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The trouble with entropy†

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An understanding of the mechanisms leading to the symmetry breaking of the dynamical description of a large system with respect to the direction of time is necessary, but not sufficient to ensure the finding of a functional of the state of the system that would satisfy the requirements placed by the Second Law of Thermodynamics upon the non-equilibrium entropy S.

1. Introduction

For theoretical physicists, the realization of the entropy concept still presents a real challenge. Only for equilibrium can a satisfactory thermodynamic entropy be obtained from dynamical considerations, via

$$\mathscr{H} = \mathrm{Tr}\,\bar{\varrho}\,\ln\bar{\varrho}, \tag{1.1}$$

where $\bar{\varrho}$ is related to the state of the system. Out of equilibrium, the connection with dynamics is still unclear, since $\bar{\varrho}$ cannot be the statistical state ϱ itself, i.e. the solution of the Liouville–von Neumann (LvN) equation

$$i\partial_t\varrho = \mathscr{L}_H\varrho, \tag{1.2}$$

because such an $\mathscr{H}$ constructed from ϱ would remain constant in time and all statistical states obtained from ρ through a unitary transformation would share this property. Thus $\bar{\rho}$ has to be some projection of ρ or the transform of ρ under some non-unitary operation. Boltzmann's $\mathscr{H}$ theorem is deservedly considered an important cornerstone: for homogeneous dilute gases, $\bar{\varrho}$ is the one-particle velocity distribution function and

$$S \propto -\mathscr{H}, \tag{1.3}$$

but its limitations appear rapidly. For systems beyond this approximation, the tentative extensions of Boltzmann's construction (using one-particle reduced distribution functions (rdf)) have led to problems preventing any identification of such $-\mathscr{H}$ with the phenomenological entropy. Among the questions that are difficult to deal with simultaneously, let us quote the extensive property and the monotonic increase with time. We wish to consider them from the viewpoint of the Brussels group.

† It is an honour for the authors to have the opportunity to dedicate this paper to Professor Radu Balescu on the occasion of his retirement from teaching. During many years of collaboration with him, they have valued his integrity and admired his commitment to high-quality standards in scientific research.

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Various members of the group have devoted much work to the derivation of an irreversible description from the reversible one, as provided by the LvN equation. An important ingredient in the original group's philosophy was the requirement of interacting many-body systems for which the thermodynamical limit could be defined (Prigogine 1962). This led to alternative formulations, which both presented suitable irreversible characteristics – one in terms of heuristic N-body operator manipulations, and one in terms of mathematically safer rdfs (Balescu 1975). Such attempts took various forms, having in common the concept of subdynamics, the first version also giving rise to the concept of non-unitary transformation to a 'physical' representation of the state (Prigogine *et al.* 1973). In all cases, the construction of an extensive $\mathscr{H}$ function from N-body density operators or sets of rdfs could not be achieved because, in spite of many attempts (involving for instance only first corrections from equilibrium distribution functions), no convincing $\mathscr{H}$ theorem could be established. Sensibly, Balescu (1975) avoids even suggesting a general entropy of any form, clearly because in his eyes the problem is not ripe for consideration.

In desperation, and in view of the importance of the Second Law of Thermodynamics, some hope has been placed in the construction of Lyapunov functions, the prototype of which is

$$\Xi = \mathrm{Tr}\,\bar{\varrho}^2, \tag{1.4}$$

but difficulties concerning extensivity do not disappear. Besides, the Ξ functions for many-body systems proposed in such contexts present additional problems about their overall non-analyticity in the coupling constant λ associated with the interactions (de Haan *et al.* 1977). The resulting scheme furthermore remains incomplete because of other, often disregarded, 'analyticity in z' difficulties (de Haan 1998). Indeed, long-time tails characteristic of the familiar mode–mode coupling are known to decrease as an inverse power of time. In the resolvent formalism, used in the Brussels approach, this implies branch points at the origin. Quite formal *ad hoc* hypotheses on the spectrum of the so-called irreducible superoperators (defined from parts of the Liouvillian), although verified at the lowest order in λ, therefore become highly doubtful for all orders. The same difficulties cast doubts on similar hypotheses for the complex spectrum of the Liouvillian superoperator itself (Petrosky and Prigogine 1996).

After some critical remarks about recent proposals for a Lyapunov function, by Petrosky, Prigogine and co-workers, we devote a section to a discussion of a new candidate, in a quantal context, for a Lyapunov function, which sheds new light on the collision process considered as the atom of irreversibility.

