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ANTS AND NONOPTIMAL SELF-ORGANIZATION: LESSONS FOR MACROECONOMICS

Alan Kirman
GREQAM
Aix Marseille University
and
EHESS

This paper suggests that we need an alternative approach to economic modeling in general, and macroeconomic modeling in particular, if we are to capture salient characteristics of recent economic developments. Rather than focusing on models built on the basis of isolated, rational, optimizing agents, we should recognize that much simpler individuals following basic rules can collectively generate complex behavior. We have lessons to learn from studying the behavior of social insects for example. Noisy systems of interactive agents can generate aggregate phenomena such as sudden changes in the state of an economy or market, with no external shock. I give two examples of simple models of financial markets to illustrate this but would argue more generally that such models are indispensible if we are to understand aggregate economic phenomena.

Keywords: Interaction, Rationality, Noise, Social Insects, Financial Markets.

> And the first thing that came to mind was something that people said many years ago and then stopped saying it: The euro is like a bumblebee. This is a mystery of nature because it shouldn't fly but instead it does. So the euro was a bumblebee that flew very well for several years. And now—and I think people ask "how come?"—probably there was something in the atmosphere, in the air, that made the bumblebee fly. Now something must have changed in the air, and we know what after the financial crisis. The bumblebee would have to graduate to a real bee. And that's what it's doing.
>
> —Speech by Mario Draghi, President of the European Central Bank at the Global Investment Conference in London, 26 July 2012

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1. INTRODUCTION

Until 2007, macroeconomists were increasingly satisfied with the models they were building. Robert Lucas (2003) asserted that the "central problem of depression prevention had been solved," and there were many papers on the "Great Moderation" attributing the decreased volatility in a number of macroeconomic variables to improved economic policy making. Given this mindset, any empirical evidence that is inconsistent with the models must be an aberration or due to some exogenous shock to the system.

Mario Draghi suggests that we should have more confidence in our models than in the evidence. But if the bumblebee does actually fly, there must surely be something wrong with the model and not with the bumblebee. In a similar way, economists should, faced with the current crisis, which is not in the set of possible outcomes of modern dynamic stochastic general equilibrium (DSGE) models, reflect on the nature of their models. Since the onset of the crisis there has been no lack of criticism of DSGE models from policy makers. Jean-Claude Trichet (2010), the ex-governor of the European Central Bank, said,in a widely quoted address, that existing analytical frameworks provided no guidance in the crisis, and Lord Turner (2011), the head of the U.K. Financial Services Authority, went further and assigned some of the responsibility for the crisis to the economists who purveyed the erroneous impression that markets "were always rational and self-equilibrating." Policy makers were lulled into a false sense of security by economists who claimed that the economy was on a steady state path, and occasionally perturbed by independent exogenous shocks. Larry Summers (2013) said recently in a speech to the IMF that he had believed the conventional wisdom that "it is all about fluctuations and what you need to do is to have less volatility." Yet there were a few important economistswho were skeptical. Among them was Buiter (2009),who now as chief economist of Citigroup has to deal with the way in which markets really function; he said that the real problem was "the uncertainty arising from the interactions of boundedly rational market participants" and not exogenous shocks. Indeed, most of the explanations for the crisis involve contagion, networks, and the development of bubbles, all of which are endogenous explanations, but not ones that are features of current macroeconomic models.

Furthermore, Summers (2013) said, the mechanisms that were supposed to bring the economy back toward steady state full employment cease to work when real interest rates become negative, as has happened. Maybe, he went on, we have to rethink our basic ideas about the self-organizing properties of the economy if we are not to leave a large part of the developed world in a state of "stagflation."But although criticism from the major policy makers was abundant, no clear alternative model has emerged. Behavioral economics emphasizes the differences between the way in which people actually make decisions and that posited in economic theory. Yet these insights have been used mainly to introduce "frictions" or "imperfections" into standard macroeconomic models. Experimental economics has shown that, even in carefully controlled situations, individuals do not behave like their

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theoretical counterparts, and the assumption of rational expectations is seriously undermined by the results from experimental asset markets [see, e.g., Hommes (2013) and Wagener (2013)] and by empirical evidence from surveys. But, despite this, not much has penetrated macroeconomics beyond the argument that including "imperfections" makes it easier to calibrate existing models to the empirical data.

What would be the basis for better models, and indeed, what is meant by "better"? Agent-based models [see, e.g., Lebaron and Tesfatsion (2008), Gaffeo et al. (2011), and Howitt (2012)] offer one alternative that views the economy as a complex system. These posit agents with heterogeneous tastes and expectations, and they model situations in which expectations are not uniform and study the impact of distributional changes. In these models, agents interact directly with each other, and they can incorporate the effects of network structure on economic outcomes. They permit the incorporation of insights from behavioral finance and, perhaps most importantly, show that major changes in macroeconomic variables can arise as a result of the self-organization of the individuals rather than as the result of some unspecified shock.

Such models relax some of the standard assumptions about individual rationality and are accused of leading us into the dangerous "wilderness of bounded rationality." It is said that if we drop the "discipline" imposed by standard models, any alternative is necessarily "ad hoc." But, for example, restricting ourselves to concave objective functions, particularly those of a specific form such as the Cobb–Douglas function, is also extremely ad hoc and in no way empirically justified.

Another argument often used against agent-based models is that they are of little use for policy purposes. Yet to be useful in this sense means that one can examine the characteristics of the equilibrium of the system and how these would be changed by a change in policy. But we have known since the mid-seventies that the standard assumptions made in general equilibrium models guarantee neither the uniqueness nor the stability of an equilibrium, both of which are necessary for such an exercise.

