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PARTITIONS OF NATURAL NUMBERS AND THEIR ORDERED REPRESENTATION FUNCTIONS

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Abstract

Let $\mathbb{N}$ be the set of all nonnegative integers. For a set $A \subseteq \mathbb{N}$, let $R_2(A,n)$ and $R_3(A,n)$ be the number of solutions of the equation $n = a_1 + a_2$ with $a_1 < a_2, a_1, a_2 \in A$ and with $a_1 \le a_2, a_1, a_2 \in A$, respectively. If $-N \le g \le N$, Yan ['On the structure of partition which the difference of their representation function is a constant', *Period. Math. Hungar.* **82** (2021), 149–152] showed that there is a set $A \subseteq \mathbb{N}$ such that $R_i(A,n) - R_i(\mathbb{N} \setminus A, n) = g$ for all integers $n \ge 2N - 1$, where N is a positive integer. In this paper, we prove that if g_1, g_2 are nonnegative integers with $g_1 \ne g_2$, then there does not exist $A \subseteq \mathbb{N}$ such that $R_i(A,2n) - R_i(\mathbb{N} \setminus A, 2n) = g_1$ and $R_i(A,2n+1) - R_i(\mathbb{N} \setminus A, 2n+1) = g_2$ for all sufficiently large integers n.

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1. Introduction

Let $\mathbb{N}$ be the set of all nonnegative integers. For a set $A \subseteq \mathbb{N}$, define the additive representation function

$$R_1(A,n) = |\{(a_1,a_2) : n = a_1 + a_2,\ a_1, a_2 \in A\}|$$

and the ordered representation functions

$$R_2(A,n) = |\{(a_1,a_2) : n = a_1 + a_2,\ a_1, a_2 \in A,\ a_1 < a_2\}|,$$
$$R_3(A,n) = |\{(a_1,a_2) : n = a_1 + a_2,\ a_1, a_2 \in A,\ a_1 \le a_2\}|.$$

For each $i \in \{1,2,3\}$, Sárközy asked whether there exist two sets $A, B \subseteq \mathbb{N}$ with infinite symmetric difference, that is,

$$|(A \cup B) \setminus (A \cap B)| = \infty$$

and

$$R_i(A,n) = R_i(B,n)$$

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for all sufficiently large integers n. For $i = 1$, Dombi [3] observed that $R_1(A, 2m)$ is odd if and only if $m \in A$. So $R_1(A, 2m) \neq R_1(B, 2m)$ for all integers $m \in (A \cup B) \setminus (A \cap B)$, and the answer is negative for $i = 1$. In 2002, Dombi [3] proved that the answer is positive for $i = 2$. In 2003, Chen and Wang [2] proved that the answer is also positive for $i = 3$. Later, many authors studied this problem by means of many different methods. In particular, Sándor [5] and Tang [6] proved the following nice results in different ways.

THEOREM 1.1 [5, 6]. *Let N be a positive integer. The equality $R_2(A, n) = R_2(\mathbb{N} \setminus A, n)$ holds for $n \geq 2N - 1$ if and only if $|A \cap [0, 2N - 1]| = N$ and $2m \in A \Leftrightarrow m \in A$, $2m + 1 \in A \Leftrightarrow m \notin A$ for $m \geq N$.*

THEOREM 1.2 [5, 6]. *Let N be a positive integer. The equality $R_3(A, n) = R_3(\mathbb{N} \setminus A, n)$ holds for $n \geq 2N - 1$ if and only if $|A \cap [0, 2N - 1]| = N$ and $2m \in A \Leftrightarrow m \notin A$, $2m + 1 \in A \Leftrightarrow m \in A$ for $m \geq N$.*

From Dombi's discussion, it can be seen that the reason why the answer for $i = 1$ is negative is because the parity of $R_1(A, n)$ and $R_1(\mathbb{N} \setminus A, n)$ is different when n is even. The authors of this paper and Cheng [9] avoided the case where n is even, and they determined the structure of the set $A \subseteq \mathbb{N}$ such that $R_1(A, 2k - 1) = R_1(\mathbb{N} \setminus A, 2k - 1)$ for all integers $k \geq K$. In addition, they obtained the following results.

