6

Alpha decay and spontaneous fission

6.1 Energy release in α-decay

We saw at the end of Chapter 4 that the binding energy per nucleon curve of the β-stable nuclei has a maximum in the neighbourhood of iron (Fig. 4.7), and that the heavier elements may be unstable to spontaneous disintegration. The principal mode of break-up is by emission of a ${}^4_2\mathrm{He}$ nucleus. Historically, the particles emitted in the decays of naturally occurring α-unstable nuclei were called α-particles before they were identified by Rutherford in 1908 as ${}^4_2\mathrm{He}$ nuclei, and the name has stayed.

The kinetic energy release $Q(A, Z)$ in an α-decay of a nucleus (A, Z) is given in terms of the binding energies of the parent and daughter nuclei by

$$Q(A, Z) = B(A-4, Z-2) + 28.3\ \mathrm{MeV} - B(A, Z), \tag{6.1}$$

(where 28.3 MeV is the experimental binding energy of the ${}^4_2\mathrm{He}$ nucleus). If the nucleus is assumed to lie on the β-stability curve, given approximately by equation (4.14), then Q may be calculated as a function of Z (or of A) from the semi-empirical mass formula, using equation (4.5). Neglecting the pairing energy term, the effects of which are small, the resulting smooth $Q(Z)$ is plotted in Fig. 6.1 for $Z > 50$. Negative values of Q imply absolute stability against α-decay. Also shown are Q values calculated from the experimentally measured masses of β-stable nuclei and their corresponding daughter nuclei. The trend of the experimental points is given correctly by the semi-empirical formula. Though the detailed predictions can be out by as much as 5 MeV, this is very small ($< 0.3\%$)

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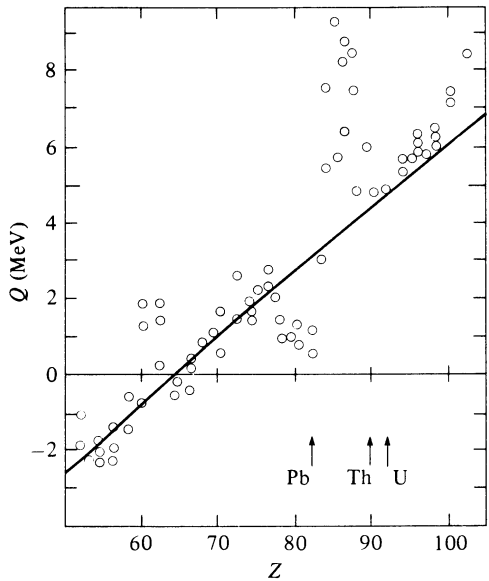


Fig. 6.1 Experimental and theoretical α-decay energies $Q = B(A-4, Z-2) + 28.3\ \text{MeV} - B(A, Z)$, as a function of the atomic number Z of the parent nucleus. The experimental points are from cases where both parent nucleus and daughter nuclei are β-stable. The theoretical curve is from equation (4.5) (neglecting the pairing energies) together with equation (4.14).

compared with the total binding energies of the nuclei in this region. The main deviation from the simple formula is due to the extra binding energy of nuclei around the double-closed shell nucleus $^{208}_{82}$Pb. This extra binding energy not only makes these nuclei more nearly stable than average, but also makes less stable the nuclei immediately above them. At higher Z, around $^{238}_{92}$U, there is another small region of relative stability.

From Fig. 6.1 it will be seen that β-stable nuclei with $Z > 66$ (and a few with $Z \leqslant 66$) are in principle unstable to α-decay. In practice the decay rate is so low as to be almost unobservable if the energy release Q is < 4 MeV. Up to Bi ($Z = 83$) the lifetimes of β-stable nuclei are many orders of magnitude greater than the age of the Earth.

6.2 The theory of α-decay

It is the electrostatic force which is responsible for inhibiting the α-decay of those nuclei for which the decay is energetically favourable. As an example, consider the decay of bismuth to thallium,

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$$^{209}_{83}\mathrm{Bi} \rightarrow {}^{205}_{81}\mathrm{Tl} + {}^{4}_{2}\mathrm{He} + 3.11\ \mathrm{MeV},$$

which is in principle possible but is not observed.