Should we, then, persist with our current framework and modify it to incorporate some of the features that have resurfaced in recent research but that have a long history in economics, or should we radically rethink the fundamentals of our models?

I suggest that we should take a wholly different approach from that currently standard, and consider the economy as an adaptive self-organizing system [see Arthur et al. (1997), Durlauf (1997), and Kirman (2010)] consisting of purposeful but not optimizing individuals, with local and limited knowledge. Such systems, just like colonies of social insects, exhibit complex aggregate behavior that cannot be represented as that of one of the individuals of whom the system is composed. I will then propose two modest examples that represent a step toward more reasonable economic models.

But why do we need to do this? Surely our models, honed to perfection over more than a century, contain the essentials of an adequate representation of the

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real economy? To understand why this is not the case, I will take a critical look at the standard assumptions on which our current models are based.

2. OUR BASIC ASSUMPTIONS

What are some of the basic assumptions underlying modern macroeconomic models and what difficulties do they present? These are the axioms that constitute the rationality that we impute to our agents, the implicit assumption that the economy is in equilibrium and the assumption that the equilibrium is stable, and finally, when there is uncertainty, the assumption that individuals have "rational expectations." The weakness of these pillars of our current macroeconomic models helps to identify the changes in direction that we need to make in our models.

2.1. The Rationality Assumptions

Most macroeconomic models are based on what have come to be called "sound micro-foundations," by which is meant that aggregate behavior is the result of the choices of isolated, rational, optimizing individuals. Yet the rationality attributed to individuals is based on the introspection of economists rather than on careful empirical observation of how individuals actually behave.[1]

Take, as an example, a particular assumption that is standard in economic models. This is that individual behavior is continuous, that is, choices or demand react continuously to prices, for example. This assumption is not made because it is plausible at the individual level, for neither the structure of the choice space nor empirical observation make this reasonable. We make it because simply adding up the continuous choice functions of individuals gives us the continuity at the aggregate level that we require for our analysis But do we need to do this?

It has long been recognized, and Cournot (1838, Chap. IV, Sect. 22)[2] said explicitly, that discontinuous individuals can generate continuous aggregate behavior. He referred to demand, arguing that if individuals each buy one unit of a good and have different reservation prices, then the resultant aggregate demand will be a good approximation of a continuous function. Cournot's idea is simple and can be formalized. Individuals have threshold prices. If their thresholds are reached they are activated and purchase. They make discrete and not continuous choices. For the overall result to be continuous, there must be a nondegenerate distribution of thresholds.

A clear indication of how discontinuous reactions at the individual level can translate into continuous aggregate behavior is given by social insects (from which I argue that we can learn a lot). To see this consider the example of bees. Bees have different functions, and one of their tasks is temperature control. When the temperature is low, bees cluster to generate heat for themselves, but when it is high, *some of them* fan their wings to circulate air throughout the hive, keeping the hive and the brood chamber at a constant temperature [see Crane (1999)]. How is this done and what is the lesson here for us? The typical macroeconomist's response

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to this phenomenon would be to consider a representative bee and suggest how its behavior responds to the ambient temperature. Wing beats of this relatively sophisticated individual would be a continuous function of the temperature. In fact this is not what happens. Bees have different threshold temperatures, and they are either *on*, beating at 11,400 beats per minute, or *off*. As the temperature rises, more bees start to beat their wings. Hence, collectively, with very simple binary rules, the bees produce a continuous response.

Thus the aggregation of simple but discontinuous individuals with heterogeneous thresholds produces continuous aggregate behavior. Such behavior is familiar in, for example, stock markets, where a stop loss order will automatically trigger the sale of a particular stock when its price falls below a certain level.

Thresholds may also be important at the collective level. In some cases a system may go through a phase change when some threshold value of a variable is reached; this will typically be the case if individuals have strongly correlated thresholds or if they are directly influenced by the choices of those with whom they interact. Although such behavior is common in physical models and is observed in financial markets, it is very infrequently found in modern macroeconomic models.

Other assumptions such as the convexity of preferences are not necessary with heterogeneous agents, and the monotonicity of preferences can be weakened if all we want to show is the existence of equilibrium. The problem arises if we want to analyze how an equilibrium is attained.

2.2. Stability

The idea that economies are systematically in an equilibrium state is highly counterintuitive to noneconomists. Indeed, early economists questioned the notion of an economy self-organizing into an equilibrium state. Already, in 1819, Sismondi said,

> Let us beware of this dangerous theory of equilibrium which is supposed to be automatically established. A certain kind of equilibrium, it is true, is reestablished in the long run, but it is after a frightful amount of suffering. [Simon de Sismondi (1819, pp. 20–21)]

Walras himself believed that there was some mechanism that was constantly trying to drive a market to equilibrium but that it never got there. He said that the market is

> Like a lake stirred by the wind, in which the water continually seeks its equilibrium without ever achieving it. [Walras (1877, p. 310)]

Why then is the assumption that the economy is constantly in equilibrium systematically made in macroeconomics? The historical basis for this is interesting. Until the results of Sonnenschein (1972), Debreu (1974), and Mantel (1974), there was a persistent hope that, with the standard assumptions on individuals, one could show that an economy starting from a disequilibrium state would tend to

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equilibrium. The failure to prove this was put down to the mathematical difficulties involved. However, the results just mentioned were proved by three of the leading mathematical economists of their time, who showed that, even under stringent and unrealistic assumptions made about individuals, one could not show that equilibria were stable. This led Morishima to remark,

