Validation of endurance estimates for battery powered UAVs

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ABSTRACT

An experimental investigation is presented to validate endurance estimates for a battery powered flight vehicle. In this study a radio controlled model aircraft was 'flown' inside a wind tunnel. Various Lithium Ion batteries were used to power the model. Battery electrical characteristics were recorded during each run to assess battery discharge behaviour. Theoretical estimates based upon battery characteristics yielded endurance estimates within 3% of that measured experimentally. The discharge characteristics of the batteries were found to be the largest source of uncertainty for endurance estimates.

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NOMENCLATURE

C battery capacity
C_D drag coefficient
C_{Do} zero lift drag coefficient
C_L lift coefficient
C_o effective initial battery capacity
E endurance
$\bar{i}$ average current during discharge
i current
k drag due to lift parameter
$k1$ constant
n Peukert constant
P power
Rt battery hour rating (in hours), i.e. the discharge time over which the capacity was determined
S planform area
t time
U flight velocity
V voltage
$\bar{V}$ average voltage during discharge
V_o battery voltage at start of discharge
W weight
η_{tot} efficiency
ρ density

Subscripts

elec electrical
expt experimental
md minimum drag
num numerical
req required
th theory

1.0 INTRODUCTION

For the aircraft designer, the ability to accurately assess the range and endurance of the vehicle under consideration is paramount. The widespread implementation of electrically and specifically battery powered unmanned aerial vehicles (UAVs) has necessitated the development of accurate methodologies that facilitate range and endurance estimates[1,2]. At present, few methods within the analysis framework that is familiar to the aircraft designer exist. In an earlier article[3], a method was presented that allows for estimation of range and endurance of battery powered flight vehicles, in a format that would be comfortable to engineers familiar with conventional propulsion formulations. As pointed out by the author[3], the method lacked validation due to an insufficiency of suitable experimental data. This article seeks to confirm the approach[3], through the implementation of an experimental program designed to provide the necessary data for validation, and then provide comparison between theory and experiment. Implementation comprised using a force balance mounted, battery powered, model aircraft that was 'flown' inside a wind tunnel at conditions corresponding to steady level flight until the battery was no longer capable of maintaining sufficient power to overcome the drag.

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2.0 EQUIPMENT AND PROCEDURE

A radio control model of a Spitfire made from foam was used for testing. The plane was assembled as per the manufacturer's instructions and equipped with an Electrify Rimfire 370 outrunner brushless electric motor. The models wing span of 0·876m coupled with the reference area of 0·141m^2 yielded an AR of 5·45. A 9″ diameter two bladed propeller was used, see inset, Fig. 1. Gaps between control elements (ailerons, etc) were sealed. The motor was controlled using an Exceed RC Proton 30A electronic speed controller. Measurements of electrical power, current and voltage were accomplished using an inline Watt's Up WU100 RC Watt Meter & Power Analyser. Prior to use, measurements from this analyser were compared with those from a Fluke 175 multi-meter. Agreement for current and voltage measurements were excellent, typically within 1·5%.

Lithium-Ion polymer batteries, manufactured by Turnigy Power Systems were evaluated, with rated capacities of 0·5C and 1C (and capable of a 20-30C discharge). The batteries were charged using an Electrify Triton EQ charger. The charger balances the individual cells while charging to maximise capacity. After a run, the batteries where allowed to cool down to the touch (typically half an hour) before re-charging. Performance characterisation of the plane was facilitated using a 6-component Pyramidal balance in conjunction with Embry Riddle Aeronautical University's 32″ by 45″ closed circuit tunnel. Testing has indicated a typical uncertainty of less than 0·07N for both lift and drag. Angle-of-attack setting is implemented using a Midori sensor mounted in the model. This yields setting accuracies better than 0·05 deg and eliminated any issues with backlash in the pitch mechanism.