2. Recent proposal for a Lyapunov function

Let us turn to the other delicate question about the entropy function, namely its monotonic behaviour in time. The quest for a better understanding of the underlying properties of the (complicated) general formalism has led to the consideration of simpler model systems where, as a first step, only an infinite-volume limit is taken. We wish to stress that the kinetic equations thus obtained do not provide suitable Lyapunov functions to link with a credible thermodynamic entropy. Our aim in the present paper is to make such criticisms more precise and to try to indicate a way out.

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Among the attempts on small systems, the Lyapunov function defined for a special sector of the Lee model appears to possess the right properties (de Haan *et al.* 1978), although its 'non-perturbative' construction is strange. The functions from which it is constructed are totally delocalized (de Haan and Henin 1973; Petrosky *et al.* 1991), so that the Ξ function starts decreasing before any physical interaction has taken place.

In several recent papers, Petrosky *et al.* have studied Lyapunov functions for the Friedrichs model (equivalent to the first sector of the Lee model) and the potential scattering model. Their attempts may be criticized from various viewpoints. We wish to concentrate here on the difficulties linked with the physical interpretation.

A characteristic of the analysis by Petrosky *et al.* (1991) is the complete disregard of the dynamical properties of the field. Indeed, the Lyapunov function proposed for the Friedrichs model decreases exponentially in time for all times. If one considers a collision process, with a localized wave packet running towards the collision centre, it is hard to accept an exponential decay even before the wave packet has actually reached its target! The interpretation of that Lyapunov function as a 'distance' from the asymptotic state therefore appears incorrect.

The use of a delocalized approach can be seen in further work by the same authors, such as their treatment of so-called *persistent* scattering (Petrosky and Prigogine 1994). There, with their proposal of a Lyapunov function, one is led to old-fashioned problems, such as the need to reinterpret the square of a Dirac delta as a Dirac delta times a volume factor, while the infinite-volume limit has already been performed previously to get a continuous spectrum. A delocalized approach seems to lead nowhere. Indeed, could a *bona fide* description of an irreversible process, such as diffusion, avoid a spatiotemporal approach?

Two important ideas have emerged from the consideration of small systems. The first is the recognition of the subdynamics concept as a way to properly derive (irreversible) kinetic equations. The second is the need for a complete description, in order to keep the essence of the physical processes. In the search for a physically acceptable entropy, the reconciliation of those two points seem difficult enough, since a subdynamics has to be derived, which is not a simple return to the original LvN equation, as the second point would seem to suggest. It is to avoid this trap that a completely new perspective has to be provided. We do not dwell here on all the considerations behind the new approach: rather we refer to the original papers.

3. In search of a Lyapunov function

In elementary quantum mechanics, the projector on the eigenstate associated with a discrete, non-degenerate eigenvalue commutes with the Hamiltonian, and defines in that sense a subdynamics, i.e. a part of the original description that is stable under the equation of motion. The knowledge of all such projectors allows the construction of a unitary transformation diagonalizing the Hamiltonian. The aim of the Brussels group was to generalize these concepts to many body systems, classical or quantal, with a continuous degenerate spectrum. The original idea for the construction was to collect all similar asymptotic time behaviour in the formal solution, defining a subdynamics governed by a kinetic equation.

A paper has recently been devoted to some criticisms of the Brussels viewpoint, as well as of the concept of a single asymptotic subdynamics as a transformation to a 'physical representation' (de Haan 1998). We have also proposed an alternative

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approach to the problem of deriving irreversible kinetic behaviour from a reversible dynamical starting point and have illustrated it with a model of potential scattering (de Haan and George 1998, hereinafter referred to as (I)). We refer to the original papers for details of notation.

The considered dynamics is first enlarged through a distinction introduced between the degrees of freedom that are of interest because of the role they play in the preparation or observation of the system, and the other degrees of freedom (the intermediate ones) that evolve through the original dynamics.

The kinetic equation in the single-vacuum subdynamics is obtained through elimination of all noncontrollable degrees of freedom. In spite of its manifestly irreversible character, it remains equivalent to the complete original dynamics once *constitutive relations* are taken into account. It also enables a step towards the construction of Lyapunov functions that are characteristic of the irreversible role played by the collision process. We first introduce a proposal for such a function and compute the contribution to it arising from the 'wave-function-like' sectors that we have studied. We consider as a first candidate the expression (1.4) in which the trace is only over the remaining controllable degrees of freedom and involves a distinction between the incoming and outcoming components of the density matrix. The trace may involve contributions from matrix elements of $\bar{\rho}$ between states pertaining to different sectors: for instance the vacuum state and a single collisional particle state (sectors (10) and (01)), as well as the expected contributions arising from matrix elements of $\bar{\rho}$ between single collisional particle states (sector (11)).

