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INFLATION STABILIZATION AND DEFAULT RISK IN A CURRENCY UNION

EIJI OKANO
Nagoya City University

MASASHIGE HAMANO
Waseda University

By developing a class of dynamic stochastic general equilibrium models with nominal rigidities and assuming a two-country currency union with sovereign risk, we show that there is not necessarily a trade-off between the prevention of default risk and stabilizing inflation. Under optimal monetary and fiscal policy, comprising a de facto inflation stabilization policy, the tax rate as an optimal fiscal policy tool plays an important role in stabilizing inflation, although not completely because of the distorted steady state. Changes in the tax rate to minimize welfare costs via stabilizing inflation then improve the fiscal surplus, and because of this and the incompletely stabilized inflation, the default rate does not increase as much.

Keywords: Sovereign Risk, European Crisis, Optimal Monetary Policy, Fiscal Theory of the Price Level

1. INTRODUCTION

How do we conduct monetary policy in a currency union amid sovereign risk premiums? How do monetary and fiscal authorities behave in this difficult situation? What clues do we have for removing the trade-off between the prevention of default risk and stabilizing inflation? In this paper, we show that there is not necessarily a trade-off between the prevention of default risk and stabilizing inflation. However, Uribe (2006) shows that there is a trade-off between the prevention of default risk and stabilizing inflation, and this result is contrary to our finding. Precisely what derives this contrary result depends on the endogeneity of the fiscal surplus.

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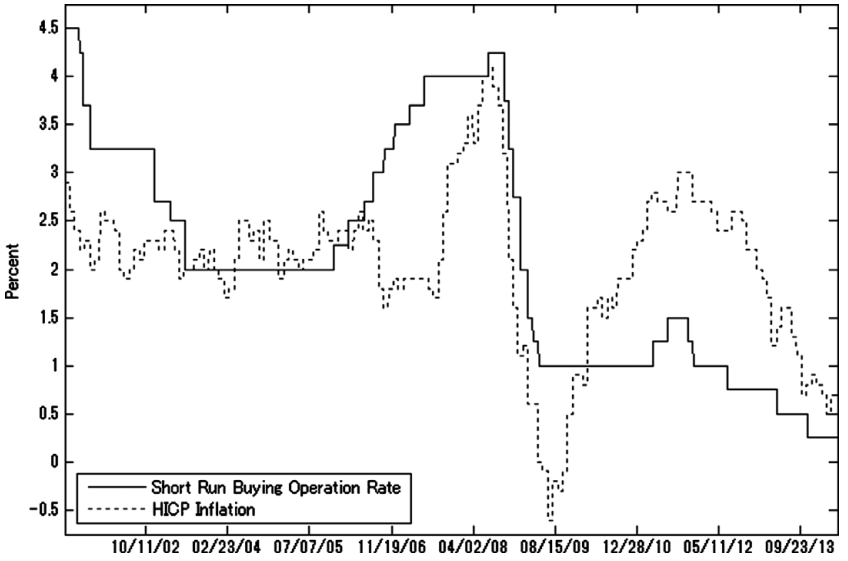


FIGURE 1. Buying operation rate and HICP inflation. *Source*: Eurostat and Bloomberg.

While Uribe (2006) assumes that the fiscal surplus is an exogenous shock, given the presence of an endowment economy, we endogenize the fiscal surplus through endogenizing production. This difference generates quite opposing results. In addition, we derive our finding that there is not necessarily a trade-off between the prevention of default risk and stabilizing inflation from a default model, including Uribe's (2006) mechanism, nor do we introduce any artificial assumptions. Thus, our finding is novel.

The European Central Bank (ECB) appears to face a trade-off between stabilizing inflation and preventing default. From January 1999 to February 2009, the ECB's official policy rate was never less than the Harmonized Index of Consumer Prices (HICP) measure of inflation, save for a period of about 21 months (Figure 1). On this basis, it seems that the ECB attempted to stabilize inflation in the Eurozone by keeping in mind a simple monetary model including the Taylor rule. However, amid the Greek crisis in March 2010, the ECB's policy rate began to fall below HICP inflation and remained low. For now, the ECB would even appear to welcome inflation, as this would help mitigate the burden associated with redeeming existing government debt. The results in Uribe (2006) may affect this policy stance. To show that there is not necessarily a trade-off between the prevention of default risk and stabilizing inflation, we develop a class of dynamic stochastic general equilibrium (DSGE) models with nominal rigidities and assume a currency union consisting of two countries. In developing our model, we draw upon earlier work in this area by Uribe (2006) and Benigno (2009). Uribe (2006) assumes that default depends on the ratio of the net present value of the real fiscal surplus to real government debt with interest payments. That is, the burden

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of government debt redemption is such that the lower the ratio, the higher the probability of default. In our model, one of two countries, which we refer to as "country F," defaults on its financial obligations following the mechanism in Uribe (2006). In the other country, referred to as country H, this ratio is strictly unity, and thus "country H" never defaults on its debt. In assuming just two countries, we in effect consider that the Eurozone currently consists of countries with and without sovereign risk.

In related work, Benigno (2009) develops a two-country model and supposes the situation where households face difficulties in purchasing foreign assets. Because of this, households obtain less remuneration from the purchase of foreign assets, a notion we include in our model. That is, we assume that country F's government has to pay additional interest because of default risk. Formally, we introduce an interest rate multiplier as an increasing function of the fiscal deficit. This interest rate multiplier decides the coupon rate of government bonds and in turn affects household decisions on holding government bonds. Although government bonds are determined in practice by the exogenous fiscal surplus and the default rate on the flow budget constraint in Uribe (2006), the government bond can affect the fiscal surplus. Therefore, by introducing Benigno (2001) into our model, which uses an earlier working paper version of Benigno (2009), we can endogenize the fiscal surplus in Uribe's (2006) framework. Through combining Uribe (2006) and Benigno (2009) in this analysis, we are able to replicate the current European debt crisis and can derive policy implications on how to conduct monetary and fiscal policy.