Generally, the induced characteristics associated with a drag polar[4] are not particularly sensitive to Re within a moderate range. For each battery capacity, tests were run at tunnel speeds of approximately 12ms^{-1} and 15ms^{-1}. To maximise accuracy (higher loads), the drag polar of the aircraft was established at $U = 15\text{ms}^{-1}$ (Re = 150,000, based on the average wing chord), the higher of the two test speeds. The propeller was removed for this test. The variation of the zero lift drag coefficient was also examined through a Re sweep (i.e. wind-tunnel tests were conducted at a range of Re numbers). Testing to estimate endurance involved setting the model at an incidence that yielded steady level flight conditions (i.e. lift = the weight of the plane = model + batteries + propeller). Consequently, after setting the tunnel to the test velocity, power to the motor was increased such that the propeller thrust equaled the aircraft drag (i.e. the drag measured by the balance was zero). Power, voltage and current were monitored at regular intervals. To maintain steady flight conditions, testing was performed at constant required power (model drag multiplied by the wind-tunnel velocity). During the course of a run, battery voltage drop necessitated a continual increase in current supplied to the motor to maintain power (otherwise drag indicated by the balance would increase from zero). Consequently, the throttle controlling the motor was continuously adjusted as needed to maintain zero indicated drag. A run was considered complete when the maintenance of constant power was not feasible. Although this is not indicative of the battery running out of capacity, as it could still discharge at lower voltage, steady level flight would not be achievable at the set flight speed. For most runs, an uncertainty of approximately 30s (0·008hrs) is indicated and is reflective of time to set the tunnel speed and bring the propeller to the required thrust level.

To provide context for the results, some background pertinent to the batteries used in this study is germane. At this time, essentially all battery powered UAVs will use a variant of a Lithium-Ion (Li-Ion) battery; consequently their characteristics will be elucidated.

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3.0 BATTERY BEHAVIOUR

Invented by Sony in the 1990s, Lithium-Ion batteries have replaced Nickel-Metal-Hydride (NiMH) and Nickel Cadmium (Ni-Cad) batteries in most applications requiring high power density. Although improper use can lead to safety concerns, Li-Ion batteries have numerous advantages over other batteries. They have low self discharge (especially if stored in a low-charge state at low temperature), are maintenance free, have no memory effects and have amongst the highest energy density (approximately twice that of a Nickel-Cadmium battery). They also have a relatively flat discharge such that their voltage does not drop significantly during a large part of the discharge cycle. Li-Ion batteries have competitive charge/discharge cycle life (similar to NiMH, approximately 500). Discharging the batteries at higher ambient temperature tends to increase their discharged capacity, while low temperatures has the opposite effect.

Data indicates that Li-Ion battery capacity drops in a linear fashion during a discharge cycle[1]. Batteries loose apparent capacity with cycling from two effects; one is a loss of active material in the cell, the other is due to an increase in resistance in the cell. With fixed charge and discharge limits, cells end up being undercharged and under-discharged due to the increased internal resistance. The apparent loss of capacity due to the increase in cell resistance may be recovered by charging to a higher cell voltage and discharging to a lower cut-off voltage, however this may not be practical if a set power is needed.

For most batteries, their rate of discharge impacts their effective capacity. Consequently, the higher the current drain, the less the effective capacity that may be expected from the battery (the cells heat up wasting energy). This is quantified in the so called Peukert effect. The capacity of a battery is typically quoted in amp hours, A.hrs, or mA.hrs. Thus a battery with a capacity of 2A.hrs would appear to be capable of supplying a current of 2 amps for one hour (battery capacity is typically based upon a 1 hour discharge for portable batteries). If the required current were 4A, the battery would not be capable of supplying this output for ½ hour, a consequence of the so called Peukert effect[5,6]. In essence, the higher the current draw, the less the effective battery capacity. Similarly, if the current draw was 1A, the battery may show an increase in effective capacity over 2A.hrs. For lead-acid batteries the Peukert effect is pronounced.

4.0 RESULTS AND DISCUSSION

4.1 Review of theoretical methodology

The theoretical method outlined in Ref. 3 will be summarised. This reference should be consulted for clarity if required. Peukert's equation, modified for discharge rates other than 1A is given by:

$$t = \frac{Rt}{i^n}\left(\frac{C}{Rt}\right)^n \qquad \ldots (1)$$

where Rt for portable batteries has a value of 1hr. The power required for an aircraft to remain in steady level flight may be extracted from the drag polar (power equals drag multiplied by velocity), which after dimensionalisation yields

$$P_{req} = \frac{1}{2}\rho U^3 S C_{Do} + \frac{2W^2 k}{\rho U S} \qquad \ldots (2)$$

[1] Lithium Ion Rechargeable Batteries Technical Handbook, Sony Corporation, [www.sony.com.cn/products/ed/battery/download.pdf], accessed 18th Dec. 2012.