Figure 6.2 shows the electrostatic potential energy of the α-particle (charge $2e$) at a distance r from a thallium nucleus ($Z_{\mathrm{Tl}} = 81$), $2Z_{\mathrm{Tl}}e^2/(4\pi\varepsilon_0)r$. Also indicated on the graph is the distance r_{s} at which the strong interaction with the thallium nucleus takes over, which we estimate as

$$r_{\mathrm{s}} = 1.1[(205)^{\frac{1}{3}} + 4^{\frac{1}{3}}]\ \mathrm{fm} = 8.23\ \mathrm{fm},$$

where we have used equation (4.2) for the radii of the Tl and He nuclei. It is around this distance, in the surface region of the parent Bi nucleus, that the α-particle can be considered to be formed.

The graph immediately explains why the α-particle finds it difficult to escape even if it is formed. At r_{s} the height of the Coulomb potential is 28.4 MeV, very much greater than the energy $Q = 3.11$ MeV of the α-particle. Classically, it cannot penetrate the barrier and is free to move only at distances greater than r_{c} where r_{c} is given by

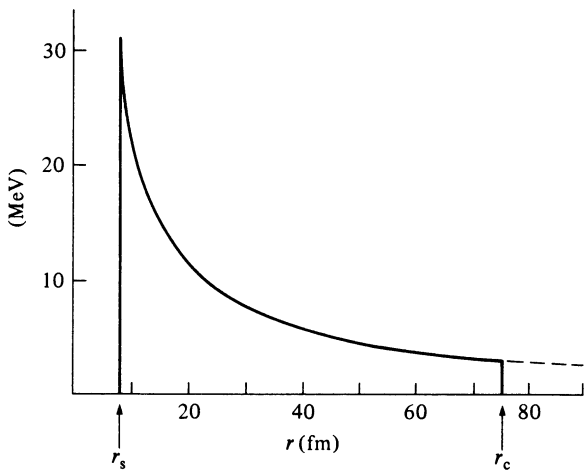


Fig. 6.2 The potential energy of an α-particle in the Coulomb field of a thallium nucleus, as a function of the separation distance r. At $r \approx r_{\mathrm{s}}$ the α-particle from the decay of bismuth is formed. At $r = r_{\mathrm{c}}$ it has penetrated through the classically forbidden region.

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$$Q = \frac{2Z_{T1}e^2}{(4\pi\varepsilon_0)r_c}; \tag{6.2}$$

r_c is the classical distance of closest approach to the nucleus of an α-particle of energy Q coming from the outside. For thallium, $r_c = 75$ fm. Thus classical mechanics forbids the α-particle to escape. Quantum mechanics, however, allows it to tunnel through, and we now estimate this important tunnelling probability.

At distances $r > r_s$, outside the range of the strong interaction, the Schrödinger equation for the radial wave-function $u(r)$ of the α-particle is

$$-\frac{\hbar^2}{2m}\frac{1}{r}\frac{d^2}{dr^2}(ru) + \left[\frac{2Z_d e^2}{(4\pi\varepsilon_0)r} + \frac{\hbar^2}{2m}\frac{l(l+1)}{r^2}\right]u = Qu. \tag{6.3}$$

Here Z_d is the atomic number of the daughter nucleus, and m is the reduced mass, i.e.,

$$m = \frac{m_\alpha m_d}{m_\alpha + m_d}, \tag{6.4}$$

where m_α, m_d are the masses of the α-particle and daughter nucleus. The use of the reduced mass takes into account the recoil of the daughter nucleus. To conserve angular momentum, the angular momentum of the α-particle and the angular momentum of the daughter nucleus must combine to give the angular momentum of the parent. Also the parity of the final state, $(-1)^l \times$ (parity of daughter nucleus), must equal to the parity of the parent nucleus. In the example $\mathrm{Bi}(\frac{9}{2}^-) \rightarrow \mathrm{Tl}(\frac{1}{2}^+)$, it is possible to conserve angular momentum with $l = 4$ or $l = 5$ (Appendix C), but parity conservation requires $l = 5$.