THEOREM 1.3 [9]. *Let $A \subseteq \mathbb{N}$, K be a positive integer and let g be an even integer with $-2K \leq g \leq 2K$. The equations*

$$R_1(A, 2k - 1) - R_1(\mathbb{N} \setminus A, 2k - 1) = g, \quad R_1(A, 2k) - R_1(\mathbb{N} \setminus A, 2k) = g + 1$$

hold for all integers $k \geq K$ if and only if

$$A = A_0 \cup \{2k : k \geq K\},$$

where $A_0 \subseteq [0, 2K - 1]$ and $|A_0| = K + g/2$.

THEOREM 1.4 [9]. *Let $A \subseteq \mathbb{N}$, let K be a nonnegative integer and let g be an odd integer with $-2K - 1 \leq g \leq 2K + 1$. The equations*

$$R_1(A, 2k) - R_1(\mathbb{N} \setminus A, 2k) = g, \quad R_1(A, 2k + 1) - R_1(\mathbb{N} \setminus A, 2k + 1) = g + 1$$

hold for all integers $k \geq K$ if and only if

$$A = A_0 \cup \{2k + 1 : k \geq K\},$$

where $A_0 \subseteq [0, 2K]$ and $|A_0| = K + (g + 1)/2$.

There are many results about the ordered representation functions $R_2(A, n)$ and $R_3(A, n)$. Cao-Labora *et al.* [1] showed that the ordered representation functions satisfy an Erdős–Fuchs-type result. Tang [7] considered Problem 6 of Erdős *et al.* [4] about locally small and locally large values of the ordered representation functions. In this paper, we consider the following problem related to ordered representation functions.

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PROBLEM 1.5. Let g_1, g_2 be nonnegative integers with $g_1 \neq g_2$. For $i \in \{2, 3\}$, do there exist infinite sets $A \subseteq \mathbb{N}$ and $T \subseteq \mathbb{N}$ such that

$$R_i(A, n) - R_i(\mathbb{N} \setminus A, n) = g_1 \quad \text{for all } n \in T$$

and

$$R_i(A, n) - R_i(\mathbb{N} \setminus A, n) = g_2 \quad \text{for all } n \in \mathbb{N} \setminus T$$

for all sufficiently large integers n?

If $-N \leq g_1 = g_2 \leq N$, the structure of a set $A \subseteq \mathbb{N}$ such that $R_i(A, n) - R_i(\mathbb{N} \setminus A, n) = g_1$ for all integers $n \geq 2N - 1$ was provided in [8], where N is a positive integer. In this paper, we prove that if $g_1 \neq g_2$ and $T = 2\mathbb{N}$, then such a set A in Problem 1.5 does not exist.

THEOREM 1.6. *Let g_1, g_2 be nonnegative integers with $g_1 \neq g_2$. Then there does not exist any set $A \subseteq \mathbb{N}$ such that*

$$R_2(A, 2k) - R_2(\mathbb{N} \setminus A, 2k) = g_1, \quad R_2(A, 2k+1) - R_2(\mathbb{N} \setminus A, 2k+1) = g_2$$

for all sufficiently large integers k.

THEOREM 1.7. *Let g_1, g_2 be nonnegative integers with $g_1 \neq g_2$. Then there does not exist any set $A \subseteq \mathbb{N}$ such that*

$$R_3(A, 2k) - R_3(\mathbb{N} \setminus A, 2k) = g_1, \quad R_3(A, 2k+1) - R_3(\mathbb{N} \setminus A, 2k+1) = g_2$$

for all sufficiently large integers k.

