalis angulus* = *rectus angulus* 'right angle'. There may, however, be a yet unnoticed terminological difference: that *normalis angulus* is the special type of *rectus angulus* which consists of two *straight* lines. In the *Expositio* pp. 100.8–101.11 L angles consisting of two straight lines are described, and in this section we find the right angle called *normalis angulus*. However, in the sections which describe angles consisting of one straight and one circular line (pp. 101.12–102.11 L) and of two circular lines (pp. 102.12–103.5 L), the right angles of these two types are never called *normales*. See especially the following passages in the *Expositio*:

43 My slightly modified version of Heath's (1926a) 153 English translation of Euclid's definition of the circle, so as to make it better suit the Latin.

44 Heath (1926b) 23.

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Rectarum ergo linearum species angulorum generis sui tres, recta ebes acuta. rectus angulus est euthygrammos, id est ex rectis lineis conprehensus, qui Latine normalis appellatur.

(*Expositio* p. 100.8–11 L)

Thus, there are three types of the kind of angles which consist of straight lines: right, obtuse, acute. The right angle is *euthygrammos* (= rectilinear), that is, contained by straight lines; in Latin it is called *normalis*.[45]

rectus ergo angulus est normalis, ebes plus normalis, acutus minus normalis.

(*Expositio* p. 101.10–11 L)

Thus, the right angle is *normalis*, the obtuse is greater than the *normalis*, the acute less than the *normalis*.[46]

Cf. the following passage in the so-called *Fragmentum Censorini*, where *normalis* (sc. *linea*) is given as a Latin equivalent to the Greek κάθετος (sc. εὐθεῖα (sc. γραμμή)) 'perpendicular (straight line)' (see Mugler (1958–9) s.v. κάθετος):

Ps. Cens. *fr.* 6.3 *si recta linea supra rectam lineam stans continuos angulos inter se pares facit, tum uterque ex paribus angulis rectus dicitur, et ea linea Graece* κάθετος, *Latine normalis dicitur.*[47]

If a straight line standing on a straight line makes the adjacent angles equal to one another, then each of the equal angles is called right, and the line is called κάθετος in Greek, *normalis* in Latin.

It should finally be noted that, granted the difference between *normalis angulus* and *rectus angulus* suggested above, the author of our text obviously did not hesitate to use the wider term *rectus angulus*, although the stricter *normalis angulus* could have been used instead; see p. 107.24 L.

Excursus: Bouma's explanation of *Ex data recta linea* … *in data linea angulos* (p. 107.12–16): Evidently Bouma's tentative and highly unconvincing description of the geometrical construction does not take the following portion of text into account: *Ex data recta linea ducere posito signo* <*> *relato in utramque partem circino* (p. 107.12–13 L).[48]

45 Note Guillaumin's (1996) 55 n. 56 comment: 'Malgré une apparente ambiguïté, le texte ne pose pas ici l'équivalence εὐθύγραμμος/*normalis*, mais celle de *rectus* et de *normalis*.' On the Greek term, see Mugler (1958–9) s.v. εὐθύγραμμος.

46 Most likely apposition is used instead of the ablative of comparison: *plus/minus normalis* = *plus/minus normali*; see Hofmann and Szantyr (1965) 110.

47 This passage is a Latin rendering of Euc. *Elementa* 1 def. 10: Ὅταν δὲ εὐθεῖα ἐπ' εὐθεῖαν σταθεῖσα τὰς ἐφεξῆς γωνίας ἴσας ἀλλήλαις ποιῇ, ὀρθὴ ἑκατέρα τῶν ἴσων γωνιῶν ἐστι, καὶ ἡ ἐφεστηκυῖα εὐθεῖα κάθετος καλεῖται, ἐφ' ἣν ἐφέστηκεν.

48 Bouma (1993) 109 with illustrating figures iv–vi (63). Bouma's description of the construction was perhaps inspired by the four circles drawn in MS J f. 70r (located between *angulos* (p. 107.16 L) and *sed* (p. 107.16 L) and reproduced in Lachmann's edition as figure 123); see the note on ***quorum*** (p. 107.15 L) above.

