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Optimal stability of a redundant mobile manipulator via genetic algorithm

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Summary
Once the path of the vehicle and the desired task of the end-effector are predefined, in order to apply the optimal stability criterion, the manipulator should be redundant. In this paper, the goal is to find the position, angular velocity, and angular acceleration of the redundant link of the manipulator such that the entire system becomes optimally stable. By considering the full dynamic interaction between the manipulator and its vehicle, the stability issue becomes more complex. There are some measures of stability and one of them is the tire's upward force. This measure tries to equalize the upward forces of the tires that lead to optimal stability and better steer-ability. Optimizing the parameters of various engineering problems is a challenging issue. In this research, the optimal position, angular velocity, and angular acceleration of the redundant link are found by the genetic algorithm approach such that the upward forces of vehicle tires are nearly equal. Furthermore, the effectiveness of this procedure is shown by presenting an example.

1. Introduction

Huang *et al*. have investigated the compensatory manipulator motion planning for increasing the stability of a mobile manipulator.[1–3] The method of ZMP path planning by a stability potential field is presented in ref. [2]. In ref. [3] a method for coordinating the vehicle motion planning considering the manipulator task constraints and the manipulator motion planning considering the platform stability is presented. Kim *et al*. have proposed an online compensation algorithm for the dynamic stability of the mobile manipulators using ZMP.[4] A genetic solution for the motion of wheeled robotic systems in dynamic environment has been considered by Leung *et al*.[5] A redundant robot for the specific applications, trajectory tracking, singularity avoidance, and obstacle avoidance is considered by Lin.[6] Tipover stability estimation for autonomous mobile manipulators using neural network has been studied by Meghdari *et al*.[7] Increasing the stability of the mobile manipulators using dynamic compensation has been studied by Meghdari *et al*.[8] A general approach to the dynamic modeling of nonholonomic mobile manipulator systems has been presented by Yu *et al*.[9]

When the path and the task of the end-effector are predefined and the vehicle is supposed to move in a specified path with constant speed (Fig. 1), the stability of the system may be endangered. In this case, the inertia forces and couples due to the dynamic interaction between the vehicle and the manipulator are very important and should be considered. In order to prevent the instability of mobile manipulator and to increase its stability, the manipulator should be of a redundant type. The purpose is to find the position, angular velocity, and angular acceleration of the redundant link of manipulator such that the stability of the entire system is optimal. For determining the stability criterion, the tire's upward force measure is employed because of its advantages. In this paper, the optimal parameters have been computed by the genetic algorithm method. The effectiveness of this method has been shown by simulation.

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2. Kinematics of the mobile manipulator

In this study, a three-wheeled vehicle has been considered. The trajectory of the vehicle will be a predefined path due to obstacle avoidance.

Figure 2 shows a schematic of the manipulator with four degrees-of-freedom and its selected frames. For extracting the kinematic equations of the system, the Denavit–Hartenberg notation is applied[10] and the parameters are shown in Table 1.

The equation of the vehicle's path is defined as:

$$y = A \sin\left(\frac{2\pi x}{X_0}\right) \tag{1}$$

where A is the amplitude and X_0 is the wavelength of motion.

3. Modeling of the vehicle

The shape of the vehicle chassis is that of an isosceles triangle. The manipulator is mounted on the base such that the rotation axis of the first joint of the manipulator is normal to the horizontal plane. The applied torques on the joints are derived by Newton–Euler formulation. For determining the stability measure, the upward forces of each of the tires are calculated.

If one of these forces approaches zero, the whole system is on the margin of instability. The best feature is the equal upward force of the tires. A cost function is defined such a way that by optimizing this function, the upward forces of the three tires become nearly optimal. The variables of

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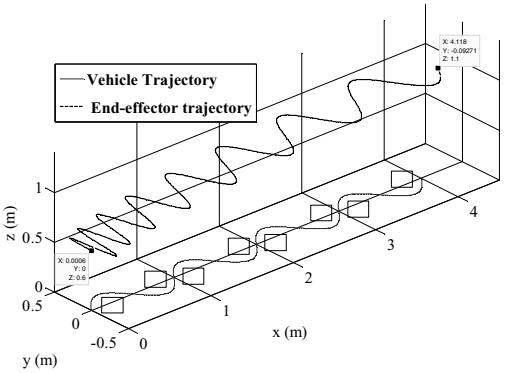


Fig. 1. Predefined vehicle trajectory on the ground and end-effector trajectory in space.

