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Breather structures in degenerate relativistic non-extensive plasma

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We examine the excitation of breather structures in a degenerate relativistic plasma consisting of non-extensive electrons and cold ions. For this purpose, the multiple time scale perturbation technique is used to obtain a nonlinear Schrödinger equation (NLSE). We then consider different localized solutions regarding analytical breather solutions of the NLSE, and examine their properties in the frame of the present plasma system, i.e. a degenerate relativistic non-extensive plasma. The results of the present investigation may be useful for the understanding of the basic features of the nonlinear excitations that may occur in dense astrophysical plasmas.

Key words: plasma nonlinear phenomena, plasma waves, space plasma physics

1. Introduction

Since Tsallis (1988) proposed the generalization of the Boltzmann–Gibbs–Shanon (BGS) entropy, by adopting a thermo-statistical theory in a way that it becomes non-additive, a great deal of attention has been paid to the Tsallis statistics (Silva, Plastino & Lima 1998; Tsallis 2001; Gell-Mann & Tsallis 2004; Martinenko & Shivamoggi 2004; Dubinova & Dubinov 2006; Tribeche, Djebarni & Amour 2010; El-Awady & Moslem 2011; Alinejad & Shahmansouri 2012; El-Taibany & Tribeche 2012; Shahmansouri & Tribeche 2012; Akhtar, El-Taibany & Mahmood 2013; Bains, Li & Tribeche 2013; Shahmansouri & Tribeche 2013; Shahmansouri & Alinejad 2013*a*,*b*; Ashraf *et al.* 2014; El-Shamy, Tribeche & El-Taibany 2014; Ourabah & Tribeche 2014; Rahman & Ali 2014; Akhtar *et al.* 2015; Behery, Selim & El-Taibany 2015; Ferdousi *et al.* 2015; Saini & Misra 2015; Saha & Chatterjee 2015*a*,*b*; Dutta, Das & Karmakar 2016; El-Tantawy 2016; Hossen, Ema & Mamun 2016; Bains *et al.* 2017; Saha 2017). In Tsallis statistics, the thermo-statistical theory has been introduced on a non-extensive entropy in the following form $S_q = k_B(1-\sum_i p_i^q)/(q-1)$, where k_B is the Boltzmann constant, p_i is the probability of the ith microstate and the real parameter q characterizes the degree of non-extensivity of the considered system as the entropic index.

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The crucial property of the Tsallis entropy comes from its non-additivity, which leads us to the fact that the entropy of a system composed of two independent subsystems, A and B, for the limit of $q \neq 1$ is not the sum of the individual entropies but it is given by $S_q(A+B) = S_q(A) + S_q(B) + (1-q)S_q(A)S_q(B)$, while the standard extensive BGS entropy is recovered for $q \to 1$.

It has been shown that non-extensive statistical mechanics is a suitable approach to describe complex systems (such as those with long-range interactions, long time memory or even systems evolving in a fractal space–time) whose properties cannot be exactly described by BGS statistics (Tsallis 2001; Gell-Mann & Tsallis 2004).

During recent years, theoretical (Plastino & Lima 1998; Silva *et al.* 1998) and experimental (Douglas, Bergamini & Renzoni 2006; Liu & Goree 2008; Pickup *et al.* 2009) efforts have shown that the Tsallis entropy possibly yields a q-deformed distribution function. In the non-extensive description, the one-dimensional equilibrium distribution function for a Fermi–Dirac plasma is given by Martinenko & Shivamoggi (1998), (Tsallis, Mendes & Plastino 1998; Ourabah & Tribeche 2014)

$$f^{(q)}(E) = \frac{1}{1 + [1 + \beta(q-1)(E-\mu)]^{q/(q-1)}}, \quad \text{with } 1 + \beta(q-1)(E-\mu) > 0 \quad (1.1)$$

in which μ is the chemical potential and β is the inverse temperature in energy units. In the limit $q \to 1$, one can obtain the familiar Fermi–Dirac distribution function from (1.1). The non-extensive plasma has received a great deal of attention due to its vital role in the description of basic phenomena in space plasma situations such as stellar polytropes, the solar neutrino deficit, hadronic matter and quark–gluon plasmas, stellar polytropes, dark-matter halos, protoneutron stars, etc. (Plastino & Plastino 1993; Tsallis, Prato & Plastino 2004; Feron & Hjorth 2008; Gervino, Lavagno & Pigato 2012).

Development in the study of nonlinear phenomena appears to be accelerating with a number of experiments and computer simulations demonstrating the basic physical phenomena of localized structures. Among the nonlinear structures, rogue waves as a short-lived nonlinear phenomenon occur spontaneously in many physical situations. The average height of these waves is a few times larger than the surrounding waves (Kharif, Pelinovsky & Slunyaev 2009; Ruban, Kodama & Ruderman 2010). It has been found that the rogue waves may form in optical systems (Shukla, Kourakis & Eliasson 2006; Solli *et al.* 2007; Eliasson & Shukla 2010; Veldes *et al.* 2013), Bose–Einstein condensates (Bludov, Konotop & Akhmediev 2009, 2010), atmospheric physics and plasma physics (Marklund & Stenflo 2009; Stenflo & Marklund 2010; El-Awady & Moslem 2011; Moslem 2011; Moslem, Shukla & Eliasson 2011; Bacha, Boukhalfa & Tribeche 2012).