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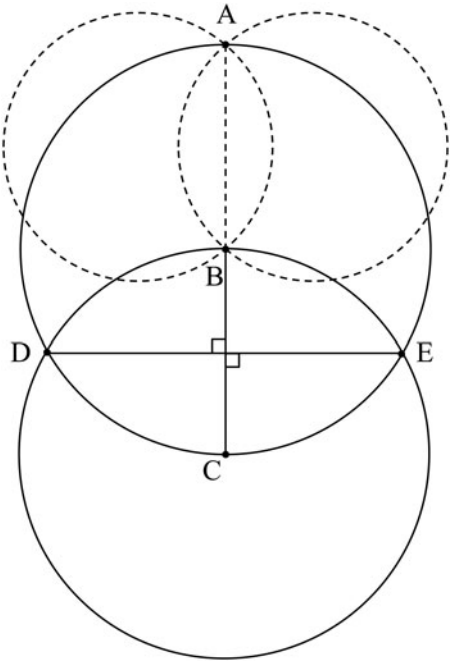


Fig. 3

Consequently he suggests that the straight line to which the perpendicular shall be dropped is first to be constructed, although it is most likely already given (*ex data recta linea*). This is done by drawing two intersecting circles with equal radii and then by drawing the straight line AB through their intersections (see Fig. 3 below). As the phrases *posito signo* and *relato in utramque partem circino* are neglected, the construction proceeds by drawing one circle with the centre at B and the radius BA and then by extending the straight line AB to C, so that another circle with the centre at C and the radius CB can be drawn. By drawing the straight line DE, finally, the intersection of DE and AC will be at right angles.

Furthermore, a great drawback is that the constructed right angles will end up at a location difficult to foresee at the beginning of the construction; this renders Bouma's construction impractical, even in theory.

Unfortunately, Bouma's explanation was uncritically adopted by Campbell in his commenting note.[49]

p. 107.16–19 *sed quo* . . . *ratione utamur*: The exact meaning of this passage is uncertain, and Lachmann obviously considered it necessary to subject the paradosis to conjectural

49 Campbell (2000) 435 n. 25.

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criticism. Guillaumin, Roth Congès and Campbell interpret the passage somewhat differently, yet they all follow Lachmann's text.

Guillaumin, to begin with, translates thus:

> Mais pour que dans une figure formée de lignes droites la ligne circulaire ne coupe pas les droites, servons-nous du procédé du chiasme, sans utiliser les circonférences.

As regards the phrase *a circumferentiarum parte*, Guillaumin states in a footnote: '[O]n peut hésiter entre deux traductions: "sans utiliser les circonférences", ou "à partir de segments de circonférences".'[50]

Roth Congès, then, interprets as follows:

> Mais là où, dans une figure à lignes droites, la ligne circulaire ne recoupera pas les droites, servons-nous du procédé du chiasme, à partir d'arcs de cercles.

In a footnote to this portion of text, Roth Congès gives the following comment: 'C'est-à-dire qu'on se contente de tracer aux endroits nécessaires deux arcs de cercle, dont l'intersection marque le point recherché. Ce sont ces traits croisés en forme de χ que l'on appelle "chiasme".'[51]

Campbell, finally, renders the passage thus:

> But in a figure consisting of straight lines, where a circular line does not intersect the straight lines, starting from part of the circumference we must use the method involving *chiasmus*.

I for my part do not profess to have found a definitive solution to this difficult passage; but in the following notes I tentatively suggest that a text at least as good – or bad – as Lachmann's can be obtained without resorting to conjectural criticism:

> *sed quoniam in rectarum linearum forma circularis linea non interuenit rectis, a circumferentiarum parte chiasmis eiusdem rationis utamur.*

This text could be translated thus:

> But since in a rectilinear figure a circular line is not combined with straight lines, we may use χ-shaped marks from segments of circumferences according to the same method.

50 Guillaumin (1996) 85 n. 95.

51 Roth Congès (1996) 372 n. 265.

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Or perhaps more freely:

> But since circular lines do not belong among the straight lines which contain a rectilinear figure, we may use χ-shaped marks according to the same method which are obtained from segments of circumferences.

p. 107.16–17 *sed quo* … *interueniat rectis*: MS J reads as follows: *sed quoniam rectarum linearum forma circularis linea non interuenit rectis*; MS V: *sed q̄m rectarum linearum forma circularis non interueniat rectis*; MS H: *sed q̄m* (*quando* MS H_L) *in rectarum linearum forma circularis linea normaliter uenit recisa*.

As stated in the section 'The textual witnesses' above, Lachmann used only MSS JV for his edition. Starting from the transmitted readings *quoniam* (MS J) and *q̄m* (= *quoniam*) (MS V),[52] Lachmann conjectured *quo in*, whereby a desired *in* was restored: *in rectarum linearum forma*. Possibly he imagined this chain of corruption: **q in* (abbreviated *quo*) > *q̄m* (MS B?, MS V) > *quoniam* (MS J). Exactly what sense of *quo* he had in mind is unclear, however. According to Guillaumin's translation, *quo* is interpreted as a final subordinating conjunction, whereas in Roth Congès' and Campbell's translations *quo* is interpreted as a pronominal adverb; see the preceding note on ***sed quo*** … ***ratione utamur*** (p. 107.16–19 L). Furthermore, Lachmann preferred *interueniat* (MS V) to *interuenit* (MS J).