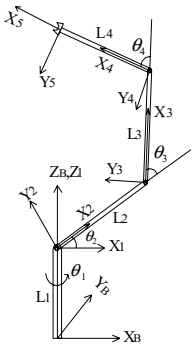


Fig. 2. Schematic of the 4R manipulator and its selected frames.

Table 1. Denavit–Hartenberg parameters.

i	α_{i-1}	α_{i-1}	d_i	θ_{i-1}
1	0	0	L_1	θ_1
2	90°	0	0	θ_2
3	0	L_2	0	θ_3
4	0	L_3	0	θ_4
5	0	L_4	0	0

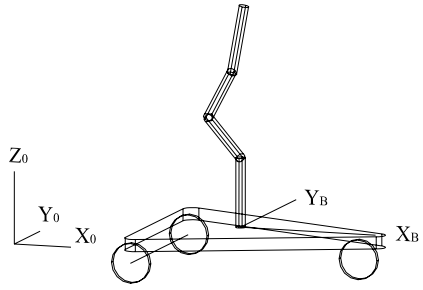


Fig. 3. Schematic of the mobile manipulator.

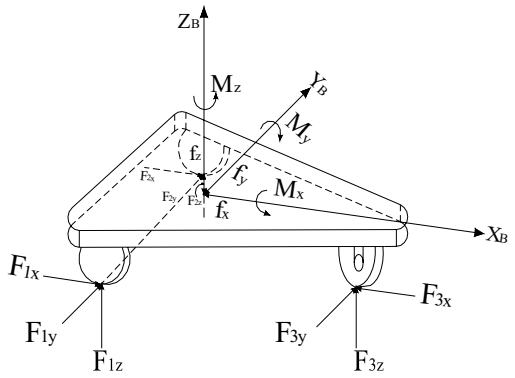


Fig. 4. Free body diagram of the vehicle.

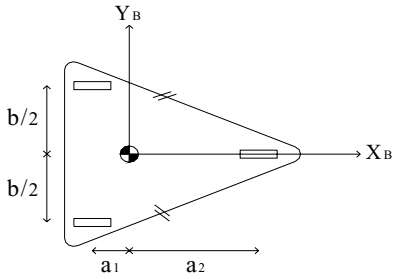


Fig. 5. Dimensions of the vehicle.

this function are the position, angular velocity, and angular acceleration of the redundant link of manipulator.

A schematic of the mobile manipulator has been displayed in Fig. 3. The free body diagram of the vehicle has been shown in Fig. 4.

The dynamic interaction between the manipulator and the vehicle has a crucial effect on the stability of the whole system. Therefore, the compensatory motions of the manipulator may cause an increase in the stability. In dynamic compensation, the internal reaction forces and couples acting on the joint, which is attached to the vehicle, affect the tire's upward force.

Dimensions of the vehicle are illustrated in Fig. 5.

In this research, as discussed before, it is assumed that the trajectory of the end-effector and the vehicle are known. In this circumstance, stabilizing of mobile manipulator is difficult. For overcoming this problem, an extra degree-of-freedom may be used. These appropriate equations are as follows:

$$\begin{aligned} F_1 + F_2 + F_3 - f_z - Mg &= 0 \\ (F_1 - F_2)\frac{b}{2} &= M_x \\ (F_1 + F_2)a_1 - F_3 a_2 &= M_y \end{aligned} \tag{2}$$

The velocity and acceleration of the vehicle are evaluated in the form of:

$$\begin{aligned} V_x &= V\cos\phi \\ V_y &= V\sin\phi \end{aligned} \tag{3}$$

$${}^0\dot{\vec{V}}_B = \ddot{X}\hat{X}_0 + \ddot{Y}\hat{Y}_0 + g\hat{Z}_0 \tag{4}$$

In Eq. (3), V denotes the tracking velocity of the vehicle and ϕ is the angle which is tangential to the path of motion. The vehicle has acceleration along the three principle axes such

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that $\ddot{X}$ and $\ddot{Y}$ may be defined as:

$$\ddot{X} = \frac{V^2}{\rho}\sin(\phi)$$
$$\ddot{Y} = \frac{V^2}{\rho}\cos(\phi) \tag{5}$$

where ρ is the radius of curvature.