For simplicity we shall only consider the case $l = 0$. In fact the angular momentum term is usually small compared with the Coulomb potential. Writing $u(r) = f(r)/r$, equation (6.4) then reduces to

$$-\frac{\hbar^2}{2m}\frac{d^2 f}{dr^2} + \frac{2Z_d e^2}{(4\pi\varepsilon_0)r}f = Qf. \tag{6.5}$$

If the Coulomb term were replaced by a *constant* potential V_0 we should have solutions

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$$f(r) = \begin{cases} e^{\pm ikr}, & Q > V_0, \quad k^2 = \dfrac{2m}{\hbar^2}(Q - V_0) \\ e^{\pm Kr}, & Q < V_0, \quad K^2 = \dfrac{2m}{\hbar^2}(V_0 - Q). \end{cases} \tag{6.6}$$

This suggests that we try to find solutions to equation (6.5) of the form

$$f(r) = e^{\phi(r)}, \tag{6.7}$$

where $\phi(r)$ is to be determined. By substitution, $\phi(r)$ satisfies

$$\frac{\hbar^2}{2m}\left[\frac{d^2\phi}{dr^2} + \left(\frac{d\phi}{dr}\right)^2\right] = \left(\frac{2Z_d e^2}{(4\pi\varepsilon_0)r} - Q\right). \tag{6.8}$$

In a constant potential V_0, $d\phi/dr = \pm ik$ $(Q > V_0)$, or $\pm K$ $(Q < V_0)$ and in both cases $d^2\phi/dr^2 = 0$. We shall assume that the Coulomb potential in (6.8) is sufficiently slowly varying for $d^2\phi/dr^2$ in this equation to be neglected compared with $(d\phi/dr)^2$. Then

$$\frac{d\phi}{dr} = \pm\sqrt{\left[\frac{2m}{\hbar^2}\left(\frac{2Z_d e^2}{(4\pi\varepsilon_0)r} - Q\right)\right]}$$

and

$$\phi(r) = \pm\int\sqrt{\left[\frac{2m}{\hbar^2}\left(\frac{2Z_d e^2}{(4\pi\varepsilon_0)r} - Q\right)\right]}dr. \tag{6.9}$$

For $r > r_c$, we can write our approximate solution in the form

$$f(r) = A\exp\left(+i\int_{r_c}^{r} k(r)dr\right) + B\exp\left(-i\int_{r_c}^{r} k(r)dr\right), \tag{6.10}$$

where

$$k(r) = +\sqrt{\left[\frac{2m}{\hbar^2}\left(Q - \frac{2Z_d e^2}{(4\pi\varepsilon_0)r}\right)\right]},$$

and for $r_s < r < r_c$ we can write

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$$f(r) = C\exp\left(+\int_r^{r_c} K(r)\mathrm{d}r\right) + D\exp\left(-\int_r^{r_c} K(r)\mathrm{d}r\right), \tag{6.11}$$

where

$$K(r) = +\sqrt{\left[\frac{2m}{\hbar^2}\left(\frac{2Z_\mathrm{d}e^2}{(4\pi\varepsilon_0)r} - Q\right)\right]}.$$

A, B, C, D are constants to be determined by the boundary conditions.

The solution (6.10) represents outgoing and incoming waves. The incoming wave would be needed in an analysis of the scattering of α-particles by a nucleus. In the problem of α-decay, only the outgoing wave is present, so that $B = 0$.

This solution must then be matched on to the exponentially increasing and decreasing functions included in (6.11). Any admixture of the second term in the expression (6.11) quickly becomes negligible as r decreases, since the fall of the exponential is very rapid for typical values of the parameters, so that in the region $r_\mathrm{s} < r < r_\mathrm{c}$ we may take

$$f(r) = C\exp\left(\int_r^{r_c} K(r)\mathrm{d}r\right). \tag{6.12}$$

At $r = r_\mathrm{s}$, we must then match this solution to the appropriate radial function of the α-particle in the region where it is subject to the strong interaction. Here, at the surface of the daughter nucleus, the description of the α-particle is really a very complicated many-body problem. But it is reasonable to assume that in a heavy nucleus the rate of formation of α-particles is a property of the nuclear surface and does not vary greatly from one nucleus to another. Given that an α-particle has been formed, the radial probability density of finding the particle at $r = r_\mathrm{c}$ relative to the radial probability density of finding it at $r = r_\mathrm{s}$ is given by

$$\frac{4\pi r_\mathrm{c}^2|u(r_\mathrm{c})|^2}{4\pi r_\mathrm{s}^2|u(r_\mathrm{s})|^2} = \frac{|f(r_\mathrm{c})|^2}{|f(r_\mathrm{s})|^2}. \tag{6.13}$$

We can therefore interpret

$$\left|\frac{f(r_\mathrm{c})}{f(r_\mathrm{s})}\right|^2 = \mathrm{e}^{-G}, \quad \text{say,} \tag{6.14}$$

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as the transmission probability, through the Coulomb barrier, for α-particles created at $r \approx r_s$. The essential correctness of this interpretation is confirmed by a more exact analysis of the wave-functions and matching conditions.