As we now are able to consult MS H, there is reason to reconsider the text. Unfortunately, however, MS H seems to offer a generally more corrupt text of this passage than MSS JV, but it might still provide us with some clues. First, it could be that *q̄m in* (= *quoniam in*) is the genuine reading, not merely a scribal emendation of a previous *q̄m*.[53] Secondly, the awkward *normaliter uenit* could, I think, speak in favour of reading the indicative *interuenit*, although, of course, scribal emendation of a previous *ueniat* cannot be totally excluded. And thirdly, the reading *recisa* (MS H) seems to support the genuineness of *rectis a* (MSS JV), as it is far more likely that *rectis a* has been corrupted to *recisa* than *vice versa*.

The intended sense of the verb *interuenire* is not perfectly clear; Guillaumin renders it with the verb 'couper', Roth Congès with 'recouper', and Campbell with 'intersect'. In my opinion, however, one may compare the following passage in the *Expositio*:

> *Aeque multas ac uarias figuras habent formae, quotiens flexuosae lineae rationalis siue recta siue circularis linea interuenit.* (*Expositio* p. 104.10–12 L)
>
> Figures have equally many varied shapes, whenever a measurable line, straight or circular, is combined with a curved line.

If the verb *interuenire* is taken in this sense, 'be combined with', one may read the passage in question with *quoniam in* and *interuenit* thus: *sed quoniam in rectarum linearum forma circularis linea non interuenit rectis* 'but since in a rectilinear figure a circular line is not combined with

52 Cf. *Expositio* p. 102.6 L *quoniam*, where MS V f. 105v has *q̄m q̄m* (dittography).

53 The scribe of MS H_L, J. J. Scaliger, expanded *q̄m* to *quando*; presumably he erroneously read *q̄m* as *q̄n*.

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straight lines'. In other words: by definition a rectilinear figure is not contained by any circular line; however, the previously described method of constructing right angles in such a figure involves the drawing of circles (p. 107.12–16 L). Consequently, the clause reminds the reader of the obvious fact that those circular lines are merely auxiliary, not constituent parts of the figure being constructed.

p. 107.18–19 *a circumferentiarum* . . . *utamur*: MS J reads as follows: *a circumferentiarum parte chiasmis eiusdem rationis utamur*; MS V: *a circumferentiarum partem chiasinis eiusdem rationem utamur*; MS H: *recisa circumferentia linearum eiusdem rationis norma utamur*.

Lachmann printed *a circumferentiarum parte* with MS J and then adopted his conjecture *chiasmi cuiusdam ratione*. In my view, however, there seems to be no need for emendation; I suggest that *chiasmis eiusdem rationis* be read with MS J. At p. 108.2 L, MS J has preserved the genuine reading *chiasmis* (the ablative is governed by *utendum est*), whereas MS V has the corrupt *chiasinis* (*in* instead of *m*); see the note on ***in hanc autem*** . . . ***utendum est*** (p. 108.1–2 L) below. Hence there is reason to assume that *chiasmis* is the genuine reading here too, where MS V also exhibits the corrupt *chiasinis*. The reading *eiusdem*, then, is supported by all three MSS, and *rationis* is supported by MSS JH; *rationem* in MS V is presumably an ungrammatical emendation introduced by the scribe (with the intention of recovering an "object" for *utamur*). The text of MS H, finally, is likely to be the result of omissions (*recisa* for *rectis a* and *parte chiasmis* omitted) and subsequent scribal emendations (*circumferentia linearum* for *circumferentiarum* and *norma* added).

As Guillaumin, Roth Congès and Campbell have noted, the word *chiasmus* here denotes a mark of intersection between two line segments in the shape of the Greek letter χ.[54] According to the *TLL* s.v. *chiasmus* 3.1007.76 ff., p. 107.18 L and p. 108.2 L are the only attestations of this Latinised Greek word (cf. LSJ s.v. χιασμός) in ancient Latin gromatical literature. They have also observed that the noun *decussis* is attested in ancient Latin in a similar sense; see *TLL* s.v. *decussis* 5:1.248.57–66 and 77–83. Cf. also *TLL* s.vv. *decusso* 5:1.249.5 ff.; *decussatio* 5:1.248.26 ff.; *decussatim* 5:1.248.13 ff.