Our novel and important finding is that monetary policy aimed at minimizing welfare cost, namely, stabilizing inflation, and the prevention of default are not inconsistent. We compare a taxation regime where the governments in both countries in the currency union conduct an optimal "fiscal" policy, in which the tax rate is a fiscal policy tool, with a nontaxation regime where only country H conducts optimal fiscal policy. (Country F under nontaxation describes taxation failure in Greece.) Intuitively, inflation is stable and welfare costs are minimized under the former rather than under the latter. However, our numerical results show that both inflation and the default rate are stable under the taxation regime, but not the non-taxation regime. This result and the associated policy implication are quite different from Uribe (2006) and this stems from differences in the fiscal deficit settings. In essence, Uribe (2006) assumes an endowment economy and the fiscal deficit is then an exogenous shock, such that there is a no way to improve the fiscal deficit to avoid sovereign default, and controlling the price level is the only way to affect the default rate. Thus, there is a simple trade-off between stabilizing the price level, namely stabilizing inflation, and suppressing the rate of default. However, in our setting, the fiscal authority should prevent an increase in the fiscal deficit, which might increase the inflation rate via an increase in the output gap, and whose loss function is mostly the quadratic term of inflation and whose policy tool is the tax rate. Thus, inflation is stabilized and default is suppressed because an increase in the fiscal deficit is prevented and there is the possibility with our

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TABLE 1. Main differences between this and previous work

Authors	We	Uribe (2006)	Corsetti et al. (2013)	Benigno (2009)	Okano (2014)
Economic structure	Currency union	Closed economy	Closed economy	Two -country	Currency union
Role of monetary policy	Stabilizing inflation	Stabilizing price or default rate	Stabilizing inflation and interest spread	Stabilizing inflation	Stabilizing inflation
Role of fiscal policy	Stabilizing inflation	Not existing	Stabilizing GDP and government debt	Not existing	Stabilizing inflation
How to conduct those policies	Minimizing welfare costs	Taylor rule or exogenous price level targeting	Taylor rule and some feedback rule	Minimizing welfare costs	Minimizing welfare costs
Sovereign risk	Existing	Existing	Existing	Not existing	Not existing
Default depending on	Endogenous fiscal balance	Exogenous fiscal balance	Exogenous probability	NA	NA
Trade-off between inflation and default	Not necessarily existing	Existing	No mention	NA	NA
Policy implication on suppressing default	Stabilizing inflation via fiscal policy	Giving up inflation stabilization	Fiscal retrenchment	NA	NA

analysis that we can simultaneously stabilize both inflation and the default rate if we adopt an endogenized fiscal deficit, which is a natural setting. Under taxation, the governments of both countries seek to stabilize inflation by manipulating the tax rate, whereas under nontaxation, only country H's government manipulates the tax rate. As a result, both inflation and the default rate are better stabilized under taxation than under non-taxation. There is not a necessarily a trade-off.

Finally, we discuss the relationship between the present work and that in related studies (Table 1). Corsetti et al. (2013) analyse the impact of strained government finances on macroeconomic stability and the transmission of fiscal policy and consider a "sovereign risk channel" through which sovereign default risk raises

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funding costs in the private sector and show that fiscal retrenchment can help curtail the risk of macroeconomic instability and, in extreme cases, even stimulate economic activity. However, unlike our paper, they assume default depending on an ad hoc exogenous probability function. They also neither analyse this problem from the viewpoint of welfare costs, nor provide any implication for the trade-off between the prevention of default risk and stabilizing inflation. The main difference between the present paper and Uribe (2006) is that he emphasizes the trade-off between preventing default and stabilizing inflation. In addition, there is a difference in the policy implication for suppressing default. While stabilizing inflation is consistent with preventing default, Uribe (2006) suggests that giving up inflation stabilization is the way to preventing default.[1]

The remainder of the paper is organized as follows. Section 2 develops the model. Section 3 solves the linear–quadratic (LQ) problem, discusses the first-order necessary conditions (FONCs) for the policy authorities and the taxation and nontaxation regimes. Section 4 calibrates the model under both regimes. Section 5 concludes the paper.

2. THE MODEL

We derive a model based on Okano (2014), who also analyzed optimal monetary and fiscal policy in a currency union. The currency union consists of countries H and F (also denoted by $i = \{H, F\}$), which together organize a monetary union. The two countries are symmetrical and the parameters are equal in both countries. The households on the interval $[0, \alpha)$ belong to country H while those on the interval $[\alpha, 1]$ belong to country F. We assume that there is default risk in country F and the default mechanism follows that in Uribe (2006), while we follow Benigno (2001) to clarify the households' choice of risky assets.

2.1. Households

The preference of a representative household is given by

$$\sum_{t=0}^{\infty} \beta^t \mathrm{E}_0 \left(U_{i,t} \right), \tag{1}$$

where $U_{i,t} \equiv \ln C_t - \frac{1}{1+\varphi} N_{i,t}^{1+\varphi}$ is the period utility , E_t is the expectation conditional on the information set at period t, $\beta \in (0, 1)$ is the subjective discount factor, $C_t \equiv [\alpha^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + (1-\alpha)^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}]^{\frac{\eta}{\eta-1}}$ is the consumption index, $C_{i,t}$ are the consumption subindexes of the continuum of differentiated goods produced in both countries, $\eta > 0$ is the elasticity of substitution between goods produced in both countries, $N_{i,t}$ is the hours of labor, and φ is the inverse of the elasticity of the labor supply.

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The aggregate budget constraint in country H is given by

$$R_{t-1}\left[B^n_{H,t-1}+B^n_{F,t-1}\Gamma\left(\widetilde{df}_{F,t-1}\right)(1-\delta_t)\right]+W_{H,t}N_{H,t}+PR_{H,t} \geq P_tC_t+B^n_{H,t}+B^n_{F,t}, \quad (2)$$