Using equation (6.12)

$$\begin{aligned} G &= 2\int_{r_s}^{r_c} K(r)\mathrm{d}r \\ &= 2\int_{r_s}^{r_c} \sqrt{\left(\frac{2m}{\hbar^2}\left(\frac{2Z_d e^2}{4\pi\varepsilon_0)r} - Q\right)\right]}\mathrm{d}r \\ &= 2\sqrt{\frac{2mQ}{\hbar^2}}\int_{r_s}^{r_c}\left(\frac{r_c}{r}-1\right)^{\frac{1}{2}}\mathrm{d}r, \end{aligned}$$

since $r_c = \dfrac{2Z_d e^2}{(4\pi\varepsilon_0)Q}$.

With the substitution $r = r_c \cos^2\theta$ the integral is easily evaluated to give

$$\begin{aligned} G &= 2r_c\sqrt{\frac{2mQ}{\hbar^2}}\int_0^{\theta_0} 2\sin^2\theta\,\mathrm{d}\theta \\ &= 2r_c\sqrt{\frac{2mQ}{\hbar^2}}[\theta_0 - \sin\theta_0\cos\theta_0], \text{ where } \theta_0 = \cos^{-1}\left[\sqrt{\frac{r_s}{r_c}}\right] \\ &= \frac{\pi}{\hbar c}\left(\frac{2Z_d e^2}{4\pi\varepsilon_0}\right)\sqrt{\frac{2mc^2}{Q}}\mathscr{G}(r_s/r_c), \end{aligned} \tag{6.15}$$

where the function

$$\mathscr{G}(r_s/r_c) = \frac{2}{\pi}\left[\cos^{-1}\left[\sqrt{\frac{r_s}{r_c}}\right] - \sqrt{\left[\frac{r_s}{r_c}\left(1-\frac{r_s}{r_c}\right)\right]}\right] \tag{6.16}$$

is dimensionless and lies between 1 and 0 for $0 < r_s/r_c < 1$ (Fig. 6.3). At low energies $r_c \to \infty$ and $\mathscr{G} \to 1$.

If the total flux of α-particles created at r_s is τ_0^{-1}, the probability per unit time of α-particle emission is $\tau_0^{-1}\mathrm{e}^{-G}$, and hence the mean life for α-decay is given by

$$\tau = \tau_0 \mathrm{e}^{G}. \tag{6.17}$$

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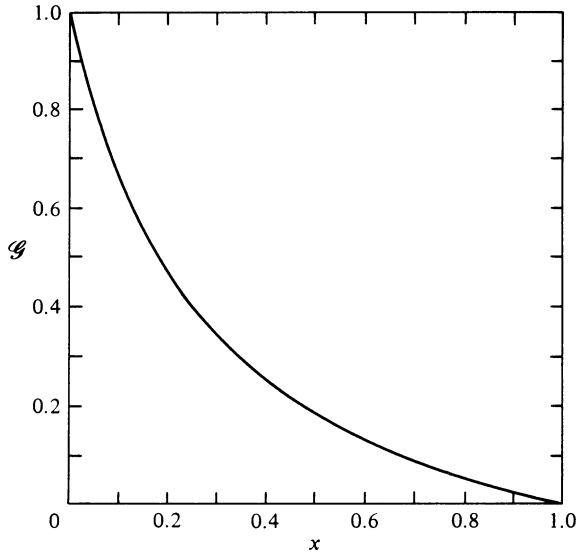


Fig. 6.3 The function $\mathscr{G}(x) = (2/\pi)\{\cos^{-1}\sqrt{x} - \sqrt{[x(1-x)]}\}$.