The contribution $\Xi^{(1)}$ arising from the simpler sectors (10) and (01) is

$$\Xi^{(1)}(t) = 2\sum_{\lambda} |\bar{\rho}_{\lambda}(t)|^2 + 2\sum_{\mu} |\bar{\rho}_{\mu}(t)|^2, \tag{3.1}$$

where the integrals involve ordinary functions of the wavenumbers and the result is independent of the quantization volume (as usual, the volume factor arising from the replacement of the sum by an integral is compensated by the inverse volume factor arising from the wave functions $|\bar{\rho}_{\lambda}(t)|^2$ and $|\bar{\rho}_{\mu}(t)|^2$).

Let us stress immediately that such an expression has no equivalent in the usual formulation of quantum theory, where a distinct treatment of the incoming and outgoing components of the density operator is not considered. The proposal (3.1) presents obviously interesting characteristics, as examplified by the consideration of the scattering of a wave packet incident upon a target. From our analysis in Sec. 4 of (I), we know that $\bar{\rho}_{\mu}(t)$ vanishes as long as the wave packet has not reached the target. During that time interval, $\Xi^{(1)}(t)$ remains constant, since the squared modulus of the incoming components $|\bar{\rho}_{\lambda}(t)|^2$ is time-independent. (The summation over λ is equivalent to the computation of the norm of the incident wave packet.)

During the collision process, outgoing components $\bar{\rho}_{\mu}(t)$ appear, and their production ceases as soon as the wave packet leaves the target. As a consequence of the completed collision process, the function $\Xi^{(1)}(t)$ has increased its value and reached a new plateau. Such behaviour is obviously welcome.

The interesting question that remains is about the nature of that increase: Is the time variation of $\Xi^{(1)}(t)$ monotonic even for the duration of the collision? The answer to this question cannot be ascertained at first sight even on physical grounds. Indeed, no assumption has been made either on the form and characteristics of the incident wave packet (in one or many pieces, adaptation of its form with respect to the target, etc.) or on the nature (or the form) of the interaction (our description

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emcompasses, among others, the possibility of the formation of stable or unstable bound states (resonances)). Whether a collision process has to be considered globally in order to provide a positive contribution to the 'entropy production' or whether it may be analysed for intermediate times is still unclear. A transient negative contribution could arise owing to a possible phase interference between the outgoing field emitted at various intermediate times, and would necessarily provide some adjustment of the entropy value. In other words, is the Second Law valid at the level of the dynamical details of one collision process or only at the level of statistics of collisions, some of them providing an intermittent negative contribution?

We do not know anything about the behaviour of the entropy production during the formation of a bound state when that process is isolated from the mechanism of production of the wave. Of course, we ought to be able to formulate some statements about the collision process when the conditions are clear-cut in that respect, i.e. when the relative structures of the incident wave and of the potential do not play a role. This is the case for the usual conditions in which scattering is described.

The solution of the kinetic equation provides the link between the incoming and outgoing components in terms of the *formation* operator (3.2.13) of (I),

$$\bar{\rho}_\mu(t) = \sum_\lambda F_{\mu\lambda}\, \bar{\rho}_\lambda(t), \tag{3.2}$$

which is expressed in terms of the t matrix (3.2.11) of (I):

$$F_{\mu\lambda} = \frac{1}{\omega_\lambda - \omega_\mu + i\epsilon}\, t^+_{\mu\lambda}(\omega_\lambda). \tag{3.3}$$

Let us focus our attention on the contribution arising from the second term in (3.1). We get

$$|\bar{\rho}_\mu(t)|^2 = \sum_{\lambda\lambda'} F_{\mu\lambda}\, F^{cc}_{\mu\lambda'}\, \bar{\rho}_\lambda(t)\, \bar{\rho}^{cc}_{\lambda'}(t). \tag{3.4}$$

With the use of (2.4.28), the time variation of $\Xi^{(1)}(t)$ is given by

$$\frac{\partial}{\partial t}\Xi^{(1)}(t) = -2i \sum_\mu \sum_{\lambda\lambda'} F_{\mu\lambda}\, F^{cc}_{\mu\lambda'}(\omega_\lambda - \omega_{\lambda'})\, \bar{\rho}_\lambda(t)\, \bar{\rho}^{cc}_{\lambda'}(t). \tag{3.5}$$