where $R_t \equiv 1+r_t$ is the gross risk-free nominal interest rate, r_t is the net interest rate, $P_t \equiv [\alpha P_{H,t}^{1-\eta}+(1-\alpha)P_{F,t}^{1-\eta}]^{\frac{1}{1-\eta}}$ is the consumer price index (CPI), $P_{i,t}$ are the producer price indexes (PPIs) in both countries, $B^n_{i,t} \equiv P_tB_{i,t}$ are the nominal government debts issued by country i' government, $B_{i,t}$ are the real government debts issued by country i' government, $W_{i,t}$ is the nominal wage, $PR_{i,t}$ is profits from the ownership of the firms, δ_t is the default rate, and $\widetilde{df}_{F,t} \equiv \frac{\frac{B^n_{F,t}}{P_t}-R_{t-1}\frac{B^n_{F,t-1}}{P_t}}{B_F(1-R)}-1$ is the percentage deviation of changes in (real) government debt with interest payments, excluding the risk premium from its steady-state value. $\Gamma\left(\widetilde{df}_{F,t}\right)$ captures the costs of issuing country F's government debt and we assume $\Gamma'(\cdot)>0$. That is, the higher the fiscal deficit in country F, the greater the remuneration for holding country F's government debt because the larger the fiscal deficit, the greater the default risk. In that case, country F has to pay additional remuneration for holding country F's government debt, which provides households with a motivation for holding country F's government debt. Although our setting on $\Gamma(\cdot)$ is hinted at by Benigno (2001), he assumes $\Gamma'(\cdot)<0$, which implies that the larger country F's government debt, the lower the remuneration for holding country F's government debt. That is, his setting captures the costs of undertaking positions in the international asset market or the existence of intermediaries in the foreign asset market. In our case, there are no intermediaries and an increase in the default rate generates the cost. To compensate for this cost, the government pays additional interest. Thus, $\Gamma'(\cdot)>0$ is applied in our paper.[2]

The representative household maximizes equation (1) subject to equation (2). The optimality conditions consist of the inter and intratemporal optimality conditions. By holding government H's debt, we have the standard intertemporal optimality condition, Euler equation. By holding country F's debt, the Euler equation still applies, although it requires an additional condition, which depicts households' motivation to hold country F's government debt and can be obtained by differentiating the Lagrangian by country F's government debt. That condition is given by

$$\mathrm{E}_t(1-\delta_{t+1})=\left\{\Gamma\left(\widetilde{df}_{F,t}\right)+B_{F,t}\Gamma'\left(\widetilde{df}_{F,t}\right)[B(R-1)]^{-1}\right\}^{-1},$$

which can be log-linearized as

$$b_{F,t}=\frac{\omega_b}{\beta}\widetilde{df}_{F,t}-\frac{\beta\omega_b^2}{\phi(1-\beta)}\mathrm{E}_t(\delta_{t+1}), \quad (3)$$

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with $\omega_b \equiv \beta(1+\gamma)-1$ and $\hat{r}_t \equiv \frac{dR_t}{R}$, where $v_t \equiv \frac{dV_t}{V}$ is the percentage deviation from an arbitrary variable V_t's steady-state value and V is V_t's steady-state value. Following Benigno (2001), we define $\phi \equiv \Gamma'(0)$, which we refer to as the interest rate spread for country F's government debt in the steady state, and assume $\Gamma(0)=1$. The unfamiliar parameter, $\gamma \equiv \frac{\Gamma''(0)}{\Gamma'(0)} > 1$ is the elasticity of the interest rate spread to a 1% change in the fiscal deficit in the steady state.[3] Equation (3) shows that an increase in the interest rate multiplier increases country F's government debt held by households, and that an increase in the expected default rate decreases country F's government debt held by households. This is because an increase in the interest rate multiplier increases the payoff for holding country F's government debt, while an increase in the expected default rate deters households from holding country F's government debt.[4]

2.2. Firms

Our setting on firms is quite similar to Gali and Monacelli (2005), except that here the tax is levied on firm sales and is not constant.[5] Each firm produces a differentiated good with a linear technology, is a monopolistic producer of one of the differentiated goods, and sets prices in a staggered fashion in the Calvo–Yun style.

2.3. Government

The government budget constraints are given by

$$B^n_{H,t} = R_{t-1}B^n_{H,t-1} - P_t SP_{H,t}; \quad B^n_{F,t} = R_{t-1}\Gamma\left(\widetilde{df}_{F,t-1}\right)(1-\delta_t)\,B^n_{F,t-1} - P_t SP_{F,t}, \tag{4}$$

where $SP_{i,t} \equiv \frac{P_{i,t}}{P_t}\left(\tau_{i,t}Y_{i,t} - G_{i,t}\right)$, $Y_{i,t}$, $G_{i,t}$, and $\tau_{i,t}$ are the real fiscal surplus, output, indexes of government expenditure, and the tax rate, respectively. The first and second equalities in equation (4) are the government budget constraints in countries H and F, respectively. The first term on the right-hand side (RHS) in the second equality shows that country F's government pays higher interest payments because of sovereign risk.

Iterating the first equality in equation (4) forward, and imposing the appropriate transversality condition for country H's government debt, we have

$$1 = \frac{\sum_{k=0}^{\infty}\beta^k \mathrm{E}_t\left(C^{-1}_{t+k}SP_{H,t+k}\right)}{C^{-1}_t R_{t-1}\frac{B^n_{H,t-1}}{P_t}}. \tag{5}$$

Equation (5) is quite similar to the central equation of the Fiscal Theory of the Price Level (FTPL). On the RHS in equation (5), the numerator is the net present value of the sum of the fiscal surplus in terms of the marginal utility of consumption, namely solvency, and the denominator is the past nominal government debt with

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interest payments divided by current CPI, namely the government's liability. The left-hand side (LHS) is unity. Thus, equation (5) implies that solvency definitely corresponds to the government's liability. If solvency is less than the government's liability, CPI increases, that is, inflation arises so that the government's liability is mitigated.