2. Proofs

In this section, we denote the generating function associated with the set A by $f_A(x)$, that is,

$$f_A(x) = \sum_{a \in A} x^a = \sum_{i=0}^{\infty} \chi_A(i)\, x^i,$$

where $\chi_A(x)$ is the characteristic function of A, that is, $\chi_A(x) = 1$ if $x \in A$ and $\chi_A(x) = 0$ if $x \notin A$. Clearly, if $B = \mathbb{N} \setminus A$, then $\chi_A(i) + \chi_B(i) = 1$ and

$$\frac{1}{1-x} - f_A(x) = \sum_{i=0}^{\infty} (1 - \chi_A(i))\, x^i = \sum_{i=0}^{\infty} \chi_B(i)\, x^i = f_B(x)$$

is the generating function associated with the set B.

LEMMA 2.1. *Let $A \subseteq \mathbb{N}$. Then*

$$\sum_{n=0}^{\infty} (R_2(A, n) - R_2(\mathbb{N} \setminus A, n))\, x^n = \sum_{n=0}^{\infty} \left(\sum_{j=0}^{n} \chi_A(j) - \chi_A\left(\frac{n}{2}\right) - \left[\frac{n+1}{2}\right] \right) x^n.$$

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PROOF. Since

$$\sum_{n=0}^{\infty} R_2(A,n)\,x^n = \frac{1}{2}(f_A^2(x) - f_A(x^2))$$

and

$$\sum_{n=0}^{\infty} R_2(\mathbb{N}\setminus A,n)\,x^n = \frac{1}{2}(f_{\mathbb{N}\setminus A}^2(x) - f_{\mathbb{N}\setminus A}(x^2)) = \frac{1}{2}\left(\left(\frac{1}{1-x} - f_A(x)\right)^2 - \left(\frac{1}{1-x^2} - f_A(x^2)\right)\right),$$

it follows that

$$\begin{aligned}
&\sum_{n=0}^{\infty}(R_2(A,n) - R_2(\mathbb{N}\setminus A,n))\,x^n \\
&\quad= \frac{1}{2}(f_A^2(x) - f_A(x^2)) - \frac{1}{2}\left(\left(\frac{1}{1-x} - f_A(x)\right)^2 - \left(\frac{1}{1-x^2} - f_A(x^2)\right)\right) \\
&\quad= \frac{1}{1-x}\left(f_A(x) - (1-x)f_A(x^2) - \frac{x}{1-x^2}\right) \\
&\quad= \frac{1}{1-x}\left(\sum_{i=0}^{\infty}(\chi_A(2i) - \chi_A(i))\,x^{2i} + \sum_{i=0}^{\infty}(\chi_A(2i+1) + \chi_A(i) - 1)\,x^{2i+1}\right) \\
&\quad= \sum_{j=0}^{\infty}\sum_{i=0}^{\infty}(\chi_A(2i) - \chi_A(i))\,x^{2i+j} + \sum_{j=0}^{\infty}\sum_{i=0}^{\infty}(\chi_A(2i+1) + \chi_A(i) - 1)\,x^{2i+j+1} \\
&\quad= \sum_{n=0}^{\infty}\left(\sum_{\substack{2i+j=n\\ i,j\ge 0}}(\chi_A(2i) - \chi_A(i)) + \sum_{\substack{2i+j+1=n\\ i,j\ge 0}}(\chi_A(2i+1) + \chi_A(i) - 1)\right)x^n.
\end{aligned}$$