The prepositional expression *a circumferentiarum parte* is somewhat obscure. It seems to go with the following *chiasmis*, stating that the intersections may be accomplished by drawing only a part, that is, segments, of the circumferences; for it is not necessary to draw the entire circumferences in order to obtain the intersections.

Consequently the whole clause *a circumferentiarum parte chiasmis eiusdem rationis utamur* could be translated thus: 'we may use χ–shaped marks from segments of circumferences according to the same method'; or more freely: 'we may use χ–shaped marks according to the same method (as described previously) which are obtained from segments of circumferences'.

p. 107.20–4 *Quod si ab* . . . *linea rectum angulum*: In this passage another method of dropping a perpendicular is described. It is an application of the so-called theorem

54 Guillaumin (1996) 85 n. 95, Roth Congès (1996) 372 n. 265, Campbell (2000) 436 n. 25.

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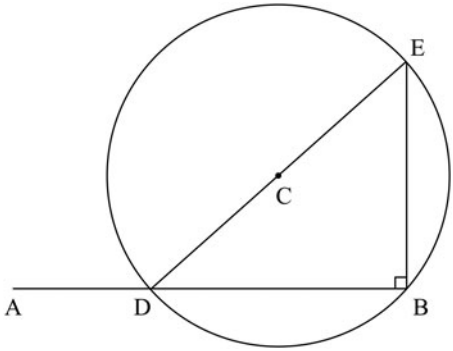


Fig. 4

of Thales, which states that the diameter of a circle always subtends a right angle at any point on the circumference of the circle.[55] Figure 4 is a modern illustration of the construction, to which I shall be referring in the following notes. Let the straight line AB be the given straight line, and the straight line EB be the constructed straight line at right angles to the given straight line. Cf. the figures given by Guillaumin, Roth Congès and Campbell.[56]

MSS JV have preserved a figure which is a simple illustration of this construction (it is reproduced in Lachmann's edition as figure 124): MS J f. 70r, between *utamur* (p. 107.19 L) and *Quod si* (p. 107.20 L); MS V f. 110r, at the bottom of the folio, after *angulum* (p. 107.24 L). The figure in the top right corner of MS H f. 47r may be a corrupt version of this figure.

p. 107.20–1 *Quod si ab* . . . *rectum angulum faciat*: The paradosis (MSS JV) is most likely sound. The phrase *recta linea* may be taken either with *ab eadem* or with *ducenda fuerit*. In either case, another *recta linea* is easily supplied in thought: *ab eadem rectā lineā* (sc. *rectā lineā*) *ducenda fuerit quae* etc. or *ab eadem* (sc. *rectā lineā*) *rectā lineā ducenda fuerit quae* etc.[57] I translate: 'But if from the same straight line (AB; see Fig. 4) a straight line (BE) shall be drawn which is to make a right angle.'

p. 107.21–2 *ex quolibet puncto* . . . *rectam lineam eicere*: MS J reads as follows: *ex quolibet iuncto qui per caput recta linea transeat rectam lineam eicere*; MS V: *ex quolibet iuncto qui per caput recta linea transceat* (lacuna after *transceat*). Lachmann obviously followed the paradosis, but rightly changed *iuncto* to *puncto* (see the note on **punctorum** (p. 107.14 L) above) and discarded the evidently erroneous reading *transceat*. Still, Lachmann's text is unsatisfactory.

55 For a historical discussion of Thales' theorem, see Heath (1921) 130–1 and 133–7.

56 Guillaumin (1996) 87 n. 96; Roth Congès (1996) 371 (fig. 40b); Campbell (2000) 436 n. 25.

57 Cf. Roth Congès (1996) 372 and 372 n. 267.

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As Lachmann's text runs, the transmitted *qui* makes no sense; it cannot refer to *puncto* either grammatically or factually. Guillaumin tentatively suggested that *et diastemate* be supplied before *qui*, so that the relative pronoun would refer to *diastemate*, and that *recta linea* be changed to *rectae lineae* (since '[...] il est inutile de préciser qu'un rayon va en ligne droite'): *ex quolibet puncto <et diastemate> qui per caput recta<e> linea<e> transeat rectam lineam eicere* 'd'un centre quelconque [C; see Fig. 4] <et avec un rayon [CB]> ' 'qui passe par l'extrémité [B] de la ligne droite [AB]' 'prolonger une droite [DCE]'.[58] However, as *diastema* ought to be a neuter noun (see the note on ***diastemate*** (p. 107.14 L) above), should *qui* not also be changed to *quod*?