Similar to equation (5), iterating the second equality in equation (4) forward and imposing the appropriate transversality condition for country F's government debt, we have

$$1-\delta_t=\frac{\sum_{k=0}^{\infty}\beta^k \mathrm{E}_t\left(C_{t+k}^{-1}SP_{F,t+k}\right)}{C_t^{-1}R_{t-1}\Gamma\left(\widetilde{df}_{F,t-1}\right)\frac{B_{F,t-1}^n}{P_t}}. \tag{6}$$

Equation (6) implies that an increase in CPI or CPI inflation does not necessarily occur if solvency is less than the government's debt. Not only inflation, but also default, mitigates the government's liability. On the one hand, suppose that CPI is constant and there is no inflation. In this situation, if the solvency is about equal to the government's liability, the RHS is less than unity. On the other hand, the LHS is definitely less than unity through an increase in the default rate. In other words, if the government falls insolvent while CPI is strictly stable, default is inevitable. Uribe (2006) pointed out that there is a trade-off between inflation stabilization and suppressing default by introducing default, namely sovereign risk, into the central equation of the FTPL. Similar to Uribe (2006), at first glance equation (6) also suggests that there is such a trade-off. However, in our setting, the fiscal surplus in the numerator is not exogenous, but rather endogenized differently from Uribe (2006). As is well known, inflation stabilization requires stabilization of the output gap in the DSGE model. Suppose that there is inflationary pressure stemming from an increase in government expenditure, which may decrease the fiscal surplus such that policy authorities seek to stabilize inflation. If the tax rate is available as a fiscal policy tool, the government hikes the tax rate to stabilize inflation because an increase in the tax rate stabilizes the output gap through a decrease in consumption. In that case, not only inflation, but also the fiscal surplus, is stabilized. Thus, there is not necessarily a trade-off between inflation stabilization and suppressing default, as long as the fiscal surplus is endogenized.

2.4. Welfare Costs

We derive the period welfare cost function from the welfare criterion. Because of the distorted steady state, we derive the welfare criterion following Benigno and Woodford (2005). We obtain the welfare cost function which is derived by subtracting a second-order approximated FONC for firms from a second-order approximated utility function to eliminate the linear terms that lead to the incorrect evaluation of welfare as follows[6]:

$$L_t^W\equiv\frac{\alpha}{2}\Lambda_\pi\pi_{H,t}^2+\frac{1-\alpha}{2}\Lambda_\pi\pi_{F,t}^2+\frac{1}{2}\Lambda_x x_t^2+\frac{\alpha\left(1-\alpha\right)}{2}\Lambda_z z_t^2, \tag{7}$$

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where $\pi_{i,t} \equiv p_{i,t} - p_{i,t-1}$ is GDP inflation, $x_t \equiv y_t - y_t^e$ is the welfare-relevant output gap, $y_t \equiv \alpha y_{H,t} + (1-\alpha)\, y_{F,t}$ is the (logarithmic) weighted-average output, y_t^e is the (logarithmic) efficient level of weighted-average output, $z_t \equiv s_t - s_t^e$ is the terms of trade (TOT) gap, $s_t \equiv p_{F,t} - p_{H,t}$ is the (logarithmic) TOT, s_t^e is the (logarithmic) efficient level of TOT, $\Lambda_\pi \equiv \frac{\varepsilon\{1-\Phi[1-\kappa\varsigma_C]\}}{\kappa\varsigma_C}$, $\Lambda_x \equiv \frac{\omega_{14}}{\varsigma_C^2}$, and $\Lambda_z \equiv \varsigma_C \eta^2 (1-\Phi)(1+\varphi) + \Phi\kappa\tilde{\Omega}_1$ are the weights of GDP inflation, the welfare-relevant output gap and the TOT gap in welfare costs, respectively, $\Phi \equiv 1 - \frac{1-\tau}{\frac{\varepsilon}{\varepsilon-1}}$ is the steady-state wedge between the marginal rate of substitution between consumption and leisure and the marginal product of labor, $\varepsilon > 1$ is the elasticity of substitution across goods, $\kappa \equiv \frac{(1-\theta)(1-\theta\beta)}{\theta}$ is the slope of the New Keynesian Philips curve (NKPC), ς_C is the steady-state share of consumption to output, and $\tilde{\Omega}_1$ and ω_{14} are combinations of the structural parameters. Notice that Λ_π, Λ_x, and Λ_z are not ad hoc, but derived through the second-order approximation of equation (1).[7]

2.5. Welfare-Relevant Output Gap and Dynamics

Combining the definition of the welfare-relevant output gap, the definitions of the (logarithmic) TOT and of the TOT gap, equations (5)–(7), and the equilibrium conditions yields

$$x_t = \frac{\varsigma_B}{\omega_1}\left\{\mathrm{E}_t\left(x_{t+1}\right) - \varsigma_C\left[\hat{r}_t + \mathrm{E}_t\left(\pi_{t+1}\right) - b_{H,t} + \hat{r}_{t-1} + \frac{1}{\beta} b_{H,t-1} + \frac{1}{\beta}\pi_t - \frac{1-\alpha}{\beta\varsigma_B} z_t - \frac{\tau}{\varsigma_B}\hat{\tau}_{H,t}\right]\right\} + \psi_{H,t}, \tag{8}$$

$$x_t = \frac{\varsigma_B}{\omega_1}\left\{\mathrm{E}_t\left(x_{t+1}\right) - \varsigma_C\left[\hat{r}_t + \mathrm{E}_t\left(\pi_{t+1}\right) + \mathrm{E}_t\left(\delta_{t+1}\right) - \frac{1}{1-\beta}\left(\tilde{\omega}_\phi b_{F,t} + \frac{\hat{\omega}_\phi}{\beta} b_{F,t-1} + \phi b_{F,t-2} + \frac{\tilde{\omega}_\phi}{\beta}\hat{r}_{t-1} + \frac{\phi}{\beta}\hat{r}_{t-2} - \frac{\tilde{\omega}_\phi}{\beta}\pi_t - \frac{\phi}{\beta}\pi_{t-1}\right) - \frac{1}{\beta}\delta_t + \frac{\alpha\omega_2}{\beta\varsigma_B} z_t - \frac{\tau}{\varsigma_B}\hat{\tau}_{F,t}\right]\right\} + \psi_{F,t}, \tag{9}$$

$$\pi_{H,t} = \beta \mathrm{E}_t\left(\pi_{H,t+1}\right) + \kappa\left\{(1+\varphi)\left[x_t + (1-\alpha)\, z_t\right] + \frac{\tau}{1-\tau}\hat{\tau}_{H,t}\right\} + \epsilon_{H,t}, \tag{10}$$

$$\Delta z_t = \pi_{F,t} - \pi_{H,t} + \frac{\omega_{21}}{(1-\tau)\,\omega_{18}}\left(\Delta\hat{\tau}_{H,t} - \Delta\hat{\tau}_{F,t}\right) + \Delta\xi_t, \tag{11}$$

with $\Delta v_t \equiv v_t - v_{t-1}$ and $\tau \equiv \tau_i$, where ς_B is the steady-state share of consumption to output, respectively, ω_1, ω_2, ω_{18}, and ω_{21}, $\tilde{\omega}_\phi$, and $\hat{\omega}_\phi$ are combinations of parameters, $\hat{\tau}_{i,t} \equiv \frac{d\tau_{i,t}}{\tau_i}$, $\psi_{i,t}$, $\epsilon_{i,t}$, and ξ_t are the tax gap, the demand shock, the cost–push shock, and the TOT shock, respectively. Equations (8) and (9) are the New Keynesian IS (NKIS) curves in countries H and F, respectively, equation

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(10) is the NKPC in country H. The NKPC in country F is analogous to equation (10).