> If economists successfully devise a correct general equilibrium model, even if it can be proved to possess an equilibrium solution, should it lack the institutional backing to realize an equilibrium solution, then the equilibrium solution will amount to no more than a utopian state of affairs which bear no relation whatsoever to the real economy. [Morishima (1984, pp. 68–69)]

But why did we give up on the stability problem? The key to the problem is the amount of information necessary for an economy to reach an equilibrium. A result of Jordan (1982) shows that no mechanism can sustain a Pareto-efficient outcome using as little information as the market mechanism uses. Unfortunately this result depends on one key assumption, which is that the economy is functioning *at equilibrium.* But, from Smale (1976) onward, we have known that getting to equilibrium from a nonequilibrium state can only be accomplished at the price of a significant increase in the amount of information used. Indeed, Saari and Simon (1978) showed that a process that would lead to equilibrium from any starting price vector would use an infinite amount of information. Many efforts were made to get round this, but none of them had a plausible economic interpretation. Thus it has become clear that there is no hope of finding a reasonable adjustment process, independent of the economy in question, that will converge to equilibrium from any price vector. This is why macroeconomists simply assume that the economy is always in equilibrium.

2.3. Rational Expectations

Now consider the missing element in this account: how we deal with uncertainty. As soon as we introduce uncertainty into the model, we have to deal with the problem that people's decisions are a function of their expectations for the future. The standard approach has been to use "rational expectations," a term coined by Muth (1961). Muth was a colleague of Simon, and both arrived at a different conclusion as to how to deal with the problem. Simon (1978) argued that in some very simple and stylized cases one could argue that people could just substitute expected values for stochastic variables. This was consistent with Muth's idea that, under certain strong assumptions, people who had a correct understanding of the process governing the evolution of the economy could argue in terms of certainty equivalents. But both warned that this was a convenient short cut and not a satisfactory explanation of economic reality.

Simon said,

> Of course, the solution though it provides optimal solutions for the simplified world of our assumptions, provides, at best, satisfactory solutions for the real world decision

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> problem. In principle, unattainable optimization is sacrificed for, in practice, attainable satisfaction. [H. Simon, Rational Decision-Making in Business Organizations, Nobel Memorial Lecture, 8 December (1978)]

Muth (1961) also warned that there is little evidence to suggest that theoretical rational expectations have anything to do with the way the economy works. Later, Simon also expressed his dissatisfaction with the rational expectations hypothesis when he said,

> A very natural next step for economics is to maintain expectations in the strategic position they have come to occupy, but to build an empirically validated theory of how attention is in fact directed within a social system, and how expectations are, in fact, formed. Taking that next step, requires that empirical work in economics take a new direction, the direction of micro-level investigation proposed by Behavioralism. [H. Simon (1984)]

This suggestion was ignored by macroeconomists, who even required that every agent should have a complete and correct understanding of the stochastic process governing the evolution of the economy.

Of course, economic theorists have wrestled with the difficulties intrinsic in the rational expectations hypothesis, and they have suggested ways in which economic agents might coordinate on common expectations. An alternative approach originally suggested by Hicks and developed by Grandmont (1983) was to consider the idea of temporary equilibria in which markets clear at one period and then reopen at the next. In this case agents have finite horizons, and this seems more reasonable than the standard infinite-horizon approach, which Poincaré (1901) already found implausible.

But to return to the standard notion of rational expectations, one of the major problems is to determine how agents might coordinate on such an outcome. Guesnerie (1992) took a more extreme position than Muth and the opposite of that which I propose, and suggested that agents might reason in a fully game-theoretic way, and thus coordinate on a rational expectations equilibrium.[3]

But as Woodford (2011) has argued, the idea that individuals will form a "correct" view of the process that governs their environment not only is implausible, but does not logically follow from the axioms postulated for individuals. He argued that even if an economist has a model in which the outcomes follow from the rational behavior of the individuals, there is no reason to believe that those agents will assume that the world is like that model. Yet the "rational expectations" hypothesis assumes that the individuals will make exactly the predictions that the model implies are correct. This logical error undermines the "efficient markets" hypothesis in asset pricing theory and "Ricardian equivalence" in macroeconomics. Woodford (1990) had already pointed out that agents may have the "wrong" model in mind, but may find that the data do not refute that model. Worse, the fact that this happens may be engendered by the actions of the agents based on their false beliefs.

Last, but far from least, a number of econometricians have pointed out that once the underlying stochastic process that governs the evolution of the economy

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exhibits "structural breaks," it is not rational to have rational expectations as usually defined. As Hendry and Mizon (2010) indicate, both of the major modern macroeconomic models based on rational expectations ignore the fact that when there are unanticipated changes, the conditional expectations used by the agents in such models are neither unbiased nor minimum meansquared error predictors, and that better predictors can be provided by robust devices.

What all of this shows us is that each of the building blocks of what has become our standard model has flaws but, more importantly, these flaws cannot easily be corrected by introducing some imperfections or frictions into the model. The beauty of the general equilibrium model was its internal consistency; introducing ad hoc frictions to better fit the data removes that consistency and, at least for the moment, provides no better explanation of the empirical facts.

3. A CHANGE OF DIRECTION

An alternative would be to replace the basic assumptions underlying the model with new ones, and to build models in which features previously considered as inconvenient imperfections become central to the analysis. This is not just an illusory dream chased by a handful of eccentric theorists. For example, Trichet (2010) argued that to understand crises we should abandon atomistic optimizing agents, and should look at the interaction between heterogeneous agents and use behavioral economics and agent-based modeling.