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The power output of the battery is given by;

$$P_{elec} = Vi \quad \ldots (3)$$

Substituting Equation (1) into (3) for i, and then noting that the electrical power multiplied by an efficiency constant (reflective of the propeller, speed controller and motor[7]) must equal the required or mechanical power (Equation (2)) yields the endurance of the aircraft as;

$$E = t = Rt^{1-n}\left[\frac{C\eta_{tot}V}{\frac{1}{2}\rho U^3 SC_{Do} + \frac{2W^2k}{\rho US}}\right]^n \quad \ldots (4)$$

The range is simply given by;

$$R = E \times U \quad \ldots (5)$$

Thus for any flight condition, the range and endurance may be estimated. Note that these equations do not represent a maximisation of either of these parameters[3] but are relevant for the analysis in this article. An issue with Equation (4) is the assumption that the battery voltage (V), typically assumed as V_o or the initial battery voltage is invariant. This is not correct. Consequently, a method is presented in Ref (3) that implements a linear voltage drop to a user specified cut-off voltage (generally when power can no longer be maintained for steady level flight at a prescribed U). The Peukert constant n, would be known for a particular battery type or established from experiment. The method is summarised below[3]:

1. Calculate the Preq for flight, Equation (2).
2. For this required power, calculate the required battery supply current, i_o at the rated full capacity battery supply voltage V_o:

$$i_o = \frac{P_{req}}{V_o\eta_{tot}} \quad \ldots (6)$$

3. For this i_o, calculate the effective initial battery capacity, C_o (given by Equation (1) multiplied by i_o):

$$C_o = i_o^{1-n}Rt^{1-n}C^n \quad \ldots (7)$$

   where C is the manufacturer's rated battery capacity for the specified discharge time Rt. Rt has a value of 1hr.
4. The following equation set is then marched in time, using specified time increments of Δt, and incrementing time using $j = 0,1,2,3..$ to N with time = $t = j\Delta t$.
5. The estimated battery voltage drop during discharge may be estimated from the manufacturer's data, or may be determined experimentally. Most constant load discharges show a small linear reduction in voltage as the battery discharges. This may be simulated using;

$$V_{j+1}(t) = V_o - k1[C_o - C_j(t)] \quad \ldots (8)$$

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$C_j(t)$ is the battery's effective capacity at each time step and is equal to C_o for $j = 0$. The parameter k1 is effectively the slope of the linear curve fit represented by Equation (8) and can be adjusted to give a desired voltage drop. k1 represents the slope of a terminal point curve fit of the experimental or manufacturers battery data.

6. As the voltage is dropping with time, this requires the battery supply current to increase to maintain the same power. The required current may be found using;

$$i_{j+1}(t) = \frac{P_{req}}{V_{j+1}(t)\eta_{tot}} \quad \ldots (9)$$

7. The remaining battery capacity, C_{j+1}, may then be estimated using Ampere-hour integration.

$$C_{j+1}(t) = i_{j+1}(t)^{1-n} Rt^{1-n} C^n - \sum_{m=1}^{j+1} i_m(t)\Delta t \quad \ldots (10)$$

This relation gives the effective battery capacity based on the current draw at this time step less the total capacity that has been discharged. During each iteration, the value for $C_{j+1}(t)$ is then substituted for $C_j(t)$ in Equation (8).

8. Steps 4 through 7 are repeated until $C_{j+1}(t) \approx 0$, indicating that the battery is discharged, or until a user specified remaining capacity is reached. At this iteration the endurance E, is given by $E = j\Delta t$. To calculate the range, the endurance is simply multiplied by the flight velocity U.

4.2 Results

Figure 1 contains a summary of longitudinal aerodynamic characteristics. The low maximum lift coefficient achieved is not atypical considering the Reynolds number, in conjunction with a somewhat arbitrary (considering the method of construction) airfoil section. The drag polar is reflective of moderate camber such that the zero lift drag coefficient is close to the minimum drag coefficient. However, for consistency, the drag due to lift parameter, as required in the formulation in Ref. 3, was extracted from the linearised polar in an approach common for cambered sections (i.e. plotted as a function of $(C_L - C_{Lmd})^2$). The slope of this polar, $k = 0{\cdot}0697$, yields an Oswald efficiency factor of 0·84. The variation of the zero lift drag coefficient indicated little variation for Re > 100,000 ($10 ms^{-1}$), for the Re range evaluated, lower plot of Fig. 1. For the tests conducted in this study, a value of $C_{Do} = 0{\cdot}0412$ was used.