3. THE LQ PROBLEM

To clarify the role of the tax gap as a policy instrument, we analyse the nontaxation regime and the taxation regime. The policy authority minimizes the weighted-average welfare costs $\mathcal{L}^W \equiv \sum_{t=0}^{\infty} \beta^t \mathrm{E}_0 \left(L_t^W\right)$ subject to the definition of the CPI inflation $\pi_t \equiv p_t - p_{t-1}$, equations (3), (8)–(11), and the NKPC in country F, the log-linearized average, and relative government budget constraints.[8] Under the nontaxation regime, the policy instruments are the nominal interest rate $\hat{r}_t$ and the tax gap in country H $\hat{\tau}_{H,t}$ and country F's government does not change the tax rate. Under the taxation regime, the policy instruments are the nominal interest rate $\hat{r}_t$, the tax gap in country H $\hat{\tau}_{H,t}$, and the tax gap in country F $\hat{\tau}_{F,t}$ and both countries' government change the tax gap to minimize welfare costs. Note that the nontaxation regime depicts the actual situation of taxation failure in Greece.

Although we analyse optimal monetary and fiscal policy, the weight on the welfare-relevant output gap Λ_x is the largest weight in the period loss function equation (7). In fact, under our plausible parameterization, discussed in Section 4.1, to calibrate the model, the weight on GDP inflation Λ_π, the weight on the welfare-relevant output gap Λ_x and the weight on the TOT gap Λ_z are 151.08, 4.28, and 45.22, respectively. Thus, we regard our analysis as inflation stabilization policy.

There are FONCs for the policy authorities for the welfare-relevant output gap, the TOT gap, GDP inflation, the nominal interest rate, the tax gap, and CPI inflation and the default rate. In our paper, the FONCs for the tax gap have an important role and account for the difference in policy implications to Uribe (2006). The FONCs for the tax gap show that the tax gap is a strong policy tool to stabilize inflation. In addition, it implies that the tax gap affects the fiscal surplus, such that an increase in the tax gap increases the (average) fiscal surplus while an increase in the tax gap in country F decreases government debt in country F compared with country H. Thus, the tax gap is not only a policy tool to minimize welfare costs via stabilizing inflation, but also controls the default rate via the effect on the fiscal surplus. Because an increase in the tax gap stabilizes inflation and subdues the fiscal deficit, a trade-off between stabilizing inflation and preventing default appears only obscurely in our model. Instead, stabilizing inflation is consistent with preventing default via controlling the tax gap, namely, conducting optimal fiscal policy. Under nontaxation, only the FONC for the tax gap in country H is available, whereas both the FONCs for the tax gaps in countries H and F are available. This implies that both inflation and the default rate are stabilized more under taxation than nontaxation.

4. NUMERICAL ANALYSIS

We run a series of dynamic simulations and adopt a benchmark parameterization mainly in line with previous work and liken Portugal, Italy, Ireland, Greece,

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TABLE 2. Parameterization

Parameter	Definition	Value	Source
Subjective discount factor	β	0.99	Ferrero (2009)
Elasticity of substitution across goods	ε	11	Ferrero (2009)
Inverse of the labor supply elasticity	φ	3	Gali and Monacelli (2005)
Steady-state share of government bonds to output	ς_B	2.4	Ferrero (2009)
Steady-state share of government expenditure to consumption	ς_C	0.724	Ferrero (2009)
Steady-state tax rate	τ	0.3	Ferrero (2009)
Elasticity of substitution between goods produced in countries H and F	η	4.5	Benigno and Benigno (2006)
Price stickiness	θ	0.75	Benigno and Lopez-Salido (2006)
Persistence of productivity shock	ρ_A	0.705	Batini et al. (2003)
Persistence of government expenditure shock	ρ_G	0.8	Poplawski-Ribeiro (2008)
Share of country H	α	0.68	GDP Share of PIIGS among 17 countries in Euro area is 0.32 in 2013Q4
Spread of the nominal interest rate	ϕ	0.1	Benigno (2001)
Elasticity of the interest rate spread to 1% change in the fiscal deficit	γ	1.145	Okano and Inagaki (2017)

and Spain (PIIGS) to country F (Table 2).[9] Because of sovereign risk, we have the elasticity of the interest rate spread to a 1% change in the fiscal deficit or government debt γ and set it to 1.145 following Okano and Inagaki (2017).[10] To examine the impulse response functions (IRFs), we consider a 1% change in productivity in countries H and F and a 1% change in government expenditure in countries H and F.

4.1. Macroeconomic Volatility and Correlation

The volatility of the default rate under taxation is 0.04, which is lower than the value under nontaxation of 0.15 (Table 3). In addition, the volatility of GDP and CPI inflation under taxation are lower than under nontaxation. That is, the volatilities of CPI inflation under taxation and nontaxation are 0.23 and 50.19, respectively, and those of GDP inflation in country H under taxation and nontaxation are 0.06 and 62.59, respectively. The volatility of GDP inflation in country F under taxation and nontaxation is 0.66 and 27.70, respectively. These results mean that there is not necessarily a trade-off between stabilizing inflation and suppressing the default rate. Underlying this result, the channel between the fiscal surplus

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TABLE 3. Macroeconomic volatility