Using the expression (3.3) for the elements of the formation operator in terms of the t-matrix, we get (cf. (4.2.5) of (I):

$$\frac{\partial}{\partial t}\Xi^{(1)}(t) = 2i \sum_\mu \sum_{\lambda\lambda'} \left(\frac{1}{\omega_\lambda - \omega_\mu + i\epsilon} - \frac{1}{\omega_{\lambda'} - \omega_\mu - i\epsilon} \right) t^+_{\mu\lambda}(\omega_\lambda) t^-_{\lambda'\mu}(\omega_{\lambda'}) \bar{\rho}_\lambda(t) \bar{\rho}^{cc}_{\lambda'}(t). \tag{3.6}$$

Using a well-known relation between the elements of the t matrix (take $k_1 = \mu$, $k' = \lambda$, $k = \lambda'$, $z' = \omega_\lambda + i\varepsilon$ in (C.11) of George *et al.* 1985) we get

$$\begin{aligned} \frac{\partial}{\partial t}\Xi^{(1)}(t) &= 2i \sum_{\lambda\lambda'} [t^+_{\lambda'\lambda}(\omega_\lambda) - t^-_{\lambda'\lambda}(\omega_{\lambda'})] \bar{\rho}_\lambda(t) \bar{\rho}^{cc}_{\lambda'}(t) \\ &= 2i \sum_{\lambda\lambda'} [t^+_{\lambda'\lambda}(\omega_\lambda) - t^-_{\lambda'\lambda}(\omega_{\lambda'})] e^{-i(\omega_\lambda - \omega_{\lambda'})t}\, \bar{\rho}_\lambda(0) \bar{\rho}^{cc}_{\lambda'}(0). \end{aligned} \tag{3.7}$$

The expression (3.5) does not enable us to make any definite statement about the sign of the time variation of $\Xi^{(1)}(t)$.

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The use of another known relation between elements of the t-matrix ((C.15) of George *et al.* 1985)†, expressing the completeness of the states belonging to the continuum and excluding the presence of bound states, does not help. An expression similar to (3.6) is obtained:

$$\frac{\partial}{\partial t}\Xi^{(1)}(t) = 2i\sum_{\mu}\sum_{\lambda\lambda'}\left(\frac{1}{\omega_\lambda - \omega_\mu + i\epsilon} - \frac{1}{\omega_{\lambda'} - \omega_\mu - i\epsilon}\right) t^{+}_{\mu\lambda}(\omega_\mu)t^{-}_{\lambda'\mu}(\omega_\mu)\bar{\rho}_\lambda(t)\bar{\rho}^{cc}_{\lambda'}(t). \tag{3.8}$$

The difference lies in the argument of the t matrices. We shall prove below that an expression with a structure like (3.8) cannot retain a definite sign at all times, independently of the form of the incident wave.

Let us therefore introduce the conditions that are usual in the study of potential scattering, and suppose that the function $\bar{\rho}_\lambda(0)$ is sharply peaked around a value k_{inc}, the distinction between peaked or not peaked arising by comparison with the wavenumbers typical of the dependence of the t matrix. We then get, approximately,

$$\frac{\partial}{\partial t}\Xi^{(1)}(t) = 2i\sum_{\mu}\left(\frac{1}{\omega_{k_{\text{inc}}} - \omega_\mu + i\epsilon} - \frac{1}{\omega_{k_{\text{inc}}} - \omega_\mu - i\epsilon}\right) \times \sum_{\lambda\lambda'} t^{+}_{\mu\lambda}(\omega_\lambda)t^{-}_{\mu\lambda'}(\omega_{\lambda'})\bar{\rho}_\lambda(t)\bar{\rho}^{cc}_{\lambda'}(t) \tag{3.9}$$

$$= 2\pi\sum_{\mu}\delta(\omega_{k_{\text{inc}}} - \omega_\mu)\sum_{\lambda\lambda'} t^{+}_{\mu\lambda}(\omega_\lambda)t^{-}_{\lambda'\mu}(\omega_{\lambda'})\bar{\rho}_\lambda(t)\bar{\rho}^{cc}_{\lambda'}(t) \tag{3.10}$$

$$= 2\pi\sum_{\mu}\delta(\omega_{k_{\text{inc}}} - \omega_\mu)\sum_{\lambda\lambda'} t^{+}_{\mu\lambda}(\omega_{k_{\text{inc}}})t^{-}_{\lambda'\mu}(\omega_{k_{\text{inc}}})\bar{\rho}_\lambda(t)\bar{\rho}^{cc}_{\lambda'}(t). \tag{3.11}$$