Figure 2 presents a summary of the measured voltage and current from the analyser, as well as the set electrical power required to achieve steady level flight. Also included is a summary plot showing common nomenclature. The discharge curves are typical for Lithium-Ion batteries and show a fairly flat mid-section discharge. The lowest included voltage in the plots is that at which required power could no longer be maintained and generally corresponds to the typical 'knee' in this type of curve. As may be seen, the reduction in voltage with battery discharge necessitates an increase in current to maintain power. This also leads to an increase in battery temperature and reduction in efficiency (due to heating losses). Also included on the plots are the 'linearised' voltage estimates associated with the numerical implementation of the method in Ref. 3, see Equation (8). The voltage was adjusted by setting the value of k1 such that the numerically estimated terminal voltage corresponded to the last measured voltage (the cut-off voltage). The discharge curves were analysed to extract the Peukert constant (n) for each test. This was accomplished in two ways. The average current was calculated during the discharge and was then substituted into

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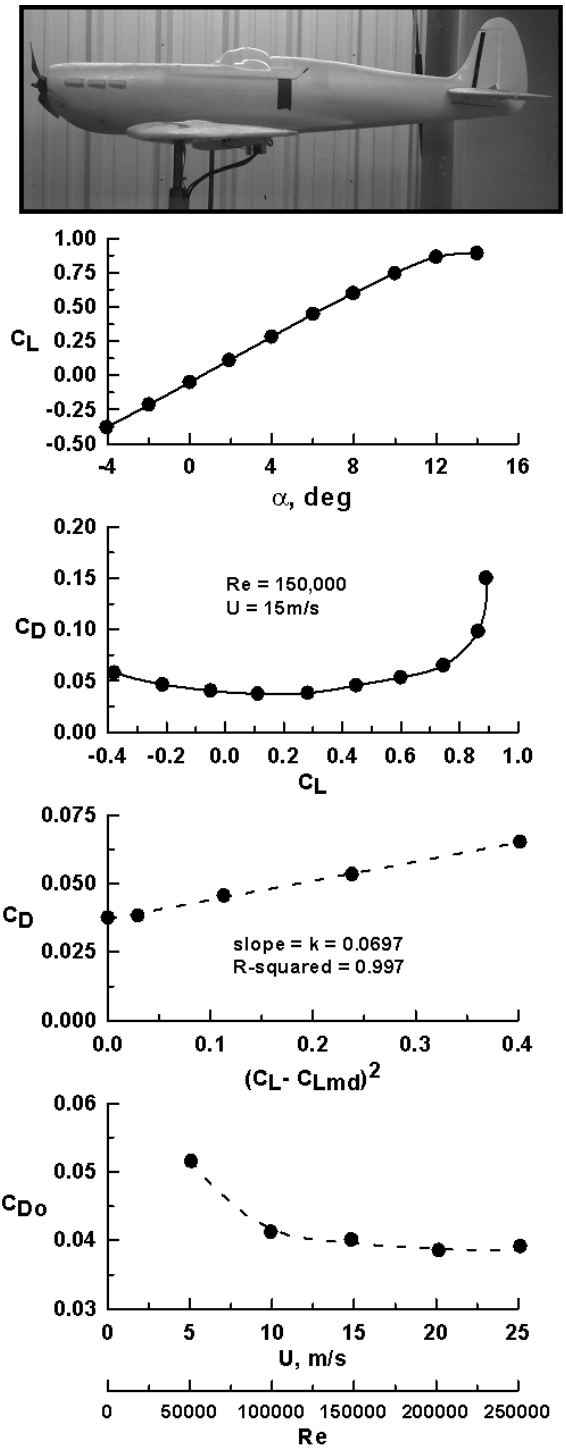


Figure 1. Summary of Spitfire aerodynamic characteristics, Re = 150,000 (U = 15ms^{-1}) for upper three figures. Inset shows an image of the model.