If $n = 2k$ is an even integer with $k \ge 0$, then

$$\begin{aligned}
&\sum_{\substack{2i+j=n\\ i,j\ge 0}}(\chi_A(2i) - \chi_A(i)) + \sum_{\substack{2i+j+1=n\\ i,j\ge 0}}(\chi_A(2i+1) + \chi_A(i) - 1) \\
&\quad= \sum_{\substack{0\le j\le n\\ j \text{ is even}}}\left(\chi_A(n-j) - \chi_A\left(\frac{n-j}{2}\right)\right) + \sum_{\substack{0\le j\le n\\ j \text{ is odd}}}\left(\chi_A(n-j) + \chi_A\left(\frac{n-1-j}{2}\right) - 1\right) \\
&\quad= \sum_{j=0}^{n}\chi_A(j) - \chi_A(k) - k = \sum_{j=0}^{n}\chi_A(j) - \chi_A\left(\frac{n}{2}\right) - \left[\frac{n+1}{2}\right].
\end{aligned}$$

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If $n = 2k-1$ is an odd integer with $k \geq 1$, then $\chi_A(n/2) = 0$ and

$$\begin{aligned}
&\sum_{\substack{2i+j=n\\ i,j\geq 0}} (\chi_A(2i) - \chi_A(i)) + \sum_{\substack{2i+j+1=n\\ i,j\geq 0}} (\chi_A(2i+1) + \chi_A(i) - 1) \\
&\quad = \sum_{\substack{0\leq j\leq n\\ j \text{ is odd}}} \left(\chi_A(n-j) - \chi_A\left(\frac{n-j}{2}\right)\right) + \sum_{\substack{0\leq j\leq n\\ j \text{ is even}}} \left(\chi_A(n-j) + \chi_A\left(\frac{n-1-j}{2}\right) - 1\right) \\
&\quad = \sum_{j=0}^{n} \chi_A(j) - k = \sum_{j=0}^{n} \chi_A(j) - \chi_A\left(\frac{n}{2}\right) - \left[\frac{n+1}{2}\right].
\end{aligned}$$

Hence,

$$\sum_{n=0}^{\infty} (R_2(A,n) - R_2(\mathbb{N}\setminus A, n))\, x^n = \sum_{n=0}^{\infty} \left(\sum_{j=0}^{n} \chi_A(j) - \chi_A\left(\frac{n}{2}\right) - \left[\frac{n+1}{2}\right]\right) x^n.$$

This completes the proof of Lemma 2.1. □

LEMMA 2.2. *Let $A \subseteq \mathbb{N}$. Then*

$$\sum_{n=0}^{\infty} (R_3(A,n) - R_3(\mathbb{N}\setminus A, n))\, x^n = \sum_{n=0}^{\infty} \left(\sum_{j=0}^{n} \chi_A(j) + \chi_A\left(\frac{n}{2}\right) - \left[\frac{n}{2}\right] - 1\right) x^n.$$

PROOF. Since

$$\sum_{n=0}^{\infty} R_3(A,n)\, x^n = \frac{1}{2}(f_A^2(x) + f_A(x^2))$$

and

$$\sum_{n=0}^{\infty} R_3(\mathbb{N}\setminus A, n)\, x^n = \frac{1}{2}(f_{\mathbb{N}\setminus A}^2(x) + f_{\mathbb{N}\setminus A}(x^2)) = \frac{1}{2}\left(\left(\frac{1}{1-x} - f_A(x)\right)^2 + \frac{1}{1-x^2} - f_A(x^2)\right),$$

it follows that

$$\begin{aligned}
&\sum_{n=0}^{\infty} (R_3(A,n) - R_3(\mathbb{N}\setminus A, n))\, x^n \\
&\quad = \frac{1}{2}(f_A^2(x) + f_A(x^2)) - \frac{1}{2}\left(\left(\frac{1}{1-x} - f_A(x)\right)^2 + \frac{1}{1-x^2} - f_A(x^2)\right) \\
&\quad = \frac{1}{1-x}\left((1-x) f_A(x^2) + f_A(x) - \frac{1}{1-x^2}\right) \\
&\quad = \frac{1}{1-x}\left(\sum_{i=0}^{\infty} \chi_A(i)\, x^{2i} - \sum_{i=0}^{\infty} \chi_A(i)\, x^{2i+1} + \sum_{i=0}^{\infty} \chi_A(i)\, x^i - \sum_{i=0}^{\infty} x^{2i}\right) \\
&\quad = \frac{1}{1-x}\left(\sum_{i=0}^{\infty} (\chi_A(i) + \chi_A(2i) - 1)\, x^{2i} + \sum_{i=0}^{\infty} (\chi_A(2i+1) - \chi_A(i))\, x^{2i+1}\right)
\end{aligned}$$