Introducing the same approximations on the form of the incident wave $\bar{\rho}_{\text{inc}}(\mathbf{r}, t)$ as in Sec. (4.2) of (I),

$$\bar{\rho}_{\text{inc}}(\mathbf{r}, t) = e^{i\mathbf{k}_{\text{inc}}\cdot\mathbf{r}}\varphi(\mathbf{r}, t), \tag{3.12}$$

with the same hypothesis on the spatial variation of $\varphi(\mathbf{r}, t)$ as in (4.2.9) of (I), we obtain by a similar computation

$$\frac{\partial}{\partial t}\Xi^{(1)}(t) = 2\sum_{\mu} L^3 2\pi\delta(\omega_{k_{\text{inc}}} - \omega_\mu)|t^{+}_{\mu k_{\text{inc}}}(\omega_{k_{\text{inc}}})|^2|\bar{\rho}_{\text{inc}}(\mathbf{r}_0, t)|^2, \tag{3.13}$$

which is positive for all times and vanishes when the centre $\mathbf{r}_0$ of the potential is outside the incident wave packet (i.e. before and after the collision process). Like $\Xi^{(1)}(t)$ itself, the expression for $(\partial/\partial t)\Xi^{(1)}(t)$ is clearly independent of the quantization volume.

This expression (3.13) is reminiscent of the variation of the Ω function considered in George *et al.* (1985, see (151)) (the kernel consists of a delta function times the squared modulus of a t-matrix element), but this latter proposal (valid for the next sector (11), since the sectors (10) and (01) considered here do not con-

† After correction of a misprint: a minus sign has to be introduced in front of the summation sign on the right-hand side of (C.15)

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tribute) provides a contribution that behaves as the inverse of the quantization volume (only the delocalized components of the field (diagonal elements) play a role, leading to the loss of a volume factor): it could only make sense as a *precursor* in a many-body problem, where the number of similar contributions would be proportional to the number of particles, providing, in the thermodynamic limit, an eventual contribution varying as the concentration of particles. Such a proposal does not take into account the relative position of the centre of collision and the actual position of the wave packet, and its physical significance remains open to question.

For the usual wave packet considered in collision theory, the present proposal (3.1) leads to a positive increase of $\Xi^{(1)}$ during the effective collision process, and $\Xi^{(1)}$ reaches a plateau when the wave packet leaves the target.

As we have said from the start, the general expression (3.1) for $\Xi^{(1)}(t)$ does not make any reference to the relative size of the potential and the incident wave packet, and no general statement seems possible on $(\partial/\partial t)\Xi^{(1)}(t)$.

There remains the possibility of considering other candidates for an entropy function. Let us show that, within the framework of quadratic expressions, no such function is possible with a monotonic increase. We restrict ourselves again to the contribution arising from the wave-function-like sectors (10) and (01). The envisaged function has to be a quadratic functional of the state, which is defined in terms of incoming and outgoing components. Since the outgoing components can be expressed as functionals of the incoming components taken at the same time, the function has to be a quadratic functional of the incoming components alone. It therefore has to be written as a double integral:

$$\Xi^{(1)}(t) = \sum_{\lambda\lambda'} O_{\lambda\lambda'}\bar{\rho}_{\lambda}(t)\bar{\rho}^{cc}_{\lambda'}(t). \tag{3.14}$$

A certain number of constraints are physically reasonable on the (still undetermined) kernel $O_{\lambda\lambda'}$, which may contain distributions. First of all, we may disregard a Dirac delta function of the two wavenumbers λ, and λ'. Indeed, the product $\bar{\rho}_{\lambda}(t)\bar{\rho}^{cc}_{\lambda'}(t)$ would then be time-independent, and the presence of such a Dirac delta function would amount to a constant contribution to $\Xi^{(1)}(t)$. Another reasonable constraint is that the kernel ought to be expressible in terms of dynamical elements (of the Hamiltonian), meaning that the dependence of the kernel on the wavenumbers is scaled by characteristics of the potential. Of course, the kernel $O_{\lambda\lambda'}$ has to be positive-definite for all possible functions $\bar{\rho}_{\lambda}(t)$. This property forbids the presence in $O_{\lambda\lambda'}$ of more general distributions such as derivatives of the delta function $\delta(\lambda - \lambda')$. In addition, $\Xi^{(1)}(t)$ has to be real, so that we have to impose the symmetry $O_{\lambda\lambda'} = O^{cc}_{\lambda'\lambda}$.