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$$n = \frac{\ln(E / Rt)}{\ln(C / [\bar{i}Rt])} \quad \ldots (11)$$

where C is the rated capacity (either 0·5 or 1A.hrs), E is the measured endurance (hrs) and $\bar{i}$ is the average current during the discharge. n was also estimated using Equation (10) with $C_{j+1} \approx 0$, i_{j+1} = last measured current reading and C = rated battery capacity (0·5 or 1A.hrs). In this instance n was adjusted such that the capacity tended to zero at the last measured endurance reading ('n, num' in Table 1). A summary of these and other calculations are presented in Table 1 and 2. As shown, agreement is excellent between the two approaches. However, what is more significant is that n is similar to that for lead-acid batteries and shows variability amongst the tests. As presented in Figure 2, and Table 2, observe the behaviour of Test 3 and 5. Both tests were ostensibly for the same case, however, the measured endurance values were 0·466hrs and 0·375hrs, a difference of 24%. The unnaturally large value of n for Test 5 would indicate that this case may be in doubt. However, there was no difference in the testing procedure. This outcome is mentioned to highlight the variability in the battery itself during charge/discharge. Temperature sensitivity during cycling, charge history, etc, may prove to be the largest source of error in endurance estimates. This is reflected in Table 1 where the value of n is seen to be variable. In the present tests, the current draw was fairly high (approximately 1·6 to 3A) compared to the C rating of the batteries, which likely had an impact on n. The battery selection was an expedient to facilitate the testing in a timely fashion, considering the 2-3 hours required for each battery recharge after a test. In addition, it was also desired to limit the duration of a test to mitigate any offset shift effects that may be present with the Pyramidal balance. Nonetheless, the current tests are germane and could represent a battery-motor implementation for a small UAV.

Table 1
Summary of Inputs

Parameter	Test					
	1	**2**	**3**	**4**	**5**	**6**
C (A.hrs)	1	0·5	1	0·5	1	0·5
V_o (Volts)	11·75	11·18	12	12·05	12	12·08
P_{elec}(W)	32·24	35·2	20·45	19·5	22·1	18·4
P_{req}(W)	12·11	11·53	6·69	6·13	6·69	6·13
Weight (N)	4·5	4·18	4·5	4·18	4·5	4·18
η_{tot}	0·376	0·327	0·327	0·314	0·303	0·333
C_{Do}	0·0412	0·0412	0·0412	0·0412	0·0412	0·0412
k	0·0697	0·0697	0·0697	0·0697	0·0697	0·0697
U (ms^{-1})	15·5	15·3	12	11·7	12	11·7
n, Eq. (11)	1·209	1·142	1·218	1·153	1·392	1·231
n, num	1·215	1·159	1·216	1·152	1·335	1·238

Table 2 presents a summary of endurance estimates using both theoretical approaches; Equation (4) and the numerical procedure outlined above, i.e. 'E_{th}(num)'. The experimentally measured endurance is denoted by 'E_{expt}' and is simply the duration of the test. Endurance estimates using Equation (4) based on the initial voltage V_o (i.e. '$E_{th}(V_o)$') are seen to be approximately 10-14%

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high compared to experiment. Using the average of the voltage during the discharge (i.e. '$E_{th}(\bar{V})$') cycle $\bar{V}$, in Equation (4) reduces the error to approximately 3%. The numerical procedure yielding 'E_{th}(num)', as seen in Table 2, also provides estimates of endurance within 3% of that measured. What the analysis implies is that the method of Ref. 3 is capable of providing endurance (and consequently range) estimates suitable for conceptual/preliminary design, but this is predicated upon accurate knowledge of the behaviour of the batteries (i.e. their start and cut-off voltage, effects of cycling, temperature, etc, on n). Note that the efficiency η_{tot}, was calculated as the ratio of the mechanical or required power, Equation (2), divided by the electrical power measured using the analyser.

Table 2
Summary of results

Parameter	Test					
	1	**2**	**3**	**4**	**5**	**6**
E_{expt}(hrs)	0·267	0·117	0·466	0·229	0·375	0·225
n, num	1·215	1·159	1·216	1·152	1·335	1·238
E_{th}(num) (hrs)	0·275	0·116	0·468	0·229	0·383	0·221
$E_{th}(V_o)$ (hrs) Equation (4)	0·295	0·122	0·523	0·258	0·443	0·252
($\bar{V}$) (Volts)	10·73	10·45	10·93	11·00	11·05	11·06
$E_{th}(\bar{V})$ (hrs) Equation (4)	0·264	0·113	0·467	0·232	0·396	0·226

Table 3
Estimated Endurance, n = 1·05

Parameter	Test					
	1	**2**	**3**	**4**	**5**	**6**
E_{expt} (hrs)	0·267	0·117	0·466	0·229	0·375	0·225
$E_{th}(V_o)$ (hrs) Equation (4)	0·347	0·145	0·571	0·291	0·527	0·310
$E_{th}(\bar{V})$ (hrs) Equation (4)	0·315	0·135	0·518	0·265	0·483	0·283