Indeed, I would argue that we should model the economy as being made up of purposeful but not necessarily optimizing individuals who manage collectively to coordinate their activities but who do not necessarily achieve an efficient state. In particular, they have limited and local information and they interact with a limited subset of the participants in the economy, very much like social insects. The sort of model that should be used is one that is genuinely dynamic and has trajectories that can exhibit sudden, major endogenous changes in aggregate outcomes. Such models are familiar to physicists and biologists and are known as "complex adaptive systems."[4] Such systems do not allow the simple passage from micro to macro behavior made by macroeconomists. We have neither theoretical nor empirical grounds for arguing that the aggregate behaves like an individual and especially a "representative" one.[5] Once we recognize that the gap between micro and macro is fundamental, we will be on the road to a more satisfactory theory. To quote two distinguished neuroscientists,

> while network properties are dependent on the properties of the neurons in the network, they are nevertheless not identical to cellular properties, nor to simple combinations of cellular properties. Interaction of neurons in networks is required for complex effects, but it is dynamical, not a simple wind-up doll affair. [Churchland and Sejnowski (1995)]

The complex system approach provides an explanation for the emergence of aggregate behavior from the interaction between simple interacting individuals.

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4. SOCIAL INSECTS

To illustrate the point, consider social insects. Nobody would ascribe "rational expectations" to ants, yet implicitly this is what we do. Thousands of years ago we were advised to consider their prudent anticipation of the future:

> Look to the ant, thou sluggard. Consider her ways and be wise. Without chief or ruler or overseer she gathers her harvest in the fall to save for the winter. [6:6–8, *Book of Proverbs, The Old Testament*]

Ants, we are told, have evolved to follow the rules that make them store food, and it is often argued that ants have adapted optimally to their environment. This argument was used by Lucas (1988) when he suggested that individuals do not consciously optimize but try simple rules and use those that are most effective. Learning replaces evolution. Yet this justification is not really theoretically founded. Although certain learning processes converge to a correct view of an exogenous process, it has not been shown that individuals learning about an environment made up of individuals who are also learning will converge to some sort of optimal outcome.

There is a more basic objection, however, which makes many uncomfortable with the comparison of human agents with social insects. As Deborah Gordon, the Stanford entomologist, says,

> The basic mystery about ant colonies is that there is no management. . . . . No individual is aware what must be done to complete any colony task. Each ant scratches and prods its way through the tiny world of its immediate surroundings. Ants meet each other, separate, go about their business. Somehow these small events create a pattern that drives the coordinated behavior of colonies. [Gordon (2010)]

To some, such as myself, this is closer to a description of economic reality than the world characterized by highly rational but independent optimizers. But is the behavior of social insects relevant for the study of human behavior? The increasing interest in what has come to be called "swarm intelligence" (SI), a means whereby a group can overcome the cognitive limitations of its members, suggests that it is. Seeley, an authority on the behavior of bees, is explicit:

> SI is a rapidly developing topic that has been investigated mainly in social insects (ants, termites, social wasps, and social bees) but has relevance to other animals, including humans. Wherever there is collective decision-making—for example, in democratic elections, committee meetings, and prediction markets—there is a potential for SI. [Thomas Seeley (2012)]

It is not necessarily true that the collective behavior of individual insects or other animals leads to an "optimal" or even a "good" collective outcome (only 10% of new nest sites found by bees are successful), and there are examples in which swarms do not have an improved, collective, cognitive ability. But, in some cases, such as that of the bees cooling their hives or ants shortening their paths to a food source, this happens. Now consider an example of how insects can help us build economic models.

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5. FLUCTUATIONS: AN ANT-BASED MODEL [KIRMAN (1993)]

Ants faced with two paths to a food source choose that which has the strongest attraction, measured by the amount of pheromone left on that trail by other ants coming from the source or by direct recruitment by the successful ants. Label the two paths red and blue. The state of the system can be described at any point in time by $k \epsilon N$, the number of ants taking the red path, or by $x = k/N$, whereas the remaining $N - k$ ants take the blue path. Identify ants by the color of the path they use; they can, over time, change color, either spontaneously (there is some noise in the system), or because they are recruited by an ant coming from the path of the other color. Because of this, the system will evolve over time.

If the noise in the system is small, the system exhibits fluctuations between two extremes with essentially all the ants on one path, and is at those extremes most of the time. If there is too much noise in the system, however, the state is, in general, one in which there is an equal division of the two groups. Thus, increasing the noise restores the mean value and eliminates herding behavior. The appropriate equilibrium notion here is then not some fixed proportion of ants on each path but rather a *limit distribution* of the underlying stochastic process. Thus, thinking of the state of the system as k/N, the proportion of ants on the blue path, we can write $f(k/N)$ for the limit distribution, and this should be viewed as the proportion of time that the system spends in any state k/N. Hans Foellmer [see Kirman (1993)] showed that if one lets N become large, and approximates f by a continuous distribution $f(x)$, where $x = k/N$ takes on values between 0 and 1, then this distribution will be a symmetric beta distribution, i.e., of the form

$$f(x) = x^{\alpha}(1 - x)^{\alpha - 1}, \tag{1}$$

and the limit distribution will be concentrated in the tails. The process spends most of its time at one of the extremes and occasionally switches between the two. Although the system is perfectly symmetric, aggregate behavior displays swings, provided there is not too much noise in the system. Note that the noise is not like the exogenous shocks in macroeconomic models. Indeed, in the limit, as N becomes large, the noise can disappear altogether. Even in the finite case, there are no large global shocks to the system that cause it to shift from one extreme to the other.