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$$= \sum_{j=0}^{\infty}\sum_{i=0}^{\infty}(\chi_A(i)+\chi_A(2i)-1)\,x^{2i+j} + \sum_{j=0}^{\infty}\sum_{i=0}^{\infty}(\chi_A(2i+1)-\chi_A(i))\,x^{2i+j+1}$$

$$= \sum_{n=0}^{\infty}\Bigg(\sum_{\substack{2i+j=n\\ i,j\geq 0}}(\chi_A(i)+\chi_A(2i)-1) + \sum_{\substack{2i+j+1=n\\ i,j\geq 0}}(\chi_A(2i+1)-\chi_A(i))\Bigg)x^n.$$

If $n = 2k$ is an even integer with $k \geq 0$, then

$$\sum_{\substack{2i+j=n\\ i,j\geq 0}}(\chi_A(i)+\chi_A(2i)-1) + \sum_{\substack{2i+j+1=n\\ i,j\geq 0}}(\chi_A(2i+1)-\chi_A(i))$$

$$= \sum_{\substack{0\leq j\leq n\\ j \text{ is even}}}\left(\chi_A\left(\frac{n-j}{2}\right)+\chi_A(n-j)-1\right) + \sum_{\substack{0\leq j\leq n\\ j \text{ is odd}}}\left(\chi_A(n-j)-\chi_A\left(\frac{n-1-j}{2}\right)\right)$$

$$= \sum_{j=0}^{n}\chi_A(j)+\chi_A(k)-(k+1) = \sum_{j=0}^{n}\chi_A(j)+\chi_A\left(\frac{n}{2}\right)-\left[\frac{n}{2}\right]-1.$$

If $n = 2k-1$ is an odd integer with $k \geq 1$, then $\chi_A(n/2) = 0$ and

$$\sum_{\substack{2i+j=n\\ i,j\geq 0}}(\chi_A(i)+\chi_A(2i)-1) + \sum_{\substack{2i+j+1=n\\ i,j\geq 0}}(\chi_A(2i+1)-\chi_A(i))$$

$$= \sum_{\substack{0\leq j\leq n\\ j \text{ is odd}}}\left(\chi_A\left(\frac{n-j}{2}\right)+\chi_A(n-j)-1\right) + \sum_{\substack{0\leq j\leq n\\ j \text{ is even}}}\left(\chi_A(n-j)-\chi_A\left(\frac{n-1-j}{2}\right)\right)$$

$$= \sum_{j=0}^{n}\chi_A(j)-k = \sum_{j=0}^{n}\chi_A(j)+\chi_A\left(\frac{n}{2}\right)-\left[\frac{n}{2}\right]-1.$$

Hence,

$$\sum_{n=0}^{\infty}(R_3(A,n)-R_3(\mathbb{N}\setminus A,n))\,x^n = \sum_{n=0}^{\infty}\left(\sum_{j=0}^{n}\chi_A(j)+\chi_A\left(\frac{n}{2}\right)-\left[\frac{n}{2}\right]-1\right)x^n.$$

This completes the proof of Lemma 2.2. □

It should be mentioned that Lemmas 2.1 and 2.2 can be obtained from the proof in [6]; we provide a different proof using generating functions.