It is found that the linear wave approach is inadequate to describe the rogue waves (Osborne 2009). One of the possible mechanisms for the rogue wave generation in the nonlinear theory is modulational instability (MI) (Benjamin & Feir 1967). Such a process can be investigated in the theoretical framework of weakly nonlinear models such as the nonlinear Schrödinger (NLS) equation, the Korteveg de-Vries (K-dV) equation the and Kadomtsev–Petviashvili (KP) equation (Shahmansouri & Borhanian 2013; Shahmansouri & Alinejad 2015), etc. Recently, Kourakis, McKerr & Ur-Rahman (2013) adopted the semi-classical relativistic fluid theory to investigate the modulational instability and rogue waves for the electrostatic envelope modes in dense electron–positron–ion (e–p–i) plasmas.

In order to clarify the importance of degeneracy pressure, let us consider the following description. In a usual fermion gas, in which the thermal effects are

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dominant, most of the available electron energy levels are unfilled, such that the electrons are free to move to these states. By increasing the particle density, electrons fill the energy levels from the lower energy states, and the other electrons have to occupy the states of higher energy even at low temperatures. In a degenerate gas, the electrons cannot move to already filled lower energy levels (due to the Pauli exclusion principle). In this case, electrons cannot give up their energy by transition to the lower energy levels, thus no thermal energy can be extracted. The Pauli principle implies a sort of pressure that keeps fermions separated. Therefore, the pressure of Fermi gas is non-zero even at zero temperature (against a classical ideal gas). This effect is called the degeneracy pressure (Dyson & Lenard 1967). Indeed the Pauli exclusion principle, through the creation of an emergent pressure, disallows compression of matter into smaller volumes of space. The electron degeneracy pressure results from the same underlying mechanism and plays a similar role in disallowing the compression of matter. Thus, the electron degeneracy pressure prevents stars from collapsing under their own weight once nuclear fusion has ceased. In the case of sufficiently large stars, the electron degeneracy pressure is not sufficient to prevent the collapse of a star and a neutron star can be formed. Consequently, since degenerate plasmas are present in many physical situations, such as in compact dense astrophysical objects (Shapiro & Teukolsky 1983) and in ordinary metals and semiconductors (Haug & Koch 2004), it is of interest to investigate such degenerate plasma systems in terms of non-extensive statistics. Thus, the aim of the present study is investigation of IA breather structures in a degenerate relativistic non-extensive plasma based on the relativistic formalism of the non-extensive Thomas–Fermi plasma proposed by Ourabah & Tribeche (2014). Here, three types of rogue waves are studied, namely Peregrine's soliton, Akhmediev's breather and the Kuznetsov–Ma breather.

The present paper is organized as follows: the basic equations governing our plasma model are presented in the next section. The modulational instability of ion acoustic waves are investigated in § 3. The localized excitation solutions are considered in § 4 and a brief discussion is provided in § 5.

2. Non-extensive Thomas–Fermi model

First let us recall some basic aspects of the non-extensive Thomas–Fermi (TF) model. According to the TF model, electrons obey the Fermi–Dirac statistics, and as a result of the Pauli exclusion principle, electrons occupy phase space with two electrons per h^3 of this space (with h being the Planck's constant). In such a system, we study the propagation properties of ion acoustic (IA) waves in a two component non-extensive TF plasma. The basic equations governing the dynamics of one-dimensional IA oscillations are as follows

$$\frac{\partial n_i}{\partial t} + \frac{\partial (n_i u_i)}{\partial x} = 0, \tag{2.1}$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} + \frac{\partial \phi}{\partial x} = 0, \tag{2.2}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_e - n_i, \tag{2.3}$$

where n_i, n_e, u_i and ϕ are, respectively, the ion number density, the electron number density, the ion fluid velocity and the electrostatic potential. Here, electrons are treated

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as a degenerate ensemble as we adopt the relativistic non-extensive number density (Ourabah & Tribeche 2014),

$$n_e = \frac{(mc)^3}{3\pi^2\hbar^3} g^3 \left[1 + 3\frac{I_1^{(q)}}{\beta mc^2}\frac{\sqrt{1+g^2}}{g^2} + 3\frac{I_2^{(q)}}{(\beta mc^2)^2}\frac{1+2g^2}{g^4} + \cdots\right], \tag{2.4}$$

in which

$$I_n^{(q)} = q\int_{-\infty}^{\infty} \frac{z^n(1+(q-1)z)^{1/(q-1)}}{1+(1+(q-1)z)^{q/(q-1)}}\,\mathrm{d}z, \tag{2.5a}$$

$$g = \sqrt{\left(\frac{\mu + mc^2 + e\phi}{mc^2}\right)^2 - 1}, \tag{2.5b}$$

where m is the electron mass, c is the light speed, and h is $h/2^*pi$.

The numerical values for the first two integrals $I_n^{(q)}$ have been estimated numerically by Martinenko and Shivamoggi (1998). We assume that the system does not deviate too far from equilibrium, thus the normalized electron number density can be approximated by the Taylor expansion, as follows

$$n_e = 1 + \varepsilon n_e^{(1)} + \varepsilon^2 n_e^{(2)} + \cdots, \tag{2.6}$$

where the coefficients are calculated to be

$$n_e^{(1)} = a_1 g_1, \quad n_e^{(2)} = a_1 g_2 + a_2 g_1^2, \tag{2.7a,b}$$

$$g_0 = \frac{\sqrt{\mu^2 + 2\mu mc^2}}{mc^2}, \quad g_1 = b_1\phi^{(1)}, \quad g_2 = b_1\phi^{(2)} + b_2(\phi^{(1)})^2, \tag{2.8a–c}$$