For obtaining the reaction forces acting on the joint attached to the vehicle, the iterative dynamical Newton–Euler algorithm is employed.

The general form of the mentioned iterative algorithm is as follows:[11]

External iterative relations:

$$\begin{aligned}
{}^{i+1}\omega_{i+1} &= {}^{i+1}_{i}R\,{}^{i}\omega_i + \dot{\theta}_{i+1}\,{}^{i+1}\hat{Z}_{i+1}\\
{}^{i+1}\dot{\omega}_{i+1} &= {}^{i+1}_{i}R\,{}^{i}\dot{\omega}_i + {}^{i+1}_{i}R\,{}^{i}\omega_i \times \dot{\theta}_{i+1}\,{}^{i+1}\hat{Z}_{i+1} + \ddot{\theta}_{i+1}\,{}^{i+1}\hat{Z}_{i+1}\\
{}^{i+1}\dot{v}_{i+1} &= {}^{i+1}_{i}R\left({}^{i}\dot{\omega}_i \times {}^{i}P_{i+1} + {}^{i}\omega_i \times \left({}^{i}\omega_i \times {}^{i}P_{i+1}\right) + {}^{i}\dot{v}_i\right)\\
{}^{i+1}\dot{v}_{Ci+1} &= {}^{i+1}\dot{\omega}_{i+1} \times {}^{i+1}P_{Ci+1} + {}^{i+1}\omega_{i+1}\\
&\quad \times \left({}^{i+1}\omega_{i+1} \times {}^{i+1}P_{Ci+1}\right) + {}^{i+1}\dot{v}_{i+1}\\
{}^{i+1}F_{i+1} &= m_{i+1}\,{}^{i+1}\dot{v}_{Ci+1}\\
{}^{i+1}N_{i+1} &= {}^{Ci+1}I_{i+1}\,{}^{i+1}\dot{\omega}_{i+1} + {}^{i+1}\omega_{i+1}\,{}^{Ci+1}I_{i+1}\,{}^{i+1}\omega_{i+1}
\end{aligned} \tag{6}$$

Internal iterative relations:

$$\begin{aligned}
{}^{i}f_i &= {}_{i+1}{}^{i}R^{i+1}f_{i+1} + {}^{i}F_i\\
{}^{i}n_i &= {}^{i}N_i + {}_{i+1}{}^{i}R^{i+1}n_{i+1} + {}^{i}P_{Ci} \times {}^{i}F_i\\
&\quad + {}^{i}P_{i+1} \times {}_{i+1}{}^{i}R^{i+1}f_{i+1}\\
\tau_i &= {}^{i}n_i^{T}\,{}^{i}\hat{Z}_i
\end{aligned} \tag{7}$$

By employing these relations and relating to Eqs. (2), the target is to minimize the cost function by defining as shown below:

$$\min = |F_3 - (F_1 + F_2)| + |F_1 - F_2| \tag{8}$$

By using the genetic algorithm, the optimal position, angular velocity, and angular acceleration of the third link, a redundant link, is found. These optimal variables will be found by GA in order to optimize the cost function Eq. (8).

A problem for a four-degree-of-freedom mobile manipulator is considered here.

The numerical parameters of the vehicle and manipulator for this example are shown in Table 2.

4. Results

For examining the feasibility of the solution, another method called "Joint Space Method" is used. In this method, one can estimate a curve in each step that compares the results with the data obtained from the first method (optimization method).

Table 2. Vehicle and arm parameters.