Roth Congès, for her part, tried, unconvincingly in my view, to retain *iuncto* (masc.) thus: *ex quolibet iuncto qui per caput recta linea transeat rectam lineam eicere* 'à partir de n'importe quelle ouverture de compas qui passe en ligne droite (CB) par l'extrémité (B), tracer une droite (DCE)'. Or: *ex quolibet iuncto qui per caput recta<e> linea<e> transeat rectam lineam eicere* 'à partir de n'importe quelle ouverture de compas (CB) qui passe par l'extrémité de la ligne droite (B), tracer une droite'.[59]

Campbell translates Lachmann's text as follows: 'from the central point (C) (of the circle) extend a straight line which passes through the extremity of the circle (D) in a straight line'. Obviously, Campbell (inadvertently?) translates as if *quae* were read, not *qui*. Moreover, he renders *per caput* as 'through the extremity of the circle (D)', which seems awkward; the circumference of a circle, which is a complete circular line, does not have any extremity. Nor is it likely that *caput* means 'circumference', as it is used in the sense of 'extremity (of a line)' at p. 107.23 L (see the note on ***caput*** (p. 107.21 L) below). In addition, if the centre C is placed at will (*ex quolibet puncto*; Campbell does not translate *quolibet*), the point D will not necessarily coincide with the extremity A of the straight line AB (see Fig. 4), although the figure preserved in MSS JV suggests this; see the note on ***Quod si ab ... linea rectum angulum*** (p. 107.20–4 L) above.

If the transmitted *qui* be regarded as genuine, one could venture to emend thus: *ex quolibet puncto <et circino> qui per caput recta<e> linea<e> transeat rectam lineam eicere* 'from whatever centre (C) and drawing/trace of the compass, which shall pass through the extremity (B) of the straight line (AB), extend a straight line (DCE)'. Cf. the following occurrences of *circinus* = 'drawing/trace of the compass' in the *Expositio*:

> *forma anguli unius ex tribus circinis, ut in opere marmoreo. duorum angulorum forma e duobus circinis, trium angulorum ex tribus circinis, quattuor angulorum ex quattuor circinis* [...].
>
> (*Expositio* p. 105.4–7 L)

> A figure containing one angle can be produced from three traces of the compass, as in marble work. A figure containing two angles can be produced from two traces of the

58 Guillaumin (1996) 87 and 87 n. 96; the references to Figure 4 are my own. Cf. Roth Congès (1996) 372 n. 268.

59 Roth Congès (1996) 372–3; 372 nn. 268 and 270; see also Roth Congès (1996) 373 n. 269 for a French interpretation of the text with *puncto* instead of *iuncto*.

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> compass, containing three angles from three traces of the compass, and containing four angles from four traces of the compass.[60]

However, in a note to this passage in the *Expositio*, Guillaumin suggests that the reason why *circinus* is used instead of *circulus* is that the traces of the compass are not complete circles, but arcs.[61] If so, one may of course object that the restoration ought to be <*et circulo*> rather than <*et circino*>: *ex quolibet puncto <et circulo> qui per caput recta<e> linea<e> transeat rectam lineam eicere* 'from whatever centre (C) and circle, which shall pass through the extremity (B) of the straight line (AB), extend a straight line (DCE)'. Moreover, the required sense of the word *circulus* here ought to be 'circle' = 'circumference of a circle', not 'circle' = 'disk'; see the note on ***conexionem*** (p. 107.15 L) above.

If the transmitted *qui* be regarded as a corruption, one could, on the other hand, emend by changing *qui* and supplying some other word(s) meaning 'circumference' attested in our text: *ex quolibet puncto <et circumferenti linea> quae per caput recta<e> linea<e> transeat rectam lineam eicere* or *ex quolibet puncto <et circumferentia> quae per caput recta<e> linea<e> transeat rectam lineam eicere* 'from whatever centre (C) and circumference, which shall pass through the extremity (B) of the straight line (AB), extend a straight line (DCE)'.

p. 107.21 *caput*: The noun *caput* in the sense of 'extremity (of a line)' occurs in our text also at p. 107.23 L. For further attestations, see *TLL* s.v. *caput* 3:2.411.83 ff.

p. 107.22 *eicere*: The verb *eicere* is used in the sense of 'extend', 'produce'; see *TLL* s.v. *eicio* 5:2.310.68 ff. Most likely it corresponds to the Greek verb ἐκβάλλειν (see Mugler (1958–9) s. v. ἐκβάλλειν), as in the following passage of the *Expositio*, which is a translation of the Euclidean definition of parallel straight lines:

> *ordinatae rectae lineae sunt quae in eadem planitia positae et* ***eiectae*** *in utramque partem in infinitum non concurrunt.* (*Expositio* pp. 98.16–99.2 L)

> Euc. *Elementa* 1 def. 23 Παράλληλοί εἰσιν εὐθεῖαι, αἵτινες ἐν τῷ αὐτῷ ἐπιπέδῳ οὖσαι καὶ **ἐκβαλλόμεναι** εἰς ἄπειρον ἐφ' ἑκάτερα τὰ μέρη ἐπὶ μηδέτερα συμπίπτουσιν ἀλλήλαις.