PROOF OF THEOREM 1.6. Suppose that there exists a set $A \subseteq \mathbb{N}$ such that

$$R_2(A,2k)-R_2(\mathbb{N}\setminus A,2k) = g_1 \quad \text{and} \quad R_2(A,2k+1)-R_2(\mathbb{N}\setminus A,2k+1) = g_2 \tag{2.1}$$

for all integers $k \geq K$. By Lemma 2.1,

$$\sum_{j=0}^{2k}\chi_A(j)-\chi_A\left(\frac{2k}{2}\right)-\left[\frac{2k+1}{2}\right] = g_1$$

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and

$$\sum_{j=0}^{2k+1}\chi_A(j)-\chi_A\left(\frac{2k+1}{2}\right)-\left[\frac{2k+2}{2}\right]=g_2 \tag{2.2}$$

for all integers $k \geq K$. Consequently,

$$g_1-g_2=1-\chi_A(k)-\chi_A(2k+1) \quad \text{for all } k \geq K. \tag{2.3}$$

Since $-1 \leq 1-\chi_A(k)-\chi_A(2k+1) \leq 1$ and $g_1 \neq g_2$, it follows that $g_1-g_2=\pm 1$. If $g_1-g_2=1$, then, by (2.3), $\chi_A(k)+\chi_A(2k+1)=0$. So $\chi_A(k)=0$ for all integers $k \geq K$. By (2.2), for all integers $k \geq K$,

$$g_2=\sum_{j=0}^{2k+1}\chi_A(j)-k-1=\sum_{j=0}^{K-1}\chi_A(j)-k-1,$$

which is impossible since g_2 and K are fixed integers and k can be sufficiently large. If $g_1-g_2=-1$, then, by (2.3), $\chi_A(k)+\chi_A(2k+1)=2$. So $\chi_A(k)=1$ for all integers $k \geq K$. By (2.2), for all integers $k \geq K$,

$$g_2=\sum_{j=0}^{2k+1}\chi_A(j)-k-1=\sum_{j=0}^{K-1}\chi_A(j)+k-K+1,$$

which is impossible since g_2 and K are fixed integers and k can be sufficiently large. Hence, there does not exist any set $A \subseteq \mathbb{N}$ such that (2.1) holds for all sufficiently large integers k. □

PROOF OF THEOREM 1.7. Suppose that there exists a set $A \subseteq \mathbb{N}$ such that

$$R_3(A,2k)-R_3(\mathbb{N}\setminus A,2k)=g_1 \quad \text{and} \quad R_3(A,2k+1)-R_3(\mathbb{N}\setminus A,2k+1)=g_2 \tag{2.4}$$

for all integers $k \geq K$. By Lemma 2.2,

$$\sum_{j=0}^{2k}\chi_A(j)+\chi_A\left(\frac{2k}{2}\right)-\left[\frac{2k}{2}\right]-1=g_1$$

and

$$\sum_{j=0}^{2k+1}\chi_A(j)+\chi_A\left(\frac{2k+1}{2}\right)-\left[\frac{2k+1}{2}\right]-1=g_2$$

for all integers $k \geq K$. Then

$$g_1-g_2=\chi_A(k)-\chi_A(2k+1) \quad \text{for all } k \geq K. \tag{2.5}$$

Since $-1 \leq \chi_A(k)-\chi_A(2k+1) \leq 1$ and $g_1 \neq g_2$, it follows that $g_1-g_2=\pm 1$. If $g_1-g_2=1$, by (2.5), then $\chi_A(k)-\chi_A(2k+1)=1$. So $\chi_A(k)=1$ and $\chi_A(2k+1)=0$ for all integers $k \geq K$, which is impossible. If $g_1-g_2=-1$, by (2.5), then $\chi_A(k)-\chi_A(2k+1)=-1$. So $\chi_A(k)=0$ and $\chi_A(2k+1)=1$ for all integers $k \geq K$,

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which is impossible. Hence, there does not exist any set $A \subseteq \mathbb{N}$ such that (2.4) for all sufficiently large integers k. □

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