$$a_1 = g_0^{-1}\left[3 + \frac{3I_1^{(q)}}{\beta mc^2}\frac{\sqrt{1+g_0^2}}{g_0^2}\frac{1+2g_0^2}{1+g_0^2} + \frac{3I_2^{(q)}}{(\beta mc^2)^2}\frac{1+2g_0^2}{g_0^4}\frac{-1+2g_0^2}{1+2g_0^2}\right] \Bigg/ \left[1 + \frac{3I_1^{(q)}}{\beta mc^2}\frac{\sqrt{1+g_0^2}}{g_0^2} + \frac{3I_2^{(q)}}{(\beta mc^2)^2}\frac{1+2g_0^2}{g_0^4}\right], \tag{2.9}$$

$$a_2 = g_0^{-2}\left[3 + \frac{3I_1^{(q)}}{\beta mc^2}\frac{\sqrt{1+g_0^2}}{g_0^2}\left(3 - \frac{(9+10g_0^2)g_0^2}{2(1+g_0^2)^2}\right) + \frac{3I_2^{(q)}}{(\beta mc^2)^2}\frac{1+2g_0^2}{g_0^4}\frac{-9+10g_0^2}{1+2g_0^2}\right] \Bigg/ \left[1 + \frac{3I_1^{(q)}}{\beta mc^2}\frac{\sqrt{1+g_0^2}}{g_0^2} + \frac{3I_2^{(q)}}{(\beta mc^2)^2}\frac{1+2g_0^2}{g_0^4}\right], \tag{2.10}$$

$$b_1 = \frac{(\mu + mc^2)k_B T_e}{mc^2\sqrt{\mu^2 + 2\mu mc^2}}, \quad b_2 = -\frac{mc^2(k_B T_e)^2}{2(\mu^2 + 2\mu mc^2)^{3/2}}. \tag{2.11a,b}$$

The normalization has been made by the following non-dimensional variables: $x \to x\omega_{pi}/C_s$, $t \to t\omega_{p_i}$, $u_i \to u_i/C_s$, $\phi \to e\phi/2E_{Fe}$, and $n_j \to n_j/n_0$, where

$$n_0 = \frac{(mc)^3}{3\pi^2\hbar^3} g_0^3 \left[1 + \frac{3I_1^{(q)}}{\beta mc^2}\frac{\sqrt{1+g_0^2}}{g_0^2} + \frac{3I_2^{(q)}}{(\beta mc^2)^2}\frac{1+2g_0^2}{g_0^4}\right]. \tag{2.12}$$

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Here $\omega_{pi} = \sqrt{n_{i0}e^2/\varepsilon_\circ M}$ is the ion plasma frequency, $\lambda_D = \sqrt{T_e\varepsilon_\circ/n_{i0}e^2}$ is the Debye length. The characteristic speed scale used for the velocity normalization is $C_s = \sqrt{2E_{Fe}/M} = hn_0/4\sqrt{mM}$, where C_s is the ion acoustic speed for a simple un-magnetized plasma.

Note that in the limit $q \to 1$, the number density function (2.4) reduces to the usual Fermi number density. The entropy non-extensivity contribution dominates the usual thermal term in the low temperature extent, which represents the importance of (2.4) in this limit. This difference is more noticeable as the entropic index q increases. It must be also noticed that in the zero temperature ($\mu/T \to \infty$), the non-extensive correction vanishes.

3. Modulational instability

We adopt the standard multiple time scale perturbation technique (Taniutiand & Yajima 1969) to investigate the nonlinear evolution of modulated IA waves in a non-extensive TF plasma. The plane-wave modulation can be considered as the carrier wave with the wavenumber k and frequency ω. By increasing of the carrier wave amplitude, the nonlinear amplitude modulation mechanism generates other harmonics. The state of the system at a given position x and a time t, may be described by an auxiliary quantity $F = (n, u, \phi)$. In the case of modulated wave packet, F is a composition of the fast carrier wave with a slow variation in amplitude. Accordingly, let us consider a small deviation from the equilibrium state, and expand the dynamical variables as

$$F = \sum_{m=1}^{\infty}\sum_{l=0}^{m} \varepsilon^m F_l^{(m)}(\xi, \tau)\exp[\mathrm{i}l(kx - \omega t)] + \text{c.c.}, \tag{3.1}$$

where c.c. refers to the complex conjugate; ω is the frequency and k is the carrier wavenumber of the given IA wave, and ε refers to a real and small parameter which measures the weakness of the dispersion or amplitude. Here we have assumed that the perturbed electrostatic potential depends on the fast scales via the phase $kx - \omega t$ only, while the slow scales enter the argument of the lth harmonic amplitude $F_l^{(m)}$. The reality condition $F_{-l}^{(m)} = F_l^{(m)^*}$ is also met by all state variables. Following the multiple time scale technique, we shall consider the following stretched (slow) space and time variables

$$\xi = \varepsilon(x - v_g t) \quad \text{and} \quad \tau = \varepsilon^2 t, \tag{3.2a,b}$$

where v_g is a characteristics velocity (to be later interpreted as the group velocity).

By substituting (3.1) and (3.2) into the basic equations of the system, and isolating the distinct orders in ε, we obtain the following equations for the first order $m = 1$ with $l = 1$ as

$$n_{i1}^{(1)} = (a_1 b_1 + k^2)\phi_1^{(1)}, \quad -\omega n_{i1}^{(1)} + k v_{i1}^{(1)} = 0, \quad -\omega v_{i1}^{(1)} + k\phi_1^{(1)} = 0. \tag{3.3a–c}$$

The dispersion relation for IA wave can be found from (3.3) as

$$\omega^2 = \frac{k^2}{a_1 b_1 + k^2}. \tag{3.4}$$

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The effect of non-extensivity of the plasma on the dispersion properties of IA waves in the TF plasma is included via the quantities a_1 and b_1 in (3.4). We can obtain the following expression for the group speed from the second order ($m=2$) with the first harmonic ($l=1$)

$$v_g = \frac{ka_1b_1}{\omega(a_1b_1+k^2)^2}. \tag{3.5}$$

The above expression represents the group velocity of IA waves and deals with an envelope wave moving at the group velocity. It is easy to show that $v_g = \partial\omega/\partial k$.