6. AN EXAMPLE: A FINANCIAL MARKET

Now consider a financial market. Replace ants following paths with traders choosing experts' recommendations about the future price of an asset. For simplicity, consider two basic types of forecasting rules, "fundamentalist" and "chartist" [see Frankel and Froot (1988)]. The fundamentalist believes that there is a fundamental price to which the asset price will return. The chartist extrapolates from past prices. Individuals have a probability of shifting from their current expert to another if they perceive that the other's recommendations are more profitable. Self-reinforcing

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success underlies the positive feedback that engenders instability and that Soros (1987) refers to as "reflexivity."

To be more precise, each agent or trader a has an excess demand at time t for the asset, which we took in Foellmer et al. (2005) to be of the form

$$e_t^a(p,\omega) = c^a\left(\hat{S}_t^a(\omega) - logp\right) + \eta_t^a(\omega), \tag{2}$$

where c^a are positive constants. Agents' forecasts $\hat{S}_t^a(\omega)$ as to next period's price generate a speculative demand for the asset, and they have some stochastic "liquidity demand" $\eta_t^a(\omega)$ which that adds some noise to the system. The idiosyncratic and independent shocks to the individuals, which can be arbitrarily small, produce the interesting dynamics of the system without global shocks. The temporary equilibrium for the logarithmic asset price is given by that price for which the aggregate excess demand is zero, i.e.,

$$S_t = \frac{1}{c_t}\sum_{a\in A} c^a S_t^a(\omega) + \eta_t, \text{ where } c_t = \sum_{a\in A} c^a \tag{3}$$

Agents choose from a finite number of forecasts $R^i{}_t$ given by m experts or "gurus," and the choices available to an agent are given by

$$\hat{S}_t^a \in \left\{R_{t\ldots}^1 R_t^m\right\}$$

The proportion of agents following expert i is then given by

$$\pi_t^i = \frac{1}{c_t}\sum_{a\in A} 1_{\{\hat{S}_t^a = R_t^i\}}, \tag{4}$$

where 1 is an indicator variable taking on the value 1 if the condition is satisfied and 0 otherwise. Given this, the price of the asset evolves as

$$S_t = \sum_{i=1}^{m} \pi_t^i R_t^i + \eta_t. \tag{5}$$

It is a weighted average of the recommendations or forecasts of the experts and of the total liquidity demand, both of which are random variables, because the weights on the experts are given by the numbers of their followers.

Assume that all experts use a combination of fundamentalist and chartist rules. Then the recommendation of expert i will be given by

$$R_t^i = S_{t-1} + \alpha^i\left[F^i - S_{t-1}\right] + \beta^i\left[S_{t-1} - S_{t-2}\right], \tag{6}$$

and the price of the asset will evolve as

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$$S_t = F\left(S_{t-1}, S_{t-2}, \tau_t\right) = \left[1 - \alpha\pi\left(t\right) + \beta\pi\left(t\right)\right] S_{t-1} - \beta\pi_2\left(t\right) S_{t-2} + \gamma(\pi_t, \eta_t),$$

$$\text{where}\, \alpha\left(\pi_t\right) = \sum_{i=1}^{m} \alpha^i \pi_t^i \quad \text{and} \quad \beta\left(\pi_t\right) = \sum_{i=1}^{m} \beta^i \pi_t^i. \tag{7}$$

Thus, the price dynamics is given by a linear recursive relation in a random environment τ_t that is given by

$$\{\tau_t\} = \{(\pi_t.\eta_t)\}.$$

If there were only fundamentalist experts, that is, if $b^I = 0$ for all I, then because $\alpha' \in (0, 1)$, prices would be mean-reverting. That is,

$$S_t = \left[1 - \alpha\left(\pi_t\right)\right] S_{t-1} + \gamma\left(\pi_t, \eta_t\right); \tag{8}$$

i.e., prices would follow an Ornstein–Uhlenbeck process in a random environment and thus fundamentalists would have a stabilizing influence. But if there were only chartists, then $\alpha_i = 0$ for all i, and the price process would be transient, unstable, and given by

$$S_t = S_{t-1} + \beta\left(\pi_t\right)\left[S_{t-1} - S_{t-2}\right] + \eta_t. \tag{9}$$

Thus, dependent on the evolution of $\pi\left(t\right)$, periods of stability will be interspersed with bubbles and crashes. To see how $\pi\left(t\right)$,the shares of followers of each guru, evolve, consider the profit G_t^i that would have been made in period t from following expert i's recommendation. This is given by

$$\begin{aligned} G_t^i &= \left(R_t^i - S_{t-1}\right)\left(e^{S_t} - e^{S_{t-1}}\right) \\ &= \left[\alpha_t^i\left(F_t^i - S_{t-1}\right) + \beta_t^i\left(S_{t-1} - S_{t-2}\right)\right]\left(e^{S_t} - e^{S_{t-1}}\right). \end{aligned} \tag{10}$$

Now the attraction of the expert i will be the sum, discounted by δ, of his past profits:

$$U_t^i = \delta U_{t-1}^i + (1 - \delta) \sum_{j=0}^{t} \delta^{t-j} G_{j+1}^i. \tag{11}$$

The probability of selecting an expert i is an increasing function of the relative attraction of that expert. For example, the logit function gives the probability of choice as

$$p_t^i = \frac{e^{\lambda U_t^i}}{\sum_{j=1}^{m} e^{\lambda U_t^j}}. \tag{12}$$

Here, each agent knows what every expert recommends, but the agent could also only update an expert's attraction from his own experience with that expert. In the model here it can be shown that the asset price process is ergodic and has a unique limit distribution, provided that the probability of becoming a chartist is not too high.[6]

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In sum, then, the price process will exhibit bubbles and crashes interspersed with calm periods. The process never settles to an equilibrium price, but in the long run, the probability of the price being in a certain interval can be calculated. This is but one step toward modeling an economic system that is constantly evolving and that has no equilibrium either in this or in the standard sense.