> Parallel straight lines are straight lines which, being in the same plane and being produced indefinitely in both directions, do not meet one another in either direction.[62]

The infinitive *eicere* is a so-called *infinitivus imperativus*; see the note on the infinitive ***ducere*** (p. 107.12 L) above.

60 Campbell's (2000) 213 translation with my modifications. For illustrating figures and further comments, see Guillaumin (1996) 73 and 75 with notes.

61 Guillaumin (1996) 73 n. 81.

62 Heath (1926a) 154.

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p. 107.22–4 *per cuius signum* … *linea rectum angulum*: Lachmann printed the text according to MS J, which, as far as I can see, is sound. Because of an omission, MS V has only preserved the last words (which agree with the testimony of MS J): *factura in data linea rectum angulum*. MS H, too, suffers from omissions; it reads: *in circumferentem lineam inclusio rectae lineae transeat factura in data linea rectum angulum*. Here *inclusio* is undoubtedly an erroneous scribal emendation.

I translate as follows: 'through/to the point (E; see Fig. 4) of which (i.e. the straight line DCE), that is, to the circumference, a straight line (BE) shall pass from the extremity (B) of the straight line (AB) and make a right angle on the given straight line (AB)'.

As my translation shows, the antecedent to *cuius* is *rectam lineam* at p. 107.22 L.

Guillaumin, Roth Congès and Campbell seem to have taken *signum* to be the antecedent to *quod*. In my view, however, *quod est* is to be understood rather as *id est* 'that is', clarifying the direction of the straight line BE.

On the terms *signum* 'point' and *caput* 'extremity', see the notes on ***signo*** (p. 107.12 L) and ***caput*** (107.21 L) above.

p. 108.1–2 *in hanc autem* … *utendum est*: Most likely, MSS JV exhibit the genuine text, except for the erroneous *chiasinis* of MS V; see the note on ***a circumferentiarum*** … ***utamur*** (p. 107.18–19 L) above. Presumably the reading *in agratione* in MS H is the result of omission (*autem*) and corruptions in the following sequence: *in hanc rationem* > **in hac ratione* > *in agratione*. The reading *thomis* in MS H is most likely an erroneous scribal emendation.

In translating this passage I mainly follow Guillaumin's interpretation:[63] 'But if the circumference is eliminated for this method, one must use χ-shaped marks.' In my view, this sentence states that if the method described at p. 107.20–4 L is applied but, for some reason, a complete circumference is not to be used, it suffices to draw arcs at the points D and E (see Fig. 4). At the point D there will be an intersection between an arc and the straight line AB; the intersection will resemble the Greek letter χ. At the point E, if the straight line DCE is produced somewhat beyond the point E, there will be an intersection between an arc and the straight line; and even if the straight line DCE stops at the point E, a mark will be accomplished which can be described as similar to the letter χ.

On the word *chiasmus*, see the note on ***a circumferentiarum*** … ***utamur*** (p. 107.18–19 L) above.

p. 108.2–4 *nam quod ad extremam* … *sex octo et decem*: Another way of constructing a right angle is compendiously presented. If the sides of a triangle are of six, eight and ten units of length, the triangle is right-angled, that is to say, one of its angles is a right angle.[64]

63 Guillaumin (1996) 87: 'Si l'on supprime le cercle pour cette opération, il faut se servir des chiasmes.' Cf. Roth Congès (1996) 373: 'Mais dans cette méthode, si l'on supprime la circonférence, il faut se servir des chiasmes.' And Campbell (2000) 215: 'If there is no circumference, we must use *chiasmi* in this manner.'

64 By using the converse of Pythagoras' theorem, it can be shown that such a triangle is right-angled: $6^2 + 8^2 = 10^2$ (36 + 64 = 100).

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So, of course, is every triangle of the same proportions (for example, if the sides are of three, four and five units of length).