It is known that the zeroth harmonic components appear due to the nonlinear self-interaction of the carrier waves. Thus, the first-order, zeroth harmonic components are obtained from the terms corresponding to ($m=2,\ l=0$)

$$\phi_0^{(2)} = A_1|\phi_1^{(1)}|^2, \tag{3.6a}$$

and ($m=3,\ l=0$)

$$n_{i0}^{(2)} = A_2|\phi_1^{(1)}|^2, \tag{3.6b}$$

$$v_{i0}^{(2)} = A_3|\phi_1^{(1)}|^2, \tag{3.6c}$$

The second harmonic modes ($m=2$) and second ($l=2$) harmonics, can also be obtained as

$$\phi_2^{(2)} = A_4|\phi_1^{(1)}|^2, \tag{3.7a}$$

$$n_{i2}^{(2)} = A_5|\phi_1^{(1)}|^2, \tag{3.7b}$$

$$v_{i2}^{(2)} = A_6|\phi_1^{(1)}|^2, \tag{3.7c}$$

where the coefficients $A_i (i=1\text{–}6)$ are given in appendix A. The above equations show that, the solution (3.1) takes the following form:

$$F = \varepsilon F_1^{(1)} \mathrm{e}^{\mathrm{i}(kx-\omega t)} + \varepsilon^2 (A_j|F_1^{(1)}|^2 + A_{j+3}(F_1^{(1)})^2 \mathrm{e}^{2\mathrm{i}(kx-\omega t)}) + \text{c.c.} \tag{3.8}$$

for $F \equiv \phi$ ($j=1$), n_i ($j=2$) and v_i ($j=3$), respectively.

Finally, at the third order ($m=3$) with the first harmonic ($l=1$), after some manipulation and by using the corresponding compatibility conditions, we obtain

$$\mathrm{i}\frac{\partial\phi_1^{(1)}}{\partial\tau} + P\frac{\partial^2\phi_1^{(1)}}{\partial\xi^2} + Q\phi_1^{(1)}|\phi_1^{(1)}|^2 = 0. \tag{3.9}$$

The aforementioned equation represents the well-known nonlinear Schrodinger (NLS) equation governing the evolution of the amplitude of the first-order electrostatic potential perturbation $\phi_1^{(1)}$. The coefficients in (3.9) are the dispersion coefficient P (which is related to the curvature of the dispersion curve) and the nonlinearity coefficient Q, respectively, given by

$$P = \frac{v_g}{2k} - \frac{v_g^2}{2\omega} - 2c_1\omega\frac{\omega^4}{k^4}, \tag{3.10}$$

$$Q = \frac{\omega^3}{2k^2}\left[3c_3 + 2c_2(A_1+A_4) - \frac{2k^3}{\omega^3}(A_3+A_6) - \frac{k^2}{\omega^2}(A_2+A_5)\right]. \tag{3.11}$$

where $c_1 = a_1b_1,\ c_2 = a_1b_2 + a_2b_1^2,\ c_3 = a_1b_3 + 2a_2b_1b_2 + a_3b_1^3$.

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Recapitulating, the dynamics of the wave fundamental harmonic amplitude $\phi_1^{(1)}$ is governed by the NLS equation (3.9), and is thus essentially the dynamics of the interplay between the dispersion and nonlinearity coefficients. Two physical phenomena that are generally modelled by this formulation are the wave collapse (due to the modulation instability) and the formation of envelope excitations. Without reproducing the whole of the existing theory (El-Labany *et al.* 2003; Dauxois & Peyrard 2005; El-Taibany & Kourakis 2006; Shahmansouri & Farokhi 2012; Alinejad, Mahdavi & Shahmansouri 2014, 2016; El-Labany, El-Taibany & Zedan 2014; Rahman *et al.* 2015; Shahmansouri & Misra 2016), we shall provide the basic information needed to understand in what follows. The dispersion relation for the disturbance of a plane wave depends on the dispersion and nonlinearity coefficients, namely $\Omega^2 = (PK^2)^2(1 - K_c^2/K^2)$, in which K_c is the threshold wavenumber below (above) it the wave is unstable (stable) under the modulation, and is given by $K_c = \sqrt{2Q|\phi_0|^2/P}$; and ψ_0 is the amplitude of perturbation. In this case the disturbance of a plane wave is of the form $\psi_0 e^{iQ\phi_0^2\tau}$. It can be seen that a plane-wave solutions to the periodic perturbation is modulationally stable for $PQ < 0$. On the other hand, the above dispersion relation shows that for $PQ > 0$ the modulation instability sets in for a wavenumber satisfying $K < K_c$, but for $K > K_c$ the wave is stable under the modulation. It must be added that the wavenumbers k and K refer to the carrier wave and an external perturbation, respectively. Also, the growth rate of the instability is given by $\Gamma = |\mathrm{Im}(\Omega(K))|$, that can reached a maximum value at $K_c/\sqrt{2}$, viz. $\Gamma_{max} = Q|\phi_0|^2$.

It appears appropriate here to discuss the non-extensivity effect, which may be traced by investigating the impact of the spectral index q. The product PQ, the threshold wavenumber K_c (below which the instability sets) and also the growth rate of instability are depicted for different values of the spectral index q, in figure 1(*a–c*), respectively. Panel (*a*) indicates the transition from the stable region to the unstable one, through the change in the sign of product. The influence on the instability region is a decrease of the instability region (figure 1*a*). It also shows how the threshold wavenumber K_c changes with the non-extensive index q (figure 1*b*). The growth rate of the modulational instability was also investigated for different values of q in panel (*c*). The growth rate is increased by the non-extensivity and thus the spectral index q may be used as a controlling parameter for the modulational instability.