Nevertheless, even this simple model captures some of the characteristics of the time series of empirical asset prices. It generates a distribution of prices with fat tails, and the price series is characterized by "long memory." Individuals in this market, like the ants in their environment, produce aggregate phenomena by their interaction that are very different from those that could be anticipated by studying individuals in isolation. Note that some noise is needed to prevent the system from being absorbed into a particular state, but in a large market, this noise can go to zero. But, and possibly more important, if there is too much noise the system will fluctuate around a mean value and switching from one extreme to another will disappear. What this analysis shows us, then, is that a market with explicit herding behavior will not necessarily converge to an equilibrium price.

Now let me give an example in which a sudden price collapse occurs despite a progressive change in the fundamentals.

7. AN EXAMPLE: THE COLLAPSE OF A DERIVATIVES MARKET

In Anand et al. (2013), we analyzed the sudden collapse of the prices of MBS (mortgage-backed securities). This collapse occurred rapidly despite the fact that the weakness of the assets, which were bundles of mortgages with a single overall rating, had increased progressively over time. Figure 1 shows the evolution of house prices over 20 years, and the developments from 2006 onward should have been a first indication that defaults on the mortgages involved in the MBS were likely to rise. The principal reason for this was that the percentage of loans that represented "positive equity," that is where the value of the house was greater than

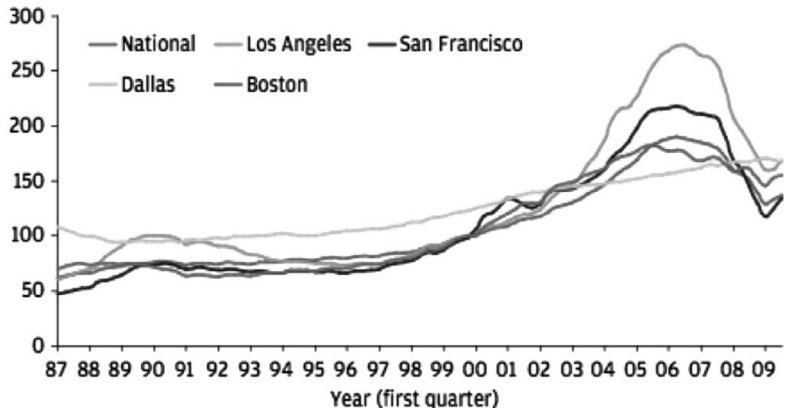


FIGURE 1. House price indices in some U.S cities, 1987–2009. *Source:* Case Shiller.

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the outstanding amount of the house loan, diminished as the increase in house prices slowed and was reversed.[7]

In Figure 2, the increase in the default rate on mortgages issued in different years, as house prices increased more slowly and then declined and mortgages were easier to obtain, is shown for the United States.

Although this information was public, the evolution of the prices of MBS did not reflect it, as can be seen in Figure 3.

Maybe this reflected the rational expectations of the whole chain of actors from the mortgagor to the investor. But as Ashcraft and Schuermann (2007), at the onset of the crisis, pointed out, there were informational frictions at each step in the chain, each of which could lead to a breakdown in the system and a collapse in the prices of MBS.

Observe first that, although the delinquency of differently rated assets was evolving similarly, as seen in Figure 2, the collapse of the better-rated assets came later. This suggests that investors' appreciation of the toxicity of the assets was influenced by the ratings, whether or not this was justified. Ashcraft et al. (2010) show that prices were strongly correlated with ratings but that ratings were very poorly correlated with default rates, indicating how imperfect the information provided was.

To understand how the sudden collapse occurred, we constructed a model of the banking system in which the participants securitize their loans and then sell them on to each other and to investors via special purpose entities (SPEs). In other words, the banks were holding derivatives, in particular MBS, the content of which was costly to evaluate. Their investor clients had neither the means nor the sophistication to evaluate whether the mortgages in the MBS did, in fact, meet the underwriting standards, as was claimed.[8] Moreover, the banks, when trading amongst themselves, were not doing due diligence. The organization, the incentives, and the interactions between the actors, in reality, did not correspond to the anonymous market inhabited by fully informed agents with rational expectations that standard models assume. The alternative model we propose is one of a complex interactive banking network in which a sudden collapse can occur. It involves agents who are not irrational in the normal sense of the word, but are not fully rational in the sense in which that term is understood in economics. They have short horizons, and their willingness to buy depends on how much checking is being done by those with whom they trade.

8. THE MODEL

The system consists of $i = 1, \ldots, N$ agents, which, in the case of the subprime crisis, for simplicity, we can think of as the banks who were both the issuers—via SPEs—and the investors in these asset-backed securities(ABS). These banks or agents are linked in a network, corresponding to the over-the-counter market, and in each period an agent draws at random another agent among her neighbors. For each agent i there is a variable $z \epsilon \{0, 1\}$ that specifies whether she follows ($z = 1$)