We have argued that the rate of formation of α-particles, and hence τ_0, is a nuclear property unlikely to vary greatly from nucleus to nucleus. In Table 6.1 we compare the formula (6.17) with experiment for a sequence of naturally occurring α-decays (all with $l = 0$) initiated by the most common isotope of uranium $^{238}_{92}\mathrm{U}$, taking τ_0 to be a constant. The value $\tau_0 = 7.0 \times 10^{-23}$ s was chosen to give a reasonable fit to this sequence of measured lives. This value is not unreasonable on a nuclear time scale of $2.6 \times 10^{-23} \times A^{\frac{1}{3}}$ s (§5.2), though it is somewhat shorter than earlier estimates because we have chosen r_s to be consistent with the modern values for nuclear radii. Early workers obtained estimates of nuclear radii from assumed values of τ_0.

The qualitative agreement of the simple theory (which was proposed in 1928 by Gamow and by Condon and Gurney) with experiment is truly remarkable. The simple quantum-mechanical formula for tunnelling comprehends time scales from as long as the age of the Earth (1.45×10^{17} s) down to times less than a microsecond. The largest discrepancy between theory and experiment occurs with $^{210}_{84}\mathrm{Po}$; this discrepancy can be associated with the closed shell $N = 126$ in this isotope.

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Table 6.1. *The α-decay series from* $^{238}_{92}$U

	Q (MeV)	r_s (fm)	r_c (fm)	$\mathscr{G}$	τ_{exp} (s)	τ_{theory} (s)
$^{238}_{92}U \rightarrow {}^{234}_{90}Th$	4.27	8.52	60.7	0.53	2.0×10^{17}	3.3×10^{17}
		($^{234}_{90}Th \rightarrow {}^{234}_{91}Pa \rightarrow {}^{234}_{92}U$)				
$^{234}_{92}U \rightarrow {}^{230}_{90}Th$	4.86	8.49	53.3	0.51	1.1×10^{13}	1.1×10^{13}
$^{230}_{90}Th \rightarrow {}^{226}_{88}Ra$	4.77	8.45	53.1	0.51	3.5×10^{12}	3.9×10^{12}
$^{226}_{88}Ra \rightarrow {}^{222}_{86}Rn$	4.87	8.41	50.9	0.50	7.3×10^{10}	7.4×10^{10}
$^{222}_{86}Rn \rightarrow {}^{218}_{84}Po$	5.59	8.37	43.3	0.46	4.8×10^{5}	4.2×10^{5}
$^{218}_{84}Po \rightarrow {}^{214}_{82}Pb$	6.11	8.33	38.7	0.43	2.6×10^{2}	1.6×10^{2}
		($^{214}_{82}Pb \rightarrow {}^{214}_{83}Bi \rightarrow {}^{214}_{84}Po$)				
$^{214}_{84}Po \rightarrow {}^{210}_{82}Pb$	7.84	8.28	30.1	0.36	2.3×10^{-4}	1.1×10^{-4}
		($^{210}_{82}Pb \rightarrow {}^{210}_{83}Bi \rightarrow {}^{210}_{84}Po$)				
$^{210}_{84}Po \rightarrow {}^{206}_{82}Pb$	5.41	8.24	43.7	0.47	1.7×10^{7}	5.8×10^{5}

The values of Q are from experiment. The intervening β-decays, which reduce the neutron-to-proton ratio as the nuclei become lighter, are given in parentheses.

This particular sequence of decays is of interest in the early history of the study of radioactivity. As is indicated in Table 6.1, the daughter nucleus of an α-decay may be unstable to β-decay. Since the mass number A decreases by four in an α-decay, and is unchanged in β-decay, we expect there to be three other similar sequences of α-decays. Two of these, based on $^{232}_{90}$Th and $^{235}_{92}$U, are also naturally occurring. The third, initiated by $^{237}_{93}$Np, is made up of comparatively short-lived isotopes which must be produced artificially for the series to be observed.

Since mean lives are dominated by the tunnelling factor, and this in turn depends principally on the value of Q, we can now understand why decay rates with $Q < 4$ MeV are so low.

It is found experimentally that α-particle emission can take place with the daughter nucleus left in an excited state. With even–even nuclei such processes usually occur with much lower probability, since the value of Q is reduced. However, the situation tends to be more complicated in the case of even–odd and odd–odd nuclei. An unpaired nucleon is less likely to take part in α-particle formation, and its state may form part of an excited configuration of the daughter nucleus. In such a case, the daughter nucleus is likely to be found in this excited state after the emission of an α-particle.

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6.3 Spontaneous fission

In α-decay, a heavy nucleus splits into a light helium nucleus and another heavy nucleus. Fission is the name given to a similar but more symmetric process of a nucleus splitting into two more or less equal masses. The two pieces are called fission fragments.