4. Localized solution of NLS equation

The occurrence of rogue waves (freak waves) associated with electromagnetic pulse propagation interacting with a plasma has been investigated by Veldes *et al.* (2013). They considered a set of non-stationary envelope solutions of the NLS equation (which governs the amplitude of the vector potential) as potential candidates for the modelling of rogue waves in beam–plasma interactions, in the form of the Peregrine soliton, the Akhmediev breather and the Kuznetsov–Ma breather. Going parallel to what has been done in Veldes *et al.* (2013), let us consider different localized solutions regarding analytical breather solutions of the NLS equation (3.9), and examine their properties in the frame of the present plasma system, i.e. a degenerate relativistic non-extensive plasma.

4.1. *Peregrine's soliton*

The Peregrine paradigm (Peregrine 1983; Dysthe & Trulsen 1999; Shrira & Geogjaev 2010) as a prototypical model for rogue waves has recently been employed

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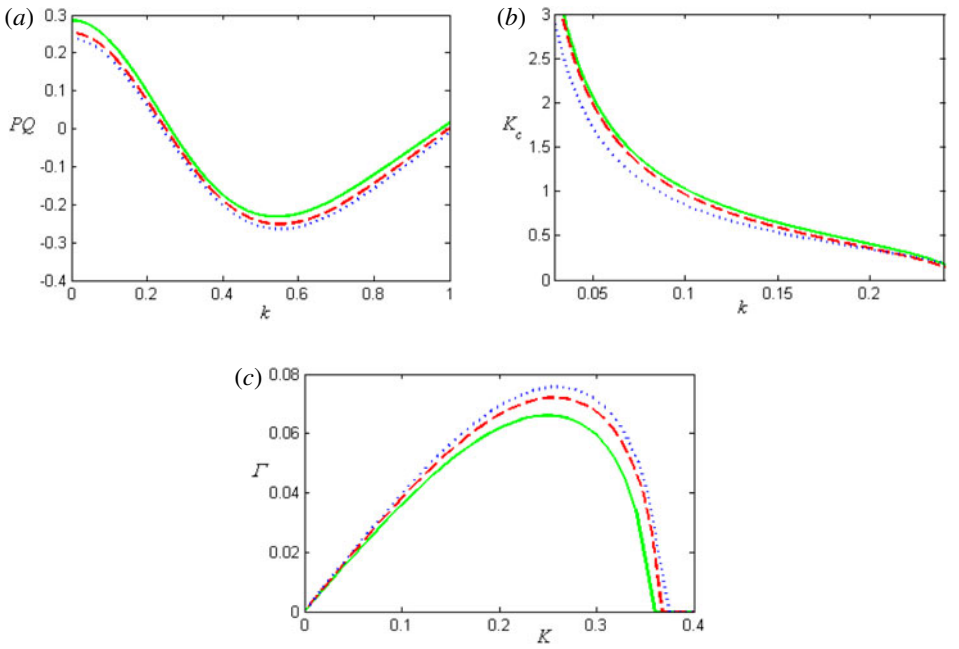


FIGURE 1. Variation of (*a*) the product PQ, (*b*) the threshold wavenumber K_c versus the carrier wavenumber k and also (*c*) behaviour of the modulational instability growth rate Γ versus the perturbation wavenumber K for different values of the spectral index q along with $q = 1.1$ (solid curve), $q = 1.2$ (dashed curve) and $q = 1.5$ (dotted curve).

successfully by Kibler *et al.* (2010) in nonlinear optics. Such a structure first appeared in the literature 30 years ago, and exhibits only a single rogue structure whose amplitude decreases in both time and space. The Peregrine solution has been examined experimentally in wave tank laboratories (Chabchoub, Hoffmann & Akhmediev 2011), in optical fibres (Kibler *et al.* 2010) and also in plasma (Bailung, Sharma & Nakamura 2011) situations. The analytical form of Peregrine's soliton is given by

$$\phi_1^{(1)}(\xi, \tau) = \left[1 - \frac{4 + 8\mathrm{i}Q\tau}{1 + 2Q\xi^2/P + 4Q^2\tau^2}\right] \mathrm{e}^{\mathrm{i}Q\tau}, \tag{4.1}$$

where P and Q refer to the coefficients of dispersive and nonlinear terms in (3.9), respectively. Recall that also the 'slow' variables (ξ, τ) were defined in (3.2). This solution shows a non-trivial behaviour over a small region of ξ and τ, but it decays to a plane-wave asymptotic background for either large ξ or τ.