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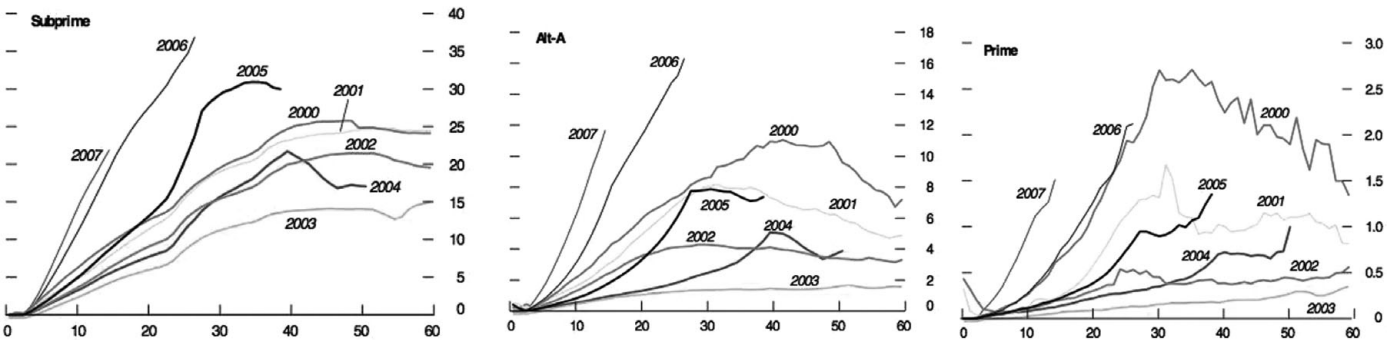


FIGURE 2. Delinquency rates on mortgages (months from origin) in the United States originating from 2004 to 2007. *Sources:* Merrill Lynch and Loan Performance.

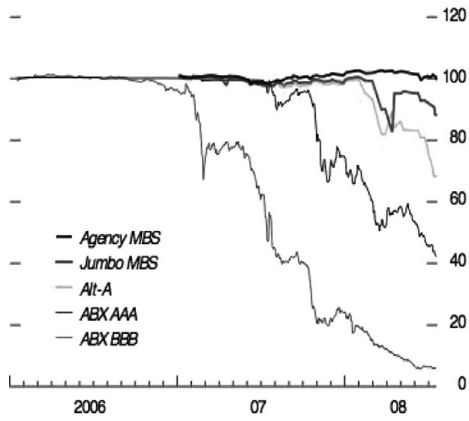


FIGURE 3. Price evolution of differently rated mortgage-backed securities (MBS). ABX is an index of credit default swaps on mortgage-related asset-backed securities. *Sources:* JPMorgan Chase & Co. and Lehman Brothers.

or not ($z = 0$) the following behavioral rule: purchase an ABS, relying on signals from the rating agencies, without evaluating the fundamental value of underlying assets. Succinctly, we write

$$z_i = \begin{cases} 1 & \text{if agent } i \text{ follows the rule} \\ 0 & \text{if agent } i \text{ does risk analysis} \end{cases}. \tag{13}$$

The rationale for adopting the rule, as we will see, is not based on the fundamentals but on the fact that others also follow the rule. If enough other participants do so, the agent is convinced, not irrationally, that the ABS is highly liquid and hence easy to trade.

Assume that the ABS is toxic with probability p. By toxic we mean, for example, either that the original borrower of the loan has already defaulted, or that he is delinquent in his payments. Assume that the cost of purchasing a security is $p0$, whereas the payoff from successfully reselling the security is $p1$, where $p1 - p0 > 0$. If the buyer checks and finds the ABM to be toxic, the price now falls to $p2$, where $p2 - p0 < 0$. The buyer can avoid this by checking at a cost of χ with p.d.f. $\Phi(\chi)$. Now normalize so that $p1 - p2 = 1$ and $p0 - p2 = c$.

Consider the expected payoff of each strategy to the seller:

$$\begin{aligned} u_i\,(z_i = 1) &= E\left[-p\left(1 - z_j\right)c\right] + \left[1 - p\left(1 - z_j\right)\right](1 - c) \\ &= 1 - p\,(1 - \bar{z}_i) - c, \end{aligned} \tag{14}$$

where $\bar{z}_i = E(z_j)$ for $j \in N_i$. That is, agent i can correctly estimate the average choice of rule by his neighbors but not the choice of each individual. Then we

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have

$$z_i = \frac{1}{k_i} \sum_{j \in N_i} z_j. \tag{15}$$

Now the expected payoff from not following the rule, and choosing $z_i = 0$, that is, from checking the value of the underlying assets, is

$$u_i\,(z_i = 0) = (1 - p)\,(1 - c) - \chi_i. \tag{16}$$

Thus if the agent checks and the assets are toxic, he pays the cost of checking, whereas, if they are not, he obtains the difference between the selling and buying price less the checking cost. The strategy, which constitutes the best reply to the strategies of the neighbors, is then given by

$$z_i = \Theta\left[u_i\,(1) - u_i\,(0)\right] = \Theta\left[p\left(\frac{1}{k_i}\right)\sum_{j \in N_i} z_j - c\right], \tag{17}$$

where the function Θ is defined as $\Theta\,(x) = 1$ if $x > 0$ and 0 otherwise. Note that the agents are assumed to know the probability of default of the underlying assets. However, in reality, the common perception of p reflected the overoptimistic evaluations of the rating agencies. For low values of p, there is one equilibrium in which all agents choose not to check, but once a critical value of the commonly perceived p is passed, another equilibrium emerges in which all agents check. This is illustrated from numerical simulations in Figure 4a.