Parameter	Value
Base mass	6 kg
Link mass 1	1 kg
Link mass 2	1 kg
Link mass 3	1 kg
Link mass 4	1 kg
L_1	0.4 m
L_2	0.5 m
L_3	0.5 m
L_4	0.5 m
a	0.6 m
b	0.5 m
Tracking velocity of the end-effector	0.05 m/s
Tracking velocity of the vehicle	0.023536 m/s

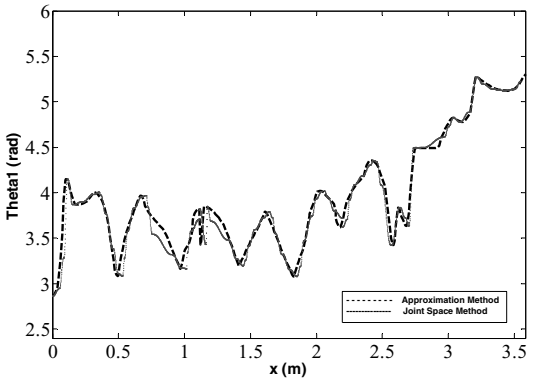


Fig. 6. Comparison of the two diagrams of position of θ_1 vs. x.

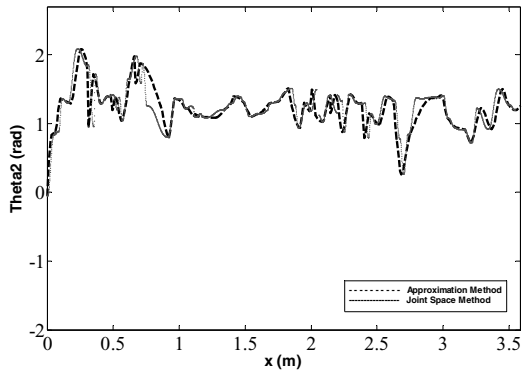


Fig. 7. Comparison of the two diagrams of position of θ_2 vs. x.

In Fig. 6, a plot of θ_1 versus x has been shown. In this figure, for 3.6 m distance from the origin, the variables of θ_1 have been obtained in two methods. The first method is that of optimization. The second method is that of joint space approach. In this technique, by employing position, angular velocity, and angular acceleration for small distances, a curve with higher order terms is extrapolated. Obviously, in each step there are no unfeasible changes between the two methods. In Fig. 7, a plot of θ_2 versus x has been drawn. Similar to Fig. 6, the two above-mentioned methods are used and compared for computing the variables of θ_2. As Fig. 7

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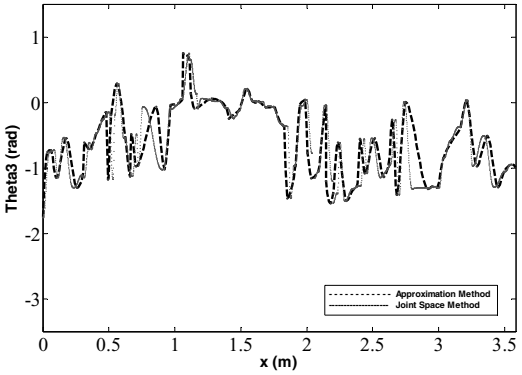


Fig. 8. Comparison of the two diagrams of position of θ_3 vs. x.

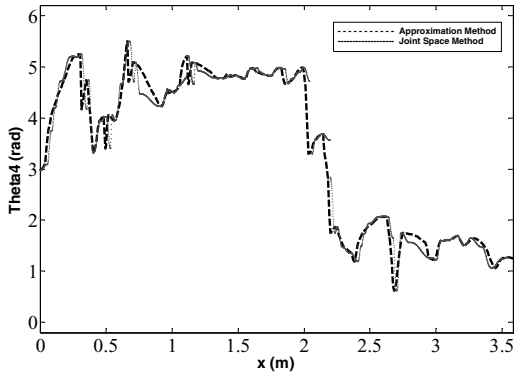


Fig. 9. Comparison of the two diagrams of position of θ_4 vs. x.