4.2. *Akhmediev's breather*

The Akhmediev breather (Akhmediev, Eleonskii & Kulagin 1987) as an exact solution of the NLS equation (3.9), describes the modulational instability in its nonlinear regime. The corresponding waveform shows a periodic structure in space but is localized in time, given by

$$\phi_1^{(1)}(\xi, \tau) = \left[1 + \frac{(2 - 4a)\cosh(bQ\tau) + \mathrm{i}b\sinh(bQ\tau)}{\sqrt{2a}\cos(w\sqrt{Q/2P}\xi) - \cosh(bQ\tau)}\right] \mathrm{e}^{\mathrm{i}Q\tau}, \tag{4.2}$$

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in which $w = 2\sqrt{1-2a}$, $b = 2\sqrt{2a(1-2a)}$ and $a \in (0,\ 1/2]$ are real parameters. The perturbation grows in the case of $1 - 2a > 0$, not surprisingly such behaviour corresponds to that obtained by performing a linear stability analysis of a Stokes wave solution of the NLS equation. The above solution (4.2) decays to a Peregrine solution in the limit of large enough spatial periods (Akhmediev *et al.* 1987). Indeed, for the large time limit the Akhmediev breather reduces to a plane wave plus a small perturbation. The absolute maximum of the solution (4.2), can be achieved after some straightforward manipulation, as

$$\phi_{max}^{(A)} = 1 + 2\sqrt{2a}. \tag{4.3}$$

4.3. *Kuznetsov–Ma breather*

The Kuznetsov–Ma breather (Kuznetsov 1977) as one of the breather solutions was originally proposed in by Kuznetsov (1977) and three years later by Ma (1979). Against the Akhmediev breather, for the large time limit the modulation for the Kuznetsov–Ma breather is never small (thus, it does not corresponds to the classical Benjamin–Feir instability). This waveform, in contrast to the Akhmediev breather, exhibits a periodic structure in time but is localized in space and is given by (Ma 1979; Veldes *et al.* 2013)

$$\phi_1^{(1)}(\xi,\tau) = \frac{\cos\left(\dfrac{s'Q\tau}{2} - 2\mathrm{i}\theta\right) - \cosh(\theta)\cosh\left(s\sqrt{\dfrac{Q}{2P}}\xi\right)}{\cos\left(\dfrac{s'Q\tau}{2}\right) - \cosh(\theta)\cosh\left(s\sqrt{\dfrac{Q}{2P}}\xi\right)}\mathrm{e}^{\mathrm{i}Q\tau}, \tag{4.4}$$

in which $s' = 2\sinh(2\theta)$, $s = 2\sinh\theta$ and $\theta \in \mathrm{Re}$. Just like the Akhmediev breather, one can find the absolute maximum of the solution (4.4), which after some straightforward algebra leads to

$$\phi_{max}^{(KM)} = 1 + 2\cosh\theta. \tag{4.5}$$

The analytical expressions (4.1), (4.2) and (4.4) show how the basic features (magnitude and periodicity) of rogue waves depend on the nonlinearity and dispersion coefficients. Recalling the functional dependence of P and Q on plasma parameters, this model allows one to investigate the parametric dependence on the spectral index q. Next, we numerically analyse the breather structures (4.1), (4.2) and (4.4) to investigate the dependences of the formation and characteristics of such localized structures on the spectral index q.

Figure 2 shows that the present plasma system supports excitation of localized structures with different periodicity characteristics in the time and space domains. According to (4.5), in figure 2(*a*), for $\theta = 0.343$ we find that the maximum amplitude corresponds to $\phi_{max}^{(KM)} \cong 3.12$. Also, for $a = 0.25$ the maximum amplitude of the Akhmediev breather corresponds to $\phi_{max}^{(A)} \cong 2.41$.

In order to investigate the effect of the spectral index q on the characteristic properties of such localized structures, we have considered a two-dimensional plot of $\phi_1^{(1)}(\xi,\tau)$ in figure 3 (space extension) and figure 4 (time extension). As clearly shown in figure 3, the increase in spectral index q decreases the extension of the excitation in the space domain in all three cases. On the other hand, an analogous trend occurs

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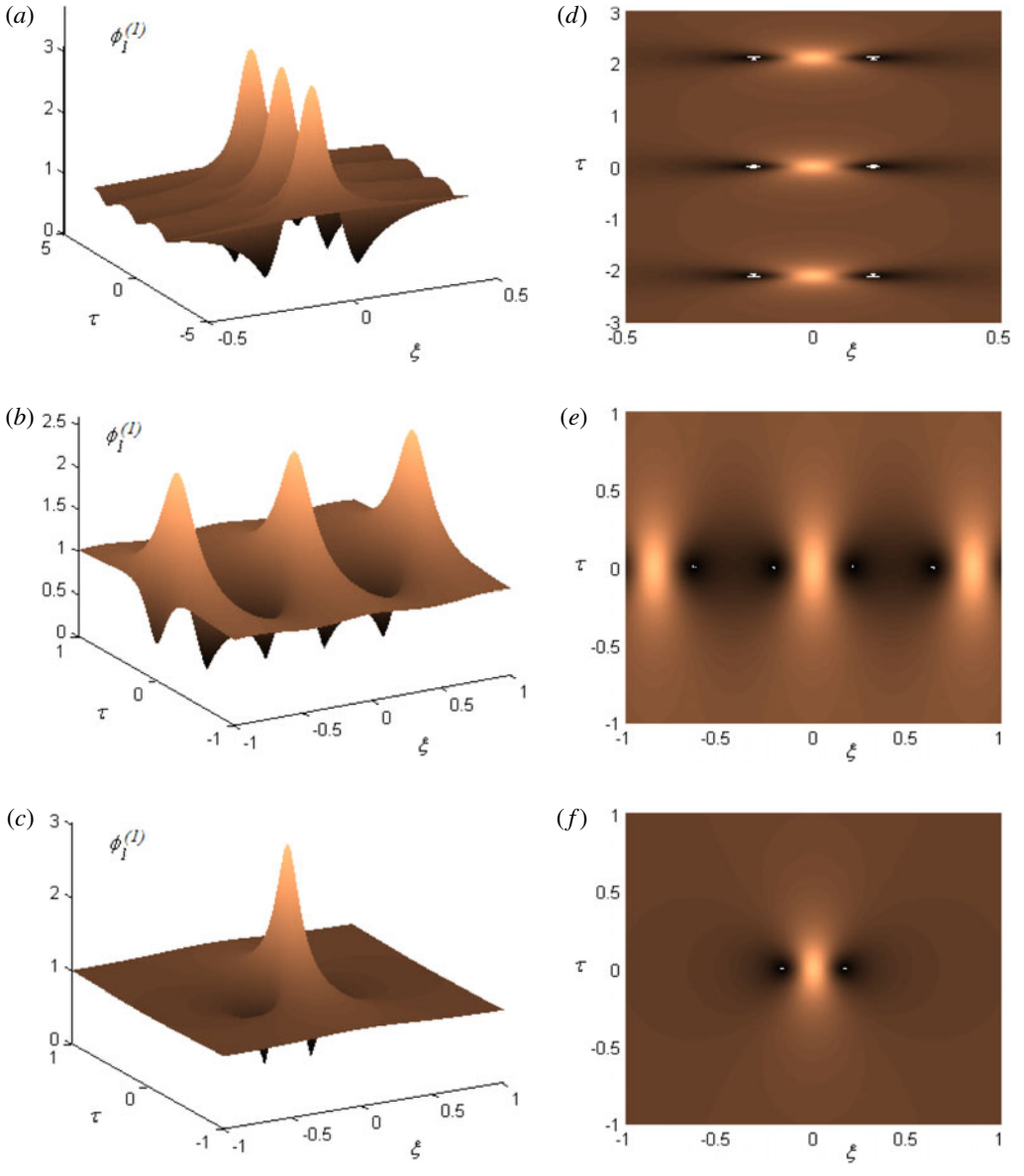