	Nontaxation			Taxation		
Variable	Benchmark	$\gamma = 1.5$	$\gamma = 2$	Benchmark	$\gamma = 1.5$	$\gamma = 2$
x_t	21.78	21.93	42.86	0.79	0.74	1.52
z_t	54.88	76.37	162.58	0.82	0.72	0.71
π_t	50.19	69.80	152.74	0.23	0.24	0.21
$\pi_{H,t}$	62.59	86.95	191.52	0.06	0.07	0.08
$\pi_{F,t}$	27.70	39.34	83.10	0.66	0.66	0.63
$\hat{r}_t$	32.48	49.44	112.18	0.28	0.28	0.72
$\hat{\tau}_{H,t}$	1,637.55	2,419.51	5,424.02	8.35	7.37	13.19
$\hat{\tau}_{F,t}$	NA	NA	NA	32.07	32.54	34.94
$b_{H,t}$	1,052.15	107.38	2,181.67	37.36	16.21	33.08
$b_{F,t}$	0.26	0.30	0.72	0.25	0.26	0.29
δ_t	0.15	0.16	0.27	0.04	0.03	0.05
$sp_{H,t}$	19,708.28	29,188.15	65,535.25	100.28	91.24	164.28
$sp_{F,t}$	1,431.79	2,040.49	4,318.08	386.30	392.11	416.52

Note: Standard deviations in percentages.

and the default rate has an important role in suppressing the latter. Lastly, the volatility of the fiscal surplus in country F under taxation is higher than under nontaxation. We calculate macroeconomic volatilities under $\gamma = 1.5$ and $\gamma = 2$. Under nontaxation, the higher the γ, the more volatile are all the macroeconomic variables. An increase in γ widens the interest rate spread so that the expected default rate rises. This makes the macroeconomy volatile. Under taxation, however, the tax gap in country F is available as a fiscal policy tool and both inflation and the default rate are well stabilized through stabilizing the fiscal surplus. Thus, the macroeconomic variables are not as severely volatile as are those under nontaxation and these volatilities are then less related to the value of γ.

We calculate the correlation between selected variables, such as CPI inflation, the default rate, and the fiscal surplus. The correlation between CPI inflation and the default rate is -0.59 under nontaxation. This implies that the higher is inflation, the lower the default rate and vice versa (Table 4). However, the correlation between the fiscal surplus in country F and the default rate is -0.69 under nontaxation. This shows that the fiscal surplus channel overwhelms the inflation channel from the viewpoint of suppressing the default rate and the fiscal surplus channel is more important than the inflation channel from this same viewpoint.

An increase in the tax gap then decreases GDP inflation through the NKIS curves and the NKPCs, as shown in equations (8)–(10) while this increase decreases the default rate via an increase in the fiscal surplus in country F, as shown in equations (5) and (6). Thus, Uribe's (2006) policy implication is not applicable in our model and the latter effect, namely the fiscal surplus channel, overcomes the former effect, namely the CPI inflation channel, in our parameterization. At the least,

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TABLE 4. Correlations between selected variables

Variable	Regime	π_t	$\pi_{H,t}$	$\pi_{F,t}$	δ_t	$sp_{H,t}$	$sp_{F,t}$
π_t	Nontaxation	1.00	0.99	0.88	−0.59	0.77	0.69
	Taxation	1.00	0.59	0.99	0.27	−0.09	0.89
$\pi_{H,t}$	Nontaxation	0.99	1.00	0.83	−0.54	0.76	0.63
	Taxation	0.59	1.00	0.44	−0.03	−0.41	0.76
$\pi_{F,t}$	Nontaxation	0.88	0.83	1.00	−0.76	0.70	0.86
	Taxation	0.99	0.44	1.00	0.31	−0.01	0.82
δ_t	Nontaxation	−0.59	−0.54	−0.76	1.00	−0.20	0.23
	Taxation	0.27	−0.03	0.31	1.00	−0.64	−0.07
$sp_{H,t}$	Nontaxation	0.77	0.76	0.70	−0.28	1.00	0.81
	Taxation	−0.09	−0.41	−0.01	−0.20	1.00	−0.03
$sp_{F,t}$	Nontaxation	0.69	0.63	0.86	−0.69	0.81	1.00
	Taxation	0.89	0.76	0.82	0.23	−0.03	1.00

in our setting stabilizing inflation is then consistent with suppressing the default rate. Under taxation, inflation, the welfare-relevant output gap and the TOT gap are well stabilized, and this feature makes the relationship between inflation, the fiscal surplus, and default obscure.

4.2. IRFs

Under nontaxation, an increase in government expenditure in country H increases the fiscal surplus in country F, increases the fiscal surplus in country H, and decreases government debt in country H (Figure 2).[11] To stabilize GDP inflation, the nominal interest rate and the tax gap in country H are increased. Because the tax gap in country F is constant, GDP inflation in both countries is not well stabilized. As a result, CPI inflation rises. Because of the increases in CPI inflation and the fiscal surplus in country F, the default rate decreases. This result is consistent with our result concerning the negative correlation between the default rate and CPI inflation and between the default rate and the fiscal surplus in country F. Both increases in the CPI inflation and in the fiscal surplus in country F mitigate default risk. An increase in government expenditure in country F then increases CPI inflation and the fiscal surplus in country F. Increases in these then decrease the default rate, similar to the case of an increase in government expenditure in country H.

Under taxation, the governments of both countries seek to minimize welfare costs. Thus, the tax gap in country F is no longer zero. An increase in government expenditure in country H provides inflationary pressure. However, because of the changes in the tax gap, GDP inflation is well stabilized in both countries (Figure 3). Thus, CPI inflation is also well stabilized. To stabilize GDP inflation, the tax gap in country F changes intricately and the changes affect the fiscal surplus in country