The fragments are often nuclei in quite highly excited states, but we can estimate the energy release in fission by considering the simple case of the symmetric fission of an even–even nucleus (A, Z) into two identical nuclei $(A/2, Z/2)$ in their ground states, and using the semi-empirical mass formula (4.5). We shall neglect the pairing energies. For a fixed ratio of protons to neutrons, the symmetry term as well as the cohesive energy term is proportional to the total number of nucleons. Thus only the surface and Coulomb energies contribute to the difference in binding energy ΔB of the two fragments and the parent nucleus:

$$\begin{aligned}\Delta B &= 2B(A/2, Z/2) - B(A, Z) \\ &= -bA^{\frac{2}{3}}[2(\tfrac{1}{2})^{\frac{2}{3}} - 1] - \frac{\mathrm{d}Z^2}{A^{\frac{1}{3}}}[2(\tfrac{1}{2})^{\frac{5}{3}} - 1].\end{aligned} \tag{6.18}$$

If ΔB is positive then this fission is energetically possible, and the fragments will acquire kinetic energy ΔB. From equation (6.18) nuclei for which

$$\frac{Z^2}{A} > \frac{b(2 - 2^{\frac{2}{3}})}{d(2^{\frac{2}{3}} - 1)}, \quad \text{i.e. } \frac{Z^2}{A} > 18, \tag{6.19}$$

are metastable with respect to fission. This condition is satisfied by β-stable nuclei heavier than ${}^{98}_{42}\mathrm{Mo}$. The energy release on the fission of the heavy elements is much larger than that in α-decay. For example, the energy release in the symmetric fission of ${}^{238}_{92}\mathrm{U}$ is $\simeq 180$ MeV. However, although this is large, the process is strongly inhibited by the tunnelling factor and spontaneous fission is only observed in the heaviest of elements.

We envisaged the process of α-decay as the initial formation of an α-particle at the surface of a nucleus and a subsequent tunnelling to freedom. It is not so easy to envisage the fission process, or to calculate the potential barrier. Figure 6.4 gives a schematic representation of the fission process, in which the nucleus is treated as a liquid drop. For the early stages of the fission of an initially spherical nucleus of radius R, it is

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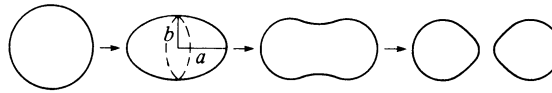


Fig. 6.4 A schematic representation of a symmetric fission in the liquid drop model.

reasonable to consider the nucleus deforming into an ellipsoid of revolution as in the first step of Fig. 6.4.

If we introduce a deformation parameter ε such that

$$\begin{aligned} a &= (1+\varepsilon)R \\ b &= R/(1+\varepsilon)^{\frac{1}{2}} \end{aligned} \tag{6.20}$$

where a, b are the major and minor semi-axes of the ellipsoid, the volume of the drop $(\frac{4}{3})\pi ab^2$ stays the same. It is not difficult to show that, for small ε, the surface area becomes to order ε^3

$$S(\varepsilon) = 4\pi R^2(1 + \tfrac{2}{5}\varepsilon^2 - \tfrac{52}{105}\varepsilon^3), \tag{6.21}$$

and the surface energy will, correspondingly, increase. (For small ε, the ε^3 term is a small correction.)

The Coulomb energy, on the other hand, decreases on deformation. A (rather lengthy) calculation for a uniformly charged ellipsoid gives the result

$$\begin{aligned} E_c &= \frac{\rho^2}{(4\pi\varepsilon_0)}\frac{1}{2}\iint\frac{\mathrm{d}^3\mathbf{r}\,\mathrm{d}^3\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|} \\ &= \frac{3}{5}\frac{(Ze)^2}{4\pi\varepsilon_0 R}(1 - \tfrac{1}{5}\varepsilon^2 + \tfrac{4}{21}\varepsilon^3) \end{aligned} \tag{6.22}$$

for small ε (cf. equation (4.6)).