FIGURE 2. Profile of (*a*) Kuznetsov–Ma breather, (*b*) Akhmediev breather and (*c*) Peregrine soliton, and (*d–f*) their corresponding false colour plots, for $q = 1.01$ and $k = 1.5$. In this case we have $Q = -4.05$ and $P = -0.074$.

for the effect of q on the time extension of the excitation structures (see figure 4). This is accompanied by increased time localization. It seems appropriate here to add that the breather solutions are related (Dysthe & Trulsen 1999). It can be seen that the Akhmediev solution transfers to the Kuznetsov–Ma solution by replacing $\sqrt{Q/P}$ by $i\sqrt{Q/P}$. On the other hand, in the limit of small values of $\sqrt{Q/P}$ (i.e. $\sqrt{Q/P} \to 0$)

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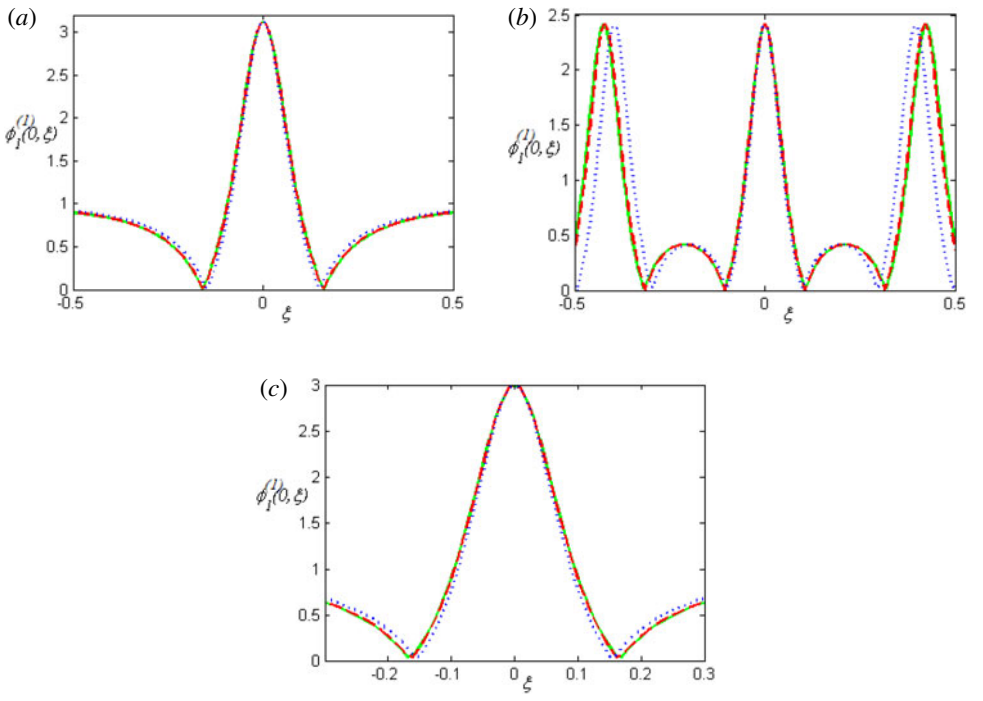


FIGURE 3. The evolution of breather solutions of the NLS (3.11) with respect to the space coordinate ξ, for the different values of the spectral index q along with $q = 1.1$ (solid curve), $q = 1.2$ (dashed curve) and $q = 1.5$ (dotted curve): (*a*) Kuznetsov–Ma breather, (*b*) Akhmediev breather and (*c*) Peregrine soliton. The parameters are: $k = 1.5$, $a = 0.25$ and $\theta = 0.343$.

the Akhmediev and the Kuznetsov–Ma solution reduce to the Peregrine solution. Also, for the large values of $\sqrt{Q/P}$ the Kuznetsov–Ma solution becomes a soliton, and for $a = 1/2$ the Akhmediev solution reduces to the plane-wave solution.