The crucial requirement is finally that the kernel must ensure that the inequality

$$-i\sum_{\lambda\lambda'} O_{\lambda\lambda'}(\omega_{\lambda} - \omega_{\lambda'})\bar{\rho}_{\lambda}(t)\bar{\rho}^{cc}_{\lambda'}(t) \geqslant 0 \tag{3.15}$$

holds for any choice of the function $\bar{\rho}_{\lambda}(t)$. Let us then decompose the kernel into its real part $O^{(1)}_{\lambda\lambda'}$ and its imaginary part $O^{(2)}_{\lambda\lambda'}$ with opposing parity properties with respect to the interchange of the two wavenumbers. We perform a similar decomposition for $\bar{\rho}_{\lambda}(t) = \bar{\rho}^{(1)}_{\lambda}(t) + i\bar{\rho}^{(2)}_{\lambda}(t)$. We may now write the inequality (3.15)

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in terms of these real and imaginary parts:

$$\sum_{\lambda\lambda'} O^{(2)}_{\lambda\lambda'}(\omega_\lambda - \omega_{\lambda'})[\bar\rho^{(1)}_\lambda(t)\bar\rho^{(1)}_{\lambda'}(t) + \bar\rho^{(2)}_\lambda(t)\bar\rho^{(2)}_{\lambda'}(t)] + \sum_{\lambda\lambda'} O^{(1)}_{\lambda\lambda'}(\omega_\lambda - \omega_{\lambda'})[\bar\rho^{(1)}_\lambda(t)\bar\rho^{(2)}_{\lambda'}(t) - \bar\rho^{(2)}_\lambda(t)\bar\rho^{(1)}_{\lambda'}(t)] \geqslant 0. \quad (3.16)$$

Such an inequality cannot hold for all choices of $\bar\rho^{(1)}_\lambda(t)$ and $\bar\rho^{(2)}_\lambda(t)$. Indeed, let us suppose that these functions present two peaks (nearly delta-like) for the values $\lambda = a$ and $\lambda = b$. Up to factors, which can be incorporated in the definitions of ρ_a and ρ_b, we may write the inequality as

$$O^{(2)}_{ab}(\omega_a - \omega_b)[\bar\rho^{(1)}_a(t)\bar\rho^{(1)}_b(t) + \bar\rho^{(2)}_a(t)\bar\rho^{(2)}_b(t)] + O^{(1)}_{ab}(\omega_a - \omega_b)[\bar\rho^{(1)}_a(t)\bar\rho^{(2)}_b(t) - \bar\rho^{(2)}_a(t)\bar\rho^{(1)}_b(t)] \geqslant 0. \quad (3.17)$$

The diagonal elements of the kernel O_{aa} and O_{bb} do not play a role. Since all products $\bar\rho^{(1)}_a(t)\bar\rho^{(1)}_b(t)$, $\bar\rho^{(2)}_a(t)\bar\rho^{(2)}_b(t)$, $\bar\rho^{(1)}_a(t)\bar\rho^{(2)}_b(t)$ and $\bar\rho^{(2)}_a(t)\bar\rho^{(1)}_b(t)$ can be given independently arbitrary real values (positive or negative), the inequality (3.17) cannot hold for any given choice of the kernel. A monotonic increase of a quadratic functional is thus not possible in the considered sectors.

4. Concluding remarks

Constructed on the basis of the distinction between incoming and outgoing components – are thus accounting for some historicity – the functional proposed in (3.1) enables a description of the possibility of the wave undergoing or not undergoing a collision. It also seems to possess the maximum of nice physical features, namely a global increase during the collision process and a monotonic increase under the usual conditions assumed in scattering theory. In the restricted sectors considered, more general entropy-like functions (such as a quartic one) could be proposed, but some nice features are at risk of being lost. We could get in trouble with respect to the order of magnitude in the quantization volume (as in the above-mentioned proposal in George *et al.* 1985) or be led to consider a completely delocalized event with the loss of a somewhat spatially and temporally localized scattering. Another reason for not making any more general proposal here is that the existence of other sectors has to be taken into account, with, at the level of Lyapunov functions, possible interference between the various sectors. This could be the subject of future publications.

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