Using the parameters of the semi-empirical mass formula, these results suggest that a small ellipsoidal deformation of a spherical nucleus gives a change in energy of

$$\varepsilon^2\left(\frac{2}{5}bA^{\frac{2}{3}} - \frac{1}{5}d\frac{Z^2}{A^{\frac{1}{3}}}\right) - \varepsilon^3\left(\frac{52}{105}bA^{\frac{2}{3}} - \frac{4}{21}d\frac{Z^2}{A^{\frac{1}{3}}}\right). \tag{6.23}$$

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The coefficient of ε^2 is negative if

$$\frac{Z^2}{A} > \frac{2b}{d} = 51. \tag{6.24}$$

Hence when this condition is satisfied the deformation energy is negative even for small ε, and fission would proceed uninhibited by any potential barrier. Thus (6.24) suggests there is an absolute upper limit for chemical elements of $Z = 144$ (using the relation between Z and A for β-stable nuclei given by equation (4.9)).

For elements of lower Z, spontaneous fission involves tunnelling through a potential barrier. We can crudely estimate the height of the barrier from the expansion (6.23). For $^{235}_{92}$U this gives the deformation energy

$$(83.35\varepsilon^2 - 159.16\varepsilon^3)\ \text{MeV}.$$

The coefficient of ε^3 is negative, which confirms that in the liquid drop model the most likely deformation is indeed a prolate, rather than oblate, ellipsoid, and the expression has a maximum of 3.4 MeV when $\varepsilon = 0.35$. The measured potential barrier is 5.8 MeV. For $A \sim 240$ barrier heights are found to be between 5 and 6 MeV. Experimental values are determined from the threshold energies required to induce fission, when the nucleus is bombarded with, for example, γ-rays. Induced fission by neutron capture also gives information on barrier heights. We shall consider induced fission in more detail in Chapter 8 and Chapter 9. It is a subject of great technological importance.

As the inequalities (6.19) and (6.24) indicate, Z^2/A is a measure of the likelihood that a nucleus will be subject to spontaneous fission. An empirical, approximately linear, relationship exists between the logarithm of the mean life for spontaneous fission and Z^2/A. This is shown in Fig. 6.5 for some even–even nuclei.

The fragments produced in spontaneous fission move apart rapidly because of their Coulomb repulsion. They are neutron rich, since the equilibrium neutron-to-proton ratio of a β-stable nucleus decreases as A decreases, and they are in highly excited states. Typically, one to four neutrons 'boil off' from the fragments in a time of 10^{-18} to 10^{-15} s. Studies of the angular distribution of these 'prompt neutrons' show that they are indeed emitted from the moving fragments, rather than at the moment of break-up of the fissioning nucleus. The resulting

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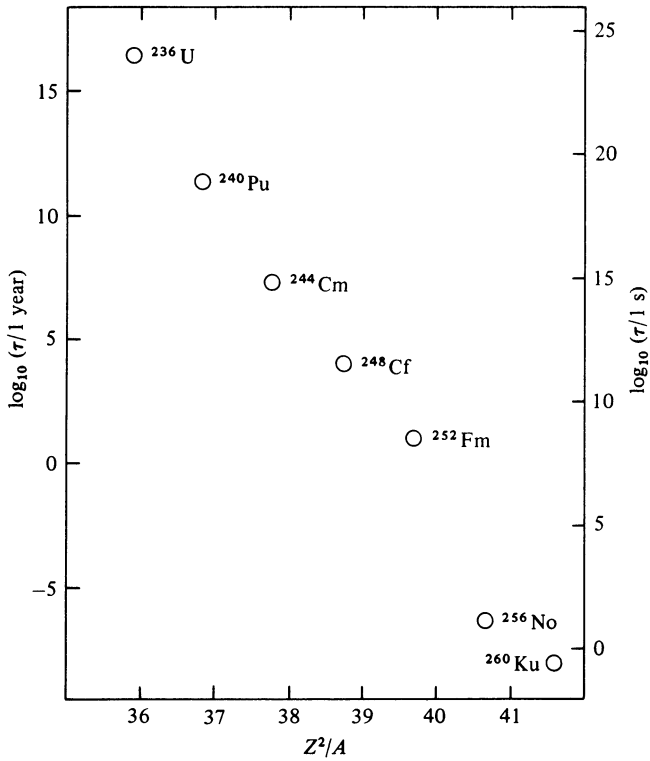


Fig. 6.5 Mean lives for spontaneous fission of some even–even nuclei. (Data from *American Institute of Physics Handbook*, 3rd ed., 1972, New York: McGraw-Hill.)