5. Conclusions

In the present study, the formation and characteristic properties of breather structures are studied in a non-extensive Thomas–Fermi plasma. Solving the fluid equations via a multiple scale technique, we have derived a NLS evolution for the amplitude of the electrostatic potential, describing the evolution of the localized structures. We have considered a set of breather-type solutions of NLS, and investigated the effect of non-extensive spectral index q on their characteristics properties. The effect of spectral index q on the dynamics has been examined by observing the influence of q on (i) the stability/instability region (the wavenumber threshold dividing the carrier wavenumber domains into the stable and unstable regions), (ii) the growth rate of modulational instability and (iii) the localized profile characteristics (space–time localization). We find that the spectral index q significantly affects the basic features of these localized excitations.

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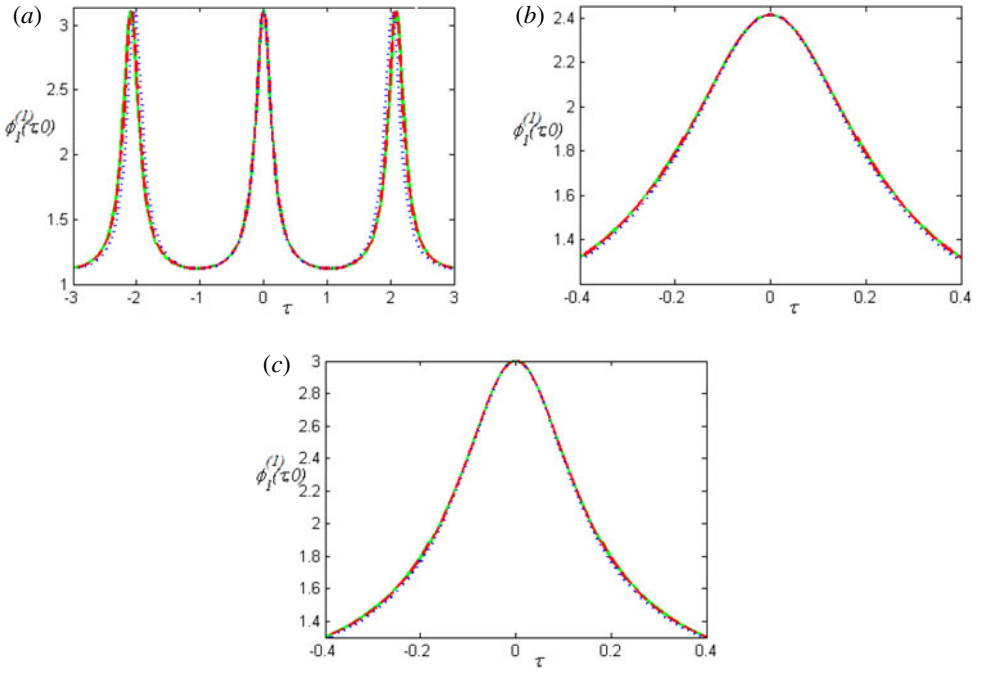


FIGURE 4. The evolution of breather solutions of the NLS (3.11) with respect to the time coordinate τ, for the different values of the spectral index q along with $q = 1.1$ (solid curve), $q = 1.2$ (dashed curve) and $q = 1.5$ (dotted curve): (*a*) Kuznetsov–Ma breather, (*b*) Akhmediev breather and (*c*) Peregrine soliton. The parameters are: $k = 1.5$, $a = 0.25$ and $\theta = 0.343$.

The influence on the instability region is a decrease of the instability region. The growth rate of the modulational instability was also investigated for different values of q. The growth rate is increased by the non-extensivity and thus the spectral index q may be used as a controlling parameter for the modulational instability.

The qualitative effect of electron non-extensivity on the localized excitation profile results in more space-extended as well as time-extended peak-shaped excitations.

The results of the present investigation must be useful to explain the basic features of nonlinear excitations associated with interstellar and space plasmas especially in stellar polytropes, protoneutron stars, hadronic matter and quark–gluon plasmas, dark-matter halos, etc. (Spruch 1991; Liolios 2003). It must be added that in the present numerical analysis, a wide range of degenerate plasma parameters was used, which are relevant for many compact astrophysical objects and cosmic environments, such as active galactic nuclei, the solar atmosphere, the primordial universe, pulsar magnetospheres, cores of giant planets such as Jupiter, neutron stars, supernovas, white dwarfs, etc. (Michel 1982; Tandberg-Hansen & Emslie 1988; Hoyos, Reisenegger & Valdivia 2008), hadronic matter and quark–gluon plasma (Gervino *et al.* 2012), stellar polytropes (Plastino & Plastino 1993), dark-matter halos (Feron & Hjorth 2008), protoneutron stars (Lavagno & Pigato 2011) etc.

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Appendix A

The coefficients of the harmonic modes in (3.6*a*)–(3.6*c*) and (3.7*a*)–(3.7*c*) are as follows

$$\left.\begin{gathered} A_1 = [(c_1 + k^2)^2(2\omega k v_g + \omega^2) - c_2 k^2 v_g^2]/(c_1 v_g^2 - 1)k^2, \\ A_2 = [A_1 k^2 + (c_1 + k^2)^2(2\omega k v_g + \omega^2)]/v_g^2 k^2, \quad A_3 = A_2 - 2\omega(c_1 + k^2)/k, \\ A_4 = \omega^2[3(c_1 + k^2)^2/2 - c_2]/[c_1\omega^2 + (4\omega^2 - 1)k^2], \\ A_5 = [3(c_1 + k^2)^2/2 - c_2]/3\omega^2 + 3(c_1 + k^2)^2/2, \\ A_6 = \omega[3(c_1 + k^2)^2/2 - c_2]/3k^3 + \omega(c_1 + k^2)^2/2k. \end{gathered}\right\} \quad \text{(A 1)}$$

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