Table 4
Estimated Endurance, n = 1·22

Parameter	Test					
	1	**2**	**3**	**4**	**5**	**6**
E_{expt} (hrs)	0·267	0·117	0·466	0·229	0·375	0·225
$E_{th}(V_o)$ (hrs) Equation (4)	0·292	0·106	0·522	0·239	0·475	0·257
$E_{th}(\bar{V})$ (hrs) Equation (4)	0·261	0·098	0·466	0·214	0·429	0·231

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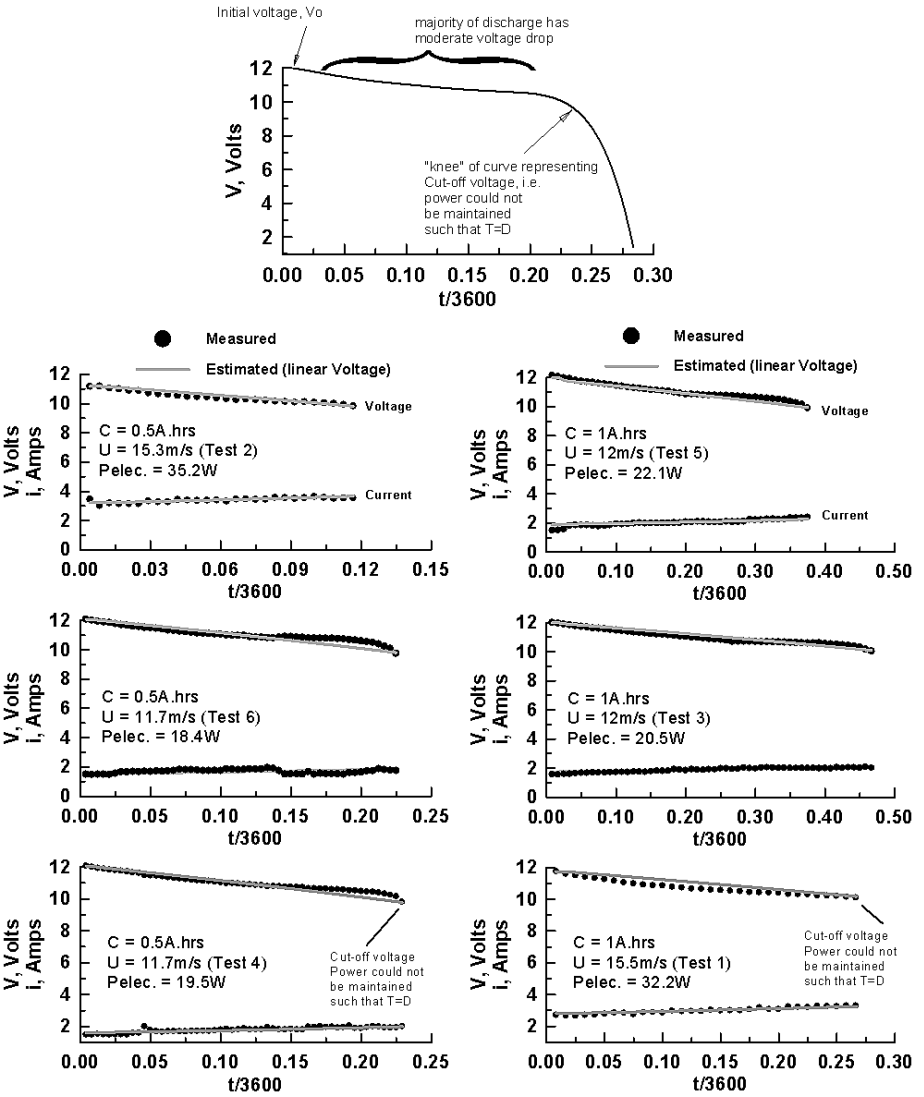


Figure 2. Measurements of battery voltage and current during discharge.

A value of n = 1·05 for the Peukert constant for Li-Ion batteries has been suggested[8]. Consequently, this value was utilised for estimates using both the initial voltage, V_o, and the average voltage, $\bar{V}$, using Equation (4). The results are summarised in Table 3. As may be seen, the endurance is significantly over-estimated, suggesting that n for these batteries was greater than 1·05. The average value of n for the six tests (= 1·22) was subsequently used to estimate the endurance using both the initial voltage (V_o) and the average over a run ($\bar{V}$). The results are presented in Table 4. Agreement is seen to be adequate for an engineering approximation based upon V_o and improve when using the average voltage.