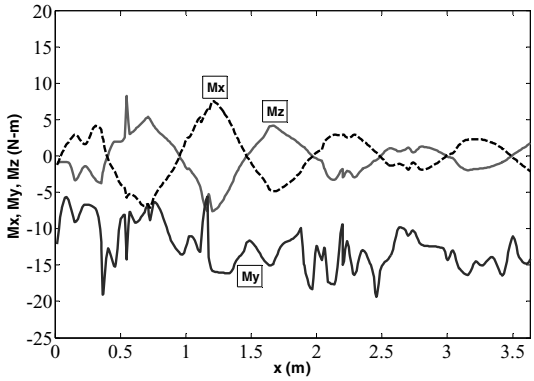


Fig. 10. Plots of M_x, M_y, and M_z vs. x.

shows, it is convenient to extrapolate the variables θ_2 by the approximation method.

Similar to Figs. 6 and 7, in Fig. 8 the variables θ_3 for a determined distance are calculated. By using the two approaches, it is seen that there are no obvious differences between these methods. Hence, to find the θ_3 variables, the first approach is used.

Figure 9 is similar to the previous figures. In all the plots for θ_1 to θ_4, the jumping rate is at the minimum value between the two near steps. Figure 10 illustrates the variables of M_x, M_y, and M_z versus x which act on the joint connecting the vehicle. These moments cause different conditions for the mobile manipulator along the motion and if these variables

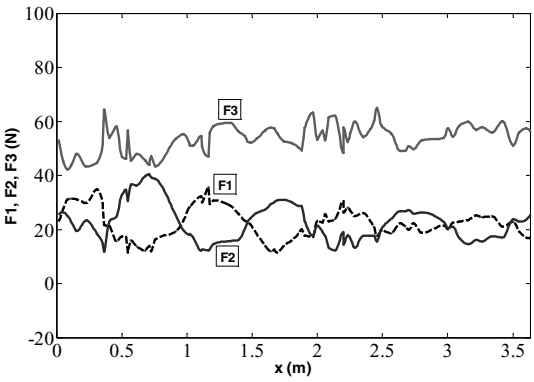


Fig. 11. Plots of F_1, F_2, and F_3 vs. x.

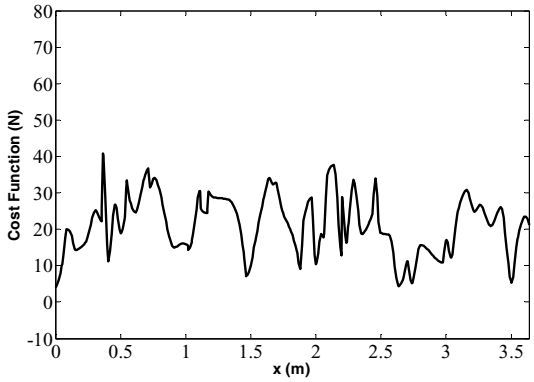


Fig. 12. Plots of cost function vs. x.

are not optimized, it may affect the stability of the system. Because of the dynamic interactions between these moments and the tire's upward force, a cost function has been defined and the goal is to minimize this cost function. In Fig. 11, the variables of the upward forces of the three tires numbered 1, 2, and 3 have been plotted. Observing the free-body diagram of the mobile manipulator, it is noticeable that for the desired purpose, the forces F_1 and F_2 should be equal and also their sum should be equal to F_3. In Fig. 12 the values of cost function have been shown along the path of motion. These variables are approximately the best ones that may appear for the mobile manipulator.

5. Conclusion

In this paper, an algorithm for increasing the stability of a mobile manipulator has been presented. When the path and task of the vehicle and the end-effector are predefined, to apply the optimal stability, the manipulator needs to be of a redundant type. It was desired to find the position, angular velocity, and angular acceleration of the redundant link such that the stability of the whole system becomes optimal. By taking into consideration the full dynamic interaction between the manipulator and its vehicle, the stability issue became complex. There were some measure of stability, but the best one was the tire's upward force because of its advantages. In this research, the optimal position, angular velocity, and angular acceleration of the redundant link had been found by the genetic algorithm. In addition, the

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effectiveness of this algorithm was shown by presenting an example.

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