MSS JV have preserved a triangle which is most likely intended to illustrate this construction: MS J f. 70r, between *angulum* (p. 107.24 L) and *In hanc* (p. 108.1 L), reproduced in Lachmann's edition as figure 125; MS V f. 110v, between *disputauimus* (p. 108.4 L) and *circuli* (p. 108.4–5 L). However, in MS J the triangle has been corrupted to an equilateral triangle, and in MS V its proportions are somewhat distorted, so that it resembles an isosceles triangle.

The MSS JV read: *nam quod ad extremam lineae normationem pertinet, uulgaris consuetudinis est sex octo et decem*. MS H reads: *nam quod ad extremae lineae normationem pertinet, uulgaris consuetudinis est VI VIII uel X numeros ponere*. On the one hand, the reading of MSS JV may be considered more likely to be genuine than that of MS H. The reading *extremam lineae normationem* (MSS JV) may be regarded as *lectio difficilior* in relation to *extremae lineae normationem* (MS H), as the adjective *extremus* is more easily understood as qualifying the concrete noun *linea* than the verbal noun *normatio*; the reading *uel* (MS H) is most likely not original (the required sense is 'and', not 'or', etc., and *uel* = *et* is not attested elsewhere in our text or in the *Expositio*); *numeros ponere* (MS H), finally, may be considered to be a later addition. On the other hand, it could be argued that *extremae lineae normationem* (MS H) is better than the corresponding reading of MSS JV and that MS H has indeed preserved an original *numeros ponere*, which makes sense of the otherwise elliptical main clause.

I for my part am inclined to think that *extremae* (MS H) should be preferred to *extremam* (MSS JV) and that *numeros ponere* is to be regarded as, at least, a plausible emendation to the text of MSS JV, which, in my view, seems too elliptical. Hence I translate thus: 'Moreover, as regards the dropping of a perpendicular to the extremity of a line, the common procedure is to employ the numbers six, eight and ten.'

See further the notes on ***extremam lineae normationem*** (p. 108.2–3 L) and ***sex octo et decem*** (p. 108.3–4 L) below.

p. 108.2 *nam*: In the sense of 'moreover', 'then'; see *OLD* s.v. *nam* 4. Cf. *Expositio* p. 92.17–18 L *nam quod ad synopsim pontium pertinet* [. . .].

p. 108.2–3 *extremam lineae normationem*: Here the verbal noun *normatio* obviously means 'dropping of a perpendicular'. Cf. *OLD* s.v. *normatio*, where another sense, inapplicable to this instance, is recorded. The genitive *lineae* is most likely objective: *lineae normatio* = 'the dropping of a perpendicular to a line'.

As stated in the note on ***nam quod ad extremam*** . . . ***sex octo et decem*** (p. 108.2–4 L) above, I tend to think that *extremae* (MS H) should be preferred to *extremam* (MSS JV). If the reading of MS H is chosen, the expression *extremae lineae normatio* means 'the dropping of a perpendicular to the extremity of a line' (the adjective *extremus* is taken partitively), which immediately makes perfect sense. If the reading of MSS JV were preferred, the expression *extrema lineae normatio* would literally mean 'the extreme dropping of a perpendicular to a line', which, in turn, ought to convey the same meaning as 'the dropping of a

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perpendicular to the extremity of a line'.[65] Although the reading of MSS JV seems to be both *difficilior* and possible, the reading of MS H is in my opinion simply more to the point and hence more likely to be genuine.

p. 108.3 *uulgaris*: The adjective *uulgaris* 'common' stands in opposition to the following *artificialis* 'being in accordance with the art (i.e. of geometry)' (p. 108.5 L). Hence, *uulgaris* does not imply that the procedure described is commonly used in the art of geometry, but that it is common in areas of applied geometry, notably land surveying; see the note on ***haec de qua*** … ***non praefinit*** (p. 108.4–6 L) below.

p. 108.3–4 *sex octo et decem*: As stated in the note on ***nam quod ad extremam*** … ***sex octo et decem*** (p. 108.2–4 L) above, I tentatively adopt the additional *numeros ponere* of MS H: *sex octo et decem numeros ponere*. This addition – genuine or scribal conjecture – remedies the awkward brevity of the reading in MSS JV.

p. 108.4–6 *haec de qua* … *non praefinit*: Here I endorse Lachmann's text, which is that of MSS JV (*quae*, however, is read only in MS J). Possibly one could also adopt the reading *autem* of MS H: *haec autem de qua* etc. The other variant readings of MS H are less attractive: *circularis* for *circuli* (banalisation/assimilation (perhaps induced by the adjectives *uulgaris* and *artificialis* in the vicinity)), *quia* for *quae* (presumably *lectio facilior*), *numerum praefinit* for *numeros non praefinit* (impossible).