To be employed as an engineering method, the present results suggest that the battery initial and cut-off voltages need to be known, as well as n. This data may be established from manufacturer

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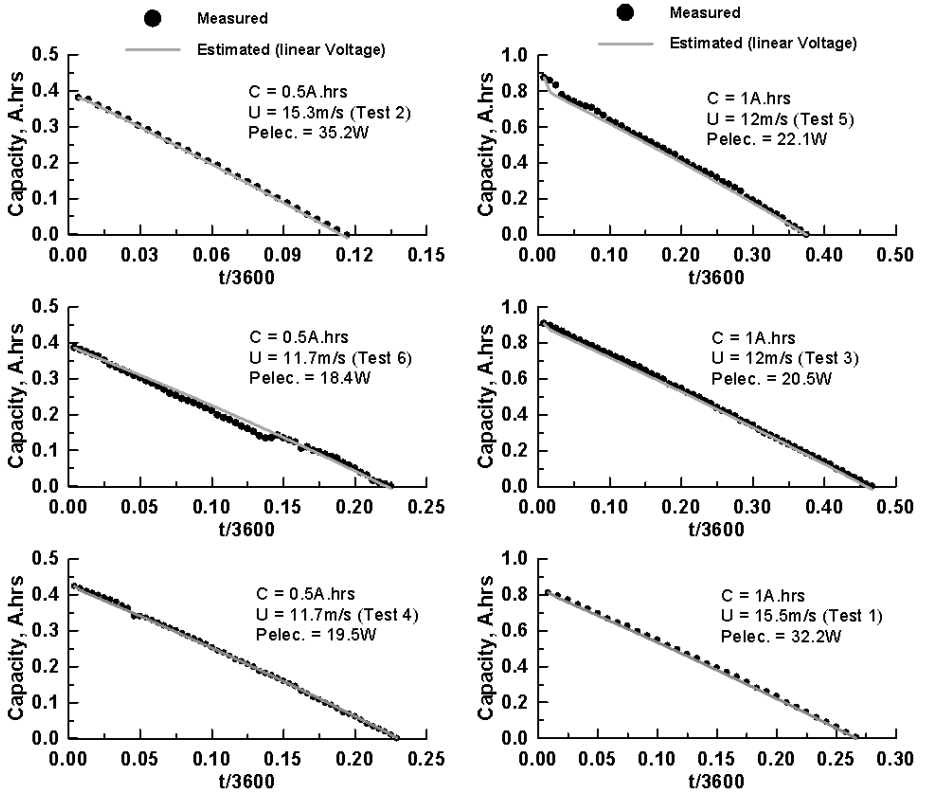


Figure 3. Effect of linear voltage reduction assumption on the predicted battery capacity during discharge.

battery performance curves or through experiment. Once endurance has been estimated, the corresponding range is established as the endurance multiplied by the flight velocity.

The linear voltage drop assumption is examined in Fig. 3. The net outcome of this assumption is its impact on the predicted remaining battery capacity which defines the endurance (i.e. when the capacity → 0). In the figure, the drop in battery capacity is presented based upon the experimental data using the measured voltage and that estimated using Equation (8). As seen, although the linear voltage drop assumption may not be representative, Fig. 2, it has little impact on the endurance estimates. The presentation of the capacity plots may be atypical from that seen in the electrical engineering community where voltage is plotted verse discharged capacity. However, within the framework of the present analysis, the time required to discharge the battery capacity available to maintain steady level flight is relevant.

5.0 CONCLUSIONS

A study is presented to validate theoretical endurance estimate relations for battery powered aircraft. The theoretical method was developed in a prior article but remained unsubstantiated. A carefully conducted experimental study was performed using a radio controlled aircraft model that was tested

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in a wind tunnel at steady level flight conditions. During testing, battery discharge characteristics were monitored. Using the experimental data to define the battery characteristics indicated that the theoretical prediction method is capable of estimates within 3% of the measured endurance. The largest source of uncertainty is in the batteries themselves, where fluctuations during cycling yielded significant variance in their effective capacity and discharge characteristics. Provided the battery initial and terminal voltage are known, along with its Peukert constant, as well as the efficiency of the propulsion system, endurance can be accurately estimated.

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