On which of the two equilibria the agents will coordinate is not clear. However, if we introduce some noise into agents' choices, as in the preceding example, then we can examine whether they coordinate on one equilibrium as p increases. Choose the logit rule as before. Then the probability of choosing $z_i = 1$ is given by

$$P\,(z_i = 1) = \frac{e^{\beta u_i(1)}}{e^{\beta u_i(1)} + e^{\beta u_i(0)}}, \tag{18}$$

where β is a parameter indicating the sensitivity of the agent to the difference between the payoffs of the two strategies. If $\beta = 0$, the agent chooses one of the two strategies at random, but if $\beta \to \infty, \infty$ then the probability of choosing the best response goes to one. With the noise in the decisions, Figure 4b shows that a continuous evolution of p, the perceived probability of default or toxicity, leads to a sudden and large change in the equilibrium state. This induces a sharp decline in the prices of the asset-backed security, as shown in Figure 3. The collapse occurred later for better-rated MBS, reflecting Ashcraft et al.'s (2010) claim that ratings did not fully reflect available data. Note that the noise plays the role of an equilibrium selection device, as does the "trembling hand" in game theory. Thus, with a model where agents are influenced by each other's decisions and their decisions are not fully "rational," we capture some important empirical facts without any specific major exogenous shock. The agents, like the insects, make

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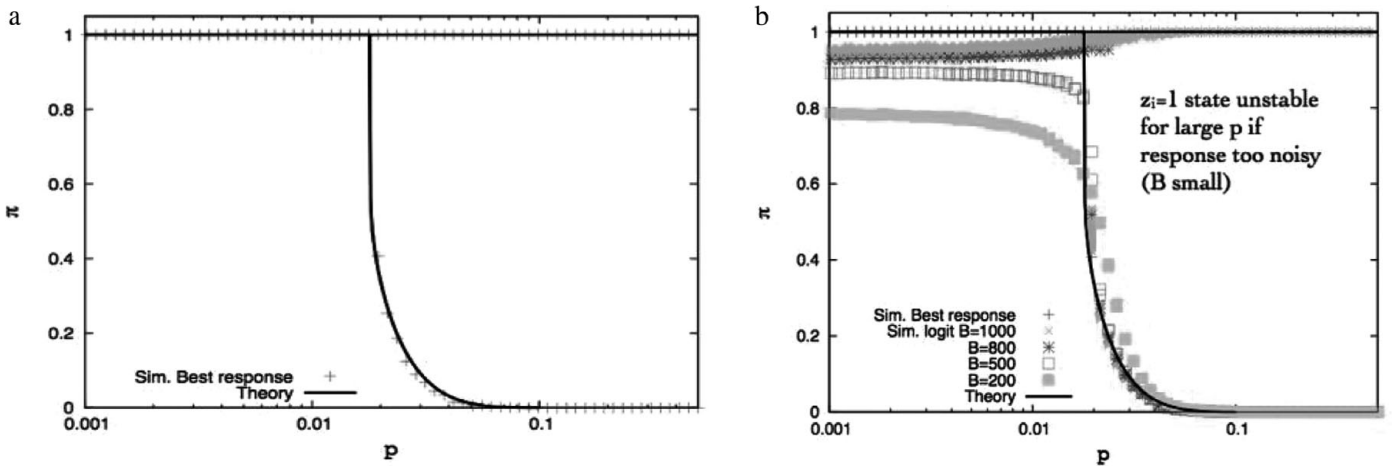


FIGURE 4. (a) The coexistence of two equilibria with all $z_i = 1$ or $z_i = 0$;(b) the evolution of theequilibrium state as p increases with noisy response. *Source:* Anand et al. (2013).

simple binary decisions, based on the actions of their partners, and this can lead to major changes in the aggregate state of the market. A more ambitious goal would be to build a model in which there are no equilibria and in which the market, its organization, and the behavior of the agents are constantly and simultaneously evolving.

9. CONCLUSION

The purpose of this paper has been to argue that our economic models, and particularly macroeconomic models, have attributed too much rationality to individuals and that collective behavior has been treated as being equivalent to that of an omniscient homo economicus. This, as I have argued, is a result of our fixation on equilibrium analysis, which, as Howitt (2012) observes, has distracted us from the problem of coordination. As he says, there is room for an ecology of approaches, "which finds room for agent-based computational economics as well as for more conventional equilibrium theories." This paper has suggested a direction in which the ecology should be enlarged.

The two most important features of the examples I gave to illustrate my point are the interaction between individuals and the extent to which there is noise in the system. Noise may undermine optimality, but it also plays an important role in the evolution of the economy and of markets, as Fischer Black (1986) noted a quarter of a century ago. I have argued that noisy systems of interacting agents, as exemplified by social insects, can provide the basis for analyzing important economic phenomena that are out of reach of our standard models.

NOTES

1. As Hildenbrand (1994) points out, Pareto, Hicks, Robbins, and Koopmans all forcefully made this argument.

2. Walras referred somewhat loosely to Cournot's argument as depending on the law of large numbers, but Trockel (1983) later gave a more precise analysis.

3. However, in his collected articles on the subject, Guesnerie (2001, 2005) does not deny the difficulties of such an approach, and he is now leading an INET project to completely reexamine the rational expectations hypothesis.

4. This terminology was introduced into economics by Arthur et al. (1997) and the analogy with statistical physics was expounded by Durlauf (1997).

5. The aggregation problem is one that has long been debated in economics, and we know that with the usual assumptions on individuals, one cannot show that the aggregate will behave like a "rational" individual [see Kirman (1992)].

6. Proofs of these statements can be found in Foellmer et al. (2005).

7. It is worth recalling that in a number of states in the United States (where loans are "nonrecourse"), the owners of a property on which they have a loan can simply turn over the house to the bank that issued the loan, without having any further financial contribution to make.

8. Worse, some of the banks knowingly misrepresented the value of the underlying assets they were selling to their investor clients, as JPMorgan Chase & Co. admitted in their November 2013 $13 billion settlement.

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