nuclei are still far from the line of β-stability. They reach their ground states through the emission of prompt γ-rays and gradually decay, by β-emission, to stable nuclei. It occasionally happens that a nucleus produced by β-decay is unstable to neutron emission, and a 'delayed neutron' may result. For example, one of the fission products of ^{236}U is $^{87}_{35}$Br. This β-decays with a mean life of 80 s to either the ground state $^{87}_{36}$Kr or an excited state $^{87}_{36}$Kr* which can lie above the threshold for neutron emission. In the latter case the rapid decay $^{87}_{36}\mathrm{Kr}^* \rightarrow {}^{86}_{36}\mathrm{Kr} + \mathrm{n}$ sometimes occurs. Thus the time scale for the emission of the delayed neutrons is determined by the long lifetimes of the β-decay processes involved. We shall see in Chapter 8 that such processes are important for the control of nuclear reactors.

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The semi-empirical mass formula predicts that the energy release in spontaneous fission is at a maximum when the two fragments are of equal mass. Notwithstanding this, it is usually found that there is a quite striking asymmetry in the mass distribution of the fission fragments. It is likely that this asymmetry is due to shell structure effects. More detailed theories of fission include these aspects of nuclear structure.

Problems

6.1 ${}^{8}_{4}\mathrm{Be}$ decays to two α-particles with a kinetic energy release of 0.094 MeV. Estimate its mean life using the tunnelling formula (6.17), and compare your estimate with the observed mean life of 2.6×10^{-17} s.

6.2 The isotope ${}^{194}_{79}\mathrm{Au}$ undergoes β-decay and has a mean life of 56 hours.

(*a*) One mode of decay is

$$ {}^{194}_{79}\mathrm{Au} \rightarrow {}^{194}_{78}\mathrm{Pt} + \mathrm{e}^{+} + \nu + 1.5 \text{ MeV}. $$

The positron in this decay is created in the nucleus and must tunnel through a Coulomb barrier to escape. Show that the barrier factor suppresses the decay rate to a positron with an energy ~ 1 MeV only by a factor of about four.

(*b*) Another energetically possible decay, which has not been observed, is

$$ {}^{194}_{79}\mathrm{Au} \rightarrow {}^{190}_{77}\mathrm{Ir} + {}^{4}_{2}\mathrm{He} + 1.8 \text{ MeV}. $$

Estimate the mean life for this mode of decay.

6.3 ${}^{238}\mathrm{Pu}$ decays by α-emission:

$$ {}^{238}_{94}\mathrm{Pu} \rightarrow {}^{234}_{92}\mathrm{U} + \alpha + 5.49 \text{ MeV}, $$

with a mean life of 128 years. The mean life of ${}^{234}\mathrm{U}$ is much longer, 2.5×10^{5} years. Space probes to the outer planets use ${}^{238}\mathrm{Pu}$ as a power source for their equipment. Estimate the mass of ${}^{238}\mathrm{Pu}$ needed to supply a minimum of 1 kW of heat for 50 years.

6.4 The intermediate members of the radioactive series stemming from ${}^{238}\mathrm{U}$ have negligible mean lives on geological time scales (Table 6.1), so that ${}^{238}\mathrm{U}$ may be said to decay to ${}^{206}\mathrm{Pb}$ with a mean life of 6.48×10^{9} years. Similarly, ${}^{235}\mathrm{U}$ decays to ${}^{207}\mathrm{Pb}$ with a mean life of 1.03×10^{9} years.

In a certain sample of uranium-bearing rock the proportions of atoms of ${}^{238}\mathrm{U}$, ${}^{235}\mathrm{U}$, ${}^{206}\mathrm{Pb}$, ${}^{207}\mathrm{Pb}$ were measured to be 1000:7.19:79.7:4.85. The rock contained a negligible amount of ${}^{208}\mathrm{Pb}$, which is usually the most common isotope of lead, indicating that all the lead in the rock came from uranium decay. Estimate the age of all rock.

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6.5 Estimate the energy release and the velocity of the fragments in the spontaneous fission

$$^{238}_{92}\mathrm{U} \rightarrow {}^{119}_{46}\mathrm{Pd}^{*} + {}^{119}_{46}\mathrm{Pd}^{*}$$

where * denotes an excited state.

Neutrons 'boil off' from the fragments. If in the frame of the moving fragments the neutrons are emitted isotropically with energy ≈ 2 MeV, describe qualitatively how the neutrons appear in the laboratory.

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