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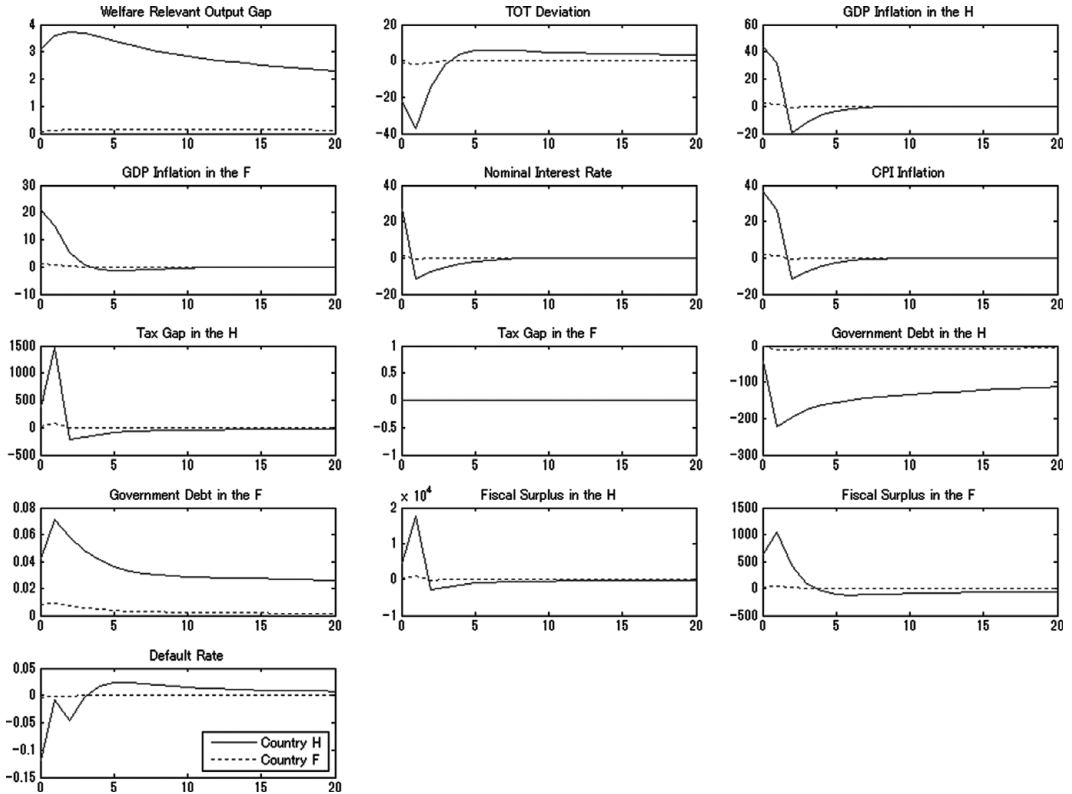


FIGURE 2. IRFs to government expenditure under nontaxation.

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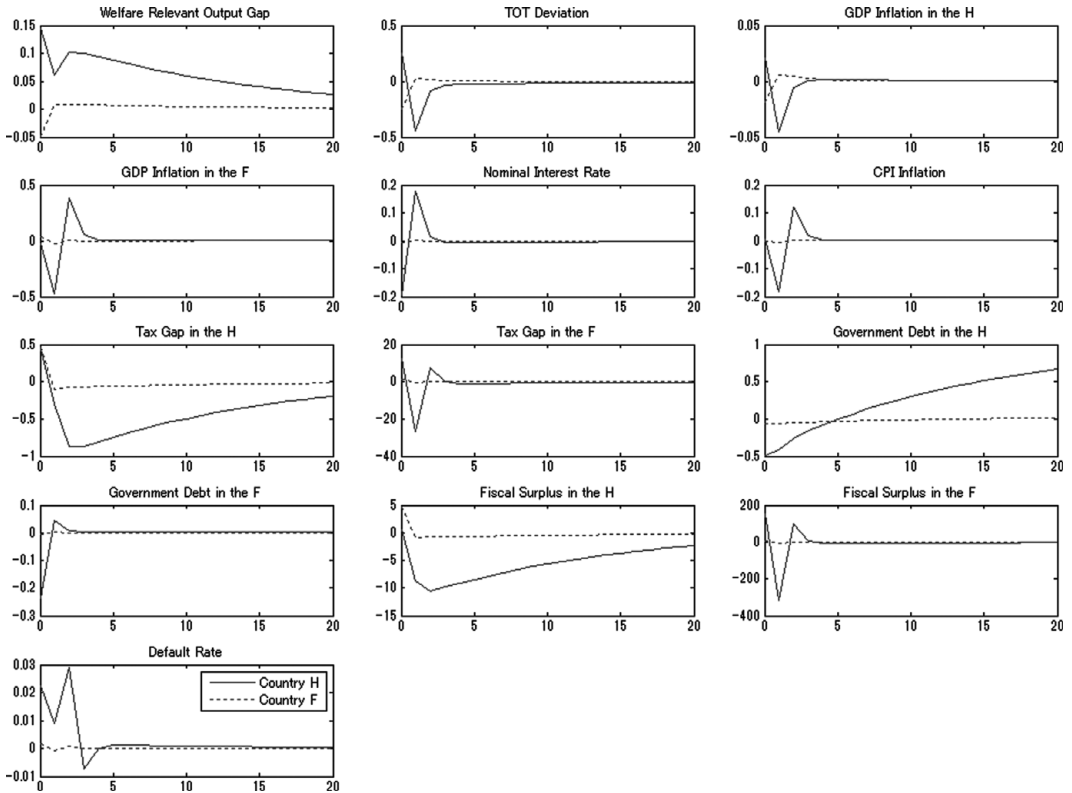


FIGURE 3. IRFs to government expenditure under taxation.

F. Inflationary pressure stemming from an increase in government expenditure in country F is well absorbed by the tax gap. As a result, CPI inflation is well stabilized via stabilization in GDP inflation. The fiscal balance does not change remarkably and the default rate is almost zero.

4.3. Welfare Costs

Following Gali and Monacelli (2005), we set $\beta \to 1$ for the weighted-average welfare costs and obtain $\mathcal{L}^W = \frac{\alpha}{2}\Lambda_\pi \text{var}\left(\pi_{H,t}\right) + \frac{1-\alpha}{2}\Lambda_\pi \text{var}\left(\pi_{F,t}\right) + \frac{1}{2}\Lambda_x \text{var}\left(x_t\right) + \frac{\alpha(1-\alpha)}{2}\Lambda_z \text{var}\left(z_t\right)$. Under our parameterization, the welfare costs under nontaxation and taxation are 2026.19 and 0.44, respectively. The welfare costs under nontaxation are then about 4,605 times those under taxation. Combined with the lower default rate, there is then no reason to avoid conducting optimal monetary and fiscal policy, which is equivalent to conducting inflation stabilization policy via optimal taxation.