I translate thus: 'The method with a circle, which we have talked about above, is more in accordance with the art (of geometry); it does not prescribe numbers.' The method referred to is evidently the one described at pp. 107.20–108.2 L.

In strict Euclidean geometry, measurement of, for example, lengths is not a valid operation in constructions and proofs. As the method of constructing a right angle by using a triangle with sides of six, eight and ten units of length (p. 108.2–4) requires numbers, that is, measuring, it is rightly pointed out that the method without measurement is *magis artificialis* 'more in accordance with the art (of geometry)'.

Finally, it should be mentioned that because of erroneous punctuation (full stop after *disputauimus*) in Guillaumin's and Roth Congès' Latin text the passage has been mistranslated and misinterpreted by Guillaumin, Roth Congès and even Campbell.[66]

p. 108.6–8 *habemus enim* … *normam facturas*: This sentence explains, with reference to Euclid, why the method described at pp. 107.20–108.2 L is more in accordance with the art of geometry. For, as Guillaumin, Roth Congès and Campbell point out,[67] it is proved in *Elementa* 3 prop. 31 that in a circle the angle in the semicircle is a right angle; that is to

65 Cf. Guillaumin's (1996) 89 translation: 'la construction d'une perpendiculaire à l'extrémité d'une ligne'. Roth Congès' (1996) 373 interprets: 'faire retourner à angle droit l'extrémité d'une ligne'. Campbell (2000) 215, finally, renders the phrase thus: 'the establishment of a perpendicular at the end of a line'.

66 Guillaumin (1996) 88–9, 89 n. 98; Roth Congès (1996) 373, 373 n. 275; Campbell (2000) 215, 436 n. 26.

67 Guillaumin (1996) 89 n. 99; Roth Congès (1996) 373 n. 277; Campbell (2000) 436 n. 27.

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say, wherever on the circumference of a circle two straight lines, one from each endpoint of the diameter, meet, they will make a right angle.

Except for the unjustified alteration *Eucliden* for the transmitted *Euclidem* (MSS JV, *Euclýdem* MS H), I here agree with Lachmann's text. The text of MS H, which, I think, suffers from omissions and erroneous scribal emendations, is as far as I can see nonsensical (it does, however, support the reading *concurrerint* of MS J against the *concurrunt* of MS V).

The words *normam facturas*, governed by *habemus*, are of course to be understood thus: (sc. *eas duas lineas*) *normam facturas* (sc. *esse*).

I translate thus: 'For we have it in Euclid that at whatever place on the circumference two (straight) lines from the (end)points of the diameter meet, they will make a right angle.'

MS J has preserved two figures which seem to serve as illustrations to this passage: MS J f. 70v, between *disputauimus* (p. 108.4 L) and *circuli* (p. 108.4 L), reproduced in Lachmann's edition as figure 126. One of them is correct, the other not. In MS V f. 110v, in the right margin next to the explicit, only the incorrect figure is found. MS H f. 47r, in the middle of the folio, has three erroneous figures, of which the third one from the left is similar to the incorrect figures found in MSS JV. Now, the incorrectness of the figures found in MSS JV seems to originate from misinterpretation of *ex signis dimensionis*, literally 'from the points of the diameter'; apparently the author of the figures interpreted 'from (whichever) points on the diameter', instead of 'from the (end)points of the diameter'.

p. 108.7 *dimensionis*: The word *dimensio* 'diameter' is attested four times in the *Expositio*: p. 101.14; 101.18; 102.3; 102.9 L. Unfortunately, this sense of the word is not recorded in *OLD* s.v. *dimensio*. And in the *TLL* the attestations of this specific sense are not singled out under a separate heading, but lumped together with attestations of other senses; see *TLL* s.v. *dimensio* 5:1.1192.72–1193.12.

p. 108.8 *normam*: The original sense of the word *norma* is 'set square' (see *OLD* s.v. *norma* 1), but here it is used in the sense of 'right angle'. The only attestation of *norma* = 'right angle' adduced in *OLD* is Vitruvius 9 praef. 6. In my view, however, that attestation is invalid, as *norma* is used there in the sense of 'set square'; in Vitruvius 1.1.4, on the other hand, *norma* = 'right angle' (so also Callebat and Fleury (1995) s.v. *norma* (col. 89)).

Although several textual and interpretative uncertainties remain, I hope that this paper may at least to some extent contribute to a better understanding of this important piece of geometry in the Roman tongue.

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