5. CONCLUSION

While sovereign risk continues to smoulder throughout the Eurozone, the ECB hesitates to stabilize inflation. However, as we show in our analysis, optimal monetary and fiscal policy itself, namely stabilizing inflation, does not worsen the default rate. Because the central bank's role is limited in a currency union, the role of fiscal policy as a tool for minimizing welfare cost, even in an asymmetric two-country currency union model, is greater than that in a simple two-country setting. In terms of policy recommendations, governments should then address taxation failure while policy authorities should not hesitate to stabilize inflation.

As a future extension, there is a possibility to solve our model with a more general period welfare cost function with approximations of orders higher than second. As mentioned by Kim and Kim (2003), welfare reversal does not occur as long as we eliminate the linear terms appropriately. Our approach is correct because we eliminate these terms following Benigno and Woodford (2005). Thus, we may not provide any new finding in terms of the welfare analysis. However, we are interested in the trade-off between preventing default and stabilizing inflation, which still exists even if we adopt such a general function.

NOTES

1. Elsewhere, we analyse and develop a two-country model similar to Benigno (2009) and Okano (2014), while Benigno (2009) does not assume a currency union. In addition, where we introduce a default mechanism following Uribe (2006), neither Benigno (2009) nor Okano (2014) assumes default risk in their models.

2. While we adopt opposite or different assumptions from Benigno (2001, 2009), we need these modifications to fit Benigno's (2001) idea to Uribe (2006). As mentioned, one different assumption is $\Gamma'(\cdot) > 0$, along with another two unlike assumptions. In our model, $\Gamma(\cdot)$'s argument is not the

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level, but the percentage deviation from the steady state. In our model, and unlike Benigno (2001), the steady-state fiscal deficit is not zero, but negative (that is, the steady-state value of the fiscal surplus is positive). If we simply assume $\Gamma(0) = 1$, similar to Benigno (2001), $\Gamma(0) = 1$ is not applicable in the steady state because in our model, the argument $\Gamma(\cdot)$ is nonzero in the steady state. Thus, $\Gamma(\cdot)$'s argument is the percentage deviation from its steady-state value. Finally, $\Gamma(\cdot)$'s argument is the fiscal deficit in our model such that it is the balance of government debt with interest payments. Corsetti et al. (2013), assume that the larger the fiscal deficit, the greater the probability of default, and vice versa. If we are given that the greater the probability of default, the higher the nominal interest rate for risky assets, our assumption that $\Gamma(\cdot)$ is an increasing function of the fiscal deficit is consistent with them. In addition, our assumption is consistent with the attempt by Okano and Inagaki (2017) to estimate the elasticity of the interest rate spread using data on government debt with interest payments and the fiscal deficit in Greece. Note that they succeed in estimating the elasticity with data on the fiscal surplus, although they find that the elasticity is not significant when using data on government debt with interest payments. This result is consistent with our assumption of $\Gamma(\cdot) > 0$.

3. We assume that $|\Gamma'(\cdot)| < |\Gamma''(\cdot)|$. By envisaging that the yields on risky assets increase at an increasing rate, we can accept this assumption.

4. Equation (3) makes a nonnegligible contribution to endogenizing the fiscal surplus. Although the exogenous fiscal surplus and the default rate on the flow budget constraint determines the government bond in Uribe (2006), the government bond balance can affect the fiscal surplus. That is, by introducing Benigno's (2001) idea, we can endogenize the fiscal surplus in Uribe's (2006) framework. Thus, equation (3) is the source of the added value from combining Uribe (2006) and Benigno (2001).

5. Unlike our setting, Gali and Monacelli (2005) assume that constant employment subsidies and monopolistic power completely disappear.

6. Benigno and Woodford (2005) derive the second-order approximated utility function without the presence of a linear term under the distorted steady state. Thus, we follow Benigno and Woodford (2005) to derive the period loss function because of the distorted steady state in our model. Note that Woodford (2003) discusses how the presence of linear terms generally leads to the incorrect evaluation of welfare. Kim and Kim (2003) propose a simple example of this result.

7. The appearance of a quadratic term for PPI inflation in equation (7) implies an important friction in the model, namely, price stickiness generates PPI inflation because firms set their prices after anticipating that they cannot choose the optimal price in future. Because of this price stickiness, GDP deviates from its efficient level and quadratic term of welfare relevant to output gap appears on equation (7). Because of the two-country setting, policy authorities have an incentive to boost consumption in their own country through manipulating the TOT. However, such manipulation invokes a beggar-thy-neighbor policy and harms the currency union's welfare. Thus, the quadratic term of the TOT gap appears on equation (7). Note that equation (7) is not specific to a currency union such as the Eurozone, but general in a two-country setting.

8. These government budget constraints are derived by log-linearizing equation (4).

9. Our value of β is widely adopted. To avoid excessive steady-state distortions, Ferrero (2009) sets ε not to the standard value of ten or less, but to eleven. On the value of φ, we follow Gali and Monacelli (2005). The value of ς_B is consistent with the Stability and Growth Pact providing that the share of government debt to GDP must be lower than 60%. Note that the timing of the model is quarterly and the yearly value of ς_B is 2.4. Based on the OECD Economic Outlook Database, Ferrero (2009) set the public spending to GDP ratio to 0.276 which corresponds to ς_C is 0.724. The average value of direct and indirect tax revenue as a percentage of GDP is 31.6% among the European Monetary Union (EMU) countries in 2003, and Ferrero (2009) sets τ to 0.3, which approximately corresponds to the data. The value of η is adopted by Benigno and Benigno (2006) based on Obstfeld and Rogoff (2000). The value of θ is set following Benigno and Lopez-Salido (2006), who estimates NKPC. ρ_A and ρ_G are set following the results in Batini et al. (2003) and Poplawski-Ribeiro (2008), respectively. To calculate the value of α, we retrieved data from Datastream. To calibrate the model, Benigno (2001) set the value of ϕ to 0.1, which approximately corresponds to the results of Okano and Inagaki (2017), who report that ϕ is 0.138.

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10. Using data on the interest spread and fiscal balance from January, 2005, to April, 2015, in Greece, they find this result through regression analysis.

11. In each figure, the vertical axis is the percentage deviation of the steady-state value and the horizontal